Monetary Union with Voluntary Participation*

William Fuchs
University of Chicago
and Stanford GSB†

Francesco Lippi
Bank of Italy and CEPR‡

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Abstract

A Monetary Union is modeled as a technology that makes a *surprise* policy deviation impossible and requires voluntarily participating countries to follow the same monetary policy. Within a fully dynamic context, we show that such an arrangement may dominate a regime with independent national currencies. Two new results are delivered by the voluntary participation assumption. First, optimal policy is shown to respond to a country's temptation to leave the union by tilting both current and future policy in its favor. This yields a non-linear rule according to which each country weight in policy decisions is time-varying and depends on its incentive to abandon the union. Second, we show that there might be conditions such that a break-up of the union, as occurred in some historical episodes, is efficient. The paper thus provides a first formal analysis of the incentives behind the formation, sustainability and disruption of a Monetary Union.

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†University of Chicago, 1126 E. 59th St., Chicago, IL 60637.

‡Research Department, Banca d'Italia, via Nazionale 91, 00184 Rome, Italy.

1. Introduction

International macroeconomic interdependence raises the possibility, first formalized in the seminal work of Hamada (1974, 1976), that non-cooperative decisions by the policy makers of different countries produce inefficient outcomes. A large body of literature has used this insight to analyze international institutions and policy cooperation.[1]

In the field of monetary economics the idea has provided a rationale for monetary unions (MU), an institutional arrangement in which countries relinquish autonomous control over national currencies to adopt a common one. Economic history offers several instances of countries that have deliberately given up monetary independence, jointly or unilaterally, to follow a common policy (Cohen, 1993). The European monetary union is the best known recent example, but the establishment of an MU is also being examined by the six states of the Gulf Cooperation Council, nine nations in South East Asia and a large group of African countries.[2] As argued by Persson and Tabellini (2000, Chapter 18), this phenomenon can be rationalized as a second-best institution-design problem when the cooperative first-best policy is not feasible. In this context, the MU may allow policy makers to alleviate the coordination problem at the expense of a reduced ability to stabilize idiosyncratic shocks.

The trade-off between coordination versus flexibility that emerges in the choice of the monetary regime has proved fruitful for the analysis of fixed exchange rate arrangements and monetary unions, e.g. Alesina and Grilli (1992), Dixit (2000), Canzoneri and Henderson (1991) and Persson and Tabellini (1995). These papers provide a useful foundation to understand the incentives to form a monetary union, but they suffer from two limitations that we try to overcome.

First, the benefits of the MU are usually discussed in comparison to the welfare achievable under the repetition of the static Nash equilibrium, given the premise

[1] For an encompassing survey of applications in the field of fiscal and monetary policy during the last two decades see Persson and Tabellini (1995). Canzoneri and Henderson (1991) use similar ideas to study international monetary arrangements.

[2] See IMF (2003) and Currency News (2003).

that the first-best coordination of policy is “not feasible”. This is not fully satisfactory. The restrictive context of one-shot games should be abandoned, to account for the fact that the underlying strategic environment is a repeated game. Dynamic provision of incentives should be properly analyzed to see what outcomes are sustainable. In practice, some degree of coordination is usually observed outside monetary unions, as one would expect if policy makers do not fully discount the future.[3] Ideally, one would like to understand why a second-best arrangement, in which countries deliberately give up policy independence, may dominate other forms of coordination which do not involve a loss of flexibility.

A second shortcoming of previous contributions concerns how the MU can be sustained. The traditional approach is to assume that countries entering the MU are not allowed to quit it, what we label “enforced participation”. In other words, countries contemplating the formation of a union face a take-it-or-leave-it offer at time zero and are given no further choices afterwards. This is unsatisfactory on both theoretical and empirical grounds.[4]

We abandon the assumption of enforced participation to shed light into how joint policy-making may make the union sustainable even in the absence of an exogenous enforcement technology. The extensions we explore deliver new insights into the sources of the welfare benefits of a monetary union and the way optimal policy responds to shocks given the countries' option to leave the union.

By modeling the union as a technology that makes a *surprise* policy deviation impossible (e.g. an unexpected exchange rate devaluation), we show that an MU may be superior to policy coordination despite the fact that it gives rise to a loss of flexibility. This occurs since a deviation from the “coordinated policy” delivers a smaller payoff when it is anticipated than when it comes as a surprise to other agents. As deviations become less tempting under the MU, better outcomes can be sustained along the equilibrium path on average.

[3] In Europe, for instance, full monetary integration between the members of the Euro area was preceded by various cooperation arrangements (e.g. the European Monetary System).

[4] Persson and Tabellini (2000, page 467) recognize the necessity to complete this analysis: “It is not enough to demonstrate that the policy outcome under cooperative policy making is superior, though, as individual countries generally have incentives to deviate from cooperative policy. The argument is therefore incomplete unless coupled with an argument as to how the suggested solution might be enforced.”

The optimal MU arrangement that emerges with voluntary participation differs markedly from the one under enforced participation. In the latter case, once the union is formed, policy is decided according to time-invariant "Pareto weights" and there are no changes in the way the benefits of belonging to the union are allocated to its members over time. In our case, instead, policy responds to the agents' incentives to leave the union by tilting both current and future policy in their favor. This finding implies that the monetary policy rule in the MU without enforcement is not guided by a time-invariant MU "average" but, in some instances, does take account of the member countries' local conditions. This point is of interest for the ongoing debate on the role that national developments play in the conduct of monetary policy in the euro area (e.g. Heinemann and Hüfner, 2004; Aksoy, De Grauwe and Dewachter, 2002).

Finally, depending on the distribution of the shocks and discount factors, our model shows that the MU might be permanent or temporary. For the latter, there are some states of the world in which the MU *always* breaks apart along the equilibrium path and countries revert to national monetary policy.[5] Intuitively, a break-up occurs because a large asymmetric shock makes it costly to follow a common policy in those states, even though it implies giving up the future benefits of the MU. The possibility that a break-up occurs along the equilibrium path highlights the importance of not assuming an "enforcement technology". Its causes, which remain largely untouched by formal economic analysis, are discussed in this paper.

The break-up phenomenon is empirically relevant. Economic historians and political scientists have long given serious consideration to the "sustainability" of currency unions.[6] Bordo and Jonung (1997) and Cohen (1993) examine the record of several monetary regimes, including various forms of currency unions, some of which successfully lasted for as long as they could (the Belgium-Luxembourg monetary union, founded in 1922, was absorbed into EMU) and others which collapsed fairly quickly (the East African Community collapsed in 1977 after about a decade from its foundation).

[5] There may also be "fragile" states of the world in which the dissolution of the MU depends on the history. See Section 4.2 for a detailed explanation.

[6] A related view was recently offered by Milton Friedman: "[...] I think that within the next 10 to 15 years the eurozone will split apart" (*Financial Times*, June 7 / June 8, 2003).

Dixit (2000) provides an earlier exploration of the idea that optimal policy in a monetary union accounts for country specific shocks. A main difference compared to this paper is that in his model countries have the possibility to lobby the central bank for their most desired policy but do not have the option to abandon the union. His model is silent on what determines the probability that lobbying is successful. In our model, the responsiveness of the common policy to individual country preferences depends on the credibility of the country's threat to abandon the union. Furthermore, we show that when shocks are sufficiently asymmetric a MU breakup may be optimal. Another difference is that the policy considered by Dixit (2000) is restricted to be history-independent.[7] Our fully dynamic analysis shows that incentives (rewards) are smoothed intertemporally under an optimal policy: when a country is hit by a large adverse shock, both current and future policy are adjusted in its favor. The history-dependent nature of optimal policy unveiled by our model captures a key feature of decision making (or, consensus-building) in supra-national institutions.

Recent contributions have revived interest in monetary unions. Alesina and Barro (2002) and Cooley and Quadrini (2003) present general equilibrium models of a currency union which allow welfare analysis to be based on the representative agent utility function. The analysis of our paper complements these studies by providing insights on the interplay of dynamic incentives that make a monetary union sustainable in the absence of an enforcement technology. In doing this, however, we abstracted from explicit microfoundations, as the basic ideas transcend a specific setting. The integration of the two approaches is a natural next step.

From a methodological point of view, our analysis relies on results from the literature on "limited commitment", pioneered by Thomas and Worrall (1988) and Kocherlakota (1996) and originally applied to a risk-sharing environment.[8] One important technical difference in comparison to those studies is that ours has an additional constraint requiring both agents to follow the same action as long as they remain in the MU. The loss of a policy instrument gives rise to a trade-off that in certain circumstances may lead to a break-up of the common policy along

[7] Policy in his setup is only allowed to depend on the current shocks.

[8] This literature has recently found fruitful applications in the international trade literature, e.g. Bond and Park (2002).

the equilibrium path.

Reversion to autarky in Kocherlakota (1996) is always Pareto dominated. It is used as a threat to provide incentives, but it is never observed along the equilibrium path. In our model too the worst possible autarchic equilibrium (implemented through the reinstatement of national currencies) is used to dissuade countries from reneging on the common policy. In addition to this role, however, in our model autarky is actually played by MU members in cases where the MU is dissolved of *mutual* accord. We show that, in such cases, the autarchic equilibria to be followed after the break-up are Pareto efficient. This result unveils the "dual" nature of the post-breakup continuation equilibrium: it serves both as a "stick" (to prevent off-equilibrium deviations) and as a (best feasible) payoff for on-equilibrium breakups.

The paper is organized as follows. The economic environment and the two monetary regimes considered are described in the next section. Section 3 provides a recursive formulation of the MU problem, that is used in Section 4 to characterize optimal policy and discuss the MU sustainability. Section 5 illustrates the key features of our model using an example economy. The main findings and conclusions are summarized in Section 6.

2. The economic environment

We consider a symmetric setup with two infinitely lived *ex-ante* identical countries, named Home and Foreign, each controlling a policy instrument $\pi, \pi^* \in \left[\underline{\pi}, \bar{\pi}\right]$ (asterisks denote foreign variables).[9]

The state of the world s in period t is determined by the realization of a discrete and i.i.d. random variable with support $S = \{s_1, s_2, ..., s_S\}$ with corresponding probabilities denoted by p_s. The state s affects the utility functions for each country in potentially different ways.[10] We assume that the distribution

[9]This assumption is for technical purposes. We will consider bounds that are so large that this constraint will not affect policy.

[10]We can think of each state s as defined by a pair of country-specific variables, as in the example of Section 5.

of these effects over individual countries is symmetric.[11] We also assume there is a payoff irrelevant random variable x_t on which countries can condition their strategies, which is independent across time and states and uniformly distributed in the [0,1] interval.

Let $U(\pi, \pi^*, s)$ and $U^*(\pi^*, \pi, s)$ be the per-period utility of, respectively, Home and Foreign in state s when the policy pair (π, π^*) is chosen. The functions $U(\pi, \pi^*, s)$ and $U^*(\pi^*, \pi, s)$ are assumed to be bounded, jointly differentiable with respect to π and π^* and to have a negative semi-definite Hessian. For there to be a coordination issue we also require some spillover between the agents' actions i.e.$U_2^*, U_2 \neq 0$. Each country maximizes the expected value of the intertemporal utility function $E_o \sum_{t=0}^{\infty} \delta^t U()$, where $\delta \in (0,1)$ is the discount factor.

The history at time t is denoted by h_t where:

$$h_t = \left(\pi_0, ..., \pi_{t-1}; \pi_0^*, ..., \pi_{t-1}^*; s_0, ..., s_t; x_0, ..., x_t\right),$$

i.e. $h_t \in H_t : \left[\underline{\pi}, \bar{\pi}\right]^{t-1} \times \left[\underline{\pi}, \bar{\pi}\right]^{t-1} \times S^t \times [0,1]^t$.

Given this general environment different games can be played depending on the monetary regime chosen. Two regimes are considered: Independent National Monetary Policy (INMP) or a Monetary Union (MU). Under the former each country has its own money printing machine and decides monetary policy unilaterally. Under the MU the individual country money prints are replaced by a commonly managed print, that is used to produce the MU single currency. The loss of a policy instrument (money print) inherent to the MU generates costs and benefits. The cost is that countries in the MU are forced to use the same policy, which may be inefficient when countries are hit by asymmetric shocks. On the other hand, the benefit arises from the fact that the single money-print makes unilateral "surprise" deviations from an agreed policy impossible. We assume that a country's decision to abandon the union (re-installing its own money print and currency) does not come as a surprise to the other country. This is a realistic assumption, justified by noting that the decision to leave the MU takes more time

[11] The purpose of this assumption is to reduce notation by keeping the environment symmetric. It can easily be relaxed.

and is more easily observed by the other parties than the decision to deviate from a plan under INMP. Since deviations no longer come as a surprise in the MU, they become less attractive. This facilitates cooperation. In the next subsections we will describe in greater detail these two monetary arrangements.

Finally, it should be stressed that the qualitative nature of the results presented below would not change if the model was modified to account for other potential benefits of forming an MU, such as a reduction in transaction costs (this can be done by adding an indicator variable to the agents' utility functions) or a fixed cost of breaking the union. We decided to overlook such effects for clarity of presentation.

2.1. Independent National Monetary Policy

When countries retain control over their monetary instrument we have the following timing of events. At the beginning of period t the state s_t and the public randomization device x_t are observed, then Home and Foreign simultaneously set monetary policy $\pi(h_t)$ and $\pi^*(h_t)$, respectively.

Let π_t denote Home's policy function, mapping any possible time-t history into a policy choice: $\pi_t : H_t \rightarrow \left[\underline{\pi}, \bar{\pi}\right]$. Similarly for Foreign $\pi_t^* : H_t \rightarrow \left[\underline{\pi}, \bar{\pi}\right]$.

A policy plan $\Pi \equiv \{\pi_t\}_{t=0}^{\infty}$ is a stochastic vector process which maps any possible history h_t into a policy choice, π_t for all t. Let P be the set of all possible plans: $\Pi \in P$.

Definition 1. *A subgame perfect policy pair $\gamma \equiv (\Pi, \Pi^*) \in P \times P$ is a pair of policy plans such that at every history h_t each country chooses a best response to the other player's strategy.*

Let $w(\gamma) \equiv E_\tau \sum_{i=1}^{\infty} \delta^{i-1} U\left(\pi_{\tau+i}, \pi_{\tau+i}^*, s_{\tau+i}\right)$ denote Home's expected utility from the subgame perfect policy pair γ (similarly $w^*(\gamma) \equiv E_\tau \sum_{i=1}^{\infty} \delta^{i-1} U^*\left(\pi_{\tau+i}^*, \pi_{\tau+i}, s_{\tau+i}\right)$ for Foreign), and indicate by W the set of all such (w, w^*) pairs. We will refer to W as the set of subgame perfect payoffs.

Proposition 1. *A policy pair γ is subgame perfect under INMP if and only if the following holds (for all $s \in S$ and $\tau = 0, 1, 2, ...$):*

$$U^*(\pi^*_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^* \left(\pi^*_{\tau+i}, \pi_{\tau+i}, s_{\tau+i} \right) \right] \geq U^*(\pi^{*d}_\tau, \pi_\tau, s_\tau) + \delta \underline{w} \quad (2.1)$$

$$U(\pi_\tau, \pi^*_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U \left(\pi_{\tau+i}, \pi^*_{\tau+i}, s_{\tau+i} \right) \right] \geq U(\pi^d_\tau, \pi^*_\tau, s_\tau) + \delta \underline{w} \quad (2.2)$$

Where π^d_τ and π^{*d}_τ stand for the optimal deviations and $\underline{w}$ is the smallest expected value attainable with a subgame perfect policy pair.

Proof: Appendix A.

We will denote the set of subgame perfect policy pairs γ with Γ.

Lemma 1. *(i) The set of subgame perfect policy pairs, Γ, is compact and convex. (ii) The set of subgame perfect payoffs, W, is compact.*

Proof: Appendix A.

Given a specific utility function and parameter values, we can use the methods developed by Abreu, Pearce and Stacchetti (1990) to find the set W. We will do this for the example economy analyzed in Section 5.

2.2. Monetary Union

As an alternative to independent monetary policies, countries can choose to form a Monetary Union, in which local currencies are replaced by a common one and policy is jointly determined.

We will focus our analysis on the optimal MU arrangement that countries would set up at the institutional design phase. The key feature of this regime is that a *common* policy $\pi = \pi^*$ must be implemented if the union is to be sustained. At every point in time, implementation of the prescribed policy, π, requires unanimity. It is assumed that countries observe the state s, the payoff irrelevant variable x (as under INMP) and then make simultaneous announcements, $a(h_t)$, $a^*(h_t)$ on whether to follow the prescribed policy ($a = 0$) or not to follow it ($a = 1$). If $a = a^* = 0$, the proposed policy π is implemented and the union is continued into the future. Otherwise, the union is dissolved and countries revert to playing some subgame perfect equilibrium of the INMP game.

Formally, the MU contract consists of three history dependent rules. The first is a "dissolution rule", D, which indicates member countries whether to dissolve the union ($D = 1$) or not ($D = 0$).[12] The second rule, Π, determines the common policy to be implemented while the union is sustained. The third rule, β, selects the equilibrium of the INMP game that is played if the union is dissolved. Denote by $w_s \equiv E_\tau \left[\sum_{i=0}^{\infty} \delta^i U\left(\pi_{\tau+i}, \pi^*_{\tau+i}, s_{\tau+i}\right) | s_\tau\right]$ the expected utility delivered to Home by the β rule conditional on the state s (similarly, $w^*_s \equiv E_\tau \left[\sum_{i=0}^{\infty} \delta^i U^*\left(\pi^*_{\tau+i}, \pi_{\tau+i}, s_{\tau+i}\right) | s_\tau\right]$ for Foreign), and indicate by W_s the set of all such (w_s, w^*_s) pairs. We summarize this scheme with:

Definition 2. *A MU contract is composed of three sequences of functions. The first prescribes whether to remain in the union, $D_t : H_t \to \{0,1\}$. The second determines the common policy for period t, $\Pi_t : H_t \to \left[\underline{\pi}, \bar{\pi}\right]$. The third selects the INMP equilibrium values to assign to each country when a break-up occurs, $\beta_t : H_t \times D \times a \times a^* \to (w_s, w^*_s) \in W_s$.*

Note that the equilibrium values prescribed by the function β upon a break-up are allowed to depend on the history prior the break and on whether the break-up was consensual or unilateral. Through this mechanism, a country that reneges on a prescribed policy and causes a break-up can be harshly punished; meanwhile, if there are states of the world in which it is optimal to dissolve the MU, then countries can play an efficient INMP equilibrium after the break-up.

Let us adopt the following:

[12] Note that since D can depend on the current realization of the public randomization device x_t the fact that $D = \{0,1\}$ is not an important restriction. We can use the dependence of D on x_t to convexify choices over $[0,1]$.

Definition 3. *A MU contract* (D, Π, β) *is sustainable if it is a best response for Home and Foreign to always follow the recommended policy (i.e.* $a_t = a_t^* = 0$*).*

(i) *For every history* h_t *with* $D_t = 0$*, given a recommended policy* π_t*, the best response for each country is to follow it.*

(ii) *For every history* h_t *with* $D_t = 1$*, it must be a best response to follow the strategies prescribed by* β*.*

Any contract (D, Π, β) will satisfy (ii) given that by definition $\beta(H_t, D, a, a^*)$ determines continuation values after break-ups that are part of the equilibrium value set in the continuation game. Therefore, to check whether a contract is sustainable, we only need to verify that the following is satisfied:

Condition 1 : *For all* h_τ *with* $D_\tau = 0$:

$$U^*(\pi_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^*\left(\pi^*_{\tau+i}, \pi_{\tau+i}, s_{\tau+i}\right) \mid (D, \Pi, \beta) \right] \geq w^*[\beta(h_\tau, 0, 0, 1)]$$
$$U(\pi_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U\left(\pi_{\tau+i}, \pi^*_{\tau+i}, s_{\tau+i}\right) \mid (D, \Pi, \beta) \right] \geq w[\beta(h_\tau, 0, 1, 0)]$$

Condition 1 requires that the welfare enjoyed by a country under the MU is not smaller than the welfare delivered by the β rule to that country in case it decides not to follow the prescribed policy (the value reported on the right hand side of the inequality).

Let Σ denote the set of sustainable MU contracts. Let $v(D, \Pi, \beta), v^*(D, \Pi, \beta)$ be the *expected* utility delivered by the (D, Π, β) contract to Home and Foreign, respectively. Let V be the set of all pairs (v, v^*) such that $(D, \Pi, \beta) \in \Sigma$. We will refer to V as the set of MU-sustainable payoffs.

Lemma 2. *(i) The set of MU sustainable contracts,* Σ*, is compact and convex. (ii) The set of MU sustainable payoffs,* V*, is compact and convex.*

Proof: Appendix A.

Note that, since countries can choose to dissolve the MU at $t = 0$, the strategies sustainable under INMP are a subset of those sustainable under the MU, which implies that $W \subseteq V$, i.e. that the MU weakly dominates the INMP regime. Note, however, that if a fixed cost was incurred by breaking up the union, or necessary to form it, then it might not anymore be true that it is always desirable to form the MU ex ante.

3. A Recursive Representation of the MU contract

3.1. Efficient frontier

To characterize the set of efficient MU contracts we need the following:

Definition 4. *A contract $(D, \Pi, \beta) \in \Sigma$ is efficient if there exists no other element in Σ that Pareto dominates it.*

We define $v_{\max}$ to be the maximal level of utility available to one of the countries from a contract in Σ. We define $v_{\min}$ as follows:[13]

$$\begin{aligned} v_{\min} &= \max_{v} v \\ \text{subject to} &: \\ (v, v^*) &\in V \\ v^* &= v_{\max} \end{aligned}$$

Proposition 2. *For all pairs $(v, v^*) \in V$ with $v^* \geq v_{\min}$ there exists an efficient allocation in Σ which delivers the payoff vector $(\bar{v}, v^*)$, where $\bar{v}$ is defined as follows:*

$$\begin{gathered} \bar{v} = \max_{q,q^*} q \\ \textit{subject to}: \\ (q, q^*) \in V \\ q^* \geq v^* \end{gathered}$$

[13] By the symmetry of the setup these values are identical for Home and Foreign. The asterisk is thus suppressed.

Proof: Appendix A.

The key of this proposition is not the existence of a solution to the maximization problem[14] but rather that in the solution the second constraint must be binding ($q^* = v^*$). That implies that the efficient frontier of the set V is decreasing in the range $[v_{\min}, v_{\max}]$. Furthermore, it implies that the Pareto frontier V is self-generating.

We can characterize the Pareto frontier as follows. Let $V(v^*)$ denote the expected utility delivered by a social planner to Home conditional on having promised an expected utility level v^* to Foreign, $V\colon [v_{\min}, v_{\max}] \longrightarrow [v_{\min}, v_{\max}]$. Then:

$$V(v^*) = \max_{(D,\Pi,\beta)} E_0 \left[\sum_{t=0}^{\infty} \delta^t U(\pi_t, \pi_t^*, s_t) \right] \tag{3.1}$$

subject to:

$$\begin{aligned} (D, \Pi, \beta) &\in \Sigma & (3.2) \\ E_0 \left[\sum_{t=0}^{\infty} \delta^t U^*(\pi_t^*, \pi_t, s_t) \,|\, (D, \Pi, \beta) \right] &= v^* & (3.3) \end{aligned}$$

Constraint (3.2) imposes that contracts must be sustainable, (3.3) is the "promise keeping" constraint i.e. it requires the contract to deliver an expected utility level of at least v^* to Foreign.

3.2. On the optimal determination of post-break-up equilibria.

There are two roles played by β. The first is to provide a punishment for a country that does not follow a prescribed policy and the second is to determine a continuation equilibrium when the MU is optimally dissolved. The following Lemma captures the optimal properties of β when performing these different roles.

Lemma 3. *(i) If a country, say Home, reneges on the prescribed policy (sets $a_t = 1$) then it is optimal to have $\beta(h_t, 0, 1, 0) = (\underline{w}_s, w^*(\underline{w}_s))$. Where $\underline{w}_s$ is*

[14] This follows from the compactness of V.

the minimum value for Home in W_s and $w^*(\underline{w}_s)$ is highest the value for Foreign consistent with this equilibrium. Similarly $\beta(h_t, 0, 0, 1) = (w(\underline{w}_s^*), \underline{w}_s^*)$

(ii) If, for a given h_t, $D(h_t) = 1$ then it is optimal to set $\beta(h_t, 1, 0, 0) = (\overline{w}_s, \overline{w}_s^*)$. Where $(\overline{w}_s, \overline{w}_s^*)$ is an element on the efficient frontier of the INMP game conditional on the state s having been realized.

Proof:

(i) Follows from noting that $Condition$ 1 is relaxed by assigning the worst possible continuation values to countries that have deviated. This allows for a greater range of policy plans to be sustainable by the MU contract. Since equilibrium policy will have no deviations, payoffs are not negatively affected.

(ii) Suppose that after a recommended dissolution β induces a Pareto dominated equilibrium with values (w, w^*). Now suppose instead we picked a pair $(\overline{w}, \overline{w}_s^*)$ that Pareto dominates (w, w^*). This would relax $Condition$ 1 since the RHS is not affected by this change but LHS is increased. Furthermore, the value of being in the MU is increased because now in the eventuality that a break-up is recommended the continuation values are weakly higher for both countries. Therefore, an optimal arrangement will always recommend an efficient equilibrium of the INMP game after an on-equilibrium dissolution.

The previous result is intuitive. When the break-up results from the deviation by one of the countries then it is optimal to punish this country as harshly as possible. By so doing, the incentives to deviate are curbed. On the other hand, when the break-up is consensual, it is in the best interest of both countries to coordinate policy in the most efficient way. We can therefore summarize the characterization of the optimal break-up rule β in the following way:

$$\beta_t(h_t, D_t, a_t, a_t^*) = \left\{ \begin{array}{ll} (\overline{w}_s, \overline{w}_s^*) & if \ D_t = 1, a_t = a_t^* = 0 \\ (\underline{w}_s, w^*(\underline{w}_s)) & if \ D_t = 0 \text{ and } a_t = 1 \\ (w(\underline{w}_s^*), \underline{w}_s^*) & if \ D_t = 0 \text{ and } a_t^* = 1 \end{array} \right\}.$$

This is not a full characterization of β since the particular points in the Pareto frontier of the INMP game that are chosen after a consensual break-up still remain to be determined. We will say more about this in Section 4.

3.3. Recursive Representation

The function $V(v^*)$ is decreasing, strictly concave and continuous.[15] Furthermore, monotonicity implies it is differentiable almost everywhere. Unfortunately the sequential representation of $V(v^*)$ $(3.1-3.3)$ is not very useful to figure out the properties of the optimal policy. Denote by the function $\overline{W}\left(\overline{w}_s^*\right)$ the Pareto frontier of the INMP game, i.e. the maximum value that can be assigned to Home when foreign is assigned a value of $\overline{w}_s^*$. Finally, let d_s denote threshold values for x_t above which a break-up is recommended in state s. The next proposition establishes a recursive formulation of the problem that is helpful to characterize the MU equilibrium.

Proposition 3. *The function V satisfies the functional equation:*

$$V(v_o^*) = \max_{(\pi_s, v_s^*, d_s, \overline{w}_s^*)} \sum_{s \in S} p_s \left\{(1-d_s)\left[U\left(\pi_s, \pi_s, s\right) + \delta V\left(v_s^*\right)\right] + d_s\, \overline{W}\left(\overline{w}_s^*\right)\right\} \tag{3.4}$$

subject to:

$$v_o^* = \sum_{s \in S} p_s \left\{(1-d_s)\left[U^*\left(\pi_s, \pi_s, s\right) + \delta v_s^*\right] + d_s \overline{w}_s^*\right\} \tag{3.5}$$

$$\underline{w}_s^* \leq U^*\left(\pi_s, \pi_s, s\right) + \delta v_s^* \qquad \forall\, s \quad s.t. \qquad d_s < 1 \tag{3.6}$$

$$\underline{w}_s \leq U\left(\pi_s, \pi_s, s\right) + \delta V\left(v_s^*\right) \qquad \forall\, s \quad s.t. \qquad d_s < 1 \tag{3.7}$$

$$v_s^* \in [v_{\min}, v_{\max}],\ \left(\overline{W}\left(\overline{w}_s^*\right), \overline{w}_s^*\right) \in W_s,\ d_s \in [0,1] \tag{3.8}$$

Proof: Appendix A.

Constraint (3.5) is the promise keeping constraint, constraints (3.6) and (3.7) are the sustainability (participation) constraints for Foreign and Home, respectively, so that they do not leave the union. Condition (3.8) imposes that promised continuation values lie in the corresponding set of sustainable and efficient MU and INMP values. Note that in this representation we have incorporated the results of *Lemma* 3. This shows up in that the LHS of the participation constraints

[15] Decreasing follows from Proposition 2. Concavity follows since we assumed the period utility function is strictly concave in π_s and the constraint set Σ is convex. Continuity is implied by the Theorem of the Maximun.

is given by $\underline{w}_s^*$ and $\underline{w}_s$. It is also embedded in the fact that we maximize over $\overline{w}_s^*$ and the value for *Home* is given by $\overline{W}\left(\overline{w}_s^*\right)$ reflecting the fact that after a consensual break-up the countries always go to a point on the Pareto frontier of the INMP game. The dissolution function, D, can therefore be expressed as:

$$D(v_0, s, x) = \left\{ \begin{array}{lll} 0 & if & x > d_s\left(v_0\right) \\ 1 & if & x \leq d_s\left(v_0\right) \end{array} \right\}$$

4. Characterization of the Equilibrium in the Monetary Union

This section establishes some results to characterize the MU equilibrium. First we study policy dynamics inside the union. Secondly we analyze the sustainability of the MU.

4.1. Optimal policy and dynamics in the MU

Let us take D as given and solve for the optimal policy inside the union. Consider the problem (3.4-3.8). For any feasible allocation that promises a value of v_o^* to Foreign, we can divide the state space in the following partition:

S_1 = states in which neither (3.6) nor (3.7) is binding
S_2 = states in which (3.6) is binding but not (3.7)
S_3 = states in which (3.7) is binding but not (3.6)
S_4 = states in which the union cannot be sustained.

The states in S_4 are such that either both countries mutually prefer to break the union or the country that prefers to remain in the union is unable (or unwilling) to provide the necessary incentives to prevent the other country from abandoning the union.[16] The optimal MU design will recommend dissolution for those states ($d_s = 1$).

[16] In Kocherlakota (1996) it is never the case that both participation constraints bind at the same time (i.e. $S_4 \equiv \varnothing$). This occurs since his contract does not restrict players' actions under the contract to be identical, hence allowing it to replicate the policy under autarky (INMP in our case). Instead, the additional constraint imposed by our problem that countries must choose the *same* policy while in the MU creates the possibility that some INMP outcomes cannot be replicated by the MU.

A useful characterization of the equilibrium properties of this problem is obtained from the Lagrangian representation of the functional equation that appeared above. Before doing so we must first address one last technical point. So far, we have shown that V is differentiable almost everywhere but, for the analysis that follows we actually need it to be differentiable everywhere. Koeppl (2003) shows how things can go wrong in the environment of Kocherlakota (1996) if V is not differentiable everywhere. He also provides sufficient conditions to guarantee differentiability of V. We will consider parameter settings such that these conditions are met. Let us write the Lagrangian:

$$\begin{aligned} V(v_o^*) \equiv & \max_{(\pi_s, v_s^*, \overline{w}_s^*)} \sum_{s\in S} p_s \left\{ \left(1-\bar{d}_s\right) \left[U\left(\pi_s, \pi_s, s\right) + \delta V\left(v_s^*\right)\right] + \bar{d}_s\, \overline{W}\left(\overline{w}_s^*\right) \right\} & (4.1) \\ & +\lambda \left\{ \sum_{s\in S} p_s \left[\left(1-\bar{d}_s\right)\left(U^*\left(\pi_s, \pi_s, s\right) + \delta v_s^*\right) + \bar{d}_s \overline{w}_s^* \right] - v_o^* \right\} & (4.2) \\ & + \sum_{d_s<1} \mu_s \left(1-\bar{d}_s\right)\left[U^*\left(\pi_s, \pi_s, s\right) + \delta v_s^* - \underline{w}_s^*\right] & (4.3) \\ & + \sum_{d_s<1} \theta_s \left(1-\bar{d}_s\right)\left[U\left(\pi_s, \pi_s, s\right) + \delta V\left(v_s^*\right) - \underline{w}_s\right] & (4.4) \end{aligned}$$

The first order conditions with respect to v_s^* give:

$$\begin{aligned} \left(p_s + \theta_s\right) V'\left(v_s^*\right) + \lambda p_s + \mu_s &= 0 \;\; if \;\; v_s^* \in \left(v_{\min}, v_{\max}\right) & (4.5) \\ &\geq 0 \;\; if \;\; v_s^* = v_{\max} \\ &\leq 0 \;\; if \;\; v_s^* = v_{\min} \end{aligned}$$

The one with respect to π_s yields:

$$\left(p_s + \theta_s\right) U_\pi + \left(\lambda p_s + \mu_s\right) U_\pi^* = 0 \tag{4.6}$$

The one with respect to $\overline{w}_s^*$ yields:

$$\overline{W}'\left(\overline{w}_s^*\right) + \lambda = 0 \tag{4.7}$$

The last result completes the characterization of the optimal β function. From Lemma 3 we know that after a consented break-up the optimal policy captured by the β function prescribes to follow "an" equilibrium on the frontier of the INMP game. Equation (4.7) together with the envelope condition $V'(v_o^*) = -\lambda$ further yields:

$$V'(v_o^*) = \overline{W}'(\overline{w}_s^*). \tag{4.8}$$

This condition completes the characterization of the β function by pinning down the equilibrium values in the post break-up game. It establishes that a higher promised value for a country in the union also implies a higher value for that same country in case the union is dissolved.

Note that, at an internal solution, (4.5) and (4.6) imply:

$$V'(v_s^*) = \frac{U_\pi}{U_\pi^*} \tag{4.9}$$

an efficiency condition equating the agents' marginal rate of substitution to the technical rate of transformation (the slope of the efficient frontier, V'). Let us study the implications of the first order conditions in the different regions of the state space:[17]

Region $\mathbf{S_1}$: Neither participation constraint binds, hence $\mu_s = \theta_s = 0$ which implies $V'(v_s^*) = -\lambda < 0$. As noted above $V'(v_o^*) = -\lambda$, which gives:

$$V'(v_o^*) = V'(v_s^*). \tag{4.10}$$

It follows from the strict concavity of V that $v_o^* = v_s^*$. Hence, when neither participation constraint binds, the expected utility promised to each country in the union is the same one with which the country entered the period, i.e. the promised value is kept constant at v_o^* for Foreign and at $V(v_o^*)$ for Home. Moreover, equations (4.9) and (4.10) show that current policy (π) in the states of this region is such that a constant ratio between the marginal utilities of Home and Foreign is maintained. Note how this last result is isomorphic to the one that emerges as the internal optimum of a planner's problem in which each country's utility function is given a time-invariant Pareto weight.

Region $\mathbf{S_2}$: The participation constraint of Foreign binds, i.e. $\mu_s > 0$, $\theta_s = 0$.

[17] The analytical derivation of the equilibrium properties in regions S_1, S_2 and S_3 is analogous to the analysis developed by Kocherlakota (1996) for a risk-sharing problem.

This yields:

$$V'(v_s^*) = V'(v_o^*) - \frac{\mu_s}{p_s} \tag{4.11}$$

which implies that $v_s^* > v_o^*$ (by the concavity of V). Hence in states of the world belonging to S_2 the promised utility to Foreign increases (the expected utility of Home decreases). It follows from equation (4.9) that the current policy choice is also closer to Foreign's preferred policy. This contrasts with the constant weighting observed in the presence of an enforcement technology (i.e. problem without participation constraints).

Region $\mathbf{S_3}$: This yields symmetric opposite results to those in Region S_2.[18]

These results illustrate the nature of optimal policy in a monetary union with voluntary participation. Policy obeys a state contingent rule which only gets revised when one of the countries has the incentive to leave the union (i.e. the participation constraint binds). When no such incentives arise, the rule is analogous to the efficient one produced by a planner who maximizes the utility of the two countries assigning each of them a Pareto weight. If one country has the incentive to leave the union, then the new policy rule for the current and future periods is closer to that country's unilateral optimal choice. The new rule increases the country's weighting in the current policy decision and the expected continuation value from remaining in the union, making the country indifferent between remaining or leaving. This rule remains in place until the next "renegotiation", i.e. until a state is again reached where one participant has an incentive to leave.

4.2. Sustainability of the Monetary Union

The general characterization of the sustainability can be given by three kind of states, first we can have no break-up states, that is, regardless of the promised value v_0^* in state s we never observe a break-up. Then we have the break-up states

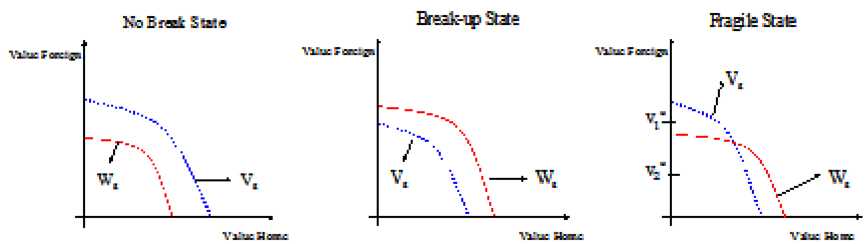
[18] Participation constraint of Home binds, i.e. $\mu_s = 0, \theta_s > 0$.

$$V'(v_s^*) = \frac{p_s}{p_s + \theta_s} V'(v_o^*)$$

which implies $v_s^* < v_o^*$ (by the concavity of V and recalling $V' < 0$). Therefore, in states of the world belonging to S_3 both the current and promised utility delivered to Foreign decrease.

for which the MU always dissolves irrespective of v_0^* and finally we have the fragile states, for which the sustainability of the MU depends on v_0^*.

The following graphs illustrate the three different type of states:



The dotted frontier V_s is the Pareto frontier achievable if the countries do not break up in state s. W_s is the frontier of values that can be achieved under INMP when the current state is s. In the leftmost figure the set of values achievable conditional on staying in the MU is higher than that under INMP for all possible values for Foreign therefore this would be a no break state. For the figure in the center the opposite is true, values in the INMP dominate those achievable under the MU for any v^* therefore a break-up will always be recommended for this state. Finally in the last case the sustainability of the MU will depend on v^*. Certain levels of promised utility can be more efficiently delivered when there are two instruments available rather than one. The value v_2^* is best to deliver it under INMP but the value v_1^* is better to deliver it under a MU. Another example that helps us understand the dependence on v^* is the case in which the shocks are almost perfectly correlated. Clearly if we want to deliver symmetric values there will be no reason to break up the union. Suppose instead we wanted to give a very high value to Foreign, this might be easier to do if we have two instruments at our disposal instead of one.

If the set of "*Fragile*" states is empty then we have separability in the sense that the Dissolution function D is only a function of s and not the whole history. This result implies that, regardless of the initial bargaining power of the two countries in the initial institution design phase of the union, they would both agree on the states of the world in which to sustain the union and on which not to. This property is quite appealing, the players will remain in the union as long as they find it mutually profitable in expectation. Though, as we showed in the previous section, their individual value of being part of the union will be changing as time goes by. From a computational standpoint this facilitates the analysis of the Pareto frontier, since we need only find one optimal set of states on which the union holds. In Appendix B we analyze two cases for which separability holds. The first case assumes that regardless if the break-up was recommended or unilateral the countries continuation value is given by the repeated static Nash Equilibrium, $\beta_t : S \times D \times a \times a^* \rightarrow \left(w_s^N, w_s^{N*}\right)$. The second case is given by assuming preferences are quasi-linear and allowing for transfers.

5. An example economy

This section utilizes a stylized economy to illustrate, by means of simple algebra and numerical computations, some of the results that were discussed above in a more general context.

Let the random variables ε_t and ε_t^* denote a time-varying policy target for, respectively, Home and Foreign. Each state of the world is characterized by the pair $s \equiv (\varepsilon_s, \varepsilon_s^*) \in S$. It is assumed that ε and ε^* follow an i.i.d. process with: $E(\varepsilon) = E(\varepsilon^*) = \bar{\varepsilon}$, $var(\varepsilon) = var(\varepsilon^*) = \sigma^2$ and covariance $cov\left(\varepsilon, \varepsilon^*\right)$. We focus on an ex-ante symmetric case, so that even though the realizations of ε_s and ε_s^* may differ, their joint distribution is symmetric.

Let Home's objectives be described by the intertemporal objective function $V = \Sigma_{t=0}^{\infty} \delta^t U_t$. The period utility function is given by:[19]

$$U\left(\pi_t, \pi_t^*, s_t\right) = (1-\delta)\left[-\frac{\left(\pi_t - \varepsilon_t\right)^2}{2} + \alpha\left(\pi_t - \pi_t^*\right)\right] \tag{5.1}$$

where π_t and π_t^* denote the control variables of, respectively, Home and Foreign (an analogous utility expression holds for Foreign). The quadratic term captures the costs incurred by a country when the policy target is not hit. The linear term $\pi_t - \pi_t^*$ posits that Home benefits from setting its instrument "above" the

[19] This specification can be seen as a indirect (or reduced form) specification of the country's motives. Canzoneri and Henderson (1991), among others, discuss policy making in interdependent economies, showing that it naturally gives rise to the tradeoff between domestic and external objectives, as posited in (5.1).

level chosen by Foreign. For concreteness we can think of π_t as denoting Home's inflation, over which policy makers have perfect control. This abstraction provides a stylized way to describe a country's motive to surprise its neighbor by means of an unanticipated monetary expansion and gives rise to a coordination problem.

5.1. Subgame perfect equilibria in the repeated game (INMP)

The repeated nature of the game allows countries to sustain reputational equilibria that dominate the static Nash equilibrium in terms of welfare. Equations (2.1) and (2.2) in Proposition 1 characterize sustainable strategies in a repeated game. The efficient welfare frontier implied by these conditions can be described through the value function $W(w_s^*)$, which traces the maximal (expected) utility attainable by Home provided the utility delivered to Foreign is w_s^*.[20] Formally, the value function $W(w_s^*)$ is defined as:

$$W(w_o^*) \equiv \max_{\pi_s, \pi_s^*, w_s^*} \Sigma_s \left[U(\pi_s, \pi_s^*, s) + \delta W(w_s^*) \right] p_s \tag{5.2}$$

$$\text{subject to :}$$

$$w_o^* = \Sigma_s \left[U^*(\pi_s^*, \pi_s, s) + \delta w_s^* \right] p_s \tag{5.3}$$

$$U^*(\pi_s^*, \pi_s, s) + \delta w_s^* \geq U^*(\pi_s^{*d}, \pi_s, s) + \delta \underline{w} \qquad \forall\ s \tag{5.4}$$

$$U(\pi_s, \pi_s^*, s) + \delta W(w_s^*) \geq U(\pi_s^d, \pi_s^*, s) + \delta \underline{w} \qquad \forall\ s \tag{5.5}$$

$$(W(w_s^*), w_s^*) \in W$$

The incentive constraints (5.4 and 5.5) impose the requirement that countries must be willing to stick to the optimal plan in each period and for all states of the world. The right hand side of these constraints states that a deviation from the optimal plan is punished in the future with the reversion to the "worst" sustainable equilibrium, which has an expected value of $\underline{w}$.[21] The credibility of this threat requires that the strategy pair that yields $\underline{w}$ is itself a subgame perfect equilibrium satisfying the participation constraints (2.1) and (2.2) (see Appendix C).

[20] The recursive formulation is achieved expressing the continuation strategy by means of its value, following Abreu, Pearce and Stacchetti (1990).

[21] The root of this idea is in the "stick and carrot" strategy first proposed by Abreu (1988).

With reputation, the first best can be sustained provided the discount factor is sufficiently large. In the example economy, it is easy to show that for a given "punishment value" $\underline{w}$, the first best is sustainable if $\delta \geq \frac{\alpha^2}{\alpha^2 - 2\underline{w}}$. For instance, if the static Nash equilibrium is chosen as a punishment for deviations ($w_N = -\frac{\alpha^2}{2}$), the first best can be sustained with reputation provided $\delta \geq \frac{1}{2}$. Even if the discount is smaller than this value, the first best can be supported by a (credible) punishment more severe than Nash. In general, finding the "best" (possibly smaller than the first-best) and the "worst" sustainable values from the solution of problems (C.1) and (C.2) can be done numerically for a given model parametrization. A few examples are discussed in Section (5.3).

5.2. Voluntary Monetary Union

In a voluntary MU the following participation constraints need to be satisfied in all states where the union is sustained :

$$
\begin{aligned}
U^*(\pi_s, \pi_s, s) + \delta v_s^* &\geq U^*(\pi_s^*, \pi_s, s) + \delta w_s^* && (5.6)\\
U(\pi_s, \pi_s, s) + \delta V(v_s^*) &\geq U(\pi_s, \pi_s^*, s) + \delta w_s && (5.7)
\end{aligned}
$$

The right hand side of these constraints denotes the INMP payoff allocated to Foreign and Home by the breakup rule $\beta(h, D, a, a^*)$, discussed in Section 2, which allows unilateral (off-equilibrium) breakups to be distinguished from consensual (on-equilibrium) breakups. As shown by Lemma 3, the efficient rule allocates "harsh punishment" to unilateral breakups and rewards consensual breakups with the reversion to efficient INMP equilibria.

A straightforward application of the results of Section 4.1 allows optimal policy in the MU to be characterized as follows:

Proposition 4. *Policy in the example economy is a convex combination of the preferred policies by Home (ε_s) and Foreign (ε_s^*):*

$$
\pi_s = \kappa_s \varepsilon_s + (1 - \kappa_s)\varepsilon_s^* \qquad (5.8)
$$

where the weight κ_s is given by:

(i) $\kappa \equiv \frac{1}{1+\lambda}$ *when neither participation constraint binds (Region* S_1*)*
(ii) $\kappa_s^F \equiv \frac{p_s}{p_s(1+\lambda)+\mu_s}$ *when Foreign's participation constraint binds (Region* S_2*)*
(iii) $\kappa_s^H \equiv \frac{p_s+\theta_s}{p_s(1+\lambda)+\theta_s}$ *when Home's participation constraint binds (Region* S_3*).*

Proof: Follows from the first order condition (4.6) and equation (5.1) by noting that, given the initial promised value v_o^**, the Lagrange multipliers* μ_s *and* θ_s *are zero in the states where their respective constraint does not bind and that* $\lambda = -V'(v_o^*)$ *in all states.*

When participation constraints do not bind, policy obeys a time-invariant weighting of the policies preferred by Home (ε_s) and Foreign (ε_s^*), with weights κ and $(1-\kappa)$, respectively.[22] This obviously replicates the outcomes of a MU in which participation is exogenously imposed (a setup that we label "MU with enforcement" and is used as benchmark in what follows). More interestingly, the proposition indicates that if a state is reached where the participation constraint of a country binds, then the optimal policy rule (5.8) prescribes that this country is given a greater weight in the decision process (note that $\kappa_s^H > \kappa$ and that $\kappa_s^F < \kappa$), which is to be maintained in future as long as participation constraint do not again bind (as discussed for the general case in Section 4.1).

5.3. Numeric examples

A few numeric examples illustrate the workings of our theory. Assume four states of the world: $s \equiv (\varepsilon, \varepsilon^*) \in \{(1,0),(0,1),(2,0),(0,2)\}$, with the probability mass of each state given, respectively, by $p_s \equiv \{0.3, 0.3, 0.2, 0.2\}$, and intertemporal discount $\delta = 0.1$. The rows of Table 1 report the welfare values of alternative subgame perfect symmetric equilibria. Each row is computed for a different value of the externality α (first column). Greater values of this parameter imply that the externality problem is more relevant, as is reflected in the worsening of the Nash equilibrium value (third column). Note that the discount factor was chosen to be sufficiently low so that the first best could not be sustained by reputation.

[22] Note that $\pi_s = \varepsilon_s$ and $\pi_s^* = \varepsilon_s^*$ is the first best (symmetric) solution of the (Ramsey) planning problem.

Table 1. Sustainable Values Under Different Regimes

	INMP			MU w. enf.	Voluntary MU
α	Worst	Static Nash	Best*	Best*	Best*
0.5	-0.23	-0.12	-0.05	-0.27	Not Sustainable
2	-3.71	-2.00	-0.70	-0.27	-0.34
4	-14.85	-8.00	-2.89	-0.27	-0.27

Note: * This column reports the value of the best *symmetric* equilibrium

The first row shows that when the externality problem is not too large (small α), the value of the best sustainable reputational equilibrium (-0.05) is very close to the value of the first best (zero). In this setting the MU collapses with probability one in the first period, we therefore write that it is not sustainable. Moreover, the best INMP dominates the value delivered by an exogenously enforced monetary union (discussed above). This parametrization thus supports the case for independent national monetary policy.[23]

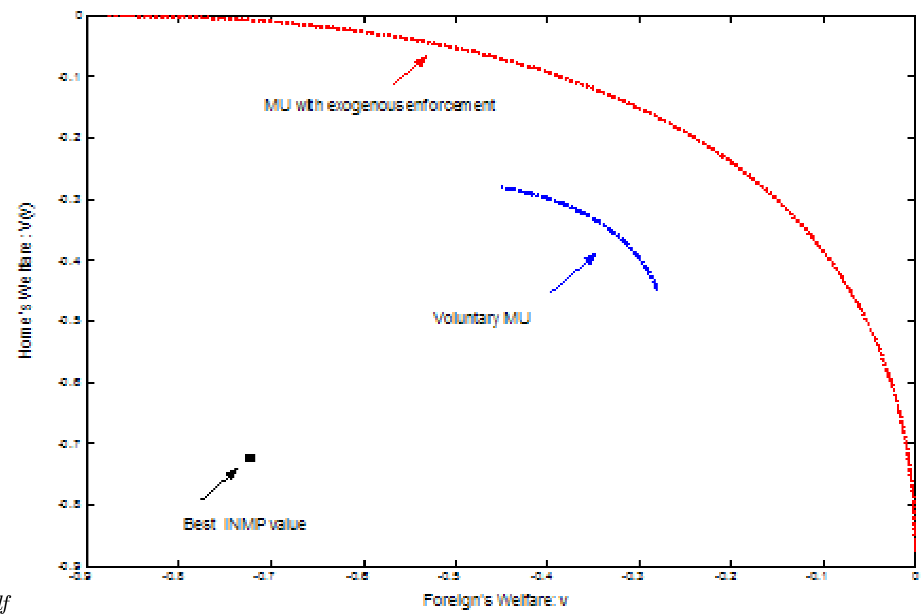
As the externality problem becomes more serious, welfare under the voluntary monetary union dominates the value of the best reputational equilibrium. The second row of Table 1 illustrates this possibility. The welfare achieved by the countries that enter the MU arrangement is superior to the welfare they achieve from the best sustainable equilibrium of the INMP regime. The lack of enforcement and the absence of breakup states make the voluntary MU slightly worse than the MU with enforcement. When α is very large participation is not a problem and the outcomes of a voluntary MU (without break-up states) coincide with those of an MU with enforcement (third row).

The welfare frontier of the MU with and without enforcement for the case in which $\alpha = 2$ are shown in Figure 1. Under the chosen parameterization, no portion of the efficient frontier is sustainable, as indicated by the fact that the frontier of the voluntary MU lies below the frontier of an MU with exogenous enforcement. This indicates that participation constraints bind, at least in some

[23] Even though, strictly speaking, the MU weakly dominates the INMP regime (as discussed in Section 2.2), this is an instance in which the MU and the INMP span the same set of values $V = W$ since the MU collapses in the first period with probability one.

states. The importance of the reputational mechanism is illustrated by the fact that welfare for Home and Foreign improves substantially, under both the INMP and the MU regimes, in comparison to the static Nash equilibrium. Moreover, the set of efficient values sustained by the MU Pareto dominates the corresponding INMP values. This point, as we mentioned in the comment of Table 1, provides a rationale for a monetary union even when better-than-Nash outcomes can be sustained by means of reputation under independent national monetary policies.

1



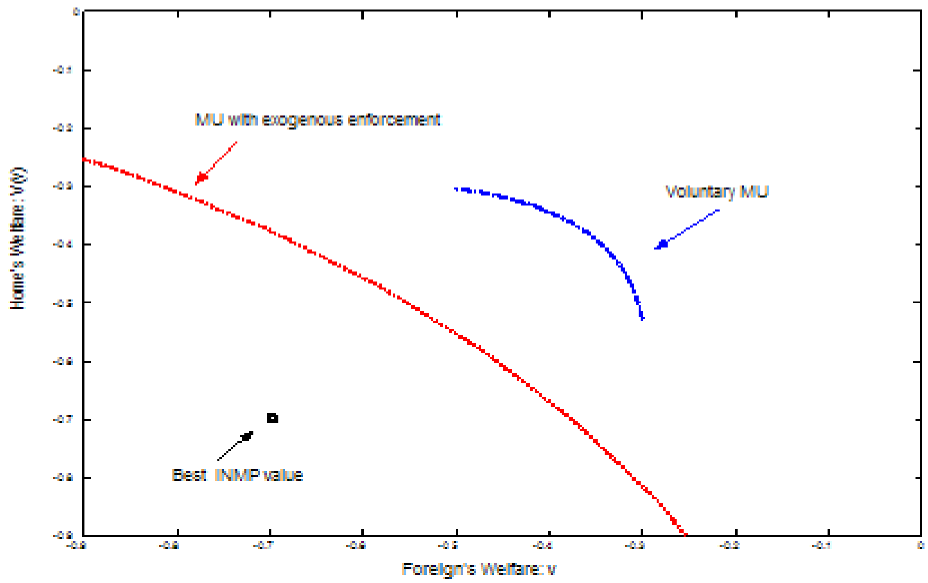
$_4$.*pdf*

Figure 1: Welfare frontiers with small asymmetric shocks

5.4. The MU with Breakups

Finally, we consider the case for a temporary MU. We modify the setup by increasing the asymmetry of the shocks in states s_3 and s_4, i.e. $s \equiv (\varepsilon, \varepsilon^*) \in \{(1,0),(0,1),(3,0),(0,3)\}$, leaving all other parameters unchanged.

2



5.pdf

Figure 2: Welfare frontiers with large asymmetric shocks

Figure 2 (for the case in which $\alpha = 2$) shows that this modification does not affect the value achieved by the INMP regime.[24] Instead, the value of the (best symmetric) MU with exogenous enforcement decreases sharply (from -0.27 to -0.52), due to the fact that the arrangement obliges countries to bear a common policy even in the presence of large asymmetries. The MU with breakups, which involves a permanent reversion to an efficient INMP if either s_3 or s_4 are hit, dominates the outcomes of both the INMP and the MU-with-enforcement regimes.

The above indicates that, in situations where shocks are strongly asymmetric, a voluntary MU involving breakups may be welfare improving ex-ante. As shown, the flexibility allowed for by the breakup option allows the MU to dominate a regime where no breakups are allowed by an exogenous "enforcement technology". In such situations countries might not want to irreversibly commit to the MU (for instance by setting an very high dissolution costs) at the institutional design stage.

[24] This is due to the linear specification of the externality term in (5.1) and the symmetry of the shocks.

6. Concluding remarks

History offers several examples of countries participating in international agreements that constrain unilateral policy actions, such as exchange rate interventions, therefore removing one adjustment mechanism otherwise available to policy makers. This paper explored the motives behind a country's choice to voluntarily adopt such a constraint, as it occurs in a monetary union.

We model the MU as a technology which precludes policy surprises (e.g. an unexpected exchange rate realignment) at the cost of foregoing a policy instrument. It is shown that this technology may dominate a coordinated system with independent national currencies, hence providing a rationale for the formation of an MU.

Departing from the previous literature on international monetary arrangements we abandon the assumption that countries are exogenously bound to the monetary union and explicitly model their incentives to remain within the union or to leave it. This leads to two novel results.

First, while optimal policy when participation is exogenously assumed obeys a time-invariant weighted average of both countries' preferred policies, optimal policy in a voluntary MU responds to a country's incentive to abandon the union by tilting current and future policy in its favor.[25] This enriches policy dynamics significantly and provides insights into the workings of decision making within supra-national institutions, such as the European Central Bank, where national interests are compounded in the choice of the common policy. Our result suggests that policy, besides depending on MU "average" economic conditions, should occasionally respond to the conditions of the member country for whom adherence to the common policy is costly. This is consistent with the findings of Heinemann and Hüfner (2004) who report descriptive and econometric evidence that national divergence from euro area averages matters for the decisions of the ECB Governing Council.

Second, we show that there might be conditions such that a break-up of the union, as occurred in some historical episodes (see Cohen, 1993), is efficient. This

[25] Hence, optimal policy is history dependent in this setting.

result stems from the fact that the MU allows a country to solve the coordination problem (as long as the union is active) without renouncing forever to the benefits of flexibility. The paper thus provides a first formal analysis of the incentives behind the formation, sustainability and disruption of a Monetary Union.

The distinguishing aspect of what we called a "union" is that, while the agents belong to it, they must choose the *same* action. Therefore, even though belonging to the union might be preferred in expectations, the lack of flexibility introduced by this constraint introduces ex-post incentives to leave the union. In some instances, a compromise regarding the common action to be taken will be reached but in others the union will be dissolved. While we focussed in this paper on monetary policy (and occasionally mentioned exchange rate policy), the key features of our analysis also appear in other settings where coordination on a single action matters, such as fiscal policies in a MU (consider e.g. the choice of the excessive deficit procedure in the EMU Stability and Growth Pact), political parties in a coalition or firms in a joint venture. Our results may find fruitful application in those fields. Other extensions might involve extending the analysis to more than two countries and to allow for one-sided lack of commitment, to model situations like exchange rate pegs or a country's decision to "dollarize". We leave these tasks for future research.

A. Appendix: Proofs

Proof of Proposition 1:

This result follows from Proposition 1 of Abreu (1988). He establishes, in more generality, that an equilibrium is sustainable (subgame perfect) if and only if, for every history, the payoff corresponding to the best current deviation plus the value from the worst sustainable continuation equilibrium is lower than the value of following the proposed equilibrium strategies.

Proof of Lemma 1:

(i) Γ is compact since it is a closed subset of $P \times P$ which is compact. Convexity follows from the concavity of $U(.)$ and the continuity of the choice sets.

(ii) W is bounded since the per period utility is bounded and $\delta \in (0,1)$. To prove compactness we therefore only need to prove that it is closed. Consider a sequence of discounted utility vectors (w_n, w_n^*) that converges to (w, w^*). For each n, let (Π_n, Π_n^*) be the policies associated with these payoffs. Since Γ is compact, there is a convergent subsequence $\left(\Pi_{n_k}, \Pi_{n_k}^*\right)$, let (Π, Π^*)denote its limit. The subsequence $\left(w_{n_k}, w_{n_k}^*\right)$ must also converge to (w, w^*). By the continuity of U over policies the payoff from (Π, Π^*) is given by (w, w^*), hence by definition it is an element of W.

Proof of Lemma 2:

(i) First let us denote by Ξ the set of all possible contracts (D, Π, β) for the MU. The set Ξ is compact since it is the product of compact sets. Σ is compact since it is a closed subset of Ξ. Convexity follows from the concavity of $U(.)$ and continuity in the choice sets.[26]

(ii) V is bounded since the per period utility is bounded and $\delta \in (0,1)$. To prove compactness we therefore only need to prove that it is closed. Consider a sequence of discounted utility vectors (v_n, v_n^*) that converges to (v, v^*). For each n, let (D_n, Π_n, β_n) be the contracts associated with these payoffs. Since Σ is compact there is a convergent subsequence $\left(D_{n_k}, \Pi_{n_k}, \beta_{n_k}\right)$, let (D, Π, β) denote its limit. The subsequence $\left(v_{n_k}, v_{n_k}^*\right)$ must also converge to (v, v^*). By the continuity of U over policies, the payoff from (D, Π, β) is given by (v, v^*), hence by definition it is an element of V.

Proof of Proposition 2:

Suppose that the constraint was not binding. This implies that there is at

[26] The public randomization device x_t is needed for the convexity of Σ since it allows us to convexify the decision with respect to break-ups.

least one state where the participation constraint is slack:

$$U^*(\pi_\tau, \pi_\tau, s_\tau) + \delta E_\tau \left[\sum_{i=1}^{\infty} \delta^{i-1} U^* \left(\pi^*_{\tau+i}, \pi_{\tau+i}, s_{\tau+i} \right) \right] > w^*[\beta(h_\tau, D_\tau, a_\tau, a^*_\tau)]$$

Now let $\bar{\pi}_\tau$ denote the optimal level of inflation that Home would choose if it could unilaterally set π for both countries. First note that if $\pi_\tau \neq \bar{\pi}_\tau$, the value to Home can be increased by bringing policy closer to $\bar{\pi}_\tau$, hence decreasing the value to Foreign until the constraint binds.

If $\pi_\tau = \bar{\pi}_\tau$ and $q^* > v^*$, future policy can be tilted towards Home's preferred policy until Foreign's promised utility (q^*) becomes $v_{\min}$.

The proof is completed by noting that it is not possible to have $q^* > v^* \geq v_{\min}$ and that for all s_τ for which $U(\bar{\pi}_\tau, \bar{\pi}_\tau, s_\tau) + \delta v_{\max} > w[\tilde{\beta}(h_\tau, D_\tau, a_\tau, a^*_\tau)]$ the following holds:

$$U^*(\bar{\pi}_\tau, \bar{\pi}_\tau, s_\tau) + \delta v_{\min} > w^*[\tilde{\beta}(h_\tau, D_\tau, a_\tau, a^*_\tau)]$$

By definition $v_{\max}$ is the upper bound of V . Since the proposed policy and continuation values $(\bar{\pi}, v_{\max})$ cannot be improved upon, they must deliver $v_{\max}$. By the definition of $v_{\min}$, this implies that $q^* = v_{\min}$,which delivers the contradiction.

Proof of Proposition 3:

Given Proposition **2,** Lemma **3** and our sequential formulation of the problem this result follows from the results by Abreu, Pearce and Stacchetti (1990).

B. Appendix: Two cases with Separability

In this section we present two examples where *separability* holds.

Repeated Static Nash after break-ups. If we considered the special case in which no matter if the break-up was recommended or unilateral then the countries continuation value is given by the repeated static Nash Equilibrium, $\beta_t : S \times D \times a \times a^* \rightarrow \left(w^N_s, w^{N*}_s\right)$.[27] Then we have separability.

Proposition 5. *If $\beta_t : S \times D \times a \times a^* \rightarrow \left(w^N_s, w^{N*}_s\right)$, the optimal set of states $H \subseteq S$ where the union is sustained is independent of the promised value v^*_o $\in [v_{\min}, v_{\max}]$.*

[27] This example is extensively analyzed in the working paper version. See Fuchs and Lippi (2003).

Proposition 5 allows the problem to be divided in two sub-problems. The first one consists in finding the optimal set H over which the union can be sustained. The second is to determine the optimal policy and continuation values (π_s, v_s^*) given this set.

Quasi-linear Preferences and Transfers. If we assumed that the countries had quasi-linear preferences and we allowed for transfers then the Pareto frontiers of the sets would be linear with a slope of -1. This implies that either $V_s \subset W_s$ or $W_s \subset V_s$. Therefore, there will be no Fragile states. For this case, the characterization of the optimal policy is even simpler since optimal policy will simply maximize the joint surplus and then transfers will be used to satisfy the participation constraints. If countries stay in the MU then π will be chosen such that $\frac{U_\pi}{U_\pi^*} = -1$. Furthermore, promised values need not change since it can be done in the current period by using monetary transfers instead.

C. Appendix: Worst and best equilibria in the INMP game

The worst sustainable value of the INMP game, $\underline{w}$, solves the following problem:

$$\begin{aligned} \underline{w} \quad &\equiv \quad \min_{\pi_s, \pi_s^*, w_s^*} \Sigma_s \left[U(\pi_s, \pi_s^*, s) + \delta W(w_s^*) \right] p_s \qquad \text{(C.1)} \\ \text{subject to} \quad &: \\ U^*(\pi_s^*, \pi_s, s) + \delta w_s^* \quad &\geq \quad U^*(\pi_s^{*d}, \pi_s, s) + \delta \underline{w} \qquad \forall\, s \\ U(\pi_s, \pi_s^*, s) + \delta\ W\left(w_s^*\right) \quad &\geq \quad U(\pi_s^d, \pi_s^*, s) + \delta \underline{w} \qquad \forall\, s \\ \left(W\left(w_s^*\right), w_s^*\right) \quad &\in \quad W \end{aligned}$$

where W is the set of sustainable payoffs, $W\left(w_s^*\right)$ is the maximum value attainable by Home conditional on the promised value w_s^* to Foreign and π^d (π^{*d}) denotes the optimal deviation from the policy plan for Home (Foreign).

A deviation from the strategy prescribed by the "worst equilibrium" is punished with the future reversion to the same equilibrium (Abreu, 1988). As is known, such punishments can be harsher than the reversion to the static Nash equilibrium and thus allow a "good" equilibrium to be sustained. The best (symmetric) sustainable equilibrium satisfies:

$$\begin{aligned}
\bar{w} &\equiv \max_{\pi_s, \pi_s^*} \Sigma_s \left[U(\pi_s, \pi_s^*, s) + \delta \bar{w} \right] p_s \qquad\qquad \text{(C.2)} \\
\text{subject to} &: \\
U^*(\pi_s^*, \pi_s, s) + \delta \bar{w} &\geq U^*(\pi_s^{*d}, \pi_s, s) + \delta \underline{w} \qquad \forall\ s \\
U(\pi_s, \pi_s^*, s) + \delta \bar{w} &\geq U(\pi_s^d, \pi_s^*, s) + \delta \underline{w} \qquad \forall\ s \\
\bar{w} &= \Sigma_s \left[U^*(\pi_s^*, \pi_s, s) + \delta \bar{w} \right] p_s
\end{aligned}$$

where the last constraint imposes the symmetry requirement. The "best" equilibrium is "self rewarding", i.e. adherence to the prescribed strategy is rewarded with the continuation of the same strategy next period.

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