Adverse Selection and Re-Trade*

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We study how present and future adverse selection costs affect the prices of assets that may be traded repeatedly. We find that adverse selection is not a trading cost on average, and therefore future trading “costs” have no direct impact on current prices. Adverse selection can lead to allocation inefficiencies, which influence prices. Specifically, prices are reduced by the present value of all future allocation costs.

We show that adverse selection affects assets of different maturities differently. Further, we study how adverse selection interacts with the macro economy.

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1 Introduction

This paper addresses the effect of repeated adverse selection. More specifically, it asks the question: How are future adverse selection costs present-valued into current prices?

Adverse selection is important in many markets and receives significant attention in the literature. Akerlof (1970) shows that adverse selection can lead to a market breakdown. If some agents have liquidity motives for trade, however, markets need not break down. In this case, adverse selection gives rise to a bid-ask spread (Bagehot (1971) and Glosten and Milgrom (1985)), and to a market impact of a trade (Kyle (1985)). The literature that follows these papers focuses on how information is revealed through trading and reflected in prices. It abstracts from the question of how future information asymmetries affect the value of the option to re-trade, and consequently the level of prices. In this literature the price is taken to be the expected liquidation value conditional on the order flow. This implies that average prices are equal to average liquidation values.

This paper departs from the above-cited literature by assuming limited market making capacity, and allowing for the possibility that a (strategic) agent may need to liquidate his security position in the presence of adverse selection. This is not the case, for instance, in Kyle (1985). There, the marketmaker can take unlimited positions, and the strategic agent is assumed to have no risk of facing an unexpected shock, such as a need for cash, inducing a need to liquidate, and can trade in a perfectly liquid market at a known final date.

Our assumptions correspond to features of the markets for durable goods, such as cars and houses, and for most financial assets. In durable-good markets, intermediaries usually do not take positions, but, rather, serve as matchmakers. In financial markets, marketmakers often have limited positions over night, which means that they have a net trade that is close to zero during the day. Further, in these markets agents might need to liquidate unexpectedly into markets plagued by adverse-selection problems. Liquidation could be induced by financial distress, hedging, a reduced need for the asset (lost service flows), unexpected relocation, and other reasons.

Given this possibility, agents anticipate future adverse selection costs. When an investor buys an asset, he must consider the costs associated with adverse selection at the future time at which he needs to sell. Moreover, the buyer at that time will face an adverse-selection problem later, when he
wants to sell, and so on for the life of the asset. This paper models how these future adverse-selection costs affect current prices.

The impact of future trading costs has been studied in a competitive framework with exogenous costs. Amihud and Mendelson (1986, 1988) show that the price of an asset with fixed costs is the present value of future dividends, reduced by the present value of all future trading costs.\(^1\) In empirical work the bid-ask spread is often taken as a proxy for this fixed cost.\(^2\) We show that if (part of) the bid-ask spread is generated by adverse selection, then reducing the price by the (entire) present value of all future bid-ask spreads is not correct.

Our results are most easily explained in the context of our basic model. This model considers one asset and \(n\) risk-neutral agents. Each period, the owner receives a dividend and then a private signal about the next dividend and another private signal about whether or not he suffers a shock that conveys a motive to sell. If the owner suffers such a shock, then he has a cost of holding the asset. We interpret the shock as a need for cash, a state of financial distress, or as a reduced need for the asset. After receiving these private signals, the owner decides whether to keep the asset or to sell it to the other investors.

The costs of adverse selection are as follows. First, there is the well-known lemons cost: If the owner sells, it is a bad signal to other market participants, because they know he may have adverse information, which leads to a discounted price. In an extension of the model, in which trades are both buyer- and seller-initiated, this effect generates a bid-ask spread. In the richer model, a buyer-initiated trade is good news to the market, and hence it is associated with a peach premium. The bid-ask spread is the sum of the lemons cost and the peach premium.

Our first key result is that the lemons cost and the peach premium are not trading costs on average, and therefore have a net present value of zero. Hence, if (part of) the bid-ask spread is generated by adverse selection, one should not reduce the level of prices by the present value of (this part of) future bid-ask spreads.

To see the intuition for this result, suppose first that the owner sells due to a need for cash. The market does not know that. Rather, it assigns a

\(^{1}\) See also Constantinides (1986), Vayanos (1998), and Vayanos and Vila (1999).

positive probability to the event that the sale is motivated by bad news, and therefore the owner is paid too little. When the owner sells because of information, on the other hand, since the market allows for the possibility of a liquidity sale, the owner is paid too much. In equilibrium, these effects balance, and therefore adverse selection is not a trading cost on average: the price paid for the asset is, on average, equal to the value of the asset.

Our second key result is that adverse selection induces a cost associated with not trading. This cost is incurred when the owner needs cash, but has such good news about the dividend that he (rationally) chooses not to sell. We call this an allocation cost. The market microstructure literature has generally abstracted from this cost. The fact that adverse selection can lead to inefficient allocations has been recognized, however, in various contexts in the literature (Akerlof (1970), Leland and Pyle (1977), Myers and Majluf (1984), DeMarzo and Duffie (1999), DeMarzo (2000), Hendel and Lizzieri (1999), and Eisfeldt (1999)). Similarly, asymmetric information can lead to imperfect risk-sharing resulting in reduced prices (Diamond and Verrecchia (1991), Wang (1993), and Easley and O’Hara (2000)). In our model, the allocation cost is the real cost of adverse selection, and we study how it affects asset prices. We show that the price of an asset is reduced by the present value of all future allocation costs. In the case of both buyer- and seller-initiated trades, what affects prices is the difference between the allocation costs for buyers and sellers.

To summarize, adverse selection has two effects on prices: a lemons cost and the present value of all future allocation costs. The relative importance of these effects depends on the maturity of the assets. The price of a short-lived asset is affected only by the lemons cost, whereas the price of a long-lived asset is affected by the future lifetime stream of allocation costs. We study the term structure of adverse-selection discounts in different economic contexts.

We consider the following extensions of our basic model: (i) a model with both buyer- and seller-initiated trades (as mentioned previously), (ii) a model in which all agents are privately informed about the asset and face the risk of a shock, (iii) a model with both fixed and informational costs, and (iv) a model with risk-averse agents. In these extensions, our main results hold: adverse selection is not a trading cost on average, but can cause allocation costs.

In our extension in which all agents are privately informed, the choice of trading mechanism is not trivial because of the double-sided asymmetric
information. Hence, one might worry that our results depend on the choice of trading mechanism. The revenue equivalence result of Gârleanu and Pedersen (1999) shows, however, that the owner’s decision to sell and the expected prices are the same for a large class of trading mechanisms.

Endowing all agents in the economy with private information further allows us to study the macro-economic effects of adverse selection. In particular, we consider how the incentives to trade on information depend on agents’ (common) risk of being in financial distress. An increase in the risk of distress diminishes the adverse selection problem because, when there is a sale, buyers ascribe a higher likelihood that this sale is due to a need for cash. When the distress risk is relatively low, an increase in this risk will lead to more informed trading because of the diminished adverse-selection problem. Hence, in times of financial distress even agents not affected are likely to sell. In a similar vein, Kyle (1985) shows that the informed agent trades more when noise trading increases.

In our economy with double-sided asymmetric information there are two (additional) costs of selling: 1) the buyers’ private information leads to imperfect competition, and 2) there is a risk of market-wide financial distress. When the risk of distress is large, the market becomes relatively thin, and these costs increase (more than the adverse selection is diminished) resulting in curbed informed trading. Hence, we find a non-monotonic relationship between the level of informed trading and the level of distress-related trading, which is due to the general-equilibrium link between the level of liquidity trading and the overall market conditions. We further show that even risk-neutral agents act as if they are risk-averse with respect to changes in the risk of distress.

Asymmetric information in competitive markets has been studied by Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985), Wang (1993), Wang (1994), and others in the rational expectations equilibrium (REE) setting. The latter two papers are especially relevant in that they consider a dynamic economy, but our focus is different. Specifically, we isolate the components of future trading costs associated with asymmetric information, and characterize the present

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3Shleifer and Vishny (1992) discuss the difficulties faced by a company in financial distress when selling assets, due to the fact that other companies in the same industry are likely to be distressed at the same time. Pulvino (1998) finds evidence for this effect for aircraft transactions. These papers do not, however, consider the implications for information-based trading.
value of these costs, comparing them across different settings. Moreover, in the REE literature, the private information structure is exogenous and the only additional learning is through prices. In our model, buying agents strategically consider the impact of becoming an owner on their private information, while non-owners draw inference (as in Kyle’s model) from selling decisions.

Several papers consider empirically how measures of liquidity affect asset prices. Amihud and Mendelson (1986, 1989) find the required rate of return on NYSE stocks to be increasing in the relative bid-ask spread. This result is questioned for NYSE stocks by Eleswarapu and Reinganum (1993), Chen and Kan (1996), and Chalmers and Kadlec (1998), but confirmed for Nasdaq stocks by Eleswarapu (1997). The required rate of return has been found to be decreasing in measures of depth (Brennan and Subrahmanyam (1996) and Amihud (2000)), and increasing in a measure of the probability of information-based trading (Easley, Hvidkjær, and O’Hara (2000)).

The rest of the paper is organized as follows. Section 2 lays out the basic model and our results concerning adverse selection, Section 3 develops our extensions, and Section 4 concludes and discusses directions for future research.

2 Basic Model

In this section we construct a simple model in which a finite set, \( N = \{0, 1, \ldots, n\} \), of identical risk-neutral agents trade one object in each of periods \( 0, 1, \ldots, T - 1 \leq \infty \). Trades are seller-initiated. Because of the assumptions of a single good and seller-initiated trades, we think of the asset as a durable good. Later, we consider a model which can be more readily thought of as a model a financial assets, in that there are multiple assets and both buyer- and seller-initiated trades.

The timing of events in any period, \( t \), is shown in Figure 1. First, the current owner of the asset, the agent denoted by \( o(t) \), receives a dividend. Then, he receives two independently distributed private signals,\(^4\) \( x_t \in \mathbb{R} \) and \( \sigma_t \in \{0, \mu\} \), where \( \mu > 0 \). We assume that \( x_t \) has mean 0, that \( \Pr(\sigma_t = \mu) = \gamma \) with \( 0 < \gamma < 1 \), and that \( (x_t, \sigma_t) \) is independent across time. The signal \( x_t \) gives the current owner private information about the dividend next period.

\(^4\)The signals \( x_t \) and \( \sigma_t \) are integrable random variables. All random variables are defined on a given probability space \( (\Omega, \mathcal{F}, \Pr) \).
Owner receives dividend

Owner decides whether to sell

Private signals received

Auction if sale

Figure 1: Time-line for period $t$.

At $t+1$. Specifically, the owner alone knows the value of the dividend next period, which is

$$\alpha + x_t.$$ 

The signal $\sigma_t$ designates whether or not the current owner needs cash. If the owner needs cash ($\sigma_t > 0$) then he has a holding cost of $\sigma_t$. Hence, the current owner’s utility from holding the asset this period is

$$\alpha + x_t - \sigma_t,$$

which is lower than the utility derived by the other agents if $\sigma_t > 0$. The parameters $\alpha$, $\mu$, and $\gamma$, as well as the distribution of $x$, are common knowledge.\footnote{We can extend the model to allow these market characteristics to change (randomly) over time. In each period, there would be a public signal about the current state of economy. See Gârleanu and Pedersen (1999) for a general model with this feature. Variations with time-dependent parameters are referred to later in this section.}

After receiving the signals, the owner decides whether to keep the asset or to sell it. If he sells, each of the other agents makes a bid, and the highest bidder receives the asset after paying his bid.\footnote{Clearly, the choice of trading mechanism is unimportant here since the buyers have no private information.} Ties are broken using a (symmetric) randomization device.
We have described the economy and agents’ possible actions. We now describe the trading game more formally and then derive a natural equilibrium.

A strategy for agent \( i \) is defined as a process \( A = (A_t)_{t=0}^{T-1} \), where \( A_t : \Omega \to \{\text{sell, keep}\} \cup \mathbb{R} \) is measurable with respect to the information, \( \mathcal{F}_t \), available to player \( i \) at time \( t \). A strategy, \( A \), for agent \( i \) is said to be feasible if \( A_t \in \{\text{sell, keep}\} \) if and only if agent \( i \) is the owner at the beginning of period \( t \).

The utility agent \( i \) derives from playing the game from time \( t \) (after the dividend has been paid) onwards, given that agent \( j \) plays strategy \( A^j \) for all \( j \in \mathcal{N} \), is

\[
\Pi_i^t(A^0, \ldots, A^n) = E \left[ \sum_{s=t+1}^{T} \delta^{s-t} \left( (\alpha + x_s) \mathbb{1}_{(i=\sigma(s))} \right) - \sigma_s \mathbb{1}_{(i=\sigma(s)=\sigma(s-1))} \right] - \sum_{s=t}^{T-1} \delta^{s-t} z_s^i \bigg| \mathcal{F}_t^i, \tag{1}
\]

where \( z_s^i \) is the net cash payment (due to sales or purchases of the asset) made by agent \( i \) at time \( s \), and where the discount factor \( \delta > 0 \) captures the agents’ time preferences.

**Definition 1** An equilibrium is a set of feasible strategies, \( A = (A^0, A^1, \ldots, A^n) \), for the respective agents in \( \mathcal{N} \), such that, for all \( i \in \mathcal{N} \),

\[
E \left( \Pi_0^i(A) \right) \geq E \left( \Pi_0^i(A^0, \ldots, A^{i-1}, A', A^{i+1}, \ldots, A^n) \right), \tag{2}
\]

for all strategies \( A' \) feasible for agent \( i \).

We are concerned in this paper only with symmetric Markov equilibria, that is, equilibria in which any agent’s strategy at time \( t \) is a function of whether

\[\mathcal{F}_t^i \text{ is the } \sigma\text{-algebra generated by } (o_0, x_0, \sigma_0, \beta_0, \ldots, o_t, x_{t-1}, \sigma_{t-1}, \beta_{t-1}, \alpha_t, x_t \mathbb{1}_{(\sigma(t)=i)}), \]

where \( \beta_s \) contains the actions at time \( s \), that is, whether or not there is a sale, and in case of sale, the bids.

\[z_s^i \text{ is defined as follows. If there is a sale at time } s \text{ and agent } j \text{ buys the asset for a price of } P_s, \text{ then } z_s^j = P_s \text{ if } i = j, z_s^j = -P_s \text{ if } i = \sigma(s), \text{ and } z_s^j = 0 \text{ otherwise. If there is no sale, } z_s^i = 0 \text{ for all } i. \]
or not he is an owner, and of \((x_t, \sigma_t)\) (if he is an owner). The agents’ optimal strategies are characterized using dynamic programming. To do that we define continuation-value functions. The continuation value at time \(t\), after the dividend is paid and before information is received, is denoted by \(S_t\) for the owner, and by \(B_t\) for the non-owners, that is,

\[
S_t = E \left( \Pi_t^i (A) \mid o(t) = i \right) \quad \text{and} \quad B_t = E \left( \Pi_t^i (A) \mid o(t) \neq i \right).
\]  

Consider first the strategies of the non-owners, given the owner’s strategy. When there is a sale, all buyers bid their identical reservation value (Bertrand competition). The reservation value is the expected next dividend, plus the value of being an owner next period, reduced by the (opportunity cost associated with the) value of being a non-owner next period. Hence, the price is

\[
P_t = \delta (\alpha + \hat{x} + S_{t+1} - B_{t+1}),
\]

where \(\hat{x} = E(x_t \mid \text{sale})\), using informal notation. (This definition of \(\hat{x}\) is made precise in Equation 7.)

Now, consider the owner’s decision of whether to keep the asset or to sell it. He sells if and only if this gives him a higher continuation value than that obtained by keeping the asset, that is, if

\[
P_t + \delta B_t \geq \delta (\alpha + x_t - \sigma_t + S_t),
\]

which is simplified using (5) to

\[
x_t - \sigma_t \leq \hat{x}.
\]

Hence, the owner sells if his news about the dividend is worse than a cut-off level, which depends on whether the owner needs cash.

Given these strategies, equilibrium is characterized by the condition that the buyers’ expectations about the dividend are consistent with the owner’s sale decision. That is, equilibrium is characterized by the condition that

\[
\hat{x} = E \left( x_t \mid x_t - \sigma_t \leq \hat{x} \right).
\]

If there is no solution to this equation, then there is no equilibrium with trade. There always exists an equilibrium in which the owner never sells, and in case of an (off-the-equilibrium-path) sale, the buyers bid minus infinity.
When $\mu$ is small, there exists no equilibrium with trade. (Similarly, Akerlof (1970) shows that adverse selection can lead to a market breakdown.) When $\mu$ is large, however, there does exist an equilibrium with trade as is seen in Corollary 2 below. We formulate our existence result in more general terms than what is needed here in order to cover more general cases considered later in this paper. To prove this existence result, we use the following lemma.

**Lemma 1** For any integrable and absolutely continuous distribution of $x_t$, and for any distribution of $\sigma_t$, there is a solution to (7) if and only if there is a number $y \in \mathbb{R}$ such that

$$y \leq E\left(x_t \mid x_t - \sigma_t \leq y\right).$$

(8)

**Corollary 2** (i) If $x_t$ is integrable and absolutely continuous, then for any $\varepsilon > 0$, there exists a number, $\mu^* \in \mathbb{R}$, such that (7) has a solution for any distribution of $\sigma_t$ with $Pr(\sigma_t \geq \mu^*) \geq \varepsilon$. (ii) If $x_t$ is integrable and absolutely continuous with support bounded below, and if $Pr(\sigma_t > 0) > 0$, then (7) has a solution.

The following proposition summarizes the structure of equilibria with trade, and characterizes the value functions.

**Proposition 3** Suppose $\hat{x}$ is a solution to (7). Then the following strategies constitute an equilibrium. The owner sells when $x_t - \sigma_t \leq \hat{x}$ and in that case all buyers bid the equilibrium price, which is given by (5). The value functions are given by

$$S_t = \sum_{s=t+1}^{T} \delta^{s-t} (\alpha - \hat{c})$$

(9)

$$B_t = 0,$$  

(10)

where $\hat{c} = \gamma Pr(x > \hat{x} + \mu) \mu$ is the allocation cost.

It should be no surprise that there is no value associated with not owning the asset in this model ($B_t = 0$). This is because non-owners have no private information and therefore cannot extract rents. In some of the extensions of the model in Section 3, however, there is a value to being a non-owner.

The value of being an owner is the present value of future dividends, reduced by the present value of future allocation costs. The allocation cost
is due to the possible misallocation of the asset — namely, it may be held by an agent with a negative private value instead of an agent with a zero private value. This happens when the owner has a need for cash and at the same time such good news about the next dividend that he chooses not to sell. The probability of this event is $\gamma Pr(x > \hat{x} + \mu)$. When this event occurs, the owner bears a holding cost of $\mu$.

Knowing the value functions, we can compute the equilibrium price explicitly as

$$P_t = \left( \sum_{s=t+1}^{T} \delta^s \alpha \right) + \delta \hat{x} - \left( \sum_{s=t+2}^{T} \delta^s \hat{c} \right).$$

(11)

The first term, $\sum_{s=t+1}^{T} \delta^s \alpha$, is the average price in a world in which $x_t$ and $\sigma_t$ are common knowledge. The other two terms are reductions in price due to adverse selection.

First, a sale signals to the market that the owner has bad news about the dividend, leading to a lemons discount, $\hat{x} < 0$, to the price. This, however, is not a trading cost to the owner, on average. Essentially, the owner is paid the value of a lemon, but this is not a cost, since indeed he is selling a lemon. Conditional on selling because of a need for cash ($\sigma > 0$), the owner is paid too little, but conditional on selling for purely information reasons ($\sigma = 0$), he is paid too much. In equilibrium these effects balance. Therefore, future lemons discounts do not affect the current price directly.

Second, the price is reduced because of the allocation costs. The allocation cost is associated with a real reduction in the value of holding the asset (and leads to a welfare loss). Therefore, the price is reduced by the present value of all future allocation costs.

We note that the relative importance to the price of the lemons cost and of the allocation cost, respectively, depends on the maturity of the asset. In particular, the price of a short-lived asset ($T = 1$) is affected only by the lemons cost, while the price of an infinitely-lived asset ($T = \infty$) is affected by a perpetuity of allocation costs. It may be misleading, though, to draw general inferences about the term structure of the adverse-selection discounts based on our basic model. Certain fundamental features of the basic model may need to be modified to fit the economic situation at hand, which may significantly change the way in which the adverse-selection discounts depend on the asset’s maturity. If the sale-inducing shock, for instance, does not represent a reduced need for the asset (in which case the the shock size can
Figure 2: This figure illustrates the lemons discount (dashed line) and the allocation cost (continuous line), normalized by the full-information price, in the following four scenarios: (i) constant shock size ($\mu = 1$) and i.i.d. dividends, which are uniformly distributed on $[-1, 1]$ (top left); (ii) shock size proportional to the full-information price ($\mu_t = 0.05 \sum_{s=t+1}^{T} \delta^s \alpha$) and i.i.d. dividends (top right); (iii) variable shock size and dividends follow a random-walk process, with increments distributed uniformly on the interval $[-1, 1]$ (bottom left); (iv) constant shock size and dividends follow a random-walk process. The other parameters used are $\alpha = 5$, $\delta = 0.98$, and $\gamma = 0.1$. 
be taken constant), but rather a need for cash, then the size of the shock should increase with the price of the asset. In this case, when the asset is far from the maturity date, the size of the sale-inducing shock is large relative to the innovation $x_t$ in the dividend. Therefore, the owner rarely keeps the asset when in liquidity need, and hence the allocation cost is small. Consequently, the allocation cost eventually decreases with maturity. For a different example, suppose that the dividends, instead of being i.i.d., form a random walk.\footnote{A random walk is a process with i.i.d. increments. Any autoregressive process can be accommodated identically.} Then, the owner’s private information is relevant for all future dividends,\footnote{We note that even if the owner’s private information concerns a persistent feature of the asset, this does not complicate the derivation of the equilibrium much, as long as the information is still short lived. By short-lived information we mean that what the owner knows privately today, is common knowledge next period. For instance, information is short lived if the owner has private information about a persistent dividend process, and past dividends are observed by everyone.} and hence the (absolute) lemons problem is higher for long-lived assets. We illustrate these points with Figure 2, where we show the term structures of the two types of discounts when allowing all four possible combinations that arise from two choices of shock sizes (constant, as in the basic model, and proportional to the full-information value of the asset) and two choices for the dividend-generating process (i.i.d., as in the basic model, and a random walk, with all agents informed of the realization of the dividend, that is, of the mean of the next dividend). The model can generate a variety of shapes of term structures of adverse selection discounts, which makes it difficult to derive testable implications. The model can also, however, identify the economic circumstances associated with each shape to the term structure.

3 Extensions

In this section we extend the basic model in various directions and show that our main conclusions apply under a wide range of circumstances.

3.1 Buyer- and seller-initiated trades

This section develops a simple model with both buyer- and seller-initiated trades.
We assume that there are $K$ assets and $N$ agents, with $K > 2$ and $N - K > 2$. Each agent may own one security or no security. Hence, we abstract from the investors’ quantity decisions (as do others, such as Glosten and Milgrom (1985)). While this is a major restriction, we think that our intuition applies more generally.

At period, $t$, one randomly chosen agent, $i$, receives private information, $x_t$ and $\sigma_t$. (Agent $i$ can be either an owner or a non-owner.) As before, we assume that the dividend in the next period is $\alpha + x_t$, with $E(x_t) = 0$, and that agent $i$’s value of holding the asset this period is $\alpha + x_t - \sigma_t$. Here, we allow a slightly richer distribution for $\sigma_t$. We assume that $\sigma_t$ takes the values $\mu$, 0, and $-\mu^- < 0$ with probabilities $\gamma$, $1 - \gamma - \gamma^-$, and $\gamma^-$, respectively. We interpret $\sigma_t = \mu$ as a need for cash, a high financing cost, or as a reduced need for the asset, and $\sigma_t = -\mu^-$ as a state of excess cash, a low financing cost, or as an extraordinary need for the asset.

After agent $i$ has received his private information, there is trade. The trading mechanism is designed to resemble, stylistically, the opening of the New York Stock Exchange.11 Every agent can submit a limit order or a market order to buy or sell one share. A limit order specifies a price at which the agent is willing to buy or sell one share (this period). A market order is interpreted as a limit order with a price of plus or minus infinity. Orders are executed as follows. First, the “specialist” determines the set of prices at which supply equals demand, or at which any excess supply or demand is due to orders at this price. It is easy to see that this set is an interval. The mid-point of this interval is denoted the clearing price.12 Then, all trades are executed at the clearing price. If there is an excess supply or demand at the clearing price, then a randomization scheme determines which orders are executed.13

The following strategies can be shown to constitute an equilibrium using arguments analogous to those in Section 2. Suppose that the informed agent,

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11Since only one agent is privately informed each period, the trading mechanism is not crucial for our results.
12This interval is bounded as long as there is at least one limit order on both the buy side and the sell side. If there are no limit orders on one side of the market, then this interval is a half line. In that case, we take the clearing price to be the end point of the half line. If there are no limit orders at all, then the clearing price is set at some pre-specified value.
13This can be interpreted as follows. Orders arrive in a random order, and priority is given based on the time of arrival.

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i, is an owner. Then he submits a market order to sell if \( x_t - \sigma_t \leq \hat{x} \), where \( \hat{x} \) is given by (7), and no order if \( x_t - \sigma_t > \hat{x} \). Similarly, if i is a non-owner then he submits a market order to buy if \( x_t - \sigma_t \geq \tilde{x} \), where \( \tilde{x} \) is given by

\[
\tilde{x} = E \left( x_t \mid x_t - \sigma_t \geq \tilde{x} \right),
\]

and otherwise he does not submit an order.

All uninformed owners submit the same limit order to sell, which we will call the ask price. The uninformed buyers submit the same limit order to buy, the bid price. The equilibrium bid and ask prices are

\[
\text{bid} = \left( \sum_{s=1}^{T} \delta^s \alpha \right) + \delta \hat{x} - \left( \sum_{s=2}^{T} \delta^s (\hat{c} - \tilde{c}) \right),
\]

\[
\text{ask} = \left( \sum_{s=1}^{T} \delta^s \alpha \right) + \delta \tilde{x} - \left( \sum_{s=2}^{T} \delta^s (\hat{c} - \tilde{c}) \right),
\]

where \( \hat{c} \) and \( \tilde{c} \) are the sell-side and buy-side allocation costs, respectively. That is,

\[
\hat{c} = \frac{1}{N} E(\sigma_t 1_{(x_t - \sigma_t > \hat{x})})
\]

\[
\tilde{c} = \frac{1}{N} E(\sigma_t 1_{(x_t - \sigma_t > \tilde{x})}).
\]

The first term, \( \sum_{s=1}^{T} \delta^s \alpha \), of the bid and ask prices is the average price if there were no private information. (We note that this is not the value of the asset when allocated efficiently since agents can have positive private values \( \sigma_t = -\mu^- \) of owning the asset. Since there is only one agent with this private benefit and there are several objects, this benefit does not, however, affect competitive prices.) The bid price is reduced by the lemons cost, \( \hat{x} < 0 \). The ask price is inflated by the peach premium, \( \tilde{x} > 0 \). The levels of both bid and ask prices are reduced by the present value of the difference between future sell-side and buy-side allocation costs.

The bid-ask spread is \( \delta (\tilde{x} - \hat{x}) > 0 \). In equilibrium, the bid-ask spread is not a trading cost on average. That is, a seller has a lemons cost, but indeed he is selling a lemon; a buyer is paying a peach premium, but indeed he is getting a peach.

The case of symmetry is an interesting benchmark:
Proposition 4 Suppose $x_t$ and $\sigma_t$ are symmetrically\textsuperscript{14} distributed around 0 and that one of the statements (i) and (ii) in Corollary 2 applies. Then, there exists an equilibrium with $\hat{c} = \tilde{c}$ and $-\hat{x} = \tilde{x}$.

This proposition implies that if $x_t$ and $\sigma_t$ are symmetrically distributed around 0, then the mid-price is the same as the price with no asymmetric information, and at the same time there is a strictly positive bid-ask spread. As discussed in the introduction, this result should be seen in contrast to the literature on exogenous transactions costs,\textsuperscript{15} which finds that the price of an asset should be reduced by the present value of all future trading costs, or, equivalently, the required rate of return should be increased by the amortized transactions costs. For stocks on NYSE the average bid-ask spread is approximately 3% and the average turnover is approximately 100%. Hence, this literature would suggest that the average required rate of return should be increased by about 3% because of trading costs, which clearly has a large impact on the level of prices. We find, on the other hand, that if bid-ask spreads are generated (partly) by adverse selection, then the price impact could be much smaller than that. Hence, our result might help explain the rather modest price impact associated with the about 40% reduction in bid-ask spreads that occurs when stock change exchange listing from Nasdaq to NYSE. See Elyasiani, Hauser, and Lauterbach (2000) and references therein.

3.2 All agents privately informed

Several interesting implications follow when one allows all agents to receive private signals, both about the dividend and about their private (liquidity) needs. We first describe the extension of the basic model and then discuss the results.

As in the basic model, the owner, $o(t)$, receives private signals, as do the other agents in this extension. Specifically, any agent, $i$, receives the signals $x_i^t$ and $\sigma_i^t$, which are mutually independent and i.i.d. across agents and time. We assume that $x_i^t$ has zero mean and a continuous, strictly increasing cumulative distribution function $F$ with a support which is a closed interval $\mathbb{X} = [\underline{x}, \overline{x}]$.

\textsuperscript{14}We say that a random variable, say $x_t$, is symmetrically distributed around 0 if $x_t$ and $-x_t$ have the distribution.

The value of the dividend to agent $i$, net of possible holding costs, is

$$ v^i_{t+1} = \alpha + x^{o(t)}_t + \beta \left( \sum_{j \neq o(t)} x^j_t \right) - \sigma^i_t. $$

Here, the parameter $\beta$ measures the importance of the non-owners’ signals, which could be different from the importance of the owner’s signal.

The structure of the game is the same as in the basic model, in that the timeline of Figure 1 still applies. Since buyers are privately informed, competition is imperfect and the choice of trading mechanism may affect prices and allocations. Our results are robust to the choice of trading mechanism. Using the Multi-period Revenue Equivalence Theorem of Gärleanu and Pedersen (1999), we show that the owner’s decision to sell, expected prices (which we denote by $P_t$), and value functions, $S_t$ and $B_t$, are the same for a large class of trading mechanisms. We refer to Gärleanu and Pedersen (1999) for a general definition of an auction mechanism and the associated repeated trading game. The class of mechanisms that we consider is characterized by the following condition.

**Condition 1** If there is a sale, and there exists $i \neq o(t)$ such that $\beta x^i_t - \sigma^i_t > \beta x^j_t - \sigma^j_t$ for all $j \neq i, o(t)$, then agent $i$ is allocated the object. A bidder with $(x^i_t, \sigma^i_t) = (\chi, \mu)$ has an (expected) payment of zero.

This condition states that the agents’ bids are increasing in $\beta x^i_t - \sigma^i_t$, and that an agent with the worst possible private signal does not pay anything (nor will he receive anything). Many standard auctions, such as the first-price auction, the second-price auction, and the ascending auction, satisfy Condition 1. In addition, our results apply for mechanisms in which a risk-neutral intermediary buys the object from the owner and sells it to the other agents in an auction that satisfies Condition 1.

The equilibria in this class of economies with double-sided asymmetric information are characterized by the following proposition.

**Proposition 5** There exists$^{16} \mu \in \mathbb{R}$ such that for $\mu > \mu$, for any mechanism

---

$^{16}$Here, $\mu$ depends on the model parameters. The reason that we need a large $\mu$ is as follows. The standard revenue equivalence results apply only when the support of the private signals is a connected set, which is not the case here because of the discreteness of $\sigma^i_t$. Revenue equivalence does apply, however, when a distressed buyer always bids less than an undistressed one, which is the case when $\mu$ is large. See Gärleanu and Pedersen (1999).
and equilibrium that satisfy Condition 1, the owner sells if $x_i^{o(t)} - \sigma_i^{o(t)} \leq \hat{x} - \beta \eta^1 - \mu \eta^2$, where $\eta^1 > 0$ and $\eta^2 > \gamma$ are functions of $\gamma$, $n$, and the distribution of $x_t$, and $\hat{x}$ solves

$$
\hat{x} = E \left( x_i^i \mid x_i^i - \sigma_i^i \leq \hat{x} - \beta \eta^1 - \mu \eta^2 \right).
$$

(13)

The expected price and value functions are

$$
P_t = \delta (\alpha + \hat{x} - \beta \eta^1 - \mu \eta^2 + S_{t+1} - B_{t+1})
$$

(14)

$$
S_{t+1} = \sum_{s=t+1}^{T-1} \delta^{s-t} \left[ \alpha - \hat{c} - \psi (\beta \eta^1 + \mu \eta^2) \right]
$$

(15)

$$
B_{t+1} = \sum_{s=t+1}^{T-1} \delta^{s-t} n \left( \beta \eta^1 + \mu (\eta^2 - \gamma) \right)
$$

(16)

where

$$
\hat{c} = \gamma (1 - F(\hat{x} - \beta \eta^1 - \mu \eta^2 + \mu)) \mu
$$

(17)

$$
\psi = (1 - \gamma) F(\hat{x} - \beta \eta^1 - \mu \eta^2) + \gamma F(\hat{x} - \beta \eta^1 - \mu \eta^2 + \mu).
$$

(18)

Here, $\hat{c}$ is the allocation cost and $\psi$ is the probability of a sale. The results of this proposition are natural and have several noteworthy features. First, the main conclusion of the basic model continues to hold: (i) Adverse selection creates a lemons discount, $\hat{x}$, but this is not a trading cost on average and it does not affect the value functions, and (ii) adverse selection (together with the other frictions) leads to an allocation cost, $\hat{c}$, which builds up in the value, $S$, of owning, and hence in prices. The former result relies to some extent on the independence of the owner’s signal and its additive impact on dividend value.

Further, in this economy there are other frictions that influence prices and allocations. The buyers’ private information leads to imperfect competition. Each buyer extracts an average rent, $\beta \eta^1 / n$, associated with his private information about the dividend, which translates into a cost for the seller of $\beta \eta^1$. Similarly, each buyer extracts an expected rent $\mu (\eta^2 - \gamma) / n$ because of his private information about his need for cash. All buyers are in financial distress with probability $\gamma$, and in this case the price is depressed by $\mu$. The total cost to the seller associated with market-wide financial distress and distress-related rents is $\mu \eta^2$.

These considerations explain why the expected price, given in (14), is reduced by the cost of selling $\beta \eta^1 + \mu \eta^2$. Further, the price is reduced due to
Figure 3: The probability that an undistressed owner sells for varying levels of market liquidity, $\gamma$. Here, $x_t$ is uniformly distributed on $[-1, 1]$, $\beta = 0.5$, $\mu = 10$, $\alpha = 10$, $\gamma = 0.1$, $n = 8$, and $\delta = 0.99$.

Figure 4: Prices of short-lived ($T = 1$) and long-lived ($T = \infty$) assets, normalized by the full-information value, $F_0 = \sum_{s=1}^{T} \delta^s \alpha$. The parameters are as in Figure 3.

all future trading costs associated with imperfect competition and market-wide distress. This price reduction happens via two channels: (i) The value of owning is reduced by the future costs of selling (incurred by any future owner — not just the current one), and (ii) the value of not owning, which stems from extracting rents when there is a sale, increases in the degree of imperfect competition.

It is interesting to consider how the common level of distress risk affects adverse selection and the incentives to trade on information. The distress risk, $\gamma$, can be regarded as a measure of the overall market liquidity. We next provide a comparative-statics analysis of $\gamma$ in the context of a numerical example. Figure 3 shows, for a range of values of $\gamma$, the tendency to trade on information. More precisely, it shows the probability that an undistressed owner sells. (For the parameters used, the owner always sells when distressed.) We see that for sufficiently low levels of risk, an increase in $\gamma$ induces the owner to trade more often, taking advantage of the higher chance that the rest of the market will perceive it as a liquidity trade. This result is striking: In times of financial distress in the market (higher $\gamma$), even an agent not affected directly is more likely to sell. This result is similar to that of Kyle (1985), who finds that information-based trading is increasing in the level of noise trading. In our model, however, high values of $\gamma$ lead to sharply increasing trading costs, which curb information-based trade until eliminating it altogether. Intuitively, if liquidity trading is generated by
financial distress, then a time of extreme liquidity trading is also a time at which the market is thin and (undistressed) counterparties are hard to find, which makes informed trading unattractive. Hence, we find that, in a general equilibrium, information-based trading is non-monotonic in the degree of distress-related trading.

These changes in $\gamma$ and the level of informed trading have effects on prices. Figure 4 shows that for short-lived assets the price is non-monotonic in $\gamma$. An increase in $\gamma$ has several effects, pulling the price in opposite directions. First, it decreases the expected number of bidders who compete for the object, thus making competition less fierce and magnifying the risk of market-wide distress. Second, it increases the frequency of (liquidity-related) trade. These effects depress the price. Third, it diminishes the degree of adverse selection, since there are more forced sales. This leads to an increased price. For short-lived assets the latter effect dominates for small values of $\gamma$. The former effects dominate for larger values of $\gamma$. For long-lived assets, the price is decreasing in $\gamma$. This is due to the importance of future trading costs and the relatively smaller importance of adverse selection (in this example). See Section 2 for a discussion of the caveats in connection with comparing effects for assets of different maturity.

The value of owning the asset is concave in $\gamma$. (Compare in Figure 4.) This means that our risk-neutral agents act as if they are risk-averse towards changes in $\gamma$. To illustrate this point, we assume that $\gamma_0 = 0.3$ and that $\gamma_t$ has outcomes $\gamma_0 \pm \Delta \gamma$, with equal probability, for all $t > 0$. Figure 5 shows that the price is decreasing in the standard deviation, $\Delta \gamma$, of $\gamma$. This result is natural: A highly volatile “market liquidity” ($\gamma$) makes it likely that one is forced to sell at the worst possible times, namely, when there are very few bidders with no shock. We note that this result would be strengthened if we allowed multiple assets. In that case, an owner would be most likely to sell at times with many sellers (and few undistressed buyers).

### 3.3 Fixed costs

In this section, we study how the presence of fixed trading costs affects adverse selection.

Suppose that the owner must pay a fixed transaction cost, $c$, when he sells. This diminishes the value received upon selling, which changes the
Figure 5: The (normalized) price of a long-lived asset as a function of the variability of the distress risk, $\gamma$. Here, $x_t$ is uniformly distributed on $[-1, 1]$, $\beta = 0.5$, $\mu = 10$, $\alpha = 10$, $n = 8$, $\delta = 0.99$, $\gamma_0 = 0.3$, and $\gamma_t$ has outcomes $\gamma_0 \pm \Delta \gamma$, with equal probability, for all $t > 0$.

equation defining the equilibrium to:

$$\hat{x} = E \left( x_t \mid x_t - \sigma_t \leq \hat{x} - c \right) .$$

Corollary 2 gives conditions for existence of equilibrium (by letting $\sigma_t - c$ play the role of $\sigma_t$), and Equation 5, Section 2, for the price still applies. The value function for owning the asset is now:

$$S_t = \sum_{s=t+1}^{T} \delta^{s-t} \left( \alpha - \hat{c} - \psi c \right),$$

(19)

where $\psi$ is the probability of a sale:

$$\psi = \gamma \Pr(x_t < \hat{x} + \mu - c) + (1 - \gamma) \Pr(x_t < \hat{x} - c).$$

Naturally, as in the basic model, there is no value in being a non-owner, that is, $B_t = 0$.

The expression (19) for the value of owning shows that our results concerning adverse selection from the basic model are unchanged. The additional effect of the fixed cost is as shown by Amihud and Mendelson (1986): The price is reduced by the present value of all future fixed costs incurred.

We note that the sale-set, whence the volume of trade, shrinks as $c$ increases, consistent with the findings of Constantinides (1986) and Vayanos (1998). Changes in the fixed transaction cost also change the severity of the adverse selection problem. First, higher fixed costs lead to larger allocation inefficiencies. Second, higher fixed costs change the average quality of the
sold asset, that is, change the lemons cost, $\hat{x}$. The lemons cost may either increase or decrease. If the lemons cost decreases, the result of the increased transaction cost is an increase in the price (paid by the buyer) of a short-lived asset, and a simultaneous decrease in the prices of long-lived assets.

One could also consider the effect of fixed transactions costs in the context of the model Section 3.1 with both buyer- and seller-initiated trades. In this case, fixed transaction costs reduces both the value of owning the asset and the value of not owning it, and there exists conditions under which future fixed transaction costs are not priced (although they always have welfare implications).

### 3.4 Risk Aversion

In this section we show that our main results apply in a particular setting with risk aversion. While the setting is special, it illustrates that our results do not hold only in the case of risk neutrality, and helps demonstrate what is needed for our results.

We assume that agents are risk-averse with respect to the service flows of the asset net of possible lost service flows (holding costs), and that their common von Neumann-Morgenstern utility function is $u(\cdot)$. Further, we assume that agents are risk-neutral with respect to payments associated with trading the asset. The interpretation of these preferences is that there are two goods in the economy: a house and apples (the numeraire). If one owns the house then one must consume its random service flows immediately. Hence, agents are risk-averse with respect to these service flows. There is a perfect market for apples, and since prices are deterministic, the agents face no risk with respect to their apple consumption. Similarly, Grossman and Laroque (1990) study an economy with assets that yield service flows that must be consumed immediately. These preferences are captured by the following utility function (which replaces Equation 1):

$$
\Pi_t^i = E \left[ \sum_{s=t+1}^{T} \delta^{s-t} u \left( (\alpha + x_s) 1_{(i=\alpha(s))} - \sigma_s 1_{(i=\alpha(s)=\alpha(s-1))} \right) \right] - \sum_{s=t}^{T-1} \delta^{s-t} z^i_s \right| \mathcal{F}_t^i ,
$$

The equilibrium of the economy with these preferences is derived similarly to the equilibrium in the basic model. If the owner sells, then all buyers bid
their reservation value,

\[ P_t = \delta (\hat{u} + S_{t+1}) , \]

where

\[ \hat{u} = E(u(\alpha + x_t) \mid \text{sale}). \]

Since the buyers are risk-averse, the owner is not paid the expected dividend, but the certainty equivalent of the dividend.

With this price, the owner chooses to sell if \( u(\alpha + x_t - \sigma_t) \leq \hat{u} \). Hence, the equilibrium condition is

\[ \hat{u} = E \left( u(\alpha + x_t) \mid u(\alpha + x_t - \sigma_t) \leq \hat{u} \right). \]

The value of owning is computed to be

\[ S_{t+1} = \sum_{s=t+2}^{T} \delta^{s-(t+1)} \left[ E(u(\alpha + x_t)) - E \left( 1_{\sigma_s = \mu, u(\alpha+x_t-\mu) > \hat{u}} \left( u(\alpha + x_s) - u(\alpha + x_s - \mu) \right) \right) \right]. \]

The value of owning is the expected utility of future dividends, reduced by the future allocation costs, namely the loss in utility when an owner in need for cash chooses not to sell. As in the basic model adverse selection is not a trading cost on average in equilibrium: The owner is just as risk-averse as the buyers, and therefore he is happy to pay for the insurance provided by the fact that the price does not depend on his information about the dividend. Hence, it is not a cost to the owner that he is paid the certainty equivalent of the dividend (instead of the conditional expected value of the dividend).

\section{Conclusion}

In this paper, we study the extent to which adverse selection is priced and the way it affects the efficiency of allocations. Our two main results are: (i) adverse selection leads to a lemons discount, but this discount is not a trading cost on average, and therefore future lemons discounts do not affect current prices directly; and (ii) adverse selection leads to allocation inefficiencies,
which depress the price by the present value of all future allocation costs. The relative importance of these effects depends on the maturity of the asset.

We also study adverse selection in the presence of other market frictions, namely imperfect competition, market-wide financial distress, and fixed transaction costs. We demonstrate that our results concerning adverse selection are robust to the presence of these other frictions, show how the other frictions affect the severity of the adverse-selection problem, and determine how the additional frictions are priced.

Our model is, however, limited. First, we assume that agents are (ex-ante) identical. This is crucial to the result that adverse selection is not a trading cost on average. While this is an interesting benchmark, which is a good description of (the big players) in some markets, it would also be of interest to consider a model with heterogeneous agents. First, consider an extension of the basic model in which there are different types of agents that differ in terms of their likelihood of financial distress, the quality of the information they learn if they own the asset, their discount rate, or their risk aversion. Suppose that an agent’s type is private information and that trade is anonymous. In such an economy, an agent of type $\tau$, say, has a value of owning of $S^\tau$. In case of a sale, a reasonable mechanism would assign the asset to an agent of the type that has the highest value of $S^\tau$. Hence, in some generic sense, only one type of agent is active in equilibrium, and we are essentially back to the basic model. This extension does show, however, who should own a very illiquid asset: agents who are likely to be informed about the asset, and who have deep pockets. To obtain an equilibrium in which multiple types of agents are active, one could consider a case in which buyers are also informed, allowing a buyer with a lower value $S^\tau$ to end up buying in the event that this agent has favorable information.

Another limitation of our model is that the agents are restricted to own zero or one unit of the asset. While it seems reasonable that agents can only take limited positions, it is restrictive to assume that agents have no quantity decision within their limits. Intuitively, our results seem to also apply when investors can trade different quantities, but it might be valuable to extend the model to encompass this possibility. This extension is especially promising if combined with a relaxation of the assumption of exogenous financial distress. In that case, one could imagine that a large security position would be associated with some combination of a high risk and a high severity of financial distress. Further, it would be interesting to consider a model with multiple classes of illiquid assets.
In our setting, the structure of asymmetric information is endogenous in that how much an agent learns privately may depend on his ownership status. The information structure can be further endogenized by assuming that agents must pay for information. In such a generalization, it would indeed be the case (in equilibrium) that the amount of information purchased would depend on ownership, and new implications may arise from studying this endogenous adverse selection problem.
A Appendix

Proof of Lemma 1:
Define \( f : \mathbb{R} \to \mathbb{R} \cup \{-\infty\} \) by
\[
    f(y) = E \left( x_t \mid x_t - \sigma_t \leq y \right),
\]
for \( y \) such that \( Pr(x_t - \sigma_t \leq y) > 0 \), and by \( f(y) = -\infty \) otherwise. First, we note that \( f \) is continuous on the set \{ \( y : f(y) > -\infty \) \}. This follows from the fact that absolute continuity of \( x_t \) implies that \( Pr(x_t - \sigma_t = y) = 0 \) for all \( y \), and by dominated convergence. Further, there exists a \( y \) such that \( y > f(y) \) since \( f(y) \to E(x_t) < \infty \) as \( y \to \infty \), and by (8) there exists a number, \( y \), such that \( y \leq f(y) \). Now, the result follows from the Mean Value Theorem.

□

Proof of Corollary 2:
We let \( f \) be the mapping defined by (A.1). By Lemma 1, all we need to do is to find a number, \( y \), that satisfies \( y \leq f(y) \).

(i) We assume without loss of generality that \( x_t \) has a mean of 0. Consider the following inequalities for any \( y \in \mathbb{R}, \mu \in \mathbb{R} \) with \( Pr(x_t < \mu + y) > 0 \), and any distribution of \( \sigma_t \) with \( Pr(\sigma_t \geq \mu) \geq \varepsilon \):
\[
    f(y) = \frac{E(x_t 1_{(x_t - \sigma_t < y)})}{Pr(x_t - \sigma_t < y, \sigma_t \geq \mu) + Pr(x_t - \sigma_t < y, \sigma_t < \mu)}
\]
\[
    \geq \frac{E(x_t 1_{(x_t - \sigma_t < y)})}{Pr(x_t - \sigma_t < y, \sigma_t \geq \mu)}
\]
\[
    \geq \frac{E(x_t 1_{(x_t - \sigma_t < y)})}{Pr(x_t < \mu + y) \varepsilon}
\]
\[
    = \frac{E(x_t 1_{(x_t - \mu < y)} 1_{(\sigma_t \geq \mu)}) + E(x_t 1_{(y - \sigma_t + \mu - x_t - \sigma_t < y)} 1_{(\sigma_t < \mu)})}{Pr(x_t < \mu + y) \varepsilon}
\]
\[
    + \frac{E(x_t 1_{(x_t - \sigma_t < y)}) 1_{(\sigma_t < \mu)}}{Pr(x_t < \mu + y) \varepsilon}
\]
\[
    \geq \frac{E(x_t 1_{(x_t - \mu < y)}) \varepsilon + \min(\mu + y, 0) - E(|x_t| 1_{(x_t < \mu + y)})}{Pr(x_t < \mu + y) \varepsilon}
\]

The latter expression depends on \( y \) and \( \mu \) only through \( \mu + y \). Hence, keeping \( \mu + y \) constant \( y \) can be decreased enough to obtain \( f(y) \geq y \), as desired.

(ii) Let \( x \) be the lower end of the support of \( x_t \). Then, since \( Pr(\sigma_t > 0) > 0, f(x) > -\infty \) is well defined, and clearly \( f(x) \geq x \).
Proof of Proposition 3:
We have already shown that the agents’ strategies form an equilibrium. The value functions remain to be derived. Since

\[
S_t = E \left( \delta [\alpha + x_{t+1} - \sigma_{t+1} + S_{t+1}] 1_{(i=o(t+1))} ight) \\
+ [P_t + \delta B_{t+1}] 1_{(i\neq o(t+1))} \mid i = o(t) \\
= \delta (\alpha - \hat{c} + S_{t+1})
\]

and

\[
B_t = E \left( [\delta (\alpha + x_{t+1} + S_{t+1}) - P_t] 1_{(i=o(t+1))} \\
+ \delta B_{t+1} 1_{(i\neq o(t+1))} \mid i \neq o(t) \right) \\
= \delta B_{t+1},
\]

we get (9)–(10) by recursion.

□

Proof of Proposition 4:
By Corollary 2 there exists a number, \( \hat{x} \), that solves (7). Let \( \tilde{x} = -\hat{x} \). Then,

\[
\tilde{x} = -E \left( x_t \mid x_t - \sigma_t \leq \hat{x} \right) \\
= E \left( -x_t \mid -x_t - (\sigma_t) > \tilde{x} \right) \\
= E \left( x_t \mid x_t - \sigma_t > \tilde{x} \right).
\]

Further,

\[
N \hat{c} = E(\sigma_t 1_{(x_t - \sigma_t > \tilde{x})}) \\
= -E(-\sigma_t 1_{(-x_t - (\sigma_t) < \tilde{x})}) \\
= -E(\sigma_t 1_{(x_t - \sigma_t < \tilde{x})}) \\
= E(\sigma_t) - E(\sigma_t 1_{(x_t - \sigma_t < \tilde{x})}) \\
= E(\sigma_t 1_{(x_t - \sigma_t \geq \tilde{x})}) \\
= N \hat{c}.
\]
Proof of Proposition 5:
First, we find the expected equilibrium price given any (anticipated) set, $X$, of signals on which the owner sells. To compute the expected equilibrium price, we find bidding strategies in a second-price auction. We work under the assumption that the second-price auction satisfies Condition 1, and provide a lower bound on $\mu$ that justifies this assumption. Then, we show that the expected equilibrium price and sale set are the same for all auction mechanisms that satisfy Condition 1. Finally, we find the equilibrium sale set.

For simplicity of notation, we drop time subscripts from now on. Clearly, $\alpha$ enters the bidding strategies and the prices additively, with the appropriate discount coefficient. Consequently, in the rest of the proof we set $\alpha = 0$.

We assume that agent 0 is the owner and agent 1 considers how much to bid. We use the notation $b^{(i)}$ to designate the $i$-th ranked among the bids, $(b(x^j,\sigma^j))_{j=2}^n$, of the other agents. At time $t$, non-owners bid as in a single-period auction in which the “prize” is the next dividend, $v_{t+1}^i$, plus the value, $S_{t+1}$, of being an owner next period, net of the opportunity of cost, which is the value, $B_{t+1}$, of being a non-owner. Hence, in a second-price auction (taking into account the discount factor) the equilibrium bidding strategy is

$$b(x^1,\sigma^1) = \delta E \left( v_{t+1}^i + S_{t+1} - B_{t+1} \mid b(x^1,\sigma^1) = b^{(1)}, x^1, \sigma^1, (x^0,\sigma^0) \in X \right),$$

where $X$ is the set of signals on which the owner sells, and where we have used that one bids conditioning on being tied for first (Milgrom and Weber (1982)). Recall that an agent with a shock always bids less than an agent without a shock. Hence, an agent with no liquidity shock conditions on the event that at least one other agent has no shock, while an agent with a shock conditions on the event that everybody has a shock. From the viewpoint of a potential buyer, let $N_1$ be the number of potential buyers, other than oneself, who do not have a shock, which is binomially distributed with parameters
Then, we get the following bidding strategies:

\[
\delta^{-1}b(x, 0) = \hat{x} + 2\beta x + \beta \sum_{m=1}^{n-1} (m-1) E[x^i|x^i \leq x] \\
+ \cdot Pr(N_1 = m|N_1 > 0) + S - B \\
= \hat{x} + 2\beta x + \beta E[x^i|x^i \leq x] \left( \frac{(n-1)(1-\gamma)}{1-\gamma^{n-1}} - 1 \right) + S - B \\
\delta^{-1}b(x, \mu) = \hat{x} + 2\beta x - \mu + \beta(n-2) E[x^i|x^i \leq x] + S - B
\]

where \( \hat{x} \) is the conditional expected value of \( x^0 \) given that a sale takes place, that is, \( \hat{x} = E(x^0 | (x^0, \sigma^0) \in X) \).

We now verify Condition 1. Since the bids increase in \( x \), it is necessary and sufficient to have \( b(\chi, 0) \geq b(\chi, \mu) \), that is,

\[
\mu \geq \beta \left[ 2(\chi - \chi) - \chi \left( \frac{(n-1)(1-\gamma)}{1-\gamma^{n-1}} - 1 \right) \right].
\]

To compute the expected price, we need to find the expected value of the second highest bid. For this, we need to consider the number, \( N \), of bidders, including bidder 1, who do not have a shock. Clearly, \( N \) is distributed binomially with parameters \((n, 1-\gamma)\). Also, we need to consider the following order statistics: \( x^{(i)} \), which designates the \( i \)-th ranked among the signals of the bidders who do not have a shock, and \( x^{<i>} \), which designates the \( i \)-th ranked among the signals of the bidders who do have a shock. The conditional expected price paid, \( P \), is (using linearity of bidding strategies):

\[
P = \sum_{m=2}^{n} E \left[ b(x^{(2)}, 0) \mid N = m \right] Pr(N = m) \\
+ E \left[ b(x^{<1>}, \mu) \mid N = 1 \right] Pr(N = 1) \\
+ E \left[ b(x^{<2>}, \mu) \mid N = 0 \right] Pr(N = 0) \\
= \delta \left( \hat{x} - c^1 + S - B \right),
\]

with

\[
c^1 = \beta \eta^1 + \mu \eta^2 \\
\eta^2 = \gamma^n + n\gamma^{n-1}(1-\gamma),
\]

and \( \eta^1 \) is a function of the distribution of \( x^i \), which can be derived in closed form if, for instance, \( x^i \) is uniformly distributed.
Now, we can compute the “value functions,” $S$ and $B$. Agent 0 considers his value as an owner as

\[
S_t = E_{t-1} \left( (\delta (v_{t+1}^0 + S_{t+1}) 1_{\text{(no sale)})} + (P_t - c + \delta B_{t+1}) 1_{\text{(sale)})} \right) \\
= \delta E_{t-1} \left( v_{t+1}^0 1_{\text{(no sale)})} + S_{t+1} + (\hat{x} - c^1 - \delta^{-1} c) 1_{\text{(sale)})} \right) \\
= \delta E_{t-1} (q^S + S_{t+1}),
\]

where the indicators use notation for the obvious events regarding $o(t+1)$, and where $q^S$ is defined as

\[
q^S = E(v_{t+1}^0 1_{\text{(no sale)})} + (\hat{x} - c^1 - \delta^{-1} c) 1_{\text{(sale)})} \\
= -\hat{c} - \psi (\beta \eta^1 + \mu \eta^2),
\]

where $\hat{c}$ and $\psi$ are given by (17)–(18). Similarly,

\[
B_t = E_{t-1} \left( ((\delta (v_{t+1}^1 + S_{t+1}) - P_t) 1_{\text{(1 wins auction))}} \\
+ \delta B_{t+1} 1_{\text{(1 loses auction))}} \right) 1_{\text{sale)}} + \delta B_{t+1} 1_{\text{(no sale))}} \\
= E_{t-1} \left( (v_{t+1}^1 - \hat{x} + c^1) 1_{\text{(sale, 1 wins auction))}} + \delta B_{t+1} \right) \\
= \delta E_{t-1} (q^B + B_{t+1}),
\]

where $q^B$ is defined as

\[
q^B(y_t) = \frac{1}{n} E \left( (v_{t+1}^1 - \hat{x} + c^1) 1_{\text{(sale))}} \mid b(x^1, \sigma^1) > b^{(1)} \right) \\
= \frac{\psi}{n} (\beta \eta^1 + \mu (\eta^2 - \gamma^n)).
\]

By induction,

\[
S_t = \sum_{s=t}^{T-1} \delta^{s-t+1} q^S \\
B_t = \sum_{s=t}^{T-1} \delta^{s-t+1} q^B.
\]

The fact that any mechanism and equilibrium satisfying Condition 1 yield the same conditional expected price, the same set of signals on which the owner sells, and the same owner and non-owner value functions follows from Theorem 4 in Gârleanu and Pedersen (1999). Condition 1 is required to
obtain the crucial condition that there exist a point in each of the two connected components of the private signal space, $\mathbb{X}$, at which the conditional probability of winning the auction is the same. That is,

$$\pi_1\left(\left(\frac{1}{2}, \mu\right), (x^2, \sigma^2), \ldots, (x^n, \sigma^n)\right) = \pi_1\left(\left(-\frac{1}{2}, 0\right), (x^2, \sigma^2), \ldots, (x^n, \sigma^n)\right).$$

To complete the proof, we need to show that there exists an equilibrium sale set of the proposed form. For a given $\hat{x}$, the owner prefers to sell if

$$x^0 - \sigma^0 < \hat{x} - c^1 - \delta^{-1} c,$$

and therefore and equilibrium is given by a number, $\hat{x}$, that solves

$$\hat{x} = E(x^0 \mid x^0 - \sigma^0 < \hat{x} - c^1 - \delta^{-1} c).$$

Now, Corollary 2 shows that there exist an equilibrium if $\mu > c^1 + \delta^{-1} c$. 

□
References


