Predictions of Expected Default Frequencies in Structural Models of Debt

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1. Introduction

This paper examines differences in the expected default frequencies (*EDF*s) that are generated by alternative "structural" models of risky corporate bonds. The objectives are to isolate key differences in assumptions about default that distinguish the models; and second, to elucidate the differences in predicted default probabilities that arise from the alternative approaches, when inputs are similar. Our comparative analysis is limited to structural models of debt with credit risk.[1] These models assume that the value of the firm's activities (loosely termed "asset value") moves randomly through time, with an exogenously specified expected return and volatility. Bonds have pre-specified senior claims on the firm's cash flow and assets. Default occurs when the firm fails to make the promised payments.

We focus on two sets of structural models that have been quite widely used in academic and/or practical applications. The first group of models is often termed the "exogenous default boundary" approach.[2] This set includes the pioneering work of Black and Scholes (1973) and Merton (1974). To exploit the analogy of corporate equity (and ultimately debt) with options, these authors largely restricted themselves to *zero-coupon* debt.[3] With zero-coupon bonds, the default boundary is zero until the bond matures, and default never occurs prior to the bond's maturity. At maturity, the firm defaults if asset value is less than the bond's principal value.

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[1] Thus, we do not make comparisons with "reduced form" models as developed (amongst others) by Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Duffie and Singleton (1999). Uhrig-Homburg (2002) provides an excellent survey of both reduced form and structural models.

[2] A "default boundary" is a level of asset value, perhaps time dependent, sufficiently low so that the firm decides to default on its debt if asset value falls beneath this level. For models whose state variable is cash flow, the default boundary is a sufficiently low level of cash flow that the firm defaults on its debt.

[3] Merton (1974) also introduced a model with infinite-maturity debt, and the ability to pay coupons from assets. Thus, the firm could drive its asset value to zero before defaulting, implying zero recovery rates. Many bond covenants restrict the liquidation of assets for paying coupons, and we shall not pursue this approach.

The primary focus here is on the default risk of *coupon-paying* bonds. The Longstaff and Schwartz (1995) model (heareafter "LS") specifies a coupon-paying bond with an exogenous default boundary that remains constant through time. This default boundary is often specified to be the principal value of debt, implying a "positive net worth" requirement. However, Huang and Huang [2002] and others have pointed out that firms often continue to operate with negative net worth, and that the implied default costs must be extremely high to explain the relatively low recovery rates on corporate bonds. Thus, a barrier that is some fraction (less than or equal to one) of debt principal is used as the representative model for examining exogenous default boundary models. A number of papers provide variations on the LS model, but we do not consider them here.[4]

Black and Cox [1976], Leland [1994] and Leland and Toft [1996] take a different approach to determining the default boundary. These "endogenous default boundary" papers presume that the decision to default is made optimally by managers, who act to maximize the value of equity. At each moment, equity holders face the question: is it worth meeting promised debt service payments? If the asset value exceeds the default boundary, the answer is "yes", and the firm will continue to meet debt service payments and will not default—even if, as is typical when asset values are low, it requires additional equity contributions. If the answer is "no", the firm does not meet the required debt service, and defaults.[5, 6] Thus the default boundary is determined by the

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[4] See the comprehensive survey of Uhrig-Homburg (2002).

[5] By assumption, and in contrast with the Merton (1974) perpetual debt case, the firm is not allowed to liquidate operating assets to meet debt service payments. As indicated, bond indentures typically have covenants restricting asset sales for this purpose.

[6] We presume that default occurs whenever the debt service offered is less than the amount promised. Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1996) introduce "strategic debt service", where bargaining between equity and debtholders can lead to lesser debt service than promised, but without formal default. We do not pursue this approach here, in part because these papers assume debt with infinite maturity, whereas we focus on debt of differing maturities.

fundamentals of asset price movement, the amount and maturity of debt issued, default costs, and corporate tax rates.

Black and Cox [1976] postulate a fixed amount of *perpetual* (infinite maturity) debt in the capital structure, as does Leland [1994] (who also considers the optimal amount of debt for a firm to issue). The optimal default boundary is shown to be constant through time.[7]

In contrast, the Leland and Toft [1996] model (hereafter "LT") considers an endogenous-boundary model in which the firm issues debt of *arbitrary* maturity. To capture the idea of a long-term or "permanent" capital structure, they presume that debt is continuously rolled over.[8] This structure assures that total outstanding principal and coupon payments, as well as average debt maturity, remains constant through time, even though each individual bond has a life that shortens with time.[9] This stationary capital structure implies that the optimal default boundary remains constant through time, although its level now depends upon the maturity of debt issued as well as the other parameters of the model.

Our objective is to contrast and compare the expected default frequencies predicted by the exogenous default boundary approach of the LS model, and the endogenous default boundary approach of the LT model. In particular, we address the questions of

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[7] Like the other models we consider here, these models assume a constant default-free interest rate and constant mean and variance of the asset value's diffusion process.

[8] To be more specific, debt is continuously offered at a maturity T, and continuously retired at maturity. In the stationary case, the total principal of outstanding debt P is constant, as are the coupons C paid and the average maturity (that equals T/2).

[9] As with the LS approach, any bond issued by the firm can be readily valued in such an environment, as long as the overall capital structure is "permanent" (or more explicitly, the default boundary is constant through time). It is assumed that all debt issues default when the boundary is reached, consistent with the cross-default provisions that typically are included in bond covenants.

(1) How well do these models capture actual default frequencies, as reflected in data provided by Moody's [1998] for bonds with different ratings over the period 1970-1997;

(2) How can the models be used to predict the impact of changes in leverage, asset volatility, debt maturity, and default costs on a firm's expected default frequency and credit rating?

An important application of structural models is provided by KMV, now a subsidiary of Moody's.[10] The KMV approach blends elements of the Merton [1974] and LS, as well as offering some important additional features. We offer some preliminary observations on their approach.

Much of the calibration used in this paper was suggested by Huang and Huang [2002]. We also recognize the guidance they have provided in locating key statistics. Nonetheless publicly available data, while voluminous, remains frustratingly incomplete for our purposes. We indicate what further data would allow more accurate testing of the alternative models' predictions.

The structure of the paper is as follows. Section 2 introduces the key parameters of structural models, and formulas for default boundaries. Section 3 shows how expected default frequencies (*EDF*s) are related to default boundaries. Section 4 introduces "base case" parameters. We show how well the LT and LS models match the observed default frequencies estimated by Moody's for A-rated, Baa-rated, and B-rated debt in Sections 5 and 6. Section 7 shows how default probabilities (and potential ratings) for the two approaches are affected by changes in a firm's leverage, volatility, debt maturity, default costs. This includes potentially testable hypotheses that distinguish the models. Section 8 provides a brief description of the KMV approach. Section 9 offers some thoughts on similarities and differences between KMV, LS, and LT models. Section 10 concludes.

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[10] KMV are the initials of its founders, Steven Kealhofer, John McQuown, and Oldrich Vasicek.

2. Structural Models and Default Risk

Prediction of corporate bond default rates has been an objective of financial analysis for decades. Moody's and Standard and Poors' have provided bond ratings for almost a century.[11] Key variables in the rating process include

Asset-liability ratios
Coverage ratios (cash flows or EBIT relative to debt service payments)
Business prospects (growth of cash flows or value of assets)
Dividends and other payouts
Business risks (volatility of cash flows or value of assets)
Asset liquidity and recovery ratios in default.

While there has been some general discussion of the use of these variables, exactly how they are balanced with one another to determine a rating remains the proprietary information of the rating agencies.[12] This lack of transparency may reflect an unwillingness to make an explicit methodology public and thus risk "gaming" of ratings (as well as loss of valuable trade secrets), or it may reflect the fact that there is no "formula", and personal judgment remains important in weighing the several factors.

The usefulness of ratings in default prediction and bond price determination, and the fees collected by bond-rating agencies, have led both academics and practitioners to attempt to quantify the

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[11] Estimating a firm's ability and likelihood to meet debt service requirements is a stated objective of bond ratings by both Moody's and S&P.

[12] See, for example, Moody's (1999) "Rating Methodology: The Evolving Meaning of Moody's Bond Ratings" (1999).

ratings process and improve predictive power. Statistical models of bankruptcy/default have been around for over thirty years.[13] More recently, practitioners as well as academics have turned to developing structural models of firms with debt. Bonds are treated as contingent claims on the firm's assets.

Structural models of debt have five primary ingredients:

(i) A specification of the default-free interest rate process (the "riskfree rate").

(ii) A state variable, typically "asset value", which follows a specified stochastic process.[14] The expected rate of return and volatility of the asset value process reflects the expected business growth less payouts and the business risk of the firm's operations, respectively.

(iii) Debt with promised coupon flow and principal repayment at maturity. The ratio of debt to asset value captures the degree of leverage.[15]

(iv) Determination of what value bondholders receive in the event of default, loosely called the "recovery ratio." This reflects the direct and indirect costs of distress/bankruptcy, the liquidity of assets in the event of default,

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[13] Statistical scoring techniques have been widely used since Altman (1968). For a recent review of this approach, see Altman (2000). More recently, the "reduced form" approach (see Footnote 1) has used hazard rates to predict default probabilities and ratings transitions.

[14] Some models (e.g., as in Goldstein, Ju, and Leland (2001)) postulate current cash flow or "EBIT" as the state variable, as an alternative to asset value as the state variable. However, because asset value is typically a fixed multiple of cash flow in these alternative models, the resulting analytics are virtually identical. Following Merton (1974) and others, we focus on models where asset value is the state variable.

[15] Dynamic restructuring models (e.g. Fischer, Heinkel, and Zechner (1989), Leland [1998], Goldstein, Ju, and Leland (2001), and Collin-Dufresne and Goldstein (2001)) allow debt (and therefore the default boundary) to be ratcheted up when asset values increase sufficiently. Their analysis requires callable, infinite-maturity debt. We focus on a static structure with noncallable finite-life debt.

and the potential leakage of value to shareholders because violation of "absolute priority" of bondholders.

(v) Specification of a "default boundary" (a level of asset value or of cash flows through time) beneath which promised payments to debt holders are not made and default occurs.[16] Default boundaries vary widely across models.

Liberally interpreted, these ingredients include most or all of the variables deemed important by the ratings agencies. Variations in how these ingredients are specified distinguish different structural models. To compare the default predictions of the alternative models considered in this review, we *standardize* aspects (i), (ii), (iii) and (iv) across models, and show how *differences in the determination of the default boundaries* (v) affect the models' predictions of default rates. While we do not focus on valuation in this review, it is straightforward to see how differences in (v) affect predicted yield spreads due to credit risk, as well.[17]

The variables underlying the five aspects above are the following:

(i) The Default-free ("Riskless") Interest Rate

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[16] As noted in footnote 6, we ignore the possibility of "strategic debt service" as in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1996).

[17] In contrast, Huang and Huang (2002) calibrate models by standardizing default rates, recovery ratios, and equity risk premiums for different levels of leverage (corresponding to different credit ratings), but allow asset volatility and asset risk premium to vary across models. Their objective is to determine the fraction of yield spreads due to credit risk. Anderson and Sundaresan (2000) assume infinite maturity debt, and allow the default barrier (and therefore default rates) to vary. Their data-fitting approach allows the volatility of asset value, payout rates, and recovery ratios to differ across models. Like our work, Ericsson and Reneby (2002) simulate models with different default boundaries but with standardized asset volatility, payout rates, and recovery ratios. Their objective, however, is to examine the efficacy of different methods of estimating the model parameters.

The continuously-compounded riskless interest rate is denoted *r*, and is assumed to be constant through time.[18]

(ii) Firm Asset Value

Following most previous work, we assume that the value of the firm's operations or "asset value" ***V*** (with initial value normalized to $V_0 = 100$), follows a diffusion process with constant drift and volatility ("Geometric Brownian Motion")

$$dV(t)/V(t) = (\mu - \delta)dt + \sigma dZ(t), \qquad V(0) = V_0, \tag{1}$$

where

$\mu = r + \lambda$ is the expected (objective) total rate of return on assets V, comprised of the riskfree return r plus

λ	the risk premium;
δ	the fractional payout rate on assets (to both debt and equity);
σ	the volatility of the rate of return on total assets; and
$\{Z\}$	a Wiener process.

All parameters are assumed constant through time. This formulation was introduced by Black and Scholes [1973] and Merton [1974], and has been used in many subsequent studies.[19] Note that the

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[18] Kim, Ramaswamy, and Sundaresan (1993), Nielsen, Saa-Requejo and Santa-Clara (1993), and Briys and de Varenne (1997) as well as Longstaff and Schwartz [1995] allow for stochastic default-free interest rates. A negative correlation between asset values and interest rates, as seems to accord with empirical evidence, reduces the estimated yield spreads, although the effect is quite small (see Longstaff and Schwartz (1995)). The endogenous default boundary model has not yet been extended to the case of stochastic interest rates. Note the default boundary would not be constant in this case, since it depends on the riskless rate.

[19] Zhou (2001) and Delianedis and Geske (2001) consider an asset value process with both a diffusion and a jump component. Keeping volatility constant, jumps tend to increase the default probabilities over short

total expected rate of return on assets μ is comprised of an "expected growth of asset value" rate $\mu - \delta$, plus a "payout" rate equal to δ.

(iii) Debt

Debt is defined by three parameters:

P the debt's principal value
C the debt's coupon flow
T the debt's maturity.[20]

Debt is assumed noncallable, and the debt variables are assumed constant through time (a "stationary" debt structure). While it is possible to extend both the LS and LT models to consider multiple classes of debt, we consider here the case where the firm issues a single class of debt.

(iv) Recovery Rates in Default

Define

α the fraction of asset value lost if default occurs.

It follows that total default costs are αV_B, where V_B is the value of assets at the default boundary. The fraction α lost in default is assumed to be a constant across default boundary levels.[21]

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periods, while default probabilities over longer periods are less affected. Duffie and Lando (2001) consider the case where asset value is observed only with noise, giving results similar to models with jumps.

[20] Thus, debt promises to pay a flow C, and thus an amount $C\Delta t$ over a time period Δt, plus an amount P after T years. We do not consider callable or convertible debt, though that can be considered within the context of extended models (e.g. Fischer, Heinkel, and Zechner (1989), Leland [1998], Goldstein, Ju, and Leland (2001), and Collin-Dufresne and Goldstein (2001)).

Costs of default not only include the direct costs of restructuring and/or bankruptcy, but also the loss of value resulting from employees leaving, customers directing business to other firms, trade credit being curtailed, and the possible loss of growth options. These "indirect" costs are likely to be several times greater than the direct costs. In addition, there may be violation of "absolute priority", implying some value is retained by equity holders despite default.

A bond's *recovery rate* upon default is typically defined as the ratio of bond value immediately after default to its principal value. The value available to bondholders after default is $(1 - \alpha)V_B$. Therefore the recovery rate R will be

$$R = (1 - \alpha)V_B / P. \tag{2}$$

(v) Default Boundaries

(v.a) The Longstaff-Schwartz (LS) Model

The LS model assumes a constant default boundary through time, proportional to some fraction of the principal value of debt:

$$V_{B-LS}(t) = \beta P \qquad \beta \leq 1. \tag{3}$$

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[21] Seniority is not a determinant of α here, since there is only one class of debt.

Their paper assumes that the default boundary equals the outstanding principal of debt: $\beta = 1$. This amounts to a "positive net worth" requirement $V \geq P$. Other authors have noted that many firms operate with negative (accounting) net worth. It has also been observed that, to be consistent with recovery rates of around 50%, the loss of value in default α would have to be high—also 50%. For this reason, the parameter β is often assumed to be less than one. Huang and Huang [2002] choose 0.60 as an approximation.

It is important to note that the specification (2) of the LS default barrier is *not* affected by time, firm risk, payout rates, the riskless rate of interest, recovery rates, or debt maturity. Only the principal value of debt will matter, when β is a constant.[22] Also note that (3) implies that the recovery rate

$$R = (1 - \alpha)\beta, \tag{4}$$

and therefore recovery rates in the LS model are invariant to firm risk, leverage, and debt maturity. In contrast, models whose default boundary is a function of firm risk, leverage, and debt maturity will have recovery rates that vary with these parameters, even though α is a constant.[23]

(v.b) The Leland-Toft (LT) Model

The LT model assumes that the decision to default (and therefore the default boundary) is chosen optimally by a manager maximizing equity value.

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[22] Nielsen et al. (1993) allow the default threshold to grow at the riskfree rate of interest.

[23] When V_B is determined endogenously, as in Leland and Toft [1996], the ratio $(1-\alpha)V_B/P$ tends to fall with P, implying R falls with leverage. There is some evidence suggesting a negative correlation between default rates (positively related to leverage) and recovery ratios: see Altman, and Resti, Sironi (2001) and Bakshi, Madan, and Zhang (2001). However, this may also reflect seniority differences.

To derive a constant default barrier, LT assume a "stationary" capital structure. Debt of a constant maturity T is continuously issued at a given rate p. As debt matures, it is replaced by new debt with equal principal value and maturity. Total debt principal will be constant at a level $P = pT$. Although the firm has an aggregate debt structure that is stationary, any particular bond can be valued as in LS, but with the default barrier $V_{B\text{-}LT}$ used for valuation rather than the default barrier $V_{B\text{-}LS}$.[24]

The default boundary in LT is given by

$$V_{B-LT} = \frac{(C/r)(A/rT) - B) - AP/(rT) - \tau Cx/r}{1 + \alpha x - (1-\alpha)B}, \tag{5}$$

where

$$A = 2ae^{-rT}N[a\sigma\sqrt{T}] - 2zN[z\sigma\sqrt{T}] - \frac{2}{\sigma\sqrt{T}}n[z\sigma\sqrt{T}] + \frac{2e^{-rT}}{\sigma\sqrt{T}}n[a\sigma\sqrt{T}] + (z-a),$$

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right)N[z\sigma\sqrt{T}] - \frac{2}{\sigma\sqrt{T}}n[z\sigma\sqrt{T}] + (z-a) + \frac{1}{z\sigma^2 T},$$

$$a = \frac{(r - \delta - (\sigma^2/2))}{\sigma^2}, \quad z = \frac{((a\sigma^2)^2 + 2r\sigma^2)^{1/2}}{\sigma^2}, \quad x = a + z,$$

where C is the coupon that results in the bond selling at par, n[•] and N[•] denote the standard normal density and cumulative distribution functions, respectively, and τ is the corporate income

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[24] Recall LT assume a constant riskless rate to derive a constant default boundary, whereas LS allow a stochastic rate but *assume* a constant default boundary. An ad hoc "hybrid" model would allow stochastic interest rates for valuing bonds a la LS, but assume V_B determined endogenously a la LT, with the central trend of interest rates used as a proxy for a constant r.

tax rate.[25] Note that, like bond prices, $V_{B\text{-}LT}$ depends only the risk-neutral drift r and not on the actual drift μ.

In general, the default boundary $V_{B\text{-}LT}$ decreases with maturity T, volatility σ, and the riskfree rate r, and increases with principal P and default costs α. The default boundary can increase (for low leverage cases) or decrease (for high leverage cases) with payout rate δ.

3. The Expected Default Frequency (EDF) with Constant Default Barrier

With a constant default barrier, the mathematics of simple barrier options is applicable.
The first-passage time probability density function for reaching a barrier V_B at time t from a starting value $V_0 > V_B$ is given by

$$f(t;b,\mu,\delta,\sigma) = \left(\frac{b}{\sigma\sqrt{2t^3\Pi}} \right) e^{-(b+(r-\delta-.5\sigma^2)t)/(2\sigma^2 t)} \tag{6}$$

where $$b = Ln(V_0 / V_B).$$

The first-passage time cumulative probability function, often termed the expected default frequency (EDF), gives the probability of default up to the time t:

$$EDF(t;b,\mu,\delta,\sigma) = N[(-b-(\mu-\delta-.5\sigma^2)t)/\sigma\sqrt{t}] + e^{-2b(\mu-\delta-.5\sigma^2)/\sigma^2} N[(-b+(\mu-\delta-.5\sigma^2)t)/\sigma\sqrt{t}]. \tag{7}$$

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[25] The coupon C(P,X) can be solved numerically as the coupon that allows the bond to sell at par, i.e. the solution to the equation D(C, V_B(P,C,X), X) = P, where D is the market value of debt given in Leland and Toft [1996, equation (3)], and X is the vector of exogenous parameters.

In the Merton (1974) model, the formula for the probability of default at time t = T (the bond maturity) is given by

(8) $N[(-b-(\mu-\delta-.5\sigma^2)t)/\sigma\sqrt{t}]$.[26]

It is immediately obvious that (8) is always less than (7) when $t = T$, reflecting the fact that default occurs only at the maturity of the zero-coupon bond. The default boundary in the Merton model is zero for all t until T, and then jumps to P at T.

4. Calibration of Models: The Base Case

We follow Huang and Huang (2002) (hereafter "HH") in many of our parameter choices, focusing on debt with a Baa rating. Base case parameters are:

Parameter	**Symbol**	**Value Assumed and Rationale**
Default free interest rate	***r***	8%, as in HH, the historical average of the default-free rate, 1973-1998.
Asset risk premium	λ	4%, the asset risk premium. Consistent with an equity premium of about 6% when the average firm has about 35% leverage.[27]

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[26] Note that the argument of (8) is similar to (negative) "d2" in the Black-Scholes option pricing formula, but with the actual drift μ rather than the risk-neutral drift *r*.

[27] As leverage increases from zero, the *equity risk* premium will increase. The equity risk premia implied by this unlevered risk premium are roughly consistent with Bhandri's (1988) regression results. Accurate

Expected asset return	μ	12%, the sum of $\boldsymbol{r} + \lambda$
Asset volatility	σ	23%. Individual stocks average about 35% volatility with a correlation of 0.20, consistent with a diversified equity portfolio risk of about 20% (The S&P500 index volatility). If the average S&P 500 firm has leverage of about 35%, average asset volatility will be about 23% per annum.[28]
Payout rate	δ	6%, as assumed by HH.[29]
Corporate tax rate	τ	15%, representing the corporate tax rate offset by the personal tax rate advantage of equity returns.[30]
Debt Principal	$\boldsymbol{P}$	43.3 in the base case, implying a leverage ratio on asset value of 43.3%, the average leverage ratio of Baa-rated firms as reported in Standard and

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estimates of expected returns and premia require very long data series, and are suspect in an environment where risks are changing through time.

[28] A recent study by KMV (2002) provided an example where the asset volatility of a low risk firm (Anheuser-Busch) was estimated to be 21%, and the asset volatility of a very high risk firm (Compaq) was estimated at 39%.

[29] More highly leveraged firms might be expected have higher payout rates. We do not attempt to estimate this, however.

[30] If the corporate tax rate is 35%, the personal tax on bond income is 40%, and the tax rate on stock returns is 20%, then the effective tax advantage of debt is 1-(1-.35)(1-.20))/(1-.40) = 0.133, or a bit less than 15%.

		Poor's (1999). We also consider leverage ratios of 32.2% (single-A rated firms) and 65.7% (single-B rated firms).
Debt Coupon	$\boldsymbol{C}$	Coupons which allow the bonds to sell at par P. These are determined numerically. Yield spread is given by $C/P - r$.
Maturity	$\boldsymbol{T}$	10 years in the base case. We also consider bond maturities of 5 and 20 years.[31]
Fractional default costs	α	30%, implying a 51% recovery rate in the base case, the recovery rate assumed by HH.[32]

5. Matching Empirical EDFs with the LT Model

For the base case in the previous section, the predicted bankruptcy barrier $V_{B\text{-}LT}$ is 31.7, the recovery rate is 51.2%, and the yield spread is 55 basis points. As is typical with structural models,

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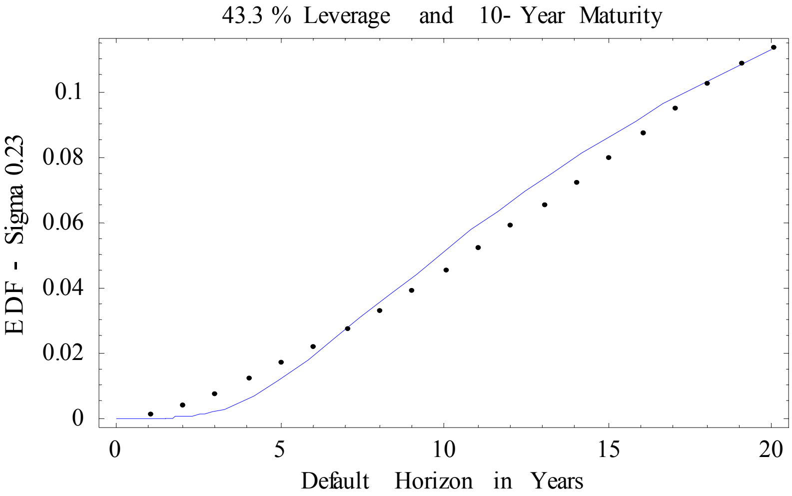
[31] In LT, T is the maturity of the bonds that are continuously being offered. Because of the stationary capital structure, bond maturities are uniformly distributed between 0 and T. Thus average maturity of the debt structure will be T/2. Stohs and Mauer (1996) find average debt maturity of 4.92 years for B-rated firms (consistent with T = 9.84 years), 4.60 years for Baa-rated firms is 4.60 years (T = 9.20), and 4.52 years for single-A rated firms (T = 9.04).

[32] Costs of default not only include the direct costs of restructuring and/or bankruptcy, but also the loss of value resulting from workers resigning, customers directing business to other potential sellers, trade credit being curtailed, and possible loss of growth options. These "indirect" costs are likely to be several times greater than the direct costs. While Andrade and Kaplan [1999] find default costs to be 10%-20% for firms that had undergone highly-leveraged buyouts previously, high leverage will typically be desirable for firms with relatively low default costs. Thus their sample is likely to have downwardly-biased default costs relative to the average firm.

the predicted yield spread is much lower than the actual average yield spread of 194 basis points. HH attribute such differences to factors other than credit risk, including liquidity, call features, and differences in taxation. We do not enter this debate, since our focus is on predicting default probabilities, not yield spreads.

The solid line in Figure 1 plots the expected default frequency (*EDF*) as a function of time horizon, as predicted by the LT model for the base case (Baa-rated bonds). The dotted line plots the actual default frequencies for Baa-rated debt as a function of horizon, as given by Moody's (1998) for the period 1970-1997. Recall that Baa-rated debt is consistent with an average leverage of 43.3% and a ten-year maturity for newly-issued debt (implying a five-year average maturity for total firm debt in the LT model).

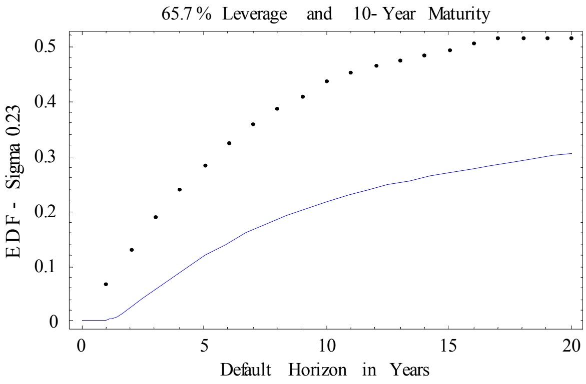
FIGURE 1



The *EDF*s predicted by the LT model seem fairly accurate for Baa-rated debt.[33] There is a systematic underestimation of default probabilities at low horizons and a slight overestimation of default in the middle ranges.[34]

When we consider riskier (single-B rated) debt, with the firm having leverage of 65.7%, the LT model does not look good in the base case:

FIGURE 2



The actual default probabilities are considerably higher for all default horizons than predicted by the LT approach. But there is no reason to think that the "base case" –which was predicated on the volatility of a typical firm—is relevant to firms issuing B-rated debt.

While Stohs and Mauer (1996) show that B-rated debt has about the same average maturity as Baa-rated debt, there is reason to believe that the firms with B-rated debt have higher asset volatility.

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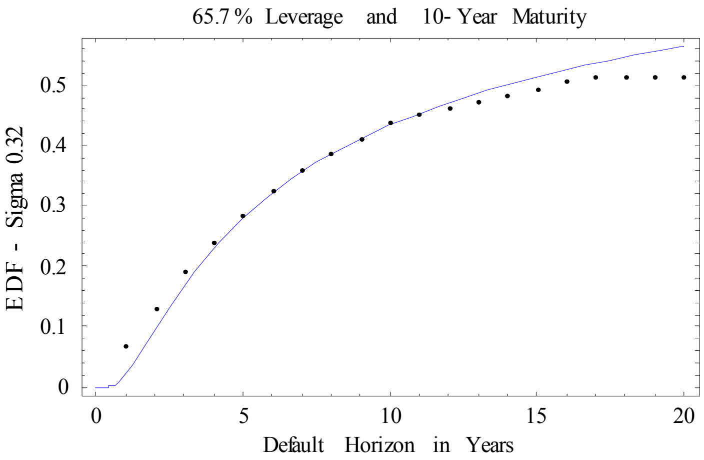
[33] In fact, a slightly better fit can be achieved by choosing a volatility 22.8% rather than 23% (see Figure 1A in the Appendix). However, we continue our comparisons with a base case volatility of 23%.

[34] There is some possibility that the data is the problem at low maturities, not the model. Default probabilities are highly convex in leverage. Thus, the average short-term default rate on a 48%-leveraged firm and a 38%-leveraged firm will be higher than the default rate of a 43%-leveraged firm. The data reflects average default rates across leverages in the Baa category; the model estimates default rates of the average leverage. Alternatively, jump models (e.g. Zhou (2001) may be able to explain higher short-term default rates.

Unfortunately, *no public data seems to be available on the volatility of the firms issuing debt with different ratings*.[35]

Using the LT approach and base case parameters (other than asset risk), an average asset volatility can be estimated by finding the volatility that best approximates actual *EDF*s. Figure 3 depicts the *EDF*s predicted by the LT model when asset volatility of firms with B-rated debt is $\sigma = 32.0\%$.

FIGURE 3



At the assumed level of volatility, the fit is again quite good.[36] Recovery is predicted to be slightly lower (50.6%) and the yield spread is estimated at 414 basis points (vs. an historical average of 408 bps for B-rated bonds in the Lehman Bond Index, over the period 1973-1993). Unfortunately, we

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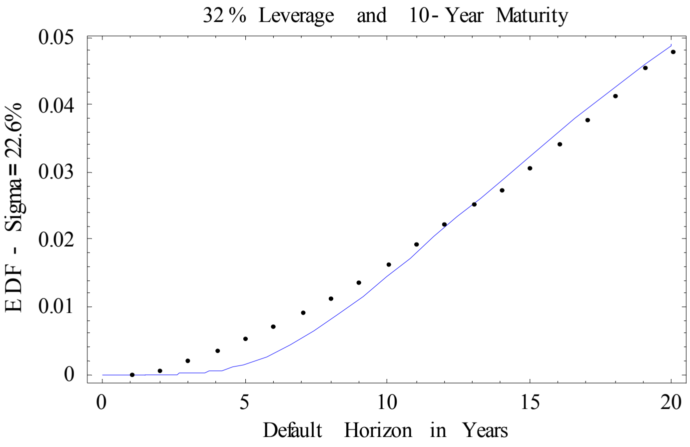
[35] It is possible that firms such as Moody's/KMV have this data, but we are unaware of publicly available data sources. HH provide estimates of asset volatilities backed out from structural models, but these vary enormously across models and many seem inconsistent with empirical estimates of average equity volatility. See the discussion of our choice of σ in the preceding section.

[36] Note that the graph of actual defaults is flat between 17 and 20 years, indicating no defaults over this interval. This seems a curiosity of the time period examined, and reflects the fact that default frequencies can vary considerably through time.

don’t know whether an asset volatility of 32.0% is empirically reasonable for firms with bonds having a B rating.

For A-rated debt, the fit of predicted and actual *EDF*s is illustrated in Figure 4 for an asset volatility of 26.7% (slightly lower than the base case):

FIGURE 4



It is clear that the LT approach is capable of reproducing the different shapes of *EDF*s. It matches default rates more accurately over longer horizons than over shorter horizons. The goodness of fit is quite sensitive to the volatility of firm assets. If, for example, the volatility of firm assets in the base case were increased to 25%, the predicted default frequency over 20 years would rise from 11% to 16%. Lowering the default cost coefficient from 30% to 15% would lower the 20-year expected default frequency from 11% to 10% (and it would also increase the recovery rate from 51.2% to 59.4%).

5. Matching Empirical EDFs with the LS Model

The LS model doesn't offer guidance on selecting the variable β in the default barrier equation

$$V_{B\text{-}LS} = \beta P.$$

Common choices for β are 1.00 and 0.60. For the base case, those coefficients yield the expected default frequencies shown in Figures 5 and 6:

FIGURE 5

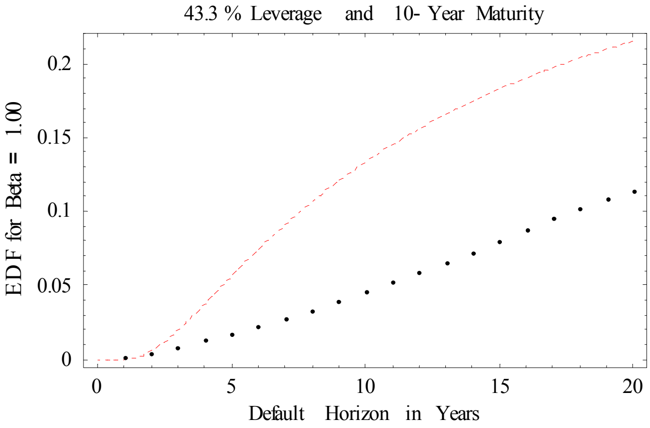
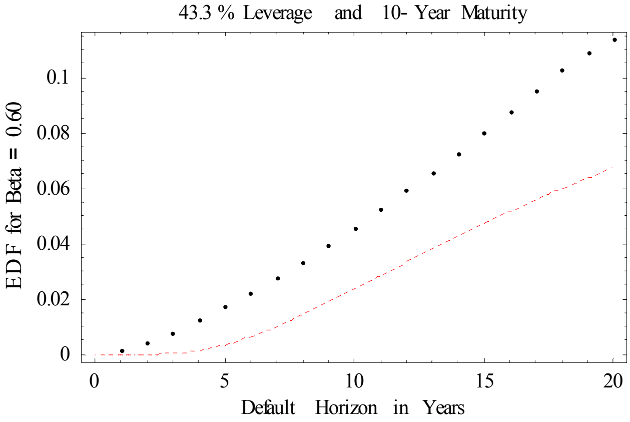


FIGURE 6



Obviously the LS approach with either β equal to 1.00 or 0.60 doesn't provide a good fit to the base case parameters. However, we can make the following

Observation 1: *for any endogenously determined boundary $V_{B\text{-}LT}$, there exists a β such that*

$$V_{B\text{-}LS} = V_{B\text{-}LT}\,.$$

That is, we can always choose a $\beta^* = V_{B\text{-}LT}/P$, in which case the exogenous boundary coincides with the boundary that is endogenously determined:

$$V_{B\text{-}LS} = \beta^* P = V_{B\text{-}LT}\,.$$

Observation 2: *the β^* above implies that the recovery rate will be the same in both LS and LT models.*

The latter observation follows from the fact that $R_{LS} = (1\text{-}\alpha)\beta^* P/P = (1\text{-}\alpha)V_{B\text{-}LT}/P = R_{LT}$. Thus the choice of β^* can also be motivated as the choice that gives a target recovery ratio equal to the LT recovery ratio..

When the default boundaries are identical, so will the *EDF*s from equation (7) when other parameters are standardized. Thus, with the exogenous boundary approach, *we can always match the EDF performance of the endogenous boundary approach* by judiciously choosing $\beta = \beta^*$.[37]

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[37] Of course the β^* so determined will create an identical boundary and *EDF*s for a *specific set of parameters*. As parameters are changed from this specific set, the models typically will differ in their default predictions. In principle, an extended LS model—rather than assuming β is constant—could specify β as a function of model parameters α, δ, σ, τ, r. The LT approach amounts to an explicit specification of such a function. In

To match the $V_{B\text{-}LT} = 31.7$ of the base case, it would be necessary to choose

$$\begin{aligned} \beta &= 31.7/43 \\ &= .731. \end{aligned}$$

As indicated, this choice for β also implies the recovery rate for the LS model is identical to the LT recovery rate (51.2%). *We shall use this β as the proxy for making comparisons between the LS and LT models.* By construction, the LS model will have the same fit as the LT model for the base case with Baa-rated debt, since they both have the same default boundary. Thus the estimated *EDF* graph for LS is identical to that in Figure 1.

Keeping β and volatility constant, the LS model also underestimates the *EDFs* for B-rated debt. Volatility must be raised for both LS and LT to have a good fit. Surprisingly, raising volatility to 32.0% produces the best fit both for the LT model and for the LS model with $\beta = .731$.

The same situation holds for A-rated debt, where leverage averages 32%. At a volatility of 22.6%, fractionally lower than the base case, both the LS model with $\beta = .731$ and the LT model find their best fits.

We conclude that empirical default frequencies are equally well predicted by LT and by LS with $\beta = 0.731$, when volatility can be adjusted as the rating class changes. Identical volatility adjustments are required by both models to give the best fit, with higher volatility required to explain the *EDF*s of lower-rated bonds. The very different shapes of investment grade and non-

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this sense, the LT model can be viewed as a subset of (extended) LS models. But what alternative specification of $\beta(\alpha, \delta, \sigma, \tau, r)$ would be reasonable is unclear.

investment grade debt *EDF*s can be quite well explained by these models.[38] Both models underestimate Moody's short-term default frequencies, but do quite well for longer horizons. Given the data currently available, there is insufficient evidence to distinguish between the models.

Yet there are important differences in the predictions of the models that are potentially testable with further cross-sectional data. The LT model predicts that the default barrier falls with asset volatility and recovery ratios decline. The LS model predicts no change in either. A change in asset volatility will have a lesser impact on default frequencies in the LT model, since changes in the default boundary will partially offset the increase in volatility.

The LS model also predicts that the default probabilities are independent of debt maturity. Casual empiricism suggests that recovery ratios decline as maturity increases, *ceteris paribus*. The LT model predicts that recovery ratios will fall with maturity. Again, further data is required to distinguish the approaches.

7. Ratings Change Predictions of Endogenous vs. Exogenous Default Models

In the previous section we found volatility and other parameters to match observed *EDF*s for bonds of different rating. Our base case parameters reflected an "average" firm with Baa-rated debt. But what if the firm we are analyzing differs from average in some notable respect?

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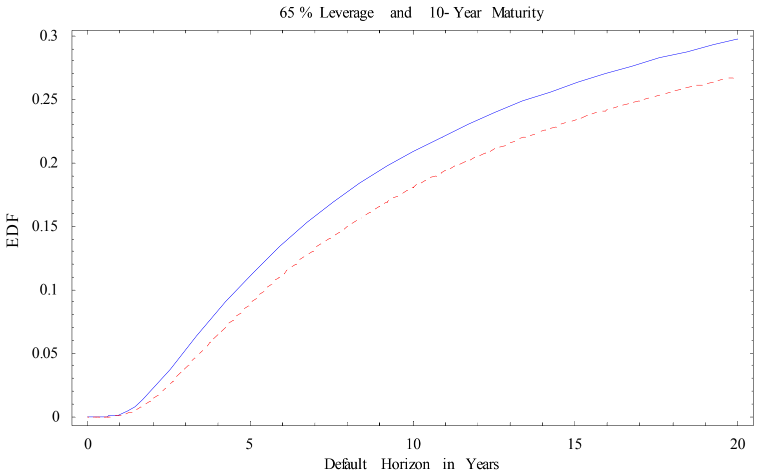
[38] While matching the LT default boundary when $\sigma = .23$ uniquely defines a β for LS, the interested reader may wonder whether a different β, σ, and α would allow the LS model to obtain a superior fit of default, yield spread, and recovery data, even though such combinations would be inconsistent with LS. Although there are combinations that provide roughly similar fits to actual data, (e.g. $\beta = 0.60$, $\sigma = .25$, $\alpha = .15$), none were superior. The preceding combination of parameters also provides a 51% recovery rate, but implies a lower yield spread (as V_B is much reduced) and an average equity volatility of around 40%.

Say the "base case" firm, with volatility $\sigma = .23$, decided to raise its leverage from 43% to 65%. How would this affect the expected default probability? And how would the bond rating be affected? Figure 7 shows the *EDF*s predicted by LT (blue) and LS (red), when principal $P = 65$.

FIGURE 7



Moody's (1998) provides *EDF*s for up to 8 years for a highly detailed set of bond classifications. The 15% default rate at 8 years projected by the LS model is consistent with the actual default frequency of a bond with Ba1 rating, while the 18% default rate of the LT model is consistent with a Ba2 rating. This compares with an initial rating of Baa2 by both models, when the firm has 43.3% leverage (the base case). Thus the LT predicts the rating will fall further with increased leverage.

Alternatively, now consider the case where the firm wishes to issue 20-year debt rather than 10-year debt in the base case. Note that the LS model, with fixed β, would predict no change in *EDF* or rating, since by assumption we are keeping the *amount P* of debt the same.[39]

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[39] If it is argued that β should change, it is necessary to specify *how*. The LS model does not provide an answer, although the LT model does.

FIGURE 8

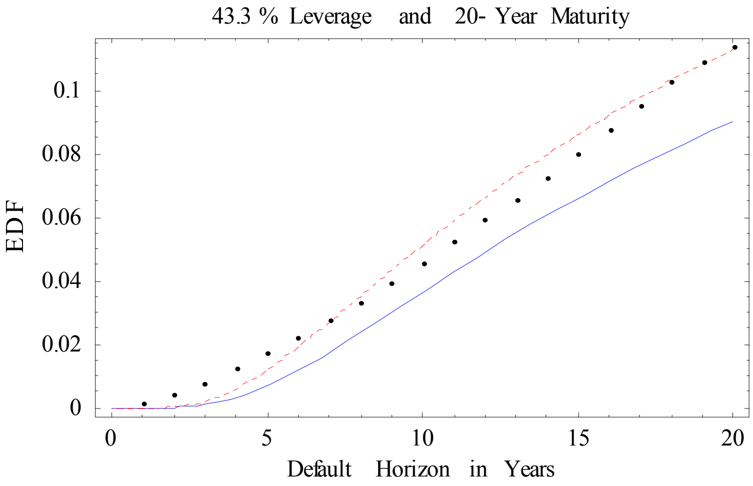

Figure 8 shows the prediction of the LT model for the *EDF* of the firm with base case parameters that decides to issue 20-year rather than 10-year debt. The lower blue line is the *EDF* predicted by LT for the 20-year debt; the red line is the *EDF* for 20-year debt predicted by LS, which is the same curve as for 10-year debt for both models (see Figure 1). The 8-year default probability estimated by the LT model falls from 3.32% for 10-year debt, consistent with a rating between Baa2 and Baa3, to 2.25% for 20-year debt, consistent with a rating between A3 and Baa1.[40]

In sum, the different models, while fitting available default data equally well, can make substantially different predictions about default probabilities when examining debt that is offered by firms whose leverage, debt maturity, and/or default costs differ from base case levels. The

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[40] The estimated yield spread on the longer debt actually rises slightly (from 55 to 63 bps), despite the fall in the default boundary and default probabilities. This reflects the fact that yield spreads depend upon the risk-neutral growth rates (r - δ) rather than objective growth rate (μ - δ) of asset value.

different default probability predictions can imply different ratings for the firm's debt. Further data and empirical work will be needed to determine which approach's predictions are better.

8. KMV's Approach

As outlined in Crosbie and Bohn (2002) and Crouhy et al. (2000), the KMV approach consists of four steps.

(i) Estimate asset value and volatility

Asset values and volatility are "backed out" from observed equity value, volatility, and leverage, using a proprietary variant of the Black and Scholes /Merton option pricing model.

(ii) Calculation of a "Default Boundary" $V_{B\text{-}KMV}$

KMV calculate the boundary as:

$$V_{B-KMV} = P_{ST} + \frac{1}{2} P_{LT}$$

where P_{ST} are short term liabilities and P_{LT} are long term liabilities. As best we understand, KMV considers a liability to be "short term" if it is due within the horizon t over which the default probability is computed. Longer horizons thus have a higher fraction of "short term" liabilities, with a consequent higher default barrier.

In the special but important case is a "homogeneous" capital structure, with total debt P and equal amounts P/T of each maturity debt, 1,..,T. If the default horizon is t years, then

(9)
$$V_{B-KMV}(t,T) = \frac{Min(t,T)}{T}P + \frac{1}{2}(1 - \frac{Min(t,T)}{T})P$$
$$= \left(\frac{1}{2} + \left(\frac{1}{2}\right)\left(\frac{Min(t,T)}{T}\right)\right)P$$

The KMV default boundary depends on the debt maturity *T*. But unlike either LS or LT, the KMV boundary also depends upon time *t*.[41] Therefore equation (7) does not apply.

(iii) Calculation of the Distance to Default (DD)

DD(t,T) is measured by number of standard deviations between the asset level and the KMV default boundary, using the asset level and asset return volatility estimated in step (i). This is given by

(10)
$$DD(t,T) = \frac{Ln(\frac{V_0}{V_{B-KMV}(t,T)}) - (\mu - \delta - .5\sigma^2)t)}{\sigma\sqrt{t}}$$

where $V_{B-KMV}\,(t,T)$ is given in equation (9).

(iv) Mapping of DD(t,T) into EDF(t,T).

KMV has an extensive proprietary data base for which DD(t)s are estimated. It then associates different levels of DD(t) with historical default frequencies over the horizon t. It thus finds a mapping *f* such that

(11) $EDF(t) = f(DD(t), t)$.

---

[41] In Section Y, we also examine term default probabilities of the KMV model for alternative horizons, when we presume that a fraction 1/T (rather than H/T) of debt principal is short term, regardless of the actual horizon H for examining default. That is, debt due within a year is classified as short term.

For given t, it is presumed that f will be monotonic in $DD(t)$. This EDF(t) can then be compared with the actual average default frequencies of bonds with different ratings, to derive an "implicit" bond rating.

In their publicly available descriptions, KMV show that *if* asset values followed the diffusion process (1), then their EDF at horizon t will be given by

$$EDF(t) = N[-DD(t)], \tag{12}$$

where N(•) is the standard normal cumulative distribution function.[42] Note that (12) is simply the probability of default for a *zero-coupon* bond with maturity t and principal $V_{B\text{-}KMV}(t,T)$ that is given in equation (8). This links KMV's approach with Merton (1974).

9. Some Preliminary Thoughts on the Relationship Between the KMV Approach and LS/LT

Given the $\boldsymbol{V_{B\text{-}KMV}(t,T)}$ function as in equation (10), and that asset values follow the diffusion process in equation (1), we can compute KMV's $EDF(t)$ from equation (12). Figure 10 plots the EDF as a function of horizon t for the base case:[43]

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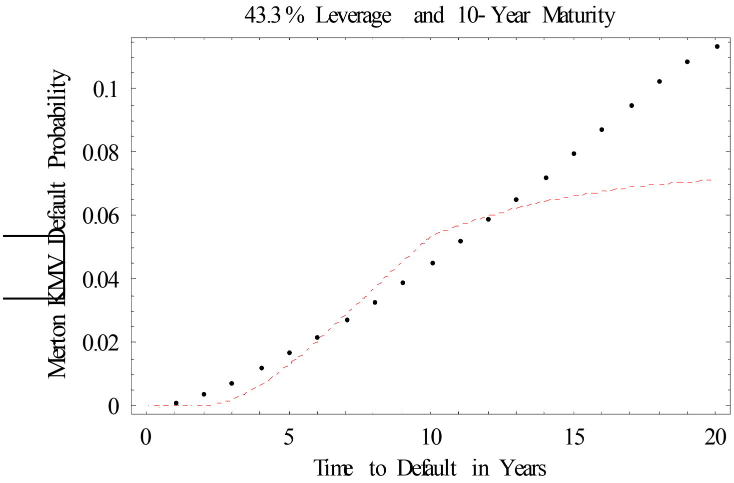
[42] We stress that KMV does not necessarily believe that asset values follow the diffusion process (1), but use it for illustrative purposes. We nonetheless use it to compare the differences in default probability estimates that the various approaches would make *under the same set of assumptions about asset value movements.*

[43] Recall that a maturity T = 10 implies an average debt maturity of T/2 = 5 years, when the capital structure is stationary.

FIGURE 10



For the first 12 years, the plot is not too dissimilar to either LT or LS with $\beta = .731$. When the horizon t reaches the term of debt maturity $T = 10$, the KMV default boundary no longer increases with t. This explains the kink at t = 10 and the flattening of the KMV *EDF*. [44]

As with LS, but in contrast with LT, the KMV *EDF* in Figure 10 (and implied bond ratings) will not depend on default costs α. As with LT, but in contrast with LS, the *EDF* declines with increased debt maturity. As with both LS and LT, the *EDF* increases with leverage and volatility. [45] Of course the sensitivities of *EDF*s to changes in parameters is different across the different models.

At a more conceptual level, having the distance-to-default *DD(t)* as the sole argument in KMV's *EDF* (whether through (12) or a more general relationship (11)) is a strong restriction. Observe that

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[44] KMV forecasts default probabilities only to a horizon of 5 years, so what happens much beyond this may be of academic interest only.

[45] The assertions in this paragraph are *not* dependent on the specific asset volatility form (1) chosen, but simply reflect properties of *DD* in equation (10) and $V_{B\text{-}KMV}(t,T)$ in equation (9) .

equation (12) gives the probability of being beneath $V_{B\text{-}KMV}(t)$ *at the horizon t only*, and *not* the probability of falling beneath the barrier $V_{B\text{-}KMV}(\tau)$ *at any time* τ *within the horizon* [0, *t*].[46]

This is in contrast with LS and LT, where the argument for *EDF* is given by equation (7). That equation has *DD* as the first term in its argument, but has a second term as well. This implies that *DD* is not a sufficient statistic for *EDF* in equation (7), and that ordinal rankings of the two criteria can differ. For example, at longer horizons, the *DD*(*t*) as computed in equation (12) can decline with *t* –i.e. the estimated default frequency can actually decrease with a longer horizon, even though this should not be possible for a cumulative default probability function. For the base case, the *EDF* is decreasing with time for time horizons *t* that exceed 25 years.[47]

10. Conclusions

We have compared structural models with exogenous (LS) and endogenous default (LT) boundaries. The theoretical models we examine are all imply a constant default boundary through time.

The endogenous model specifies the default boundary as a function of the expected return and volatility of asset value, the risk free rate of interest, leverage, debt maturity, and default costs. It fits actual default frequencies for longer time horizons exceeding quite well for a reasonable "base case" parameters, although the predicted default frequency is too low for short maturities.

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[46] Crosbie and Bohn (2002) write that KMV presumes "the probability of default is the probability that the market value of the firm's assets will be less than the book value of the firm's liabilities by the time the debt matures." (p. 16). Their subsequent equation for this probability is identical to (12).

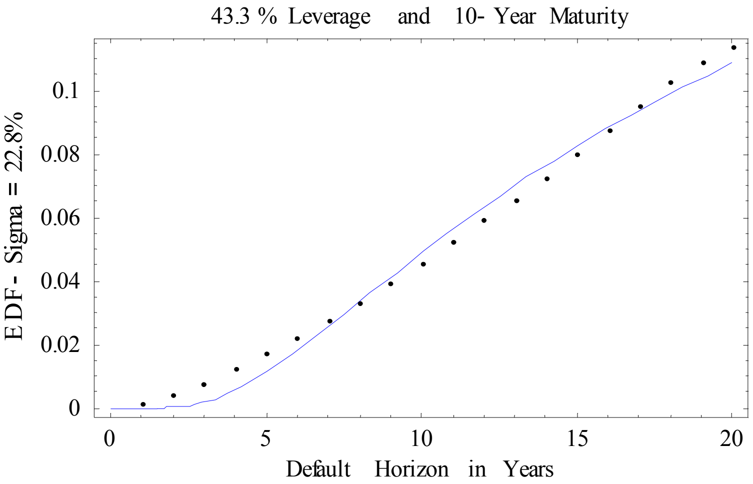
[47] Because KMV limits forecasts of *EDF*s to 5 years, this is a theoretical but not a practical difficulty. Furthermore, the fact that *f* in equation (11) can depend separately on *t* can offset the decline in *DD(t)*.

Exogenous models typically specify the default boundary as a fixed fraction β of principal value P. This fraction can be chosen to match the default boundary of the endogenous model for any specific set of parameters. When β is chosen to match the endogenous default boundary in the base case, the exogenous model gives identical predicted default frequencies. Alternative choices of β do not result in more accurate predicted default frequencies in that case.

The predicted default frequencies of the LS and LT models differ as parameters change from the base case. The LT model implies default frequencies rise with default costs and fall with bond maturity. The LS model implies default frequencies are invariant to these parameters. The LS model implies default frequencies are more sensitive to changes in the volatility of asset value than the LT model. Unfortunately, data is not publicly available to test the relative accuracy of these predictions.

The KMV approach is a structural approach with differences from either LS or LT. Its default boundary, like LT, declines with asset maturity. Like LS, the default boundary is invariant to default costs. Its technique for determining default probabilities differs in two ways from LS and LT. First, it introduces a measure *DD* that (as in the Merton model) determines the probability that the asset value exceeds the boundary only at the horizon time t, and not the probability that the asset value exceeds the boundary at all times until t. Second, it uses an empirically-estimated relationship between DD and the expected default frequency, rather than the relationship implied by a geometric Brownian motion of asset values.

FIGURE 1A: 22.8% Volatility



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