On Purely Financial Synergies and the Optimal Scope of the Firm:
Implications for Mergers, Spinoffs, and Off-Balance-Sheet Finance

Hayne Leland and Jure Skarabot

Abstract

We consider multiple activities with imperfectly correlated cash flows and zero operational synergies. These activities may be separated (through incorporation or special purpose vehicle), allowing each to optimize its financial structure, or merged into an entity with a single optimal financial structure. In the presence of corporate taxes and default costs, mergers can realize positive financial synergies due to reduced risk and the potential for greater leverage, but at the cost of losing separate debt/equity choices and separate limited liability. Closed-form measures of financial synergies of combining (or separating) activities are developed. Characteristics of activities that gain from combination or separation are identified, and the magnitudes of gains and their distribution of gains between stockholders and bondholders are examined. The results have direct application to the desirability of mergers, spinoffs, and off-balance-sheet techniques such as project finance and asset securitization.

JEL Classification Codes: G32, G34

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1. Introduction

Decisions that alter the scope of the firm are amongst the most important faced by management, and amongst the most studied by academics. Mergers and spinoffs are classic examples of such decisions. More recently, equity carve-outs, joint ventures, project finance, and off-balance-sheet special purpose vehicles (“SPVs”) have been used alter the boundaries of the firm.¹

Positive or negative operational synergies are often cited as the prime motivation for decisions that change the scope of the firm. A rich literature addresses the roles of economies of scope and scale, market power, incomplete contracting, property rights, and agency costs in determining the optimal boundaries of the firm.²

But such operational synergies are not the focus of this paper. Rather, the analysis considers purely financial synergies. To facilitate this focus, it is assumed that the operational cash flow of the combined activities is simply the sum of the operational cash flows of the separate activities, at the time the decision on scope is made. If operational synergies exist, their effect will be incremental to the financial synergies examined here.³

In a Modigliani-Miller [1958] world without taxes, bankruptcy costs, informational asymmetries, or agency costs, there will be no purely financial synergies. Capital structure is irrelevant to total firm value. But in a world with taxes and bankruptcy costs, it is well known that capital structure matters. Changes in the

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¹ Equity carve-outs differ from spin-offs in that the originating firm retains considerable ownership: see Vijh (1999), Vijh (2001), and Hulburt, Miles, and Woolridge (2002). Project finance is defined in Kleimeir and Megginson (1999) as “limited or non-recourse financing of a new project through the establishment of a vehicle company (separate incorporation).” Special purpose vehicles or entities (SPVs/SPEs) are typically trusts that have separate, limited-recourse financing secured by the cash flows and assets of a specific activity. Skarabot (2002) examines off-balance-sheet financing techniques such as asset securitization.

² Classic studies include Coase (1937), Williamson (1975), and Grossman and Hart (1986). See also Holmstrom and Roberts (1998) and the references cited therein.

³ Note that the assumption that operational cash flows are additive rules out “agency” costs related to changes in scope or financial structure.
scope of the firm will affect optimal capital structure, which in turn will create purely financial synergies. Financial synergies will affect the optimal boundaries of startup firms as well as extant firms: should an entrepreneur with two or more projects structure them as a single firm, or as separate firms?

Keeping activities separate allows each to have its appropriate capital structure, with an optimal amount of debt and equity. Each firm or SPV has limited liability, and its debt has recourse only to its cash flow and assets. If activities are notably different in terms of risks, default costs, and taxes, their optimal leverage ratios may vary significantly. In contrast, a firm merging multiple activities must have a single leverage ratio. And equity in the merged firm loses the multiple bankruptcy shelters of activities that are individually incorporated.

Countering these financial advantages to separate incorporation is the fact that, when activities’ cash flows are imperfectly correlated, risk can be lowered via a merger or initial consolidation. Lower risk will reduce expected default costs and interest expense. Leverage can then be increased with potential tax benefits, as first suggested by Lewellen (1971). However, Lewellen asserts that the purely financial benefits of mergers always are positive, which is shown to be incorrect.

This paper examines four questions:

(i) How do the financial advantages of separate versus combined activities depend upon the riskiness and correlation of cash flows of the separate activities, as well as upon payout rates, tax rates, default costs, and relative size?

(ii) How important are the magnitudes of potential financial synergies?

(iii) How does the existence of outstanding debt affect the ability of current equity holders to capture the gains of mergers or spinoffs?

(iv) How does the opportunity to hedge risks affect the desirability of merging activities?

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4 The analysis considers a single class of debt for each separate firm. Multiple seniorities of debt within a firm will not affect the results as long as all classes have potential recourse to the firm’s total assets.
We derive formulas that provide quantitative estimates of financial synergies, as well as identifying the capital structures that maximize these synergies. While operational synergies may exceed financial synergies in many mergers or spin-offs, financial synergies can be sizable in situations that are identified in this paper. And financial synergies may be the principal reason behind off-balance-sheet financing techniques such as project finance or asset securitization.5

2. Previous Work

A number of theoretical and empirical studies have considered the effects of conglomerate mergers and diversification. Only Lewellen (1971) and Stapleton (1982) consider purely financial synergies. Lewellen correctly argues that combining imperfectly correlated assets, while not value-enhancing per se, has a coinsurance effect: it reduces the risk of default, and thereby increases a measure of debt capacity. But he then asserts that higher debt capacity (by his measure) will create greater value for the merged firm.6 Our model quantifies the coinsurance effect, and shows that it does not always overcome the disadvantage of forcing a single financial structure onto multiple activities. When the latter dominates, separation rather than merger is desirable.

Many papers have considered the potential impact of firm scope on operational synergies. Flannery, Houston, and Venkataraman (1993) consider investors who issue external debt and equity to invest in risky projects, and must decide upon separate or joint incorporation of the projects. They show that joint incorporation is more valuable when project returns have similar volatility and lower correlation. Operational synergies are key to their conclusions: investment and therefore the cash flows of the merged firm will be different from the sum of investments and cash flows of the separate firms. John (1993) uses a related approach to analyze spinoffs, while Chammanur and John (1996) use managerial ability and control issues to

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5 Recent experience suggests “cooking the books” as a consideration in some off-balance-sheet financing. Our results demonstrate the potential value of legitimate use of these vehicles.

6 Lewellen (1971, p. 533). Stapleton (1982) uses a slightly different definition of debt capacity but also states that mergers always have positive financial benefits. He notes that bondholders may reap some of the gains. Neither paper formally considers optimal capital structure.
explain project finance and the scope of the firm. Other papers have focused on negative operational synergies arising from managerial agency costs, including the misallocation of internal capital, that lead to a “conglomerate discount.”\textsuperscript{7} There is a rich but still inconclusive empirical literature testing whether such a discount exists.\textsuperscript{8}

Without denying their potential importance, we abstract from managerial agency costs and other potential sources of operational synergies, and focus uniquely on the purely financial effects resulting from alternative boundaries of the firm. To obtain quantitative as well as qualitative results, we utilize an extension of the continuous-time approach of Leland [1994] and Goldstein, Ju, and Leland [2001] to provide closed-form solutions for valuation and capital structure. These models, appropriately extended, provide the analytical underpinnings for this paper.

3. The Model

3.1 Individual Activities’ Cash Flows and Values

Consider two separate activities generating cash flows (“EBIT”) $X_i(t)$, $i = \{1, 2\}$, that follow lognormal dynamic processes:

$$dX_i = \mu_i X_i \, dt + \sigma_i X_i \, dZ_i,$$


\textsuperscript{8} Martin and Sayrak (2003) provide a useful summary of this research. Berger and Ofek (1995) suggest conglomerate discounts on the order of 15%. This and related results have recently been challenged on the basis that diversified firms trade at a discount prior to diversifying: see Lang and Stulz (1994), Campa and Kedia (1999), and Graham, Lemmon, and Wolf (2002). Mansi and Reeb (2002) conclude that the conglomerate discount of total firm value is insignificantly different from zero, when debt is priced at market rather than book value.
with $\mu_i$ the (constant) instantaneous risk-adjusted expected growth rate of the cash flow, i.e. the drift rate
under the risk neutral measure $^{9}$, $\sigma_i$ is the (constant) volatility of the growth rate, $dZ_i$ is the increment to a joint
Brownian motion with (constant) correlation $\rho$, i.e.

$$E(dZ_i dZ_j) = \rho dt.$$ 

Extending this notation beyond two dimensions is straightforward.

We assume a riskless asset exists with a constant rate of return $r$. Under these assumptions, Goldstein, Ju, and Leland (2001) show that the total value to all claimants (including the government) of the cash flows at time $t$ will be

$$F_i(X_i(t)) = \frac{X_i(t)}{r - \mu_i}. \quad ^{10}$$

The value of the unlevered activity or firm, after paying corporate income taxes, will be given by

$$V_i(X_i(t)) = \frac{(1 - T_i)X_i(t)}{r - \mu_i},$$

where $T_i$ is the effective tax rate for the separate activity $i$. \(^{11}\) Observe that the payout rate (dividends and coupons, assuming the entire after-tax cash flow is distributed) on this market value will be

$$\delta_i = \frac{(1 - T_i)X_i(t)}{V_i(X_i(t))} = r - \mu_i.$$

The dynamics of value $V_i$ under the risk-neutral measure are given by Ito’s Lemma, with

$$dV_i = \mu_{V_i} V_i dt + \sigma_{V_i} V_i dZ_i,$$

where $\mu_{V_i} = r - \delta_i = \mu_i$ and $\sigma_{V_i} = \sigma_i$. Note that, inclusive of the payout rate $\delta_i$, the total (risk-adjusted) rate of return to owning shares with value $V_i$ is $\delta_i + \mu_{V_i} = r$, as is expected under the risk-neutral measure.

\(^9\) Alternatively, $\mu_i$ is the actual expected growth rate of cash flows in an economy where all investors are risk neutral.

\(^{10}\) Note that this is simply the “Gordon growth formula”, where $\mu_i$ is the risk-adjusted growth rate rather than the actual growth rate.

\(^{11}\) We assume a constant corporate tax rate applied to all levels of $X_i(t) \geq 0$, when there is no corporate debt. More complex modeling would allow for nonlinear tax schedules, e.g. as in Leland (1994).
3.2 The purely-additive merged firm

If the activities of each separate firm or activity were combined into a single firm with no operating
synergies at time $t$, the cash flow $X_M$ of the merged firm will satisfy

$$(7) \quad X_M(\tau) = X_1(\tau) + X_2(\tau), \quad \text{for all } \tau \geq t.$$ 

An immediate implication of (7) is that there is no change in the value of operational cash flows at the time of
merger:

$$(8) \quad F_M(X_M(t)) = F_1(X_1(t)) + F_2(X_2(t)).$$

Note that $X_M(\tau) = X_1(\tau) + X_2(\tau)$ is not a lognormal dynamic process. From Ito’s lemma,

$$(9) \quad \frac{dX_M(\tau)}{X_M(\tau)} = h_1(\tau) \frac{dX_1(\tau)}{X_1(\tau)} + h_2(\tau) \frac{dX_2(\tau)}{X_2(\tau)}$$

where the relative weights on the individual return processes equal relative cash flows

$$(10) \quad h_i(\tau) \equiv \frac{X_i(\tau)}{X_1(\tau) + X_2(\tau)} \quad i = \{1,2\}$$

and are stochastic for $\tau \geq t$. Thus the drift and volatility of the return process (9) are stochastic, and the
valuation formula (3) does not hold. This reflects the fact that the sum of two lognormal return processes is
not itself a lognormal return process. Numerical methods are required and comparative statics cannot be
readily provided in this case.

3.3 An Alternative Dynamic Process for Merged Firm Cash Flows

Consider now a return process for the post-merger cash flows that evolves as a constant
rather than stochastic mix of the individual return processes, with constant weights $w_i$:

$$(11) \quad \frac{dX_M(\tau)}{X_M(\tau)} = w_1 \frac{dX_1(\tau)}{X_1(\tau)} + w_2 \frac{dX_2(\tau)}{X_2(\tau)} \quad \tau \geq t.$$ 

This implies that future merged cash flows follow a lognormal dynamic process, with
\[
\frac{dX_M(\tau)}{X_M(\tau)} = \mu_M d\tau + \sigma_M dZ_M, \quad \tau \geq t
\]  

(13) \quad \mu_M = w_1 \mu_1 + w_2 \mu_2; 

(14) \quad \sigma_M = (w_1^2 \sigma_1^2 + 2 \rho \sigma_1 \sigma_2 w_1 w_2 + w_2^2 \sigma_2^2)^{1/2}.

(15) \quad X_M(t) = X_1(t) + X_2(t).

Note that (15), in contrast with (7), holds at the time of merger only. After the merger, the cash flow process follows (11) rather than (9). \(^{12}\)

The weights \(w_i\) that determine the merged firm’s dynamics still need to be determined. Equations (7) and (8) imply that the merger creates neither operational cash flow synergies nor value change when the merger occurs. Similarly, in addition to (15), there should zero operational value change at the time of the merger, i.e. equation (8) must also hold at \(t\) for the process (11).

**Proposition 1: (Relative Value Weighting)**

Given the dynamic specifications (12)- (14), equation (8) will hold and there will be a zero operational value change at the time of merger if and only if the weights \(w_i\) are proportional to the relative values of the individual activities’ operational cash flows at the time of the merger, i.e.:

(16) \quad w_i = \frac{F_i(X_i(t))}{F_1(X_1(t)) + F_2(X_2(t))}, \quad i = \{1, 2\}.

**Proof:** In Appendix A.

\(^{12}\) There are two ways to consider process (11)-(14). First, it can be viewed as an approximation to the process (9) - (10). This is examined in Appendix B. Alternatively, it can be viewed as the result of (nonsynergistic) post-merger management that continuously re-allocates resources between divisions to keep them in constant relative weights \(w_i\), as specified in (16) below. The process (11)-(14) is similar to the return process of a portfolio of two assets, each following a lognormal dynamic process, which is continuously rebalanced to keep asset proportions constant. A portfolio managed in this way will itself follow a lognormal process given by equations (11)-(14), where \(w_i\) is the proportion of asset \(i\) in the portfolio. Observe that, in assuming individual firms’ cash flows (which most likely combine the cash flows of different sub-activities) are themselves lognormal, we may have (implicitly) already made this assumption at the individual firm level.
Proposition 1 shows that the constant weights \( w_i \) that determine the drift \( \mu_M \) and volatility \( \sigma_M \) of the post-merger cash flows depend on the relative values \( F_i \) at the time of the merger \( t \), and are different from the random weights \( h_i(\tau) \) in equation (9) that are based on relative cash flows at \( \tau \geq t \).

Appendix B compares the distribution of future unlevered merged-firm values that result from the cash flow process (11) with weights (16), with the distribution of future values that result from the cash flow process (9) with weights (10). It can be seen in Figures 12A and 13A that the probability distributions of the two are very close, for parameters in base cases 1 and 2.

**Proposition 2: (No After-tax Value Synergies)**

Given the dynamic process (10)-(13) for merged cash flows, with weights as in equation (16), there will be a zero after-tax operational value change at the time of merger, i.e.

\[
V_M(X_M(t)) = V_1(X_1(t)) + V_2(X_2(t)),
\]

iff the corporate income tax rate on the merged firm’s cash flows, \( T_M \), satisfies

\[
T_M = w_1T_1 + w_2T_2
\]

for arbitrary tax rates \( T_i \) on the individual activities’ cash flows.

**Proof:** The proof is immediate by substitution, using (4), (12), and (18).

The tax rate for the merged activities given by (18) will be termed the value-neutral tax rate. Tax rates on merged firm’s cash flows that are higher (lower) than the value-neutral tax rate produce will negative (positive) financial synergies.

Finally, recall from (4) – (6) that the drift \( \mu_M \) of \( V_M \) is the same as the drift of combined cash flow \( X_M \) and pre-tax value \( F_M \), and the payout rate is \( \delta_M = r - \mu_M \).

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\(^{13}\) If \( \mu_1 = \mu_2 \), \( w_i = h_i(t) \) and the relative value and cash flow weights will be the same at the time of merger (although \( h_i(\tau) \) will subsequently move randomly). If \( \mu_1 \neq \mu_2 \), the expected future growth rate of combined cash flows will exceed the growth rate based on cash flow weights \( h_i(t) \) at the time of the merger, since more-quickly growing activities will have a greater expected cash-flow weight in the future. Thus it is appropriate to use current value weights \( w_i(t) \), which through (3) will accord greater weights to activities with higher growth rates.
4. Optimal Capital Structure for Separate and Merged Activities

We adopt the environment assumed by Leland (1994). Firms issue debt with long (infinite, in fact) maturity. Firms (indexed i = 1, 2, M) are described by the following variables:

\[ V_i(\tau) \text{ value of after-tax operational cash flows at time } \tau, \text{ given by equation (4)} \]

\[ \delta_i \text{ (constant) payout rate as fraction of after-tax asset value } V_i \]

\[ \mu_i \text{ (constant) drift of } \frac{dV_i(\tau)}{V_i(\tau)}, \tau \geq t, 15 \]

\[ \sigma_i \text{ (constant) volatility of } \frac{dV_i(\tau)}{V_i(\tau)}, \tau \geq t, \]

\[ \alpha_i \text{ fractional costs to after-tax value } V_i \text{ in the event of default,} \]

\[ C_i \text{ coupon flow rate to long-term (infinite life) debt,} \]

\[ P_i \text{ principal value of long-term debt issued by firm,} \]

\[ R_i \text{ rate of interest on debt issued by firm } ( = C_i/P_i), \]

\[ T_i \text{ tax rate on operational cash flows less coupon payments,} \]

\[ v_i(\tau) \text{ total value of the levered firm at time } \tau, \]

\[ D_i(\tau) \text{ value of the firm’s debt at time } \tau, \]

\[ E_i(\tau) \text{ value of equity at time } \tau, \]

\[ V_{Bi} \text{ value which, if reached, the firm defaults on debt.} \]

When the bankruptcy value \( V_{Bi} \) is determined endogenously, Leland (1994) derives closed-form expressions for the optimal financial structure of the firm. (Optimized variables are starred.) These include

\[ 14 \text{ Absent agency costs, Leland and Toft (1996) show that firms will prefer to issue debt of maximal maturity. Dynamic extensions of the debt model here are considered by Fischer, Heinkel, and Zechner (1989), Leland (1998), and Goldstein, Ju, and Leland (2001). Such models generally create somewhat higher benefits to leverage, and somewhat lower (initial) leverage. Both positive and negative financial synergies are likely to be somewhat higher in such models.} \]

\[ 15 \text{ Recall from equation (6) and following } \mu_{\bar{V}_i} = \mu_i = r - \delta_i, \text{ and } \sigma_{\bar{V}_i} = \sigma_r. \]
\[ C_i^* = V_i \left( (1 + x_i)h_i \right)^{-1/s_i} \]

\[ D_i^* = C_i^* \left( 1 - k_i \left( (1 + x_i)h_i \right) \right)^{-1/r} \]

\[ v_i^* = V_i \left( 1 + (T_i/r) \left( (1 + x_i)h_i \right)^{-1/s_i} \left( x_i/(1 + x_i) \right) \right) \]

\[ E_i^* = v_i^* - D_i^* \]

where

\[ h_i = (1 + x_i + \alpha_i (1 - T_i) x_i / T_i) m_i \]

\[ k_i = (1 + x_i - (1 - \alpha_i)(1 - T_i) x_i) m_i \]

\[ m_i = \left( \frac{(1 - T_i) x_i}{r(1 + x_i)} \right)^{s_i} \left( 1 + x_i \right)^{-1} \]

\[ x_i = \left( r - \delta_i - 5\sigma_i^2 \right) + \left( (r - \delta_i - 5\sigma_i^2)^2 + 2\sigma_i^2 r \right)^{1/2} / \sigma_i^2. \]

Purely financial synergies will be positive if and only if \( v_M^* > v_1^* + v_2^* \), i.e. the optimally leveraged merged firm has greater value than the sum of the optimally-leveraged separate firms.

5. Measuring Financial Synergies

Financial synergies can be measured in a number of ways. An obvious measure is

**Measure 1.** The percentage increase in total firm value:

\[ (v_M^* - (v_1^* + v_2^*)) / (v_1^* + v_2^*). \]

Competition may induce an acquiring firm to bid an amount that reflects total synergies, including financial synergies.\(^{16}\) We adopt the convention that firm 1 is the acquiring firm, realizing none of the benefits, and firm 2 is the acquired or target firm, realizing all the benefits of the merger. This suggests that an alternative

\(^{16}\) Numerous studies (e.g. Andrade, Stafford, and Mitchell (2001) suggest that acquiring firms realize little or no increases in their market value. Firms being acquired, however, realize substantial value premiums.
A measure of financial synergies is the percentage premium over current value of firm 2 that firm 1 could pay, due solely to financial synergies.

**Measure 2.** The percentage increase in value of the acquired firm:

\[
\frac{(v_M^* - (v_1^* + v_2^*))}{v_2^*}.
\]

A final yardstick, Measure 3, considers the case of an optimally-leveraged target firm. It is assumed that all benefits will accrue to the *stockholders* of the levered firm.\(^{17}\)

**Measure 3.** The percentage increase in equity value of the acquired firm:

\[
\frac{(v_M^* - (v_1^* + v_2^*))}{E_2^*}.
\]

Note Measure 1 < Measure 2 < Measure 3 when these measures are positive, whereas the opposite inequalities hold when separation is optimal and the measures are negative.

### 6. Identifying Sources of Financial Synergies

#### 6.1 Base Case 1: Symmetric Firms

**Base Case 1** assumes two symmetric firms, identical in size and in the nature of their cash flows, but imperfectly correlated. The objective is to determine the magnitude of financial synergies for a merger of equals, and then see how these synergies depend on the firms’ correlation, drift, volatility, tax rate, and default costs.

**Table 1** lists parameters for Base Case 1. (Unlevered) risk \(\sigma\) of 25%, total payout (including to debtholders) \(\delta\) of 5% of firm value, and correlation \(\rho\) between firms of 0.20 chosen are chosen to approximate a “typical” mature firm. Default costs \(\alpha\) of 20% of default value fall within a range of recent empirical estimates.\(^{18}\)

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\(^{17}\) The distribution of merger benefits between stockholders and bondholders is considered in Section 7.

\(^{18}\) Anderson and Betker (1995) estimate a median \(\alpha = 34.7\%\), and Gilson (1997) estimates \(\alpha = 45.3\%\). Andrade and Kaplan (1998) suggest a range of default costs from 10% to 20%, based on studies of firms undergoing highly leveraged takeovers (HLTs). However, firms subject to HLTs are likely to have lower-than-average default costs, since high leverage will only be optimal for firms with this characteristic.
The net tax rate of 20% reflects both the corporate income tax rate and the tax advantage of equity.\textsuperscript{19} Tax benefits for the optimally-leveraged separate firms in the base case are consistent with recent studies.\textsuperscript{20}

Table 2 calculates financial effects of mergers using the formulas listed in Section 4.

\textbf{TABLE 2:}
\textbf{SEPARATE AND MERGED FIRM VARIABLES}

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Merged Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-tax Operational Value</td>
<td>$V$</td>
<td>80.00</td>
<td>80.00</td>
</tr>
<tr>
<td>Post-tax Cash Flow</td>
<td>$(1-T)X$</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>Optimally-levered Value</td>
<td>$v^*$</td>
<td>86.74</td>
<td>86.74</td>
</tr>
<tr>
<td>Equity</td>
<td>$E^*$</td>
<td>33.99</td>
<td>33.99</td>
</tr>
<tr>
<td>Coupon</td>
<td>$C^*$</td>
<td>3.64</td>
<td>3.64</td>
</tr>
<tr>
<td>Debt</td>
<td>$D^*$</td>
<td>52.75</td>
<td>52.75</td>
</tr>
<tr>
<td>Leverage Ratio</td>
<td>$L^*$</td>
<td>60.8%</td>
<td>60.8%</td>
</tr>
<tr>
<td>Interest Rate on Debt</td>
<td>$R^*$</td>
<td>6.91%</td>
<td>6.91%</td>
</tr>
<tr>
<td>Value Triggering Default</td>
<td>$VB/V^*$</td>
<td>33.7%</td>
<td>33.7%</td>
</tr>
<tr>
<td>Implied Recovery Rate</td>
<td>$RR^*$</td>
<td>40.9%</td>
<td>40.9%</td>
</tr>
</tbody>
</table>

\textbf{SYNERGIES}

| Measure 1 | 1.09% |
| Measure 2 | 2.18% |
| Measure 3 | 5.56% |


\textsuperscript{20} Graham [2000] estimates that leveraging the firm optimally increases value by about 8.2 percent. Goldstein, Ju, and Leland (2001), using a dynamic extension of the model considered here, have an 8.3 percent benefit for their closely-related base case. Our model implies maximal leverage benefits of about 8.4 percent in Base Case 1.
Financial synergies are positive, and of modest magnitude. Measure 3, which always yields the highest value, indicates that Firm 1 could make an offer for Firm 2’s equity at a premium exceeding 5.5 percent, based purely on financial synergies. As measured relative to the total value (debt and equity) of the two firms, Measure 1, the value increase from financial synergies is 1.09 percent.

Table 2 indicates that there are two sources of financial synergy in this case, both related to lower operational risk after the merger. First, optimal leverage is slightly increased after the merger. Second, the merged firm can issue debt at a lower credit spread than was required by the separate firms: 129 basis points, versus 191 basis points. Note that, despite the increase in leverage, the annual coupon C paid by the optimally-levered merged firm, has actually declined relative to the total coupons paid when the firms are separate. This reflects the lower yield spread of the less-risky merged debt.

We now consider the sensitivity of the three benefit measures to changes in the parameters, still assuming the merged firms are symmetric. Perhaps the single most important parameter is the correlation $\rho$ between the individual firms’ cash flows. Figure 1 examines the financial benefits to merger as a function of the firms’ cash flow correlation $\rho$. Observe that the financial benefits decline from about 1.5% for Measure 1 (7.7% for Measure 3) to zero, as correlation increases from zero to one. (Benefits would be higher if cash flows were negatively correlated).

Table 3 considers the effect on merger benefits of altering the parameters for both the separate firms. The middle row in each cell is the base case parameter. Financial benefits are never negative in these cases, because the separate firms are by assumption symmetric, and there is no advantage to having different leverage ratios through separate incorporation. For the symmetric case, Lewellen’s (1971) assertion that there will be positive purely financial synergies to mergers is correct. Yet the magnitudes of such synergies are modest. Even in an “extreme” case, where parameters reflect low payout rates (high growth), low volatility,

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21 This is the “credit spread”. As suggested by Huang and Huang [2002], actual spreads relative to Treasuries may be higher still, reflecting liquidity and tax differences.
low correlation, low default costs, and high taxes, purely financial benefits are limited to 2.47% by Measure 1, although Measure 3 rises to 28.8% reflecting high leverage and therefore relatively small equity value $E_2$.

**TABLE 3: COMPARATIVE STATICS OF MERGER BENEFITS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payout Rate $\delta$</td>
<td>2.00%</td>
<td>1.68%</td>
<td>3.37%</td>
<td>9.35%</td>
</tr>
<tr>
<td></td>
<td>5.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>8.00%</td>
<td>0.64%</td>
<td>1.27%</td>
<td>3.06%</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
<td>10.00%</td>
<td>1.17%</td>
<td>2.34%</td>
<td>8.24%</td>
</tr>
<tr>
<td></td>
<td>25.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>40.00%</td>
<td>0.86%</td>
<td>1.72%</td>
<td>3.91%</td>
</tr>
<tr>
<td>Default Cost $\alpha$</td>
<td>10.00%</td>
<td>0.95%</td>
<td>1.89%</td>
<td>5.99%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>1.19%</td>
<td>2.37%</td>
<td>5.21%</td>
</tr>
<tr>
<td>Tax Rate $T$</td>
<td>10.00%</td>
<td>0.58%</td>
<td>1.17%</td>
<td>2.13%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>1.09%</td>
<td>2.18%</td>
<td>5.56%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>1.57%</td>
<td>3.14%</td>
<td>10.14%</td>
</tr>
<tr>
<td>Most Extreme:</td>
<td></td>
<td>2.47%</td>
<td>4.94%</td>
<td>28.80%</td>
</tr>
<tr>
<td>$\delta = 2%$, $\sigma = 10%$, $\alpha = 30%$, $T = 30%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Besides low correlation, low payout rates (implying higher risk-adjusted growth rates), low volatility, and high tax rates are consistent with greater benefits to merger. These environments are generally consistent with higher optimal leverage ratios. But an exception to this principle is default costs. The risk reduction from diversification particularly benefits high-default cost firms, even though optimal leverage ratios are smaller.

**6.2. Base Case 2: Asymmetric Firms**

Differences *between* firms will also affect potential financial synergies. Recall that if firms (or activities) are dissimilar, it may be advantageous to keep them separated. This is because capital structures can optimized separately.
**Base Case 2** considers firms that differ according to the optimal leverage that might be utilized. Firm 1 is assumed to be a “mature” firm, with modest growth, P/E ratio, risk, and default costs. Firm 2 is a smaller firm with high growth, P/E ratio, risk, and default costs. Parameters are given in Table 4.

### Table 4: Base Case 2 Parameters

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Merged Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT Value ( F )</td>
<td>100.00</td>
<td>10.00</td>
<td>110.00</td>
</tr>
<tr>
<td>Cash flow (EBIT) ( X )</td>
<td>5.00</td>
<td>0.20</td>
<td>5.20</td>
</tr>
<tr>
<td>Payout ( \delta )</td>
<td>5.00%</td>
<td>2.00%</td>
<td>4.73%</td>
</tr>
<tr>
<td>Drift ( \mu )</td>
<td>0.00%</td>
<td>3.00%</td>
<td>0.27%</td>
</tr>
<tr>
<td>Volatility ( \sigma )</td>
<td>10.00%</td>
<td>40.00%</td>
<td>9.79%</td>
</tr>
<tr>
<td>Tax Rate ( T )</td>
<td>20.00%</td>
<td>20.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>Default Costs ( \alpha )</td>
<td>20.00%</td>
<td>80.00%</td>
<td>25.45%</td>
</tr>
<tr>
<td>Correlation ( \rho )</td>
<td></td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td></td>
<td></td>
<td>5.00%</td>
</tr>
</tbody>
</table>

As with drift, volatility, and tax rates, the default cost fraction of the merged firm is a value-weighted combination of the individual default cost fractions. Table 5 computes key variables for Base Case 2.

### Table 5: Base Case 2 Separate and Merged Firm Variables

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Merged Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-tax Operational Value ( V )</td>
<td>80.00</td>
<td>8.00</td>
<td>88.00</td>
</tr>
<tr>
<td>Post-tax Cash Flow ( (1-T)X )</td>
<td>4.00</td>
<td>0.16</td>
<td>4.16</td>
</tr>
<tr>
<td>Optimally-levered Value ( v^* )</td>
<td>90.39</td>
<td>8.21</td>
<td>99.16</td>
</tr>
<tr>
<td>Equity ( E^* )</td>
<td>25.67</td>
<td>6.06</td>
<td>29.74</td>
</tr>
<tr>
<td>Coupon ( C^* )</td>
<td>3.56</td>
<td>0.16</td>
<td>3.79</td>
</tr>
<tr>
<td>Debt ( D^* )</td>
<td>64.72</td>
<td>2.15</td>
<td>69.42</td>
</tr>
<tr>
<td>Leverage Ratio ( L^* )</td>
<td>71.6%</td>
<td>26.2%</td>
<td>70.0%</td>
</tr>
<tr>
<td>Interest Rate on Debt ( R^* )</td>
<td>5.50%</td>
<td>7.23%</td>
<td>5.46%</td>
</tr>
<tr>
<td>Value Triggering Default ( VB^*/V )</td>
<td>52.0%</td>
<td>10.3%</td>
<td>50.7%</td>
</tr>
<tr>
<td>Implied Recovery Rate ( RR^* )</td>
<td>51.4%</td>
<td>7.6%</td>
<td>48.0%</td>
</tr>
</tbody>
</table>

**SYNERGIES**

<table>
<thead>
<tr>
<th>Measure</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure 1</td>
<td>0.56%</td>
</tr>
<tr>
<td>Measure 2</td>
<td>6.69%</td>
</tr>
<tr>
<td>Measure 3</td>
<td>9.22%</td>
</tr>
</tbody>
</table>

---

22 Andrade, Mitchell, and Stafford (2001) estimate the median target firm was about 11.7% the size of the acquiring firm, in a sample of mergers over the period 1973-1998. We assume 10% for Base Case 2.
Notice the substantial difference in the initial capital structures of the two firms. The optimal leverage and implied recovery rate for Firm 1 are considerably higher than for Firm 2, which has both high volatility and high default costs. Despite these differences, there are still positive financial synergies to merging the firms, due to the lower risk created through diversification. But the synergies become negative when the \textit{correlation} increases, as seen in Figure 2: Separation is preferred to merger when $p > 0.60$. Conglomerate firms that combine the two activities would benefit from spinoffs in this case. This is in contrast with the symmetric activities of Base Case 1, and with Lewellen’s (1971) conclusion that the purely financial synergies of mergers are always positive.

Although the benefits of merger by Measure 1 are smaller here than those of Base Case 1, observe that the benefits relative to the size of the acquired Firm 2 are quite substantial. Measures 2 and 3 are more than twice as large in Base Case 1, indicating that the acquiring Firm 1 could bid a substantial premium above the current value of the target Firm 2, based on financial synergies alone.\footnote{Stafford, Mitchell, and Andrade (2001) find that, over the period 1973-1998, the 3-day abnormal returns to acquired firms is 16\% of value, and to acquiring firms is slightly negative but insignificantly different from}

\begin{table}[h]
\centering
\caption{Comparative Statics of Merger Benefits - Base Case 2}
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Parameter} & \textbf{Value} & \textbf{Measure 1} & \textbf{Measure 2} & \textbf{Measure 3} \\
\hline
Payout Rate $\Delta$ & 2.00\% & 0.57\% & 7.00\% & 9.66\% \\
& 5.00\% & 0.56\% & 6.69\% & 9.22\% \\
& 8.00\% & 0.36\% & 4.24\% & 5.84\% \\
\hline
Volatility $\sigma$ & 5.00\% & 0.32\% & 3.97\% & 5.47\% \\
& 10.00\% & 0.56\% & 6.69\% & 9.22\% \\
& 40.00\% & -0.09\% & -1.07\% & -1.48\% \\
\hline
Default Cost $\xi$ & 10.00\% & 0.42\% & 5.12\% & 7.06\% \\
& 20.00\% & 0.56\% & 6.69\% & 9.22\% \\
& 30.00\% & 0.64\% & 7.60\% & 10.48\% \\
\hline
Tax Rate $T$ & 10.00\% & 0.61\% & 7.57\% & 10.43\% \\
& 20.00\% & 0.56\% & 6.69\% & 9.22\% \\
& 30.00\% & 0.51\% & 5.90\% & 8.13\% \\
\hline
\end{tabular}
\end{table}
Tables 6 and 7 provide comparative statics in Base Case 2, for both Firm 1 (the acquiring firm) and Firm 2 (the acquired firm). The middle row of each cell is the base case parameter.

**TABLE 7: COMPARATIVE STATICS OF MERGER BENEFITS - BASE CASE 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Measure 1</th>
<th>Measure 2</th>
<th>Measure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payout Rate $\delta$</td>
<td>0.00%</td>
<td>0.76%</td>
<td>9.05%</td>
<td>12.88%</td>
</tr>
<tr>
<td></td>
<td>2.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>4.00%</td>
<td>0.35%</td>
<td>4.26%</td>
<td>5.71%</td>
</tr>
<tr>
<td>Volatility $\sigma$</td>
<td>20.00%</td>
<td>0.59%</td>
<td>6.74%</td>
<td>13.39%</td>
</tr>
<tr>
<td></td>
<td>40.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>60.00%</td>
<td>0.16%</td>
<td>1.94%</td>
<td>2.38%</td>
</tr>
<tr>
<td>Default Cost $\Theta$</td>
<td>40.00%</td>
<td>0.64%</td>
<td>7.56%</td>
<td>13.44%</td>
</tr>
<tr>
<td></td>
<td>80.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>100.00%</td>
<td>0.48%</td>
<td>5.82%</td>
<td>7.51%</td>
</tr>
<tr>
<td>Tax Rate $T$</td>
<td>10.00%</td>
<td>0.09%</td>
<td>0.95%</td>
<td>1.07%</td>
</tr>
<tr>
<td></td>
<td>20.00%</td>
<td>0.56%</td>
<td>6.69%</td>
<td>9.22%</td>
</tr>
<tr>
<td></td>
<td>30.00%</td>
<td>0.91%</td>
<td>11.86%</td>
<td>20.42%</td>
</tr>
</tbody>
</table>

Beyond the negative relationship between correlation and merger benefits seen in Figure 2, observe that

(i) Lower payout rates $\delta$ for both the acquiring and target firms favor mergers. This is because there are greater benefits to risk reduction via merger when optimal leverage ratios are relatively high, which in turn is the case when payout rates are low. For the same reason, higher payout activities are more likely to benefit from remaining separate.

(ii) Relatively high volatilities, or very low volatilities, of both the acquiring and target firms tend to disfavor merger. Such differing volatility, *ceteris paribus*, leads to differing optimal zero. Our Base Case 2 estimates are consistent with financial synergies explaining over half of realized merger gains.
leverage ratios for the separate activities, which in turn favors keeping them separate.\textsuperscript{24}

\textit{Figure 3} demonstrates this result for very high or very low volatilities of the target Firm 2. (iii) Higher default costs of Firm 1, and lower default costs of Firm 2, tend to favor merger. This is because the two default costs started at very different levels in Base Case 2. Greater similarity of default costs implies closer initial leverage ratios for the separate activities, which in turn enhances the benefits to merger.

(iv) Lower tax rates to Firm 1 and higher tax rates to Firm 2 would offset the initial disparities in optimal leverage ratios, again increasing the benefits of merger.

When mergers are beneficial, the optimal leverage of the merged activities typically is greater than the value-weighted average of the optimal leverage of the separate activities.\textsuperscript{25} The theory therefore suggests that financial leverage will increase following mergers, consistent with empirical results.\textsuperscript{26}

The purely financial synergies reported above assume that the taxes of the merged firm represent a value-weighted combination of the taxes of the separate activities. Additional financial benefits (costs) would occur if taxes for the merged firm are lower (higher) than the rates assumed.

Finally, \textit{Figure 4} demonstrates that size matters. As the relative size of the target firm changes, both the magnitude and sign of the measures of financial synergy will change. Measure 1 synergies increase as the size of the target rises to about 20\% of the acquiring firm. This is the “ideal” target size for merger, given the parameters in Base Case 2. As the target becomes larger, synergies decrease and become negative when the target size exceeds 55\% of the potential acquirer. Thus relative size, as well as the characteristics of the separate firms, affects merger desirability.

\textsuperscript{24} The reader may have noticed from Table 6 that synergies are negative when the volatility of Firm 1 equals the volatility of Firm 2, yet positive when $\sigma_1 < \sigma_2$. But in fact the optimal leverage ratios of the separate firms will be closer when the volatilities differ, since the volatility difference offsets the assumed difference in default costs $\alpha$.

\textsuperscript{25} While true in the majority of cases, there are minor exceptions. And the magnitude of leverage increases is not monotonically related to the net financial benefits (Measure 1).

\textsuperscript{26} See, for example, Kim and McConnell (1977) and Ghosh and Jain (2000).
7. Who Realizes the Benefits of Mergers?

If equity holders are to capture the full financial benefits of merger, bondholders must be prevented from participating in the gains. This poses no problem for start-up firms, where there is no initial debt. The entrepreneur decides whether to incorporate activities jointly or separately, and subsequently leveres the firm(s) optimally by issuing debt at a price that fairly reflects firm risk(s). If the separate firms have extant debt that is callable at par, or otherwise can be retired at a price that reflects pre-merger risks, again the separate firms’ bondholders will not participate in windfall gains from a merger.

Problems potentially arise when the extant debt is noncallable.  In Base Case 1, bond prices will rise dramatically when the merger is announced. The optimally-levered merged firm value will rise by $1.89 (1.09% by Measure 1) relative to the optimally-leveraged separate firms. But the extant debt of the firm will jump in value by $9.07, reflecting the lower risk of the merged firm. Shareholders will actually suffer a substantial loss of value in this case, 4.0% by Measure 1, and 10.3% of the initial equity values of the separate firms. Even in the extreme case considered in Table 3, the total benefits of the merger are $5.93 compared with bondholder gains of $6.70, again indicating that shareholders will lose value if debt cannot be retired prior to the merger. Of course, these statistics assume that the separate firms are optimally levered before the merger takes place.

Base Case 2 also leads to bond price increases that exceed merger benefits, when the latter are positive. The numbers are less extreme than in Base Case 1, with debt rising in value by $1.10, when merger

---

27 The potential of debtholders to benefit from mergers was first suggested by Stapleton (1982). Our approach permits quantification of these effects.

28 We assume the extant debt of the separate firms is assumed by the post-merger firm, and is priced based on the amount of coupons paid \((C_1^* + C_2^*)\) and the post-merger firm risk. Optimally, some debt may now be retired to reduce the coupon to \(C_M^*\) (see Table 2). This could benefit extant bondholders even more than $9.07, if the debt buy-back is at a price reflecting the lower optimal leverage.
benefits are $0.55 (0.56% by Measure 1). Indeed, we have found no combination of parameters where positive merger benefits exceed the rise in extant debt value.

These results underscore the reasons for callable debt. Such debt enables corporations to reduce or eliminate windfall gains to bondholders that could prevent the firm from pursuing valuable opportunities. Alternatively, the “leakage” of value to bondholders can be reduced when the firm uses short-term debt, since subsequent rollovers will be at a price that reflects post-merger risks. But short-term debt limits the potential gains from leverage: Leland and Toft (1996) show that optimal leverage ratios, and consequent tax benefits, are considerably reduced by the use of short-term debt.

8. Off-Balance-Sheet Finance

Nonfinancial firms’ increasing use of project finance, asset securitization, and other off-balance-sheet financing tools raises the question: why? Off-balance-sheet financing is claimed beneficial both for activities with high-risk cash flows (e.g., from major investment projects) and activities with low-risk cash flows (e.g., from assets that are securitized). Purely financial reasons are often cited for the use of off-balance-sheet financing for both types of activities, but little formal analysis has examined the validity of such claims.

In our framework, off-balance-sheet finance represents a financial separation of new or extant activities from the Parent Firm. The tools developed in Section 5 can be used to examine the types of activities that benefit. Below we examine two particular forms of off-balance-sheet financing: Project finance, and asset securitization.

---

29 These results are generally consistent with the findings of Mansi and Reeb (2002), who conclude that although the value of equity in conglomerate mergers drops, there is a significant increase in the value of debt above par (book value). They find that the net change in value is insignificantly different from zero.

30 Chemmanur and John (1996) examine several of the issues considered here. But their focus is not on the cited financial synergies, but rather on operational synergies resulting from differential managerial abilities across projects, and different benefits of control.
8.1 Project Finance

Project finance is characterized as the formation of a separate corporation (or SPV) to undertake a specific new investment project, rather than undertaking the project within the extant firm. The parent corporation typically retains all, or a substantial fraction of, the equity in the corporation formed to undertake the project. The formation of a separate financial entity ensures that debt financing for the project has recourse only to the project’s cash flows and assets.

Large and risky investments such as oil pipelines, power plants, or transportation infrastructure are regularly financed by this technique. Commonly-cited justifications for separate rather than Parent-firm funding include greater total financing ability, cheaper financing for assets that remain in the firm, and preserving core firm assets from bankruptcy risk.

Our project finance example considers a Parent Firm (before undertaking the project) that is identical to the Firm 1 in Base Case 1. It is a relatively high-payout, low risk firm. The Investment Project (“Firm 2”) is large (50% of the size of the Parent Firm, “Firm 1”) and also has a high payout rate (5%), and similar default costs (20%). It has a similar 20% corporate income tax rate, and a cash flow correlation of 0.20 with the Parent firm cash flow. By assumption, operational cash flows of the project are independent of whether it is financed internally by the Parent firm, or separately through an SPV.

Figure 5 graphs the financial synergies from using project finance, vs. financing the project inside the firm. It can be observed that project finance is preferred whenever the standard deviation of the project’s cash flows exceeds 35% (or 30%, if the project is the same size as the Parent Firm). Higher risk brings greater benefits to project finance. When the annual standard deviation of the project’s cash flow is 60%, the benefits are significant: 4.5% of project value (Measure 2), or 7.3% of the Parent Firm’s equity value. Benefits rise to 11.1% of the Parent Firm’s equity if the project is the same size as the parent, and to 12.4% if in addition the correlation between the project and parent cash flows is 0.60.

---

31 Kleimeier and Megginson (1999) and Esty, Harris, and Krueger (1999) provide further descriptions of project finance.
Observe that the commonly-cited reasons justifying project finance are valid for high-risk investments. Compared with parent-firm financing, the use of project finance allows greater additional debt financing (22 vs. 16), and keeps the credit spread of the Parent firm at 50 bps (vs. 160 bps if it finances internally). The project uses 53 percent leverage, but its high risk incurs a high credit spread for its debt (740 basis points).

8.2 Asset Securitization

In recent years, financial and industrial firms have transferred trillions of dollars of mortgages, commercial loans, accounts receivables, and other cash flow sources to special purpose vehicles or entities. These vehicles are bankruptcy-remote from the originating (or “Parent”) firm. Debt (typically in tranches with differing seniority) and equity (often termed the “residual tranche”) are then issued by the SPV, with the proceeds used to compensate the Parent firm for the assets transferred. As such, a bankruptcy-remote SPV with separate, limited-recourse financing has the key features of a separate firm from our analytical perspective.32 We retain the zero operational synergy assumption: pre-tax cash flows from the assets transferred from parent to an SPV are not affected by the transfer itself.

Asset securitization is frequently justified by the assertion that separate, low volatility assets offer lower cost financing and can be more effectively leveraged. Other reasons cited for securitization have included relaxation of capital constraints (for financial institutions), and reduced informational asymmetry, agency costs, taxes, and default costs.33

Securitized assets such as accounts receivable typically have relatively low cash flow volatility (compared with the firms’ other activities), high cash-flow payouts, and utilize high leverage. As an example of asset securitization (e.g., securitization of accounts receivable) we use Base Case 1 parameters for the Parent firm prior to securitization. We assume that the securitized assets have one-quarter the value of the

32 Guarantees from a Parent firm to an SPV would reduce this parallel, and our analysis assumes such guarantees are not present. We also assume a single class of debt, rather than tranches. Finally, we require that the value received from the sale of all SPV securities be used to reduce outstanding debt to the (new) optimal level, and the remaining funds be distributed to Parent firm shareholders.

33 See, for example, Greenbaum and Thakor (1987), Rosenthal and Ocampo (1988), and Lockwood, Rutherford, and Herrera (1996).
Parent firm. The SPV containing the securitized assets has high payouts ($\delta_2 = 5\%$), relatively low default costs ($\alpha_2 = 5\%$), and a correlation with the Parent firm’s other cash flows of 0.50.

Figure 6 plots the benefits of asset securitization versus keeping the assets inside the firm, as a function of the volatility ($\sigma_2$) of the securitized assets. The solid (green) line indicates the benefits of securitization as a percent of the value of the assets secured. This is simply the negative of Measure 2, as defined in Section 5. For annual cash flow volatilities of the secured assets that are less than 16%, securitization is desirable. At a 5% (2%) annual cash flow volatility, benefits amount to approximately 340 (560) basis points of the value of assets secured. If the Parent firm’s cash flows have a 30% annual volatility before securitization, the benefits rise to 420 (640) basis points.

These results validate the first reason cited above for asset securitization. When securitized cash flow has annual volatility of 5%, the SPV optimally uses 83% leverage and pays a credit spread on its borrowings of only 20 basis points (bps). After securitization, the Parent firm optimally reduces leverage from 61 percent to 54 percent, but pays a higher credit spread (280 basis points vs. 190 bps) reflecting the higher risk of the remaining Parent activities. Securitization leads to a slightly greater total leverage (62% vs. 61% of value).

Both project finance and asset securitization can be desirable, despite their very different risk characteristics, precisely because the risks (and perhaps default costs) of the separated activities are significantly different from the risks of their Parent firms’ other activities. Separation is beneficial, once again because it allows substantial differences in the leverage appropriate for the separate entities. Although we do not do so here, the effects of different tax and default cost assumptions for the SPV could be examined.

8.3 Who Realizes the Benefits of Off-Balance-Sheet Finance?

Parallel to our discussion of mergers, all value gains will flow to shareholders of the Parent Firm if extant debt is retired at its fair market value prior to project finance or asset securitization, and the firm is optimally levered thereafter. If the original Parent firm debt remains outstanding, the situation becomes more complex.
With project financing, original debt holders will not experience a value shift, since the new project is undertaken independently. If the project is undertaken internally, and is riskier than current activities, debt holder and shareholder value depends on the balance of new debt and equity financing. Heavier use of debt will typically hurt debt holders (and help shareholders).

The effects of asset securitization depend on what the Parent firm does with the cash from financing the securitized assets. If it invests in assets with cash flows that are less risky than those of the securitized assets, extant bondholders will likely benefit. If the Parent firm invests in assets with riskier cash flows, or pays out a substantial fraction to shareholders via dividends (or stock repurchases), extant bondholders could lose value.

Because these issues are complex, they are not examined further in this paper. They may well affect the distribution of gains between shareholders and bondholders. But the distribution of gains does not affect the size of the gains calculated previously.\(^{34}\)


Thus far the analysis has not addressed the impact of hedging opportunities on the relative benefits of merger vs. separation. But because the risks of the separate and merged firms importantly condition our conclusions, it is reasonable to speculate that the opportunities to hedge may affect the results derived above. Here, some preliminary observations are offered, under the conditions that

(i) hedges are fairly priced, implying the net present value of hedge returns less hedge price is zero; and

(ii) the hedged cash flows of the separate and merged firms continue to follow a logarithmic dynamic process, as in equations (1) and (12).\(^{35}\)

---

\(^{34}\) By assumption, we are abstracting from possible agency costs, where the distribution of gains might affect operational cash flows.

\(^{35}\) If risks were reduced proportionately by hedging, this condition would hold.
Under these two conditions, the analysis developed previously applies when $\sigma_1$ and $\sigma_2$ are the after-hedging risks of the two activities, $\rho$ is the correlation of the two hedged cash flows, and the hedging done after the merger is the same as the hedging done before the merger.36

Base Case 2 can be used to illustrate the impact of hedging. Assume a zero-cost hedging instrument is available to Firm 2 that reduces operational risk but does not change correlation. We consider two situations: where the hedging instrument can reduce $\sigma_2$ from 40% to 25%, and where it can reduce $\sigma_2$ from 40% to 5%.37

In the first situation, an independent Firm 2 can increase value by increasing leverage from 27.5% to 41.9% when the hedge is in place. But the merger of the hedged Firm 2 with Firm 1 will be even more valuable than before, creating an increase in value of 7.6% (by Measure 2) compared with 6.7% in the case without hedging. This can be observed in Figure 3. With hedging, the optimal leverage of Firm 2 is now closer to the optimal leverage of Firm 1. Independent capital structures are less useful, and the value increment to merging rises. After the merger, the combined firm will wish to continue the hedge—if it did not, the combined firm’s optimally-levered value would fall by 0.33%.

In the second (admittedly extreme) situation, a separate Firm 2 will be able to increase its leverage to 88.5%, and its optimally leveraged value will increase substantially. Large tax shields from debt provide this increment to value. Now, however, the merger will not be beneficial. Even though the value of the merged firms rises relative to the non-hedged case, the value of the two firms kept separate rises more. The extreme differences in initial leverage (resulting from the hedging opportunities of Firm 2) imply that the firms should

36 In the environments studied, the optimal zero-cost hedge will be chosen to reduce cash flow volatility by the maximum, since (ceteris paribus) the value of the levered firm will be greatest when $\sigma$ is minimized. (Froot and Stein’s (1998) Proposition 1 formalizes this result in a somewhat different context.) For fixed correlation $\rho$, the risk of the merged firm $\sigma_M$ will be minimized by minimizing the separate firm risks. Thus the hedging after the merger will remain unchanged. Our analysis could be extended to consider hedging instruments that also change the correlation between the hedged cash flows. Such considerations are beyond the scope of this paper, however.

37 Like Froot and Stein (1998), we assume that some risks remain after hedging. Although the cost of distress in their model is related to the potential need for costly outside financing rather than default costs, the motivation of their optimal capital structure is similar to ours.
be kept separate. Again this can be observed in Figure 3, where the benefits to merger become negative when $\sigma_2 < 8\%$. When $\sigma_1 = 25\%$ (rather than 10\%), the firms should remain separate whenever hedging can reduce Firm 2 volatility to less than 23\%.

Thus the potential to hedge will affect the extent of purely financial synergies, and the optimal scope of the firm. Results are reasonably intuitive when hedging can reduce risks without changing correlation. Our approach is sufficiently general to explore the effects of hedges that also change correlation. But we do not pursue such an extension here.

10. Conclusions

We have developed quantitative measures of purely financial synergies that result from combining (or separating) multiple activities with stochastic cash flows. To focus explicitly on financial benefits, the model abstracts from operational synergies, including agency costs. Thus our measures of financial benefits are incremental to possible operational benefits or costs.

Contrary to Lewellen’s (1971) contention, we show that purely financial synergies can be negative as well as positive. Combining activities into a single firm offers the advantage of risk-reduction from diversification, whereas keeping firms separate offers the advantage of different capital structures. In contrast with previous work, we develop closed-form expressions to quantify net benefits.

The financially-optimal scope of the firm is determined by a complex interaction of cash flow risks and correlation, payouts, tax rates, default costs, and the relative size of the activities. Activities whose optimal leverage ratios are quite similar, and whose cash flows have low correlation, are likely to realize financial benefits from merger. As a general (but not exact) rule, benefits will be positive if the optimal leverage of the merged firm exceeds the value-weighted average of the optimal leverages of separate firms.

Activities whose separate optimal leverages are quite different, and whose cash flows are more positively correlated, will typically realize financial benefits from separation, whether through spinoffs or the use of SPVs. Purely financial considerations seem to explain many of the features of off-balance-sheet
financing techniques such as asset securitization or project finance. The theory developed here predicts that large investments with high risk relative to the Parent Firm will benefit from project finance—and empirical evidence suggests that typical projects using this type of financing are indeed large and risky. Similarly, the theory predicts that asset securitization will be desirable for assets with stable cash flows and low default costs relative to the Parent firm—again consistent with observed practice.

Our measures of purely financial synergies are generally quite modest in size. But they are still capable of explaining a substantial fraction of realized merger or spinoff gains, even if they are not the primary motivation. And financial synergies may well explain off-balance-sheet financing decisions, where there are less likely to be operating synergies, and value gains of a few hundred basis points on asset value are significant.

If the debt of the separate firms cannot be retired prior to a merger, debt holders may well benefit from value increases that exceed the total purely financial benefits, leaving shareholders worse off. Thus, in considering changing the optimal scope of the firm, the distribution as well as the size of gains must be considered. More work remains to be done on the distributional consequences of changes in firm scope.
FIGURE 5: PROJECT FINANCE
Percentage Financial Benefit to Project Finance

FIGURE 6: SECURITIZATION
Percentage Financial Benefit to Asset Securitization
References


APPENDIX A:

Proof of Proposition 1:

Sufficiency:

\[
F_M(X_M(t)) = \frac{X_M(t)}{r - \mu_M} \\
= \frac{X_1(t) + X_2(t)}{r - \mu_M} \\
= \frac{X_1(t)}{r - \mu_1} + \frac{X_2(t)}{r - \mu_2} \\
= F_1(X_1(t)) + F_2(X_2(t)),
\]

where the first line follows from equation (3) and equation (11), the second line follows from equation (14), the third line from equations (12) and (16) (with simplification), and the last line from equation (3).

Necessity follows from the fact that the third line (and therefore the fourth line) will not equal the second line (and therefore the first line) whenever \( \mu_1 \neq \mu_2 \) and (16) is violated.

■
APPENDIX B. Comparison of the risk-neutral probability density function (p.d.f.) of the value of the merged firm from equations (12)–(14), with the p.d.f. of the sum of separate firm values.

The figures below compare the (risk-neutral) probability density functions (p.d.f.s) of firm value twenty years\(^{38}\) from the time of merger, for

(a) the merged firm whose value follows the geometric process in equations (12) - (14)  
   (the blue dotted line); 

(b) the sum of the values of the two separate firms, each of which follows a geometric process  
   (the solid red line).

The p.d.f. for case (a) is derived directly from the lognormal distribution. The p.d.f. for case (b) is computed numerically, and is the density function of the sum of two lognormal random variables.

We first examine Base Case 1 described in Tables 1 and 2. The sum of the two values (of EBIT) at the time of merger is 2.00, and the default value \( V_B = .986 \).

**Figure 12A** gives the p.d.f.s (a) and (b) for this case:

**FIGURE 12A: Base Case 1: PDFs of Separate and Merged Firms After 20 Years**

Note the p.d.f.s look almost identical. However, for debt value and optimal leverage purposes, it is the left tail (up to \( V_B \)) of the risk-neutral p.d.f. that is crucial. Note that the red p.d.f. lies slightly above the blue p.d.f. for low values. Thus the risk-neutral (cumulative) probability that the sum of values (solid red line) is less than the default value \( V_B = .768 \) after twenty years (16.07\%) is slightly higher than the corresponding risk-neutral probability (15.07\%) for the process (12) merged value (dotted blue line). That is, the benefits of leverage determined in Base Case 1 and based on the merged firm’s process, reflect a slightly lower risk of

\(^{38}\) 20 years (= 1/\( r \)) is the duration of (riskless) infinite-life debt, given a riskfree rate of 5\%. Even though the duration of risky debt is shorter than 20 years, we err on the side of conservatism in using this longer horizon, as the p.d.f.s will be more similar at shorter horizons.
default than would be the case if the merged firm’s cash flow were actually the sum of the two separate firms’ cash flows. This could lead to an overestimate of optimal leverage and the potential benefits of merger.

A way to correct for this slight bias is to choose a slightly higher volatility than given by (14) for the merged firm’s stochastic process, such that the risk-neutral probability of default (i.e. $V \leq V_B$) is the same as for the sum of the separate firms (16.07%). This is achieved by increasing the annual volatility of the merged process to 15.89%, rather than the 15.49% computed using (14). Since the drift term remains the same, it follows from equation (3) that the value of the firm at the time of merger is not affected by the assumed change in risk.

Figure 12B re-plots the p.d.f. of the merged firm (blue dotted line), with the higher volatility that gives equal risk-neutral probabilities of default. A slight leftward-shift can be observed, relative to that in Figure 12A:

![Figure 12B: Base Case 1: PDFs of Separate and Merged Firms After 20 Years](image)

With the higher risk, the optimal amount of leverage is reduced from 70.1% to 69.8%, and Measure 1 benefits are slightly lower, 1.26% rather than the initially estimated 1.38%. Note that this adjustment is for a long time horizon—20 years. The required volatility adjustment will be smaller when shorter horizons are considered.

Figure 13A gives the p.d.f.s equivalent to Figure 12A, for the Base Case 2 considered in Tables 3 and 4. The sum of the two values of EBIT at the time of merger is 1.10, and the default point $V_B = .558$.

The p.d.f.s of the merged firms still conform quite closely, although there are considerable differences in the firms being merged. (The disparity would be somewhat greater if the firms were of more equal size).
FIGURE 13A: Base Case 2: PDFs of Separate and Merged Firms After 20 Years

Again, the left tail is crucial. The risk-neutral cumulative probability that \( V \leq V_B = .558 \) is slightly higher (9.68%) for the sum of values of the separate firms than the probability computed for the merged firm (9.05% when \( \sigma_M = 10.44\% \)). Increasing the volatility for the merged firm to \( \sigma_M = 10.75\% \) gives an equal risk-neutral probability (9.68%) that \( V \leq V_B = .558 \). With the higher volatility, optimal leverage for the merged firm falls from 70.0% to 69.6%, and the Measure 1 benefits to merger fall from 0.56% to 0.41%.

The difference again would be even smaller if we looked at time horizons shorter than 20 years.

Figure 13B shows the effect of the slight increase in volatility on the p.d.f. of the merged firm.

FIGURE 13B: Base Case 2: PDFs of Separate and Merged Firms After 20 Years

From these examples, we conclude that the calculated financial benefits of mergers, based on the lognormal dynamic process (12) – (14), provide a close approximation to the benefits that would be derived under the value process of the (non-lognormal) sum of the individual firms’ values. Our approximation allows closed-form estimates of these financial benefits.