How to Calculate Present Values

Chapter 3

Slides by
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Topics Covered

- Valuing Long-Lived Assets
- PV Calculation Short Cuts
- Compound Interest
- Interest Rates and Inflation
- Example: Present Values and Bonds
Present Values

Discount Factor = DF = PV of $1

- Discount Factors can be used to compute the present value of any cash flow.
Discount Factor = DF = PV of $1

\[ DF = \frac{1}{(1+r)^t} \]

- Discount Factors can be used to compute the present value of any cash flow.
Present Values

Discount Factors can be used to compute the present value of any cash flow.

\[ PV = DF \times C_1 = \frac{C_1}{1 + r_1} \]

\[ DF = \frac{1}{(1+r)^t} \]
Replacing “1” with “t” allows the formula to be used for cash flows that exist at any point in time.
Present Values

Example

You just bought a new computer for $3,000. The payment terms are 2 years same as cash. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?
Present Values

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\[ PV = \frac{3000}{(1.08)^2} = $2,572.02 \]
PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots.$$
Present Values

- Given two dollars, one received a year from now and the other two years from now, the value of each is commonly called the Discount Factor. Assume $r_1 = 20\%$ and $r_2 = 7\%$. 
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\[
DF_1 = \frac{1.00}{(1+.20)^1} = .83
\]

\[
DF_2 = \frac{1.00}{(1+.07)^2} = .87
\]
Example

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 7% required rate of return, create a present value worksheet and show the net present value.

Year 0 | Year 1 | Year 2
-------|-------|-------
-150,000 | -100,000 | +300,000
**Example - continued**

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 7% required rate of return, create a present value worksheet and show the net present value.

<table>
<thead>
<tr>
<th>Period</th>
<th>Discount Factor</th>
<th>Cash Flow</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>−150,000</td>
<td>−150,000</td>
</tr>
<tr>
<td>1</td>
<td>(\frac{1}{1.07} = .935)</td>
<td>−100,000</td>
<td>−93,500</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{(1.07)^2} = .873)</td>
<td>+300,000</td>
<td>+261,900</td>
</tr>
</tbody>
</table>

\[ NPV = Total = \$18,400 \]
Sometime there are shortcuts that make it very easy to calculate the present value of an asset that pays off in different periods. These tolls allow us to cut through the calculations quickly.
Perpetuity - Financial concept in which a cash flow is theoretically received forever.

\[
\text{Return} = \frac{\text{cash flow}}{\text{present value}}
\]

\[
r = \frac{C}{PV}
\]
**Perpetuity** - Financial concept in which a cash flow is theoretically received forever.

\[
PV = \frac{C_1}{r}
\]

- \(PV\) is the present value of the cash flow.
- \(C_1\) is the cash flow at the first period.
- \(r\) is the discount rate.
Annuity - An asset that pays a fixed sum each year for a specified number of years.

PV of annuity = \( C \times \left[ \frac{1}{r} - \frac{1}{r(1 + r)^t} \right] \)
Annuity Short Cut

Example
You agree to lease a car for 4 years at $300 per month. You are not required to pay any money up front or at the end of your agreement. If your opportunity cost of capital is 0.5% per month, what is the cost of the lease?
Example - continued

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\[
\text{Lease Cost} = 300 \times \left[ \frac{1}{.005} - \frac{1}{.005(1 + .005)^{48}} \right]
\]

\[
\text{Cost} = $12,774.10
\]
### Compound Interest

<table>
<thead>
<tr>
<th>Periods per year</th>
<th>Interest per period</th>
<th>APR (i x ii)</th>
<th>Value after one year</th>
<th>Annually compounded interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6%</td>
<td>6%</td>
<td>1.06</td>
<td>6.000%</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>$1.03^2 = 1.0609$</td>
<td>6.090</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>6</td>
<td>$1.015^4 = 1.06136$</td>
<td>6.136</td>
</tr>
<tr>
<td>12</td>
<td>.5</td>
<td>6</td>
<td>$1.005^{12} = 1.06168$</td>
<td>6.168</td>
</tr>
<tr>
<td>52</td>
<td>.1154</td>
<td>6</td>
<td>$1.001154^{52} = 1.06180$</td>
<td>6.180</td>
</tr>
<tr>
<td>365</td>
<td>.0164</td>
<td>6</td>
<td>$1.000164^{365} = 1.06183$</td>
<td>6.183</td>
</tr>
</tbody>
</table>
Compound Interest

Number of Years vs. FV of $1

- **10% Simple**
- **10% Compound**
Inflation - Rate at which prices as a whole are increasing.

Nominal Interest Rate - Rate at which money invested grows.

Real Interest Rate - Rate at which the purchasing power of an investment increases.
1 + real interest rate = \frac{1 + \text{nominal interest rate}}{1 + \text{inflation rate}}
Inflation

\[ 1 + \text{real interest rate} = \frac{1 + \text{nominal interest rate}}{1 + \text{inflation rate}} \]

approximation formula

Real int. rate \approx \text{nominal int. rate} - \text{inflation rate}
Example

If the interest rate on one year govt. bonds is 5.9% and the inflation rate is 3.3%, what is the real interest rate?
Inflation

Example

If the interest rate on one year govt.
bonds is 5.9% and the inflation rate is 3.3%, what is the real interest rate?

\[
1 + \text{real interest rate} = \frac{1+.059}{1+.033}
\]

\[
1 + \text{real interest rate} = 1.025
\]

real interest rate = .025 or 2.5%
Example

If the interest rate on one year govt. bonds is 5.9% and the inflation rate is 3.3%, what is the real interest rate?

\[
1 + \text{real interest rate} = \frac{1+0.059}{1+0.033}
\]

\[
1 + \text{real interest rate} = 1.025
\]

\[
\text{real interest rate} = 0.025 \text{ or } 2.5\%
\]

Approximation \(= 0.059 - 0.033 = 0.026 \text{ or } 2.6\%

Savings

Bond
Valuing a Bond

Example

If today is October 2000, what is the value of the following bond?

- An IBM Bond pays $115 every Sept for 5 years. In Sept 2005 it pays an additional $1000 and retires the bond.
- The bond is rated AAA (WSJ AAA YTM is 7.5%).

Cash Flows

<table>
<thead>
<tr>
<th>Sept 01</th>
<th>02</th>
<th>03</th>
<th>04</th>
<th>05</th>
</tr>
</thead>
<tbody>
<tr>
<td>115</td>
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</table>
Example continued

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- An IBM Bond pays $115 every Sept for 5 years. In Sept 2005 it pays an additional $1000 and retires the bond.
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\[
PV = \frac{115}{1.075} + \frac{115}{(1.075)^2} + \frac{115}{(1.075)^3} + \frac{115}{(1.075)^4} + \frac{1150}{(1.075)^5}
\]

\[
= $1,161.84
\]
Bond Prices and Yields

- 5 Year 9% Bond
- 1 Year 9% Bond