How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk? A New Calibration Approach\textsuperscript{1}

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Abstract

The existing literature on credit risk valuation does not have a consensus on whether credit risk can explain the observed corporate-Treasury yield spreads — models with different economic assumptions have generated very different estimates of credit yield spreads. We propose a simple calibration approach based on historical default data and estimate how much of the corporate-Treasury yield spread is due to credit risk. For corporate bonds of each credit rating, we calibrate a large class of structural models to be consistent with the empirically observed frequency of default and average loss rate upon default. This approach ensures that the credit risk represented by each model is commensurate with the empirically observed level of credit risk experienced by bondholders. Application of such approach to a large class of structural models shows that the calculated credit yield spreads are highly stable across both different models with very different economic assumptions, and different parameter choices. The stability of our estimates allows us to conclude that credit risk accounts for only a very small portion of the yield spread for investment-grade bonds of all maturities, but a higher fraction of the yield spread for junk bonds.

JEL Classification Numbers: G0, C0

Key words: Credit spreads, yield spreads, and pricing corporate bonds.
1 Introduction

Corporate bonds typically trade at higher yields than Treasury bonds of comparable maturities. The yield spread is partly due to the credit risk of corporate bonds, and is thus frequently referred to as “credit spread.” Credit risk, however, is only one of the factors contributing towards the corporate-Treasury yield spread; other factors include liquidity, call and conversion features of some corporate bonds, and asymmetric tax treatments of corporate and Treasury bonds. In this paper, we propose a simple new calibration approach and ask how much of the corporate-Treasury yield spread is actually attributable to credit risk. This question is important since its resolution can help us understand how well the corporate bond market is integrated with the equity market.

Any attempt to address this question needs to start from a theoretical framework on how credit risk should be priced.\footnote{This creates a joint hypothesis problem. When a calculated credit yield spread based on a credit-risk model fails to match the empirically observed corporate-Treasury yield spread, some may conclude that the model fails to explain the data. This conclusion, however, relies on an implicit assumption that all of the corporate-Treasury yield spread should be due to credit risk. In this paper, we take the view that credit risk is only one of the factors behind the corporate-Treasury yield spread, and prefer to interpret the calculated credit yield spread as the part of the observed yield spread attributable to credit risk. Such an interpretation does demand that the calculated credit yield spreads should be stable across model and parameter choices. The calibration approach in this paper—as we will argue—indeed provides stable results.} The most widely employed framework of credit risk valuation is perhaps the structural approach pioneered by Black and Scholes (1973) and Merton (1974); and we conduct our analysis within this framework as well.\footnote{Another widely used approach of credit risk valuation is the reduced-form approach (see, for example, Jarrow and Turnbull (1995) and Duffie and Singleton (1999)). Our calibration method, however, is not well-suited for this valuation approach because reduced-form models are designed to take prices of defaultable bonds as input.} Over the last few decades, many researchers have applied and extended this valuation framework to study the pricing of credit risk. In particular, many studies have analyzed whether the structural approach can explain the observed corporate-Treasury yield spread. In our view, however, there are still two issues on which the existing literature has not yet provided completely satisfactory answers.
First, the literature does not have a consensus on how much of the corporate-Treasury yield spread is due to credit risk, especially for medium- and long-maturity corporate bonds. An early work by Jones, Mason, and Rosenfeld (1984) showed that the credit yield spreads predicted by Merton-type models are far below the empirically observed corporate-Treasury yield spreads. Other researchers, however, have argued that the structural credit risk approach can effectively explain the observed yield spreads (for medium- and long-maturity bonds) if it is extended to allow for realistic economic considerations. A list of such considerations — not all of which necessarily leading to higher credit spreads — includes stochastic interest rate, bankruptcy costs, and violation of the absolute-priority rule (see, for example, Kim, Ramaswamy, and Sundaresan (1993) and Longstaff and Schwartz (1995)); endogenous low default boundaries (Black and Cox (1976), Leland (1994, 1998) and Leland and Toft (1996)); strategic defaults by equity holders (Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996), and Mella-Barral and Perraudin (1997)); and mean-reverting leverage ratios (Collin-Dufresne and Goldstein (2001)). Recent direct tests of structural models conducted by Anderson and Sundaresan (2000), Lyden and Saraniti (2000), and Eom, Helwege, and Huang (2002) show mixed results on the ability of structural models to explain observed corporate yield spreads.

The lack of consensus on how much of the observed corporate-Treasury yield spreads can be explained by credit risk through the structural approach is partly because the predicted yield spreads are very sensitive to different economic assumptions — different models incorporating different realistic economic considerations tend to generate very different yield spreads. In addition, for each model, the calculated yield spreads can be very sensitive to choices of key parameters such as asset return volatility.

Secondly, the literature on the structural approach on credit risk has so far paid much less attention to available historical data of actual default experiences — both frequency of default and severity of loss upon default — than to data

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3For short-maturity corporate bonds (with maturity less than one year), the consensus seems to be that structural models, especially those assuming that firm asset values evolve as diffusion processes, cannot explain the observed yield spreads. But this is understandable since considerations such as incomplete accounting information (see Duffie and Lando (2001)) and liquidity premia are more likely to have a larger impact on prices of bonds with shorter maturities.
on market prices and yields. Few studies have systematically tested the implications of structural models on both the prices of defaultable bonds and the (real) probability of default on such bonds. This bias of attention seems unwarranted to us. In theory, structural models are almost equally capable of making predictions on default probabilities as they are on bond prices; while bond prices can be easily calculated by using the risk-neutral valuation approach, real-world default probability can be calculated by analyzing the firm’s value process under the real probability measure. In practice, corporate bond investors do care about potential losses associated with default, and their quantitative assessment of potential default loss does affect their required rate of return for holding such bonds.

In this paper, we hope to contribute to the literature in the above two dimensions by proposing a simple calibration approach that makes use of historical data on actual default experience. Our approach first calibrates a large class of structural models according to empirically observed default frequency, and then use such calibrated models to calculate required credit yield spread, which can then be compared against empirically observed yield spreads. An advantage of such an approach is that the estimated credit spread is generated by a model that indeed describes the actual severity of default risk — both in likelihood of default and in loss rate upon default — for bondholders. We will show that the results obtained through this simple calibration approach are highly stable both across different models with very different economic assumptions, and across different parameter choices. This allows us to reach robust conclusions on issues such as how much of the observed corporate-Treasury yield spreads is due to credit risk.

Default experience data, however, are available only through credit rating agencies, such as Moody’s and Standard and Poor’s, for each class of credit ratings. Limited by this, we conduct our analysis by treating firms in groups according to the credit ratings of bonds issued by them. While doing so, we try our best to structure our calibration to be consistent with the way in which rating agencies calculate their default data.

To illustrate our calibration approach, it is useful for us to present a simple

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4KMV, for example, has explored this theoretical implication of structural models in building financial tools for default probability prediction (see Kealhofer and Kurbat (2001)).

5We address issues of concerns related to use of such data later in the paper.
example of applying our approach to a structural model to calculate the credit spread of a 10-year corporate bond issued by Baa-rated companies. Our approach first calibrates the model to a generic Baa-rated firm so that (i) the firm’s initial leverage ratio is 43.28%, consistent with the historical average leverage ratio of Baa-rated firms (Standard & Poor’s (1999)); (ii) the expected equity premium for the firm is 6.55%, consistent with empirical evidences on average equity premium and on how expected equity returns depend on leverage ratio; (iii) the firm’s probability of default over the next 10 years is 4.39%, consistent with historical default frequencies reported by Moody’s (Keenan, Shtogrin, and Sobenhart (1999)); and (iv) the recovery rate upon default is set to be 51.31%, consistent with the historical average recovery rate for senior unsecured bonds reported by Moody’s (Keenan, Shtogrin, and Sobenhart (1999)). We then use the calibrated model to calculate the required yield spread commensurate with the level of credit risk corresponding to a generic Baa-rated bond. The result of course depends on the structural model used. Take, for example, the Longstaff and Schwartz (1995) model. The above calculation would give 64.5 basis points. Compared with the average historical Baa-Treasury yield spread of 194 basis points, our result would suggest that roughly 33% of the Baa-Treasury yield spread is due to credit risk, according to the Longstaff-Schwartz (1995) model.⁶

We apply this calibration approach to a large class of structural models. Specifically, we consider the two-factor model of Longstaff and Schwartz (1995), the strategic default models of Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996), and Mella-Barral and Perraudin (1997), the endogenous default boundary models of Leland (1994) and Leland and Toft (1996), the mean-reverting leverage ratio model of Collin-Dufresne and Goldstein (2001), and a model that takes into account of a predictable time-varying equity risk premium. While these previous studies have shown that different economic considerations can lead to very different predictions of credit yield spreads — in particular, considerations such as strategic defaults and mean-reverting leverage ratios can help generate credit yield spreads comparable to those observed in the market — the estimated credit yield spreads in our approach are not at all sensitive to choices of

⁶See footnote 1 for the joint hypothesis problem and the existence of an alternative interpretation of this result.
different models with different economic assumptions. In addition, our results are insensitive to parameter choices as well.

The stability of our estimates for credit yield spreads allows us to reach the conclusion that, according to structural models, credit risk can only explain a very small portion of the observed corporate-Treasury yield spreads for investment-grade corporate bonds with long maturities, but it does explain a relatively larger portion of the observed spreads for junk bonds. Specifically, for 10-year bonds with ratings at or above A, credit risk accounts for about 20% of the observed yield spreads. The number is about 30-35% for Baa-rated bonds, 60-80% for Ba-rated bonds, and close to 100% for B-rated bonds.

It is useful to understand why our approach generates stable estimates for credit yield spreads while previous calculations show large variations. The credit spread of a corporate bond depends on three factors: (i) probability of default; (ii) loss rate upon default; and (iii) “credit risk premium.” (The credit risk premium is defined as the required risk premium for bearing credit risk; it is the difference between required average realized return for holding corporate bonds minus the riskfree rate of return.) In our calibration approach, we require that each of our calibrated models for bonds of a given credit rating match both the frequency of default and the average loss rate upon default for the credit rating. Results calculated from different models are different only because different economic considerations may generate different predictions of credit risk premia. In comparison, previous studies that use structural models to estimate credit yield spread typically did not impose the requirement that the chosen model and parameter be consistent with actual level of default risk experienced by bondholders, and therefore might have generated high credit yield spreads by predicting counter-factually high default probabilities.

It is interesting to note that many different models with very different assumptions predict similar credit risk premia under our calibration approach, which is why they generate similar credit yield spreads. We will provide intuitive discussions on why these different models generate similar predictions of credit risk premia. We will also point out that models which incorporate empirical evidence that equity premium is predictable and counter-cyclical can help narrow the gap between our calculated credit yield spreads and empirically observed corporate
yield spreads for investment-grade bonds, but not enough quantitatively to affect our conclusion.

Some other studies are also related to our work. Collin-Dufresne, Goldstein, and Martin (2001) and Campbell and Taksler (2002) use regressions to study whether economic variables that determine default risk can explain the time variation of the corporate-Treasury yield spreads, with the former paper showing that credit-risk-related variables can explain only a small portion of the corporate-Treasury yield spread movements. Elton, Gruber, Agrawal, and Mann (2001) study whether credit risk and other factors such as asymmetric tax treatment can fully explain the observed corporate-Treasury yield spreads. Fons (1994) applies a statistical approach to default probability and recovery rate data to understand the term structure of corporate-Treasury yield spread curve. In contrast to the above two papers, we study credit risk with structural models, which allow for a more rigorous treatment of the pricing of credit risk.

The rest of the paper is organized as follows. Section 2 briefly reviews the class of structural models that we consider; section 3 describes our calibration method; section 4 presents and discusses our calibration results from various models; and section 5 concludes.

2 Review of Structural Models

Our calibration approach uses a large class of structural models. We first outline the general setup of this class of structural models of credit risk. Most of the structural models, including all of the models that we consider in this paper, belong to this class.

All models that we consider share the following common general structure:

- **Firm value process:** the firm asset value evolves according to a diffusion process with a constant volatility:

  \[ dV_t = (\lambda^V_t + r_t - \delta_t)V_t\, dt + \sigma_t V_t\, dZ_t, \]  

  where \( r_t \) is the riskfree interest rate, \( \lambda^V_t \) is the asset risk premium, \( \delta_t \) is the rate of cash payout to holders of all of the firm’s securities, as a fraction of
the firm asset value, and $\sigma_v$ is the constant volatility of the firm’s asset value process. Both $r_i$ and $\lambda^v_i$ can be either deterministic or stochastic. As usual, $Z$ denotes a standard Brownian motion under the real probability measure. In the risk neutral measure, the firm value process can be written as

$$dV_t = (r_i - \delta_t)V_t dt + \sigma_v V_t dZ^Q_t,$$

(2)

where $Z^Q$ is a standard Brownian motion under the risk-neutral probability measure.

- **Default boundary**: default occurs when the firm value $V_t$ falls to the level of $V_t^*$ for the first time. That is, default occurs at $\tau = \min\{t : V_t \leq V_t^*\}$. The default boundary $\{V_t^*\}$ can be constant or stochastic, exogenously specified or endogenously derived.

- **Default settlement rule**: at default, bondholders receive a payoff of $\Pi(V_\tau, F)$, where $F$ denotes the face value of the bond.

The time-$t$ market value of a bond issued by the firm, $B_t$, can be calculated using the risk-neutral pricing method. For the class of models that we consider in this paper, we have:

$$B_t = B(V_t, \sigma_v, r_t, \Theta),$$

(3)

where $\Theta$ denotes a vector of additional structural parameters in the model such as the default recovery rate and parameters in the interest-rate process.

This class of models also generate quantitative predictions of default probability for bonds. The firm value process under the real measure, shown in equation (1), and the default boundary specification allow us to obtain the real-world default probability. Let $\Pr(t, T)$ denote the cumulative real default probability over time interval $[t, T]$ estimated at time $t$. We have

$$\Pr(t, T) = \Pr(V_t, \sigma_v, r_t, \lambda^v_t, \Theta).$$

(4)

Note that the real default probability depends on the firm’s asset risk premium, although the bond price does not.

The structural models that we will calibrate in this paper are the two-factor model of Longstaff and Schwartz (1995) (referred to as LS), the strategic default
models of Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996), and Mella-Barral and Perraudin (1997) (collectively referred to as AST-MBP), the credit risk models of Leland (1994) and Leland and Toft (1996) (collectively referred to as LT), the stationary leverage model of Collin-Dufresne and Goldstein (2001) (referred to as CDG). We also calibrate a model with a time-varying asset risk premium. All these models are special cases of the above class of general models.

Undoubtedly, the list of models that we consider misses some important structural models. We choose these models because they collectively dealt with some of the main economic considerations involved in credit risk valuation within the structural framework while maintaining analytical tractability.

For completeness, we review each of these models briefly. While each of these models was developed to address a wide range of economic issues on credit risk, our review here focuses only on the technical setup of such models for the purpose of credit risk valuation.

**Longstaff and Schwartz (1995) (LS)**

In the LS model, interest rate is stochastic and is described by the Vasicek (1977) model under the risk-neutral measure:

$$dr_t = \kappa_r (\theta - r_t) dt + \sigma_r \, dW_t^Q,$$

where $\kappa_r$, $\theta$, and $\sigma_r$ are constant parameters, and $W^Q$ is a standard Brownian motion under the risk-neutral measure with a constant correlation with $Z^Q$ denoted as $\rho_{rY}$. For the interest rate process under the real probability measure, we follow the convention and assume that the interest rate risk premium is of an affine form in $r_t$, such that

$$dr_t = \bar{\kappa}_r (\bar{\theta} - r_t) dt + \sigma_r \, dW_t,$$

where $\bar{\kappa}_r$ and $\bar{\theta}$ are constant parameters, and $W$ is a standard Brownian motion under the real probability measure.

The default boundary $\{V^*_t\}$ is exogenously specified to be constant, and is typically taken to be equal to $F$. In the event of default, each holder of a coupon bond is assumed to continue to receive with certainty an exogenously specified fraction of all future payments according to the original time schedule.
In the Longstaff-Schwartz model, the payout parameter $\delta_t$ is assumed to be zero. Here we consider a more general case where $\delta_t = \delta$ can be greater than zero. This allows for the cash outflow that is required for the firm to pay bond coupons and dividends.

The formulas for bond price $B_t$, for the general case of $\delta > 0$, can be found in Appendix A. We also derive the formula for the real cumulative default probability $Pr(t, T)$ for the case of constant $\lambda^v_t$ in Appendix B.


In the AST-MBP models, equity holders default strategically to extract concessions from bondholders. Default boundary and payoff upon default are obtained endogenously. If equity holders have all the bargaining power, then at the endogenously determined default boundary, the payoff for bondholders will be equal to the firm asset value minus the bankruptcy costs.

Both interest rate $r_t$ and asset payout rate $\delta_t$ are constant.

For the simple case of a perpetual constant flow coupon bond, under the assumption that the bankruptcy costs are a constant fraction $(1 - w)$ of the firm value plus a fixed cost of $K$, the default boundary is constant at:

$$V^*_t = \frac{cF/r + K}{w (1 + x^{-1})} \quad (7)$$

where $c$ is the coupon flow rate and $x$ is defined as

$$x \equiv \frac{r - \delta}{\sigma^2_v} - \frac{1}{2} + \left[ \left( \frac{r - \delta}{\sigma^2_v} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2_v} \right]^{1/2}. \quad (8)$$

The value of the perpetual bond is

$$B_{AST-MBP}(t) = \frac{cF}{r} \left[ 1 - \left( \frac{V_t}{V^*_t \text{AST-MBP}} \right)^{-x} \right] + \max(wV^*_t \text{AST-MBP} - K, 0) \left( \frac{V_t}{V^*_t \text{AST-MBP}} \right)^{-x}. \quad (9)$$

Assuming that the asset risk premium $\lambda^v_t = \lambda^v$ is constant, then the cumulative real default probability over $[t, t + \tau]$, estimated with information available at $t$, is
given by

$$P_{t+\tau} = N\left(-\frac{b + \gamma}{\sigma_v \sqrt{\tau}}\right) + e^{-\frac{2\phi}{\sigma_v^2}} N\left(-\frac{b + \gamma}{\sigma_v \sqrt{\tau}}\right)$$ (10)

where $b = \ln(V_t/V_{\text{AST-MBP}}^*)$, $\gamma = \lambda v + r - \delta - \sigma_v^2/2$, and $N(\cdot)$ denotes the cumulative standard normal function.

Leland (1994) and Leland and Toft (1996) (LT)

The LT model, following Black and Cox (1976), assumes that firms would issue equity to service debt in order to avoid default, and that default occurs when the value of equity goes to zero. This allows for the default boundary to be determined endogenously. In the event of default, equityholders get nothing and bondholders receive a constant fraction $(1 - w)$ of the firm asset value, with $w$ denoting the fractional bankruptcy costs.

Both interest rate $r_t$ and asset payout rate $\delta_t$ are constant.

As in AST-MBP, for analytic tractibility, firms in LT model has a perpetual bond outstanding with a constant coupon flow.\footnote{Leland and Toft (1996) also provide a valuation model for annuity-like debt. We use the simpler perpetuity case here for ease of comparison with other models.}

Under the simplifying assumption that the fixed portion of the bankruptcy costs, $K$, is zero, the endogenous default boundary is given by

$$V_{\text{LT}}^* = \frac{cF}{r} \frac{1}{1 + \lambda - 1},$$ (11)

where $\lambda$ is as defined in (8).

The solutions for both the debt value and the real default probability in the LT model take the same form as those in AST-MBP (as in (9) and (10)), except that $V_{\text{AST-MBP}}^*$ is replaced by $V_{\text{LT}}^*$.

Collin-Dufresne and Goldstein (2001) (CDG)

In the CDG model, firms adjust their outstanding debt levels in response to changes in firm value, which makes the stochastic leverage ratio mean-reverting. The structure of the model is very similar to that of the Longstaff-Schwartz model,
except that the default boundary \( V_t^* \) is assigned to be mean-reverting according to:
\[
d \ln V_t^* = \kappa_t (\ln V_t - \ln V_t^* - \nu) \, dt.
\]
(12)
For the case of constant interest rate, the “log-leverage ratio” \( \ell_t \equiv \ln(V_t^*/V_t) \) follows
\[
d\ell_t = \kappa_t (\bar{\ell} - \ell_t) \, dt - \sigma_v \, dZ_t,
\]
(13)
where the “long-run average leverage ratio” \( \bar{\ell} \) is related to other parameters through
\[
\bar{\ell} = \frac{- (\lambda^u + r) + \delta + \sigma_v^2/2}{\kappa_t} - \nu,
\]
(14)
where the asset payout rate \( \delta_t \) and the asset risk premium \( \lambda_t^u \) are both taken to be constant. As in LS, CDG also allows for the interest rate to be stochastic as described by the Vasicek (1977) model (see (5)).

The formulas for bond price \( B_t \) and for the real cumulative default probability \( Pr(t,T) \) can be found in the appendix.

**A Model with Time-Varying Asset Risk Premium**

Our calibration approach puts an emphasis on the need to compare the prediction of structural models with real world default data. Since the predicted real default probability of a bond depends on the expected return of the firm’s asset, we need to model the asset risk premium \( \lambda_t^u \) more carefully.

Many empirical studies have shown that expected aggregate equity market excess return is time-varying and predictable (see Campbell, Lo, and MacKinlay (1997) for a textbook treatment).\(^8\) Time-variation and predictability of aggregate equity market risk premium implies that individual stock risk premium should also be time-varying and predictable (see Vuolteenaho (2002) for some evidence). Existing structural models of credit risk, however, have not addressed time-variation and predictability of asset risk premium. The models shown above either do not make any explicit assumption about \( \lambda_t^u \) (as they focus on bond pricing using the risk neutral valuation method), or assume it is constant.

\(^8\)However, there are still on-going debates about the validity of such findings (see, for example, Goyal and Welch (2002)).
Our calibration will consider a model with a time-varying and predictable asset risk premium. Specifically, we assume the following dynamic process for the firm’s asset risk premium:

\[ d\lambda_t^v = \kappa_\lambda(\bar{\lambda}^v - \lambda_t^v)dt + \sigma_\lambda dY_t, \]  

(15)

where \( \kappa_\lambda, \bar{\lambda}^v, \) and \( \sigma_\lambda \) are constant parameters, and \( Y \) is a standard Brownian motion under the real probability measure, which has a correlation coefficient of \( \rho_{\lambda V} \) with the firm value process. Later, we will discuss how to fit the parameters according to existing studies of equity premium predictability.

For analytic tractability and ease of comparison, we follow the Longstaff-Schwartz model in making all other assumptions, such as default boundary and default settlement rule. This effectively extends the LS model to allow for a time-varying asset risk premium.

The analytic formulas for bond price \( B_t \) and for the real cumulative default probability \( Pr(t, T) \) can be derived for this extended model, and are shown in the appendix.

3 Calibration Method

The idea behind our calibration approach is that the model (with the associated choice of all of its parameters) that we use to calculate credit yield spread should reflect the severity of default risk — both frequency of default and the average loss rate upon default — actually experienced by bondholders. So our calibration method should be designed around the availability of data on default experience.

Data on default probability and loss rate, however, are available only for each level of credit ratings as classified by credit rating agencies such as Moody’s and Standard and Poor’s. To make use of such data, we focus our analysis on all companies with the same credit rating at a given point in time, rather than on any individual company. Specifically, we consider companies that have the following credit ratings by Moody’s: Aaa, Aa, A, Baa, Ba, and B. This allows us to take advantage of the historical default data of bonds of each level of credit rating from Moody’s. Since we make use of data from both Moody’s and Standard and Poor’s, we make a reasonable assumption that the two rating systems have the following
one-to-one mapping: Aaa = AAA, Aa = AA, A = A, Baa = BBB, Ba = BB, B = B.

We use Moody’s as our source of data on historical default frequency and average loss rate upon default (see Keenan, Shtogrin, and Sobenhart (1999)).\(^9\) Since both credit rating and loss rate upon default for a bond depend on the seniority of the bond, it is important that the historical default frequency and average loss rate of default which we calibrate our models against correspond to bonds with the same seniority. The historical default probability estimates reported by Moody’s correspond to senior unsecured bonds.\(^10\) So for default recovery rate, we should also use the average recovery rate upon default for senior unsecured bonds, which was reported to be 51.31%. Altman and Kishore (1996), using a different method to measure recovery rate upon default, arrived at similar numbers for recovery rate.

When we calibrate a structural model for a given credit rating, we imagine a “generic” firm which has a senior unsecured bond outstanding, and assume that the bond issued by our generic firm has the same probability of default with the historical probability of default for bonds of the same seniority and credit rating. The loss rate upon default for the bond is assumed to be the same as the historical average loss rate for bonds of the same seniority and credit rating.

Table 1 shows, among other things, the cumulative default probability and average loss rate of default for each credit rating from Moody’s.

It should be pointed out that, although we treat companies with senior unsecured bonds of the same credit rating as a group, it does not mean that we need to know the criterion used by the rating agencies for each credit rating. All we need to assume is that bonds with different credit ratings represent bonds with different level of credit risk and therefore different required yield spreads. Given that we consider only 6 broad levels of credit rating, this assumption is likely to

\(^9\)See Carey and Hrycan (2001) for a careful discussion of issues on using credit rating to predict default probability.

\(^10\)Of course, in arriving at such estimates, Moody’s needs to use a procedure to “back out” from each subordinated and senior secured bond an implied senior rating. Such a procedure can create biases, which is why we should take caution when using the reported default probability. Later in the paper, we will do sensitivity analysis with our results by altering the estimates of historical default probability.
hold.

When we calibrate a model to each credit rating, in addition to default probability and loss rate upon default, we should also take into account balance sheet information for companies with different bond credit ratings. Standard and Poor’s (1999) provides the average leverage ratio for companies with bonds of a given credit rating. In our approach, we calibrate our model for each credit rating to this reported average leverage ratio, which is shown in column 2 of Table 1.

One other variable that our model should be calibrated against is equity premium for companies with bonds of each credit rating. This variable is important because the predicted bond default probability (in the real world as opposed to the risk-neutral world) depends on the firm’s asset risk premium, which in turn is related to the firm’s equity premium. Since companies with bonds of different credit ratings in general have different leverage ratio and therefore different equity premium, we need to calibrate our model to different equity premium for companies with bonds of different credit ratings. To choose equity premia for companies with different bond credit ratings, we start from Bhandari (1988), which through regressions shows how cross-sectional equity returns are related to leverage ratios. Using the coefficients from Bhandari’s regression, and noting that the average equity premium and leverage ratio for S&P 500 companies are, respectively, roughly 6% and 35%, we arrive at a set of estimates for equity premia of companies with bonds of different credit ratings. The results of such estimates are shown in column 3 of Table 1.

Table 1 also shows the historical average yield spreads for bonds of each credit rating against Treasury bonds of similar maturities. Investment-grade bond spreads are based on the Lehman bond index data from 1973-1993, with the time period chosen to be concurrent with the time period for which Moody’s reports default frequency. Junk bond yield spreads are based on Altman (1990). Note that these yield spreads are not part of input parameters for our calibration; they are only to be compared with our calculated credit yield spreads.

To summarize, for bonds of each credit rating, we need to calibrate structural models to match four target quantities: the initial leverage ratio, the equity premium, the cumulative default probability over a given future time horizon, and recovery as a fraction of face value upon default. Table 1 contains all such infor-
mation.

We now turn to structural models and talk about how we choose our model parameters.

Model Calibration

To specify any structural model quantitatively, we need to choose the following set of parameters (or processes in case it is stochastic): the initial firm value $V_0$ (as a multiple of the total face value of bonds outstanding), the interest rate $r$, the asset risk premium $\lambda^v$, the asset payout rate $\delta$, the asset volatility $\sigma_v$, the default boundary $V^*$, and the default settlement rule.

Here is how we choose our model parameters. We choose $r$ according to historical average (for constant interest rate model) or according to fitted interest rate models (for stochastic interest rate process). We choose $\delta$ to be consistent with historical average asset payout rate. For $V^*$, some models determine it endogenously. For other models, we choose $V^*$ by convention or by making sure that it is consistent with the fact that, after bankruptcy costs and violation of absolute priority rule take away about 15% of the firm value $V^*$ from bondholders, they still recover about 51.31% of the total face amount.

Four important parameters, $V_0$, $\lambda^v$, $\sigma_v$, and default settlement rule, are still to be determined. For each model calibrated to each credit rating, we choose these four parameters such that the model matches the four target parameters of initial leverage ratio, equity premium, cumulative default probability, and recovery rate upon default. During such calibration, we need to know formulas that allow us to calculate bond price and default probability. Such formulas for all models considered in this paper can be found in section 2 or appendix.

Specific numerical choices of parameters will be discuss in the next section when we present results.

Before we present results and discussions, we want to point out that in our calibration approach, as we average over historical time periods as well as over companies with bonds of similar credit ratings, we ignore the cyclicality and predictability of default rates. Ideally, one would like to calibrate our models for each time period separately so that the models are consistent with each time period’s expected default probability. But given the limited historical data on default and
given the noisy nature of default rates over any short period of time, it is very
difficult to have any precise estimate of the expected default probability if we limit
ourselves to a short time period.

4 Results and Discussions

In this section, we present and discuss our calibration results from various struc-
tural models. We organize our presentations and discussions by the economic
considerations proposed by these models, rather than by models themselves.

We choose the Longstaff-Schwartz model to present our base case around which
we introduce different economic considerations. The analytic flexibility of, and the
ease of extensions from, the LS model allow us to analyze the effects of stochas-
tic interest rate, mean-reverting leverage ratio, and time-varying and predictable
equity risk premium on our calibration results.

4.1 The Base Case

Our “base case” uses the LS model with constant interest rate. Stochastic
interest rate will be considered later. The base case also requires a full set of
parameters as inputs. We now specify the choice of all parameters.

First, we need to choose target parameters for each credit rating. For the base
case, the target initial leverage ratio (defined as the ratio between the market value
of debt and the market value of firm asset), the target expected equity premium,
the target cumulative default probability, and the recovery rate upon default are
all chosen according to Table 1.

Second, we need to choose parameters for credit risk models. Four model
parameters—initial firm value $V_0$, asset risk premium $\lambda^v$, the asset volatility $\sigma_v$, and the recovery rate upon default $w$—will be determined by calibration to match
the above target parameters. This leaves three parameters to be determined: the
asset payout rate, the interest rate, and the default boundary.

For the asset payout rate, we use $\delta = 6\%$ for all firms. In principle, this payout
rate should represent the average total net annual payments to holders of all of
the firm’s securities (which includes all coupon, principal, and dividend payments,
plus proceeds from all security issuances, minus all share repurchases) as a fraction

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of the firm’s asset value. As a rough estimate, we can take the weighted averages between the average dividend yields (which is close to 4% according to Ibbotson Associates (2002)) and the average historical coupon rate (which is close to 9% during 1973-1998, the time period for which we calibrate our models), with weights given by the average leverage ratio of all firms (which is close to 35% for S&P 500 firms). This rough estimate gives a payout rate that is close to 6%.

We do not assume different asset payout rates for firms with different credit ratings. Firms with junk bonds may have more debt and higher coupon rates than firms with investment-grade bonds, but they are likely to pay much less dividends. In any case, we will show later that our calibration results are not sensitive to this choice.

For interest rate, we will choose a continuously compounded constant rate of $r = 8\%$. This is close to the historical average interest rate during 1973-1998.

The last parameter we need to choose is the boundary of default. In the LS model, default is assumed to occur when the firm value falls to the bond face value. It is arguable that this assumption is not very reasonable given that bondholders on average recover only about 51% of the face value upon default, and it is difficult to believe that 49% of the firm value is lost due to bankruptcy costs or violation of the absolute priority rule. So we also consider a lower default boundary $V^*$ at 60% of the bond face value.\footnote{In order to make the assumption that the default boundary in the LS model can be below the bond face value $F$, we need to reinterpret the model and let $F$ denote the total amount of liabilities. If the firm has only one corporate bond maturing in 10 years, then the default boundary has to be close to the face value close to bond maturity.} Upon default, bondholders recover about 51% of the face value, which is about 85% of the firm value at default. The 15% loss rate (for firm’s asset) is consistent with previous findings that bankruptcy costs are about 10-20% of firm value (see, for example, Andrade and Kaplan (1998)) and that violation of absolute priority rule represents only a small fraction of the firm’s value. Although we frequently show results for both default boundaries, our preferred choice for the base case is $V^* = 0.6F$. The calculated credit yield spreads are not very sensitive with respect to this choice.

Third, we need to choose the coupon rate and maturity for the corporate bonds. We choose 10-year maturity for our base case. Structural models are known to have difficulty explaining short-term corporate bond yield spreads, and liquidity
and other considerations tend to have larger effects on shorter maturity bonds. We focus on 10-year maturity bonds since there is much less consensus on whether structural credit risk models can explain the corporate yield spreads at long maturities. We do, however, show results for short maturity bonds as well later on.

We choose the coupon rate to be 8.162%, which is the par coupon rate for risk-free bonds with semi-annual coupon payments when the continuously compounded interest rate is 8%. One obvious alternative is to choose a different coupon rate for bonds of each credit rating so that the corporate bond is initially priced at par. We made our choice so that the LS default settlement rule matches the way default settlement data is recorded by Moody’s. While Moody’s reports default recovery rate as a fraction of the face value received on the day of default, the LS model assumes that upon default, each holder of a coupon bond is assumed to continue to receive with certainty an exogenously specified fraction of all future payments according to the original time schedule. When we choose the coupon rate to be the par coupon rate for an otherwise riskfree bond, the bondholder in the LS model effectively receives the specified fraction of the face value, in terms of market value, on the day of default. This makes it possible for us to calibrate the LS model to Moody’s default data. To correct for the fact that this choice of coupon rate may underestimate the coupon rate for junk bonds, we calculate all leverage ratios by using market value, rather than face value, of bonds.

The above choices of parameters and terms of the coupon bond allow us to calculate the results for the base case, which is shown in Table 2. We have two panels in the table, corresponding to the above two choices of the default boundary.

Table 2 shows that, for 10-year investment-grade bonds (those with a credit rating higher than or equal to Baa), the calculated credit spreads only account for a very small fraction of the observed total corporate-Treasury yield spreads. For bonds rated at or above “A,” the base case model suggests that credit risk only accounts for less than 20% of the observed yield spreads. For 10-year junk bonds (with ratings at or below Ba), however, credit risk does account for a much larger fraction of the total corporate-Treasury yield spreads.

Table 3 shows results for 4-year bonds. According the model and parameter choice in the base case, credit risk accounts for an even smaller fraction of the observed yield spreads for investment-grade bonds with 4-year maturity. For 4-
year junk bonds, however, credit risk does account for a higher fraction of the observed yield spreads than for 10-year junk bonds. These results are not surprising as many previous studies have shown that structural models with a firm value evolving as a diffusion process tend to generate very low yield spread for short-term investment grade bonds but high spread for short-term junk bonds. But our calculation strengthens such conclusion from previous studies by showing that calibrating structural models to historical default experience data results in the same conclusion.

Overall, the base case seems to do a much better job explaining the junk bond spreads than investment-grade bonds. Since structural models clearly have difficulty explaining short-term investment grade corporate bond yield spreads, we put more focus on 10-year corporate bonds in the remainder of the paper because structural models offer much less consensus on how well long-maturity corporate-Treasury yield spreads can be explained by credit risk.

It is important to know whether models that introduce other economic considerations would change our results, and whether our results are sensitive to choice of parameters. We take up these issues next. Specifically, we ask whether our results will be significantly different when we consider that: (i) interest rate may be stochastic (the Longstaff-Schwartz two-factor model); (ii) default boundary should be endogenously determined as firms issue equity to avoid default (Leland-Toft); (iii) firms may default strategically to extract concessions from bondholders (Anderson-Sundaresan, Anderson-Sundaresan-Typhon, and Mella-Barral and Perraudin); (iv) firms adjust their leverage ratio, making it mean-reverting (Collin-Dufresne and Goldstein); and (v) equity premium may be time-varying and predictable. In addition, we also make sensitivity analysis with different parameter choices.

As we will show below, the quantitative results obtained in our base case remain very robust, and are sometimes even further strengthened, when we consider different economic assumptions on credit risk valuation, as well as when we choose different parameters. Before we show the results, we discuss the intuition behind
the robustness of results from our calibration.

4.2 Discussion of Intuition

Before we leave the base case, however, it is useful for us to discuss why the results in the two panels in Tables 2 and 3 are so similar despite the fact that they corresponds to two very different default boundary levels. The discussion can help us build up intuition that allows us to understand results from other model and parameter choices.

In standard applications of structural models for bond valuation, we first choose appropriate model parameters such as initial leverage ratio and asset volatility, and then calculate the yield spread under such parameter choices. The result from such an approach should of course depend very strongly on the default boundary $V^*$ — holding constant the default recovery rate (as a fraction of bond face value), the lower the default boundary, the less likely the firm will default, and the lower the credit spread.

In our approach, however, we choose model parameters like the asset volatility such that our model matches the empirically observed default probability. When we choose a lower default boundary $V^*$, the calibration produces a higher asset volatility such that the model continues to match the observed default probability. While the lower default boundary reduces credit spread, the higher asset volatility increases credit spread, and the net result is not very sensitive to the choice of the default boundary.

There is also an intuitive alternative to this mechanical explanation. Credit yield spread depends on the average loss rate due to default and an additional "credit risk premium." The credit risk premium is defined as the required risk premium for bearing credit risk; it is the difference between required average realized return for holding corporate bonds minus the riskfree rate of return. For different model or parameter choices, our calibration approach always ensures that the calibrated model matches the same empirically observed average loss rate due to default. So different models or different parameter choices produce different credit yield spreads only because they generate different credit risk premium.

Why, then, do our calibrated models with different default boundary levels generate similar credit risk premium, as shown in Tables 2 and 3? Recall that
investors should require credit risk premium because loss associated with credit risk tends to be negatively correlated with market return; default occurs only after the firm asset value has experienced very severe negative shocks in returns, and such negative shocks to individual firm asset returns are more likely to happen when the aggregate market experiences negative shocks given that a typical firm’s asset return is positively correlated with the aggregate market return.

Our first reaction might be that the model with the lower default boundary should generate higher credit risk premium – a lower default boundary means that default occurs only after the firm asset return has experienced a more severe negative shock. But a more precise measure of the severity of a negative shock in return is how many standard deviations such a negative return is away from the mean asset return. Following the KMV approach (see Kealhofer and Kurbat (2001)), we call this “distance to default” (DTD), and it is defined as

$$ DTD \equiv \left[ \ln \left( \frac{V^*}{V_0} \right) - \left( \lambda^v + r - \delta - \frac{\sigma^2}{2} \right) T \right] \sigma_v \sqrt{T} $$  \hspace{1cm} (16)

In our calibration approach, for a lower default boundary, the asset volatility needs to be calibrated to a higher level such that the model’s predicted default probability is consistent with the empirically observed default frequency. So the model with a lower default boundary also has a higher asset volatility, and the DTD quantity, as well as the credit risk premium, may not be higher than those with a higher default boundary.

The above discussion suggests that the credit risk premium is mostly driven by the DTD variable. But since the real-world default probability for bonds is mostly determined by the DTD variable as well (see Kealhofer and Kurbat (2001)), when we calibrate different models with different parameter choices to the same observed default frequency, these different models are very likely to generate similar credit risk premium as well. This intuition explains why our calculated credit spreads are relatively stable as we vary the default boundary dramatically from Panel A to Panel B in Table 2. It also underlies the robustness of our results as we go into
different models of credit risk below.

4.3 Stochastic Interest Rate

One realistic consideration ignored in the base case is that interest rate is stochastic. To study the effect of stochastic interest rate on our calculation, we calibrate the original two-factor LS model with a stochastic interest rate given by the Vasicek (1977) model (see (5) and (6)). The parameters for the interest rate process are chosen as:12

\[
\kappa_r = \bar{\kappa}_r = 0.226; \; \theta = 11.3\%; \; \bar{\theta} = 6.2\%; \; \sigma_r = 4.68\%
\]

The correlation coefficient between the interest rate process and the firm value process is assumed to be \(\rho_{rv} = -0.25\), following LS. The initial interest rate at time zero is set to be 8%.

Table 4 presents the results of this calculation for 10-year bonds. The results show that, across all credit ratings, the calculated credit spread from our calibration under the assumption of a stochastic interest rate is much smaller than those in the base case. This further strengthens our conclusion that, for investment-grade bonds, credit spread is only a very small portion of the observed corporate-Treasury yield spreads.

The reason that stochastic interest rate has a negative impact on our calculated credit spread is as follows. First, we make an observation that the long-term mean interest rate in the risk neutral measure is always higher than in the real probability measure. In our above choice, for example, \(\theta = 11.3\% > \bar{\theta} = 6.2\%\). This is not unique to the Vasicek model — any term structure model that tries to explain the positive premium for holding long-term bonds inevitably assumes that the long-term mean for the interest rate process under the risk-neutral measure is higher than under the real probability measure so that long-term bond prices are “fair” relative to short-term bonds.

This observation allows us to understand why stochastic interest rate reduces the credit spread calculated in our calibration approach. Note that, holding other model parameters constant, a higher interest rate leads to a lower default probability, as the firm value has a higher drift rate. If the interest rate is constant,

12These parameters were obtained from Qiang Dai.
then the interest rates under both the real and risk-neutral measures are the same constant. But a stochastic interest rate (appropriately calibrated to term structure data) generates a higher long-term mean under the risk-neutral measure than under the real measure, and this drives the risk-neutral default probability lower relative to the real default probability. In our approach, since our model is always calibrated to generate the same empirically observed real default probability, so stochastic interest rate drives down the risk neutral probability of default as well as the calculated credit spread.

4.4 Endogenous Default Boundary Set By Zero Equity Value

One issue with the LS model is that the default boundary is specified exogenously. Ideally, we would like to know whether an endogenously derived default boundary, by imposing more structure on the model, may have a large quantitative effect on our calculated credit spread. Unfortunately, not many models of this type are analytically tractable for calibration purpose. Two groups of structural models have endogenously derived default boundaries under some restrictive simplifying assumptions. The first group, by Black-Cox (1976), Leland (1994), and Leland-Toft (1996), assumes that firms delay default as much as possible by issuing equity to service debt coupon payments, and default occurs when equity value is zero. We consider this group of models here. The second group assumes strategic defaults, which we will take up next.

The calibration approach for the Leland-Toft (LT) model and for all parameter choices are the same as in the base case, except for differences in bond maturity and in modelling. Results are shown in Table 5.

On appearance, the LT model seems to generate higher credit spreads for investment-grade bonds than the LS model did in our base case. This, however, is due to the fact that the LT model considered here has a perpetual bond, which should have a higher credit spread than 10-year bonds for investment grades. The only other difference between the LT and the LS model, given that we have calibrated the two models in the same way, is in their default boundaries—the endogenously determined default boundary in the LT model is in general different from the exogenously specified default boundary in the LS model. This, however, cannot explain the difference in credit spreads as Table 2 has already shown.
that credit spreads calculated in our calibration approach are not very sensitive to default boundary specification.

To check that the higher investment-grade credit spread calculated in the LT model is indeed due to the infinite maturity of the perpetual bonds in LT, we take the exact same asset risk premium, default boundary, and asset volatility which the LT model generated in Table 5 for Aaa- and Aa-rated bonds, and then plug these parameters into the Longstaff-Schwartz model to calculate the credit yield spreads of a 10-year bond. Such calculation shows a spread of only 8.9 bp and 12.4 bp, respectively, for Aaa- and Aa-rated 10-year bonds, which are much lower than given by the LT model. Note that this is a precise comparison as both models use the same underlying firm value process and our calibration ensured that all other parameters are identical.

One nice feature of the LT model is, as the calibration results show, that it does endogenously generate default boundaries that are much lower than the bond face value. This is reasonably consistent with the fact that historical average default recovery rate is only about 51% of the bond face value even though bankruptcy costs should be smaller than 20%. Occasionally, however, the default boundary can be even lower than 51% of the face value, making it impossible for the model to match the 51% average recovery rate. In such cases, we cap the recovery to be whatever the firm value is at default.

4.5 Strategic Default

Another set of models of credit risk with endogenous default boundary are given by Anderson-Sundaresan (1996), Anderson-Sundaresan-Tychon (1996), and Mella-Barral and Perraudin (1997) (collectively AST-MBP). While the LT model assumes that firms try to delay default as much as they can by issuing equity to service debt if necessary, the AST-MBP approach assumes that firms default strategically to extract concessions from bondholders whenever possible.

One of the important conclusions from the strategic default approach in AST-MBP is that firms are more likely to default than the traditional approach in Merton (1973) and the model in Longstaff-Schwartz (1996) would suggest. As a result, AST-MBP showed that their approach does a better job at explaining the empirically observed high corporate-Treasury yield spreads for investment-grade
bonds.

Our calibration calculation, however, arrives at a different conclusion. Table 6 shows the results of our calibration.\textsuperscript{13} As in the LT model, on appearance, the AST-MBP model seems to generate much higher credit spreads for investment-grade bonds than the LS model did in our base case. This, however, is again due to the fact that the AST-MBP model considered here has a perpetual bond, which should have a higher credit spread than 10-year bonds for investment grades. To check that this is indeed the case, we take the exact same asset risk premium, default boundary, and asset volatility which the AST-MBP model generated in Table 6 for Aaa-rated bond, and then plug these parameters into the Longstaff-Schwartz model to calculate the credit yield spread of a 10-year bond. Such calculation shows a spread of only 12.8 bp, which is much lower than given by the AST-MBP model. Again, note that this is a precise comparison as both models use the same underlying firm value process and our calibration ensured that all other parameters are identical.

Comparing the AST-MBP model with the LT model, we note that endogenous defaults tend to occur at firm value above the face value of the debt. This may make it difficult to understand why bondholders receive, on average, only roughly 51% of the face value upon default while bankruptcy costs are smaller than 20% of the firm asset value.

**4.6 Firms with Mean-Reverting Leverage Ratio**

Collin-Dufresne and Goldstein (2001) (CDG) proposed a model that incorporates the idea that firms may adjust their outstanding debt levels in response to changes in firm value, which makes the stochastic leverage ratio mean-reverting. They argue that such considerations generate credit yield spreads for high quality investment-grade bonds with long maturities that are comparable to those observed in corporate bond data, as such firms tend to increase their leverage ratio over time by increasing debt issuance, and that the calculated yield spreads are reasonably consistent with the observed corporate yield spreads.

We apply our calibration approach to the CDG model. The results are shown

\textsuperscript{13}We do not show results for junk bonds because we cannot find a set of parameters that satisfy our calibration requirements.
in Table 7. For parameters in the CDG model (see (12) and (13)), we choose
the mean-reversion coefficient $\kappa_t = 0.2$, and the long-term average leverage ratio
$\bar{\ell} = 38\%$. These numbers are similar to those chosen in CDG, and also comparable
to those found in empirical studies (see Fama and French (2002a)).

The results in Table 7 show that, the calculated credit yield spreads in the CDG
model under our calibration approach are slightly smaller than, but very similar to,
the results in our base case. The higher credit yield spread generated in the original
CDG approach is due to the higher default probability driven by the assumption
that firms with high credit quality may increase their debt outstanding, rather
than due to a higher credit risk premium. Once we force the model to match the
empirically observed default frequency, we are still left with the conclusion that,
according to the CDG model, credit risk accounts for only a small portion of the
observed corporate-Treasury yield spreads for investment-grade bonds with long
maturities.

4.7 Predictable Time-Varying Equity Premium\textsuperscript{14}

Our discussions above suggest that models that generate high credit yield
spread mainly by considering economic factors that lead to high default proba-
bility are unlikely to produce high credit spread under our calibration approach.
Since our approach requires that each model match the empirically observed de-
fault loss rate, only economic considerations that produce high credit risk premium
can generate higher credit yield spread under our approach.

One such consideration is that equity premium is time-varying and predictable.
Previous empirical studies show that the aggregate equity market risk premium
is time-varying and predictable (see, for example, Campbell, Lo, and MacKinlay
(1997)). Specifically, business cycle variables such as market dividend yield can
help predict equity premium; a higher aggregate market dividend yield (which
tends to show up at bottoms of business cycles), for example, predicts higher
future equity premium.

This consideration will, at least qualitatively, lead to a higher credit yield
spread under our calibration approach. A firm’s asset return is typically positively

\textsuperscript{14} We thank Sheridan Titman for pointing out that this consideration should increase our
estimated credit yield spread.
correlated with the aggregate market return. Such a correlation implies that, if a firm’s asset return experiences a negative shock, then it is more likely that future risk premium is higher (for both aggregate market and individual firms’ asset return). This, in turn, implies that the real default probability of such firms should be lower when equity returns are counter-cyclical. But since bond prices will not be affected by changes in asset risk premium due to risk-neutral pricing, the net effect of a predictable equity premium is that corporate bonds can have low prices (and high yields) while experiencing low default probability.

To investigate whether this effect can make a significant quantitative difference in our calibration calculation, we apply our approach to the model with a time-varying asset risk premium introduced in section 2.

We first need to assign numerical values for parameters $\kappa_\lambda$, $\bar{\lambda}^v$, and $\sigma_\lambda$ for the asset risk premium process in (17). We start with the following aggregate equity market risk premium process as calibrated in Campbell and Viceira (1999) for annual S&P 500 data from 1890 to 1993:

$$d\lambda^M_t = \kappa_M (\bar{\lambda}^M - \lambda^M_t) \, dt + \sigma_M \, dY^M_t,$$

where

$$\kappa_M = 0.202; \quad \bar{\lambda}^M = 4.165\%; \quad \sigma_M = 3.10\%.$$  

The correlation between the market risk premium process with the market return process is $\rho_{\lambda M} = -0.701$. To come up with a model for our individual firm asset risk premium, we combine the Campbell-Viceira calibration with the CAPM model,

$$R_{i,t} = r_f + \beta_i (R_{M,t} - r_f) + \epsilon_{i,t},$$

where $\epsilon_{i,t}$ represents firm $i$’s idiosyncratic risk that is uncorrelated with all other variables.

The final parameter choices for the individual firm asset risk premium process are

$$\kappa_\lambda = 0.202; \quad \bar{\lambda}^v = 4.165\%; \quad \sigma_\lambda = 3.10\%$$

where we have assumed that the beta for a typical firm’s asset is roughly 1 in arriving at the estimate for $\bar{\lambda}^v$. The correlation between a firm’s asset risk premium and its asset return process is given by

$$\rho_{\lambda w} = \rho_{\lambda M} \times \rho_{wM} \approx -0.701 \times 0.5 = -0.351$$
where we have used the fact that a typical firm’s asset return correlation with the market return is about 0.5.

With these parameter choices, the results of our calibration calculation are shown in Table 8. As expected, the predictability of equity risk premium does produce a higher credit yield spread, but the quantitative effect is limited — credit risk still only accounts for a small fraction of the observed yield spread for investment-grade bonds, according to the model.

4.8 Sensitivity Analysis on Parameter Choices

We now consider whether our conclusions are sensitive to the choices of our parameters. We start from our base case, and then change various parameter choices. To be conservative, we give more considerations to parameter choices that would lead to higher calculated credit yield spread.

**Equity premium**

Higher equity premium should lead to higher credit yield spread in our calibration approach. The intuition behind this is similar to that behind why predictable equity premium leads to higher credit yield spread; higher equity premium does not affect bond prices because of risk-neutral pricing, but it does make it possible for our models to generate low default probability.

One way to make sure that our results are not very sensitive to our choice of equity premium is to look at the effect of a higher equity premium. We start from the base case, and then add 2% to the equity premium for firms of each credit rating. Given that some researchers believe that the true equity premium may be smaller than 6% (see Fama and French (2002b)), and that the realized equity premium during 1973-1998 is also less than 7% for firms with average leverage ratio,\(^\text{15}\) this choice of higher equity premium is conservative enough.

Results in Table 9 show that the calculated credit yield spread is indeed rather

\(^{15}\)One may even argue that it is better to use the realized equity premium to calibrate our model since we use realized default probability over this time period.
sensitive to the equity premium, but the total quantitative effect is still small.

**Average leverage ratio for each credit rating**

It is possible that the average leverage ratios of bonds with different credit ratings as given in Standard and Poor’s (1999) may not be precise. In Table 10, we deviate from our base case by assuming that the reported leverage ratio for each credit rating may be off by 30% (of the reported value). Results in both panels of Table 10 show that our calculated credit yield spreads are not sensitive to the choices of these parameters.

**Average asset payout ratio δ**

We chose the asset payout ratio to be δ = 6% in our base case. To make sure that our results are not sensitive to this choice, we looked at two other choices: 0% and 8%. Results in Table 11 show that the calculated credit spreads are not sensitive to the choice of this parameter as well.

**Default probability for each credit rating**

Since our calibration approach puts an emphasis on matching historically observed default probability, it is important that our results are not overly sensitive to the default probability estimate that we use, especially given that there are reasons to believe that the estimated default probability by Moody’s may not be accurate.

In Table 12, we show results of our calculation when we deviate from our base case by increasing the assumed target default probability of each credit rating by 50% (of the base case value). Our calculated credit yield spreads are sensitive to this change, but the overall quantitative effect remains too small to alter our conclusions.

**Average recovery rate upon default**

Table 13 shows the results of our calculation when we change the average recovery rate for senior unsecured debt from 51.31% (as estimated by Moody’s) in our base case to 45%. This change increases our calculated credit spread, but the effect is again not large.
5 Conclusions

The existing literature on credit risk valuation does not have a consensus on whether credit risk can explain the observed corporate-Treasury yield spreads — models with different economic assumptions have generated very different estimates of credit yield spreads. In addition, few studies have systematically incorporated historical default experience data in testing structural models.

In this paper, we have proposed a simple calibration approach based on historical default data and estimate how much of the corporate-Treasury yield spread is due to credit risk. For corporate bonds of each credit rating, we calibrate a large class of structural models to be consistent with the empirically observed frequency of default and average loss rate upon default. This approach ensures that the credit risk represented by each model is commensurate with the empirically observed level of credit risk experienced by bondholders. Application of such approach to a large class of structural models shows that the calculated credit yield spreads are highly stable across both different models with very different economic assumptions, and different parameter choices. The stability of our estimates allows us to conclude that credit risk accounts for only a very small portion of the yield spread for investment-grade bonds of all maturities, but a higher fraction of the yield spread for junk bonds.

Our results show that, based on many of the proposed structural models of credit risk and estimates of market equity premium in the literature, the equity market and corporate debt market may not be very well integrated in terms of the pricing of firm-asset related risk.\textsuperscript{16} One alternative conclusion from our results is that these two markets may be well integrated, but the existing structural models may not accurately capture how credit risk are priced in the market.

\textsuperscript{16}This has implications on corporate capital structure decisions; see Titman (2002) for a discussion of the importance of integrated markets with costless financial intermediation on Modigliani and Miller theorem.
References


Standard & Poor’s, 1999, “Corporate Ratings Criteria.”


Appendix

A Formulas of Defaultable Bond Prices

For completeness, all the formulas of bond prices used in our implementation of the LS and CDG models are given in this appendix.

Consider a defaultable bond with maturity $T$ and unit face value that pays semi-annual coupons at an annual rate of $c$. For simplicity, assume $2T$ is an integer. Let $T_n, n = 1, \ldots , 2T$, be the $n$th coupon date. In both LS and CDG, the value of a defaultable bond that pays semi-annual coupons is given by

$$P(0, T) = \left( \frac{c}{2} \right) \sum_{i=1}^{2T-1} D(0, T_i)[1 - w_tQ(0, T_i)] + \left( 1 + \frac{c}{2} \right) D(0, T)[1 - w_TQ(0, T)]$$

(18)

where $D(\cdot)$ denotes the value of a default-free zero-coupon bond given by the Vasicek (1977) model, $Q(0, T_i)$ represents the time-0 default probability over $(0, T_i]$ under the $T_i$-forward measure, and $w_t$, the loss rate, equals $1 - w$. In CDG, the loss rate on coupons is 100%. Here we follow LS to use the same loss rate on both coupons and principal. It is easy to see that in order to use (18), one needs to determine $Q(0, \cdot)$.

Let $V_t, V_t^*$, and $r_t$ be the time-$t$ values of the firm’s assets, total liabilities, and the risk-free interest rate, respectively. Define a new process $X = (V_t/V_t^*)_{t \geq 0}$. Assume

$$d \ln X_t = \left[ r_t - \delta - \sigma^2_r/2 - \mu_k(r_t, \ln X_t) \right] dt + \sigma_r dZ_t^Q$$

(19)

$$dr_t = (\alpha - \beta r_t) dt + \sigma_r dW_t^Q$$

(20)

where $\mu_k$ is affine in the two state variables, $\sigma_v, \delta, \alpha, \beta$, and $\sigma_r$ are constants, and $Z^Q$ and $W^Q$, two one-dimensional standard Brownian motion processes under the risk-neutral measure $Q$, are assumed to have a constant correlation coefficient of $\rho$. For comparison, we use the LS notation here. To be consistent with the notation used earlier, we have $\beta = \kappa_r, \theta = \alpha/\beta$, and $\rho = \rho_{rv}$.
Default probabilities $Q(0, \cdot)$ can be calculated using an approach in the spirit of LS. Namely,

$$Q(t_0, U) = \sum_{i=1}^{n} q(t_i; t_0), \quad t_i = iU/n, \quad U \in (0, T],$$

where for $i = 1, 2, \ldots, n$,

$$q(t_i; t_0) = \frac{N(a(t_i; t_0)) - \sum_{j=1}^{i-1} q(t_{j-\frac{1}{2}}; t_0) N(b(t_i; t_{j-\frac{1}{2}}))}{N(b(t_i; t_{i-\frac{1}{2}}))}$$

$$a(t_i; t_0) = -\frac{M(t_i, T|X_0, r_0)}{\sqrt{S(t_i|X_0, r_0)}}$$

$$b(t_i; t_j) = -\frac{M(t_i, T|X_{t_j})}{\sqrt{S(t_i|X_{t_j})}}$$

and where the sum on the RHS of (22) is defined to be zero when $i = 1$, and

$$M(t, T|X_0, r_0) \equiv E_{0}^{F_{t}}[\ln X_{t}]; \quad S(t|X_0, r_0) \equiv \text{Var}_{0}^{F_{t}}[\ln X_{t}];$$

$$M(t, T|X_{u}, r_0) = M(t, T|X_0, r_0) - M(u, T|X_0, r_0) \frac{\text{Cov}_{0}^{F_{t}}[\ln X_{t}, \ln X_{u}]}{S(u|X_0, r_0)}, \quad u \in (t_0, t)$$

Notice that we follow CDG to discretize at $t_{j-\frac{1}{2}}, j = 1, \ldots, i - 1$, on the RHS of (22). In order to implement this approach, it is sufficient to know $M(t, T|X_0, r_0)$ and $\text{Cov}_{0}^{F_{t}}[\ln X_{t}, \ln X_{u}], \forall u \leq t \leq T$.

### A.1 The Collin-Dufresne and Goldstein Model

In the CDG model,

$$\mu_{k}(r_{t}, \ln X_{t}) = \kappa_{t} [\ln X_{t} - \nu - \phi r_{t} - \theta]$$

where $\kappa_{t}$, $\nu$, and $\phi$ are constants. The above approach is used in CDG to implement the one-factor version of their model (see their Proposition 1). To implement their two-factor model using the same approach, consider under the $T$-forward measure:

$$d \ln X_{t} = [(1 + \phi \kappa_{t})r_{t} + \kappa_{t}\nu - \kappa_{t} \ln X_{t} - \rho \sigma_{\nu} \sigma_{r} B(t, T)] dt + \sigma_{\nu} dZ_{t}^{F_{t}}$$

$$dr_{t} = (\alpha - \beta r_{t} - \sigma_{r}^{2} B(t, T)) dt + \sigma_{r} dW_{t}^{F_{t}}$$
where

\[ \bar{\nu} \equiv (\nu - \phi \theta) - (\delta + \sigma_v^2/2)/\kappa_t \]  
\[ B(t, T) = \frac{1}{\beta} \left( 1 - e^{-\beta(T-t)} \right) \]  

Integrating (30) and (31) from \( t_0 \) to \( t \) yields

\[ e^{\kappa t} \ln X_t = \ln X_0 + \bar{\nu} \left( e^{\kappa t} - 1 \right) + \int_0^t \left( 1 + \phi \kappa_t \right) r_u - \rho \sigma_v \sigma_r B(t, T) \int_0^t e^{\kappa u} du \]

\[ + \int_0^t \sigma_v e^{\kappa u} dZ_u^v \]  

\[ r_t = r_0 e^{-\beta t} + \left( \frac{\alpha}{\beta} - \frac{\sigma_r^2}{\beta^2} \right) (1 - e^{-\beta t}) + \frac{\sigma_r^2}{2 \beta^2} e^{-\beta t} (e^{\beta t} - e^{-\beta t}) \]

\[ + \sigma_r e^{-\beta t} \int_0^t e^{\beta u} dW_u \]  

\[ = E_0^{\mathbb{P}_T} [r_u] + \sigma_r e^{-\beta t} \int_0^t e^{\beta u} dW_u \]  

(35)

It follows from (34) that

\[ e^{\kappa t} E_0^{\mathbb{P}_T} [\ln X_t] = \ln X_0 + \bar{\nu} \left( e^{\kappa t} - 1 \right) + \int_0^t \left( 1 + \phi \kappa_t \right) e^{\kappa u} E_0^{\mathbb{P}_T} [r_u] du \]

\[ - \rho \sigma_v \sigma_r \beta \left[ \frac{e^{\kappa t} - 1}{\kappa_t} - \frac{e^{\beta T} e^{(\kappa + \beta) t} - 1}{\kappa_t + \beta} \right] \]  

(36)

Eq. (35) can then be used to evaluate explicitly the integral on the RHS of (36) and hence to arrive at \( M(t, T|X_0, r_0) \).

To calculate \( \text{Cov}_0^{\mathbb{P}_T} [\ln X_t, \ln X_u], \) we have from Eqs. (34) and (35) that

\[ \text{Cov}_0^{\mathbb{P}_T} [\ln X_t, \ln X_u] e^{\kappa (t+u)} = \]

\[ \sigma_v^2 E_0^{\mathbb{P}_T} \left[ \int_0^t e^{\kappa v} dZ_v^v \int_0^u e^{\kappa v} dZ_v^v \right] \]  

(\( \equiv I_1 \))

\[ + \sigma_v (1 + \phi \kappa_t) E_0^{\mathbb{P}_T} \left[ \int_0^t e^{\kappa v} dZ_v^v \int_0^u e^{\kappa v} r_v dv \right] \]  

(\( \equiv I_2 \))

\[ + \sigma_v (1 + \phi \kappa_t) E_0^{\mathbb{P}_T} \left[ \int_0^u e^{\kappa v} dZ_v^v \int_0^t e^{\kappa v} r_v dv \right] \]  

(\( \equiv I_3 \))

\[ + (1 + \phi \kappa_t)^2 \text{Cov}_0^{\mathbb{P}_T} \left[ \int_0^t e^{\kappa v} r_v dv, \int_0^u e^{\kappa v} r_v dv \right] \]  

(\( \equiv I_4 \))

37
One can show that $\forall t \geq u$,
\[
I_1 = \frac{\sigma_v^2}{2\kappa_t} (e^{2\kappa_t u} - 1)
\]
\[
I_2 = (1 + \phi_t) \frac{\rho \sigma_v \sigma_r}{\kappa_t + \beta} \left[ \frac{e^{2\kappa_t u} - 1 - e^{(\kappa_t - \beta)u} - 1}{2\kappa_t} \frac{1 - e^{(\kappa_t - \beta)t}}{\kappa_t - \beta} + \frac{e^{2\kappa_t u} - 1}{2\kappa_t} + e^{(\kappa_t + \beta)u} \frac{e^{(\kappa_t - \beta)t} - e^{(\kappa_t - \beta)u}}{\kappa_t - \beta} \right]
\]
\[
I_3 = (1 + \phi_t) \frac{\rho \sigma_v \sigma_r}{\kappa_t + \beta} \left[ \frac{1 - e^{(\kappa_t - \beta)t}}{\kappa_t - \beta} \frac{1 - e^{(\kappa_t - \beta)u} - 1}{\kappa_t - \beta} + \frac{e^{(\kappa_t + \beta)u} - 1}{\kappa_t - \beta} \right]
\]
\[
I_4 = (1 + \phi_t)^2 \frac{\sigma_r^2}{2\beta} \left[ - \frac{\beta e^{2\kappa_t u} - 1}{\kappa_t} + \frac{1}{\kappa_t^2 - \beta^2} \left( 1 - 2e^{(\kappa_t - \beta)u} + e^{2\kappa_t u} \right) \right]
\]

### A.2 The Longstaff-Schwartz Model

LS can be nested within CDG. The LS price of a defaultable bond can be obtained by setting $\kappa_t$ to zero in CDG. The resultant formulas are omitted here for brevity.

### B Formulas of Real Default Probability

Assume that under the physical measure denoted by $\mathbb{P}$, one has
\[
\begin{aligned}
&d \ln X_t = [(r_t + \lambda_v^v - \delta - \sigma_v^2/2 - \mu_k(r_t, \ln X_t)]dt + \sigma_v dZ_t \\
&dr_t = \kappa_t (\tilde{\theta} - r_t) dt + \sigma_r dW_t \equiv (\alpha_t - \beta r_t) dt + \sigma_r dW_t \\
&d\lambda^\nu_t = \kappa_\lambda (\tilde{\lambda^\nu} - \lambda^\nu_t) dt + \sigma_\lambda dY_t \equiv (\alpha_\lambda - \beta_\lambda \lambda_t^\nu) dt + \sigma_\lambda dY_t
\end{aligned}
\]  

where $\alpha_\lambda, \beta_\lambda$, and $\sigma_\lambda$ are constant, $Y$ is a one-dimensional standard Brownian motion under $\mathbb{P}$, and $Z$ and $Y$ have a constant correlation coefficient of $\rho_{\lambda v}$. Recall from the assumption of a constant interest risk premium that
\[
\alpha = \alpha_t + \lambda_t \sigma_r
\]

Since the drift function in (37) is affine in $\lambda^\nu$, which, like $r$, is a Gaussian process, Eqs.(22)-(28) can still be used to calculate the $\mathbb{P}$-default probability provided that all the expectations are done under $\mathbb{P}$.
B.1 Constant Asset Risk Premium

Assume $\lambda_t^v = \lambda_0^v \forall t$, a constant. It follows from (37) and (38) that (under the $\mathbb{P}$ measure)

$$e^{\kappa_t} \ln X_t = \ln X_0 + (\lambda_0^v + \bar{\nu} \kappa_t) e^{\kappa_t} - 1 + \int_0^t (1 + \phi \kappa_t) r_u e^{\kappa_t} du + \int_0^t \sigma_v e^{\kappa_t} dW_u \quad (41)$$

It is easy to see from (40) and (41) that

$$Cov_0[\ln X_t, \ln X_u] = Cov_{\mathbb{P}}[\ln X_t, \ln X_u]$$

$$e^{\kappa_t} E_0[\ln X_t] = \ln X_0 + (\lambda_0^v + \bar{\nu} \kappa_t) e^{\kappa_t} - 1 + \int_0^t (1 + \phi \kappa_t) e^{\kappa_t} E_0[r_u] du
\begin{align*}
\quad + (1 + \kappa \phi) \left( r_0 - \frac{\bar{\kappa}}{\beta} \right) \frac{e^{(\kappa_t - \beta) t} - 1}{\kappa_t - \beta}
\end{align*} \quad (43)$$

B.2 Mean-Reverting Asset Risk Premium

For simplicity, we assume in this subsection that the interest rate is constant. In this case, $\phi$ is zero.

$$e^{\kappa_t} \ln X_t = \ln X_0 + (r_0 + \bar{\nu} \kappa_t) e^{\kappa_t} - 1 + \int_0^t \lambda_0^v e^{\kappa_t} du + \int_0^t \sigma_v e^{\kappa_t} dZ_u \quad (44)$$

$$\lambda_t^v = \lambda_0^v e^{-\beta \lambda t} + \frac{\alpha \lambda}{\beta \lambda} (1 - e^{-\beta \lambda t}) + \sigma_\lambda e^{-\beta \lambda t} e^{\beta \lambda t} dY_u \quad (45)$$

It is easy to see from (40) and (41) that

$$e^{\kappa_t} E_0[\ln X_t] = \ln X_0 + \left( r_0 + \bar{\nu} \kappa_t + \frac{\alpha \lambda}{\beta \lambda} \right) e^{\kappa_t} - 1 + \left( \lambda^v - \frac{\alpha \lambda}{\beta} \right) \frac{e^{(\kappa_t - \beta) t} - 1}{\kappa_t - \beta} \quad (46)$$

C Relating Equity Premium to Asset Risk Premium

In this section, we briefly describe how we relate asset premium to equity premium. We use the case of a semi-annual coupon bond for illustration.
Given the asset premium $\lambda^v$, the following formula is used to back out the equity premium $\lambda^e$

$$\lambda^v V_t = \lambda^e (V_t - B_t) + \lambda^b B_t$$  

(47)

where $\lambda^b$ denotes the bond risk premium. To relate $\lambda^v$ to $\lambda^e$, we need to know $\lambda^b$.

Below we use a simple method to estimate the risk premium of a risky bond. Let $\nu_{t_0, t_i}$ denote the cumulative (real) default probability over $[0, t_i]$, where $i$ indexes the coupon dates. Let $y_b$ denote the expected rate of return of a bond. We have

$$B_0 = \sum_{i=1}^{2T} \frac{\nu_{t_0, t_i}(1 - u_{0, t_i}) + w \frac{C}{2} u_{0, t_i}}{(1 + y_b/2)^i} + \frac{F(1 - u_{0, T}) + w F u_{0, T}}{(1 + y_b/2)^{2T}}$$

(48)

$$= \sum_{i=1}^{2T} \frac{\nu_{t_0, t_i}(1 - w) u_{0, t_i}}{(1 + y_b/2)^i} + \frac{F[1 - (1 - w)u_{0, T}]}{(1 + y_b/2)^{2T}}$$

where $C$ is the annual coupon payment and $w$ is the recovery rate. Given the bond price, the above equation can be used to solve for $y_b$. The spread of $y_b$ over a comparable default-free bond is then used as a proxy for the risk premium of a bond.
Table 1: The Base Case: Parameter Value Choices

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
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<th>(9)</th>
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<td>Credit rating</td>
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<td>Equity premium (%)</td>
<td>Default rates (%)</td>
<td>Recovery rate (%)</td>
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</table>

Table 1 shows the set of base parameter values used in numerical analysis. Data on leverage ratio are from Standard & Poor’s (1999). Equity premia estimates are based on regression results in Bhandari (1988). Historical default rates and recovery rate are from Moody’s report by Keenan, Shtogrin, and Sobehart (1999). Average yield spreads for investment-grade bonds are based on the monthly Lehman bond index data from 1973-1993. (The Lehman Bond Index data does go beyond 1993, but we choose the 1973-1993 time period so that the time period for our bond yield data is commensurate with the time period of Moody’s default data.) Average yield spreads for non-investment-grade bonds are based on Altman (1990).
Table 2: The Base Case: 10-Year Bonds with the Longstaff-Schwartz Model

Panel A: Default boundary $V^* = F$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Premium (%)</th>
<th>Cum. Default Probability (%)</th>
<th>Asset Volatility (%)</th>
<th>Asset Premium (%)</th>
<th>Credit Spread (bp)</th>
<th>Yield Spread (bp)</th>
<th>% of Default</th>
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<td>0.77</td>
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Panel B: Default boundary $V^* = 0.6F$

<table>
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<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Premium (%)</th>
<th>Cum. Default Probability (%)</th>
<th>Asset Volatility (%)</th>
<th>Asset Premium (%)</th>
<th>Credit Spread (bp)</th>
<th>Yield Spread (bp)</th>
<th>% of Default</th>
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<tr>
<td>Aaa</td>
<td>13.1</td>
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<td>0.77</td>
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<td>0.99</td>
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<td>1.55</td>
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Table 2 reports credit yield spreads of defaultable coupon bonds over default-free bonds computed using the Longstaff and Schwartz (1995) model. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads due to credit risk. Coupon rate is 8.162%. The payout ratio $\delta = 6\%$. The base parameters for the interest rate process are $r_0 = 8\%, \kappa_r = 0, \theta = 0.113, \sigma_r = 0,$ and $\lambda'_d = -0.248$. Recovery rate is fixed at 51.31%.
Table 3: Four-Year Bonds with the Longstaff-Schwartz Model

Panel A: Default boundary $V^* = F$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob. (%)</th>
<th>Cum. Asset (%</th>
<th>Asset Risk Prem (%)</th>
<th>Cache Credit (%)</th>
<th>Avg Yield (%)</th>
<th>Spread (bp)</th>
<th>% of Default</th>
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</thead>
<tbody>
<tr>
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Panel B: Default boundary $V^* = 0.6F$

<table>
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<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob. (%)</th>
<th>Cum. Asset (%</th>
<th>Asset Risk Prem (%)</th>
<th>Cache Credit (%)</th>
<th>Avg Yield (%)</th>
<th>Spread (bp)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
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<td>0.04</td>
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<td>4.95</td>
<td>1.1</td>
<td>55</td>
<td>2.1</td>
<td></td>
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<td>5.60</td>
<td>0.23</td>
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<td>4.90</td>
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<td>5.99</td>
<td>0.35</td>
<td>29.8</td>
<td>4.85</td>
<td>9.9</td>
<td>96</td>
<td>10.3</td>
<td></td>
</tr>
<tr>
<td>Baa</td>
<td>43.3</td>
<td>6.55</td>
<td>1.24</td>
<td>28.9</td>
<td>4.91</td>
<td>32.0</td>
<td>158</td>
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<td>8.51</td>
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<td>5.29</td>
<td>172.3</td>
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<td>6.25</td>
<td>445.7</td>
<td>352</td>
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</tr>
</tbody>
</table>

Table 3 reports credit yield spreads of defaultable coupon bonds over default-free bonds computed using the Longstaff and Schwartz (1995) model. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulting credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio $\delta = 6\%$. The base parameters for the interest rate process are $\gamma = 8\%, \kappa_r = 0, \theta = 0.113, \sigma_r = 0,$ and $\lambda_0 = -0.248$. Recovery rate is fixed at 51.31%.
Table 4: Stochastic Interest Rate with the LS Model

Panel A: Default boundary $V^* = F$

<table>
<thead>
<tr>
<th>Rating</th>
<th>Leverage Ratio (%)</th>
<th>Eq. Prem (%)</th>
<th>Default Prob. (%)</th>
<th>Cum. Asset Prob.</th>
<th>Asset Volatility (%)</th>
<th>Prec. Asset Risk (%)</th>
<th>Calc Credit Spread (bp)</th>
<th>Avg Yield Spread (bp)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
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<td>5.38</td>
<td>0.77</td>
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<td>0.99</td>
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<td>4.51</td>
<td>9.5</td>
<td>91</td>
<td>10.5</td>
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<td>5.99</td>
<td>1.55</td>
<td>17.6</td>
<td>4.25</td>
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<td>12.9</td>
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<td>4.39</td>
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<td>4.07</td>
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<tr>
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<td>7.30</td>
<td>20.63</td>
<td>20.4</td>
<td>4.32</td>
<td>168.4</td>
<td>299</td>
<td>56.3</td>
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</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
<td>43.91</td>
<td>23.5</td>
<td>5.11</td>
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<td>408</td>
<td>96.1</td>
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</table>

Panel B: Default boundary $V^* = 0.6F$

<table>
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<th>Rating</th>
<th>Leverage Ratio (%)</th>
<th>Eq. Prem (%)</th>
<th>Default Prob. (%)</th>
<th>Cum. Asset Prob.</th>
<th>Asset Volatility (%)</th>
<th>Prec. Asset Risk (%)</th>
<th>Calc Credit Spread (bp)</th>
<th>Avg Yield Spread (bp)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
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<td>0.77</td>
<td>31.5</td>
<td>4.99</td>
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<td></td>
</tr>
<tr>
<td>Aa</td>
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<td>5.60</td>
<td>0.99</td>
<td>27.5</td>
<td>4.95</td>
<td>8.6</td>
<td>91</td>
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<td>5.99</td>
<td>1.55</td>
<td>24.5</td>
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<td>14.5</td>
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<td>11.8</td>
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<td>24.7</td>
<td>5.05</td>
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<td>7.30</td>
<td>20.63</td>
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<td>5.47</td>
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<td>299</td>
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<td>83.8</td>
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</tbody>
</table>

Table 4 reports results for the base case plus a stochastic interest rate, implemented by using the Longstaff and Schwartz (1995) model. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio $\delta = 6\%$. The base parameters for the interest rate process are $r_0 = 8\%, \kappa_r = 0.226, \theta = 0.113, \sigma_r = 4.68\%$, and $\lambda_0 = -0.248$. The correlation coefficient $\rho_{r,V}$ is chosen to be $-0.25$. Recovery rate is fixed at 51.31%.
Table 5: Endogenous Default with the Leland-Toft Model

<table>
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<th>Target</th>
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<tbody>
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<td>(1)</td>
<td>(2)</td>
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<tr>
<td>Credit Rating</td>
<td>Leverage Ratio (%)</td>
<td>Equity Prem (%)</td>
</tr>
<tr>
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<td>13.1</td>
<td>5.38</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
</tr>
<tr>
<td>Baa</td>
<td>43.3</td>
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<td>7.30</td>
</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
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</tbody>
</table>

Table 5 reports credit yield spreads of defaultable coupon bonds over default-free bonds computed using the Leland and Toft (1996) model. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio ($F/V$) for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8%. The payout ratio $\delta = 6\%$. The interest rate $r_0 = 8\%$. Recovery rate is fixed at 51.31\%. 

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Table 6: Strategic Default with the Anderson-Sundaresan-Tychon and Mella-Barral-Perraudin Models

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio</th>
<th>Equity Prem</th>
<th>Default Probability (%)</th>
<th>Fixed Distress Cost (%)</th>
<th>Recovery Rate (%)</th>
<th>Asset Volatility (%)</th>
<th>Asset Risk Prem (%)</th>
<th>Calc Credit Spread (bp)</th>
<th>Avg Spread (bp)</th>
<th>% of Spread due to Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>0.77</td>
<td>2.40</td>
<td>54.64</td>
<td>25.00</td>
<td>4.73</td>
<td>31.22</td>
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</tr>
<tr>
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<td>5.60</td>
<td>0.99</td>
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<td>22.00</td>
<td>4.52</td>
<td>33.33</td>
<td>91</td>
<td>36.6</td>
</tr>
<tr>
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<td>5.99</td>
<td>1.55</td>
<td>17.23</td>
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<td>19.00</td>
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<td>4.39</td>
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<td>100.00</td>
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<td>4.10</td>
<td>59.41</td>
<td>194</td>
<td>30.6</td>
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</tbody>
</table>

Table 6 reports credit yield spreads of defaultable coupon bonds over default-free bonds computed using the Anderson-Sundaresan-Tychon (1996) and Mella-Barral and Perraudin (1997) models. The setup is otherwise identical to that in the base case. Column 2 through 4 show respectively the target firm leverage ratio \( F/V \), the target equity risk-premium, and the target cumulative real default probability over a given horizon. Columns 5 through 8 represent respectively the implied fixed bankruptcy cost \( K \) (normalized by the initial firm value \( V_0 \)), recovery rate, asset volatility, and asset risk-premium. Columns 9-11 report respectively the calculated credit spreads, the total yield spreads, and the percentage of total yield spreads due to credit risk. Coupon rate is 8%. The payout ratio \( \delta = 6\% \). The interest rate \( r_0 = 8\% \).
<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage</th>
<th>Equity Prem</th>
<th>Default Prob.</th>
<th>Cum. Asset</th>
<th>Credit Asset Vol.</th>
<th>Risk Prem</th>
<th>Spread Calc</th>
<th>Avg Spread</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
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<td>13.1</td>
<td>5.38</td>
<td>0.77</td>
<td>24.1</td>
<td>4.69</td>
<td>11.9</td>
<td>63</td>
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</tr>
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<td>5.60</td>
<td>0.99</td>
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<td>91</td>
<td>16.6</td>
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<td>1.55</td>
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<td>4.15</td>
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<td>6.55</td>
<td>4.39</td>
<td>22.7</td>
<td>3.94</td>
<td>50.5</td>
<td>194</td>
<td>26.0</td>
<td></td>
</tr>
<tr>
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<td>7.30</td>
<td>20.63</td>
<td>27.4</td>
<td>4.21</td>
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<td>8.76</td>
<td>43.91</td>
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<td>4.90</td>
<td>403.1</td>
<td>408</td>
<td>98.8</td>
<td></td>
</tr>
</tbody>
</table>

Table 7 reports credit yield spreads of defaultable coupon bonds over default-free bonds computed using the one-factor version of the Collin-Dufresne and Goldstein (2001) model. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio \(F/V\) for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8%. The payout ratio \(\delta = 6\%\). The interest rate \(r_0 = 8\%\). \(\kappa_I = 0.2\), and the long-term mean leverage is around 38%. Recovery rate is fixed at 51.31%. 

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Table 8: Predictable Equity Risk Premium

Panel A: Default boundary $V^* = F$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Target</th>
<th>Implied</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Leverage Ratio (%)</td>
<td>Equity Prem (%)</td>
</tr>
<tr>
<td>Aaa</td>
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<td>5.38</td>
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<tr>
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<td>5.60</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
</tr>
<tr>
<td>Baa</td>
<td>43.3</td>
<td>6.55</td>
</tr>
<tr>
<td>Ba</td>
<td>53.5</td>
<td>7.30</td>
</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
</tr>
</tbody>
</table>

Panel B: Default boundary $V^* = 0.6F$

<table>
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<tr>
<th>Credit Rating</th>
<th>Target</th>
<th>Implied</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Leverage Ratio (%)</td>
<td>Equity Prem (%)</td>
</tr>
<tr>
<td>Aaa</td>
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<tr>
<td>A</td>
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<td>5.99</td>
</tr>
<tr>
<td>Baa</td>
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<td>6.55</td>
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<tr>
<td>Ba</td>
<td>53.5</td>
<td>7.30</td>
</tr>
<tr>
<td>B</td>
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<td>8.76</td>
</tr>
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</table>

Table 8 reports credit yield spreads of defaultable coupon bonds over default-free bonds computed using the Longstaff and Schwartz (1995) model, with the additional consideration that the asset risk premium is time-varying and predictable. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio $\delta = 6\%$. The interest rate is fixed at $r_0 = 8\%$. Parameters in the risk-premium process are $\kappa_\lambda = 0.202$, $\sigma_\lambda = 3.1\%$, and $\rho_\lambda = -0.35$. Recovery rate is fixed at 51.31%.
Table 9: Sensitivity Analysis: Equity Premium Higher By 2%

Panel A: Default boundary $V^* = F$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob (%)</th>
<th>Cum. Asset Risk (%)</th>
<th>Implied Asset Risk (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Yield Spread (bps)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>7.38</td>
<td>0.77</td>
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Panel B: Default boundary $V^* = 0.6F$

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<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob (%)</th>
<th>Cum. Asset Risk (%)</th>
<th>Implied Asset Risk (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Yield Spread (bps)</th>
<th>% of Default</th>
</tr>
</thead>
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<td>0.99</td>
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<td>19.8</td>
<td>91</td>
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</tbody>
</table>

Table 9 reports results with the equity premia for all credit ratings assumed to be 2% higher than those assumed in the base case. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio $\delta = 6\%$. The base parameters for the interest rate process are $r_0 = 8\%$, $\kappa_r = 0$, $\theta = 0.113$, $\sigma_r = 0\%$, and $\lambda^0 = -0.248$. Recovery rate is fixed at 51.31%.
Table 10: Sensitivity Analysis: Different Leverage Ratios

Panel A: Leverage ratio = 0.7 * Base leverage ratio

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Target Leverage Ratio (%)</th>
<th>Target Equity Prem (%)</th>
<th>Target Default Prob. (%)</th>
<th>Cum. Asset Risk (%)</th>
<th>Implied Asset Risk (%)</th>
<th>Implied Credit Spread (bps)</th>
<th>Calc Yield Spread (bps)</th>
<th>Avg Spread Due to Credit Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>5.5</td>
<td>5.38</td>
<td>0.77</td>
<td>35.3</td>
<td>5.08</td>
<td>9.2</td>
<td>63</td>
<td>14.6</td>
</tr>
<tr>
<td>Aa</td>
<td>8.9</td>
<td>5.60</td>
<td>0.99</td>
<td>31.9</td>
<td>5.12</td>
<td>13.0</td>
<td>91</td>
<td>14.2</td>
</tr>
<tr>
<td>A</td>
<td>13.4</td>
<td>5.99</td>
<td>1.55</td>
<td>29.5</td>
<td>5.22</td>
<td>21.3</td>
<td>123</td>
<td>17.3</td>
</tr>
<tr>
<td>Baa</td>
<td>18.2</td>
<td>6.55</td>
<td>4.39</td>
<td>30.4</td>
<td>5.46</td>
<td>52.1</td>
<td>194</td>
<td>26.9</td>
</tr>
<tr>
<td>Ba</td>
<td>22.5</td>
<td>7.30</td>
<td>20.63</td>
<td>38.4</td>
<td>6.00</td>
<td>180.6</td>
<td>299</td>
<td>60.4</td>
</tr>
<tr>
<td>B</td>
<td>27.6</td>
<td>8.76</td>
<td>43.91</td>
<td>47.4</td>
<td>7.11</td>
<td>366.2</td>
<td>408</td>
<td>89.8</td>
</tr>
</tbody>
</table>

Panel B: Leverage ratio = 1.3 * Base leverage ratio

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Target Leverage Ratio (%)</th>
<th>Target Equity Prem (%)</th>
<th>Target Default Prob. (%)</th>
<th>Cum. Asset Risk (%)</th>
<th>Implied Asset Risk (%)</th>
<th>Implied Credit Spread (bps)</th>
<th>Calc Yield Spread (bps)</th>
<th>Avg Spread Due to Credit Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
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<td>5.38</td>
<td>0.77</td>
<td>29.6</td>
<td>4.84</td>
<td>10.6</td>
<td>63</td>
<td>16.9</td>
</tr>
<tr>
<td>Aa</td>
<td>16.5</td>
<td>5.60</td>
<td>0.99</td>
<td>25.7</td>
<td>4.70</td>
<td>15.2</td>
<td>91</td>
<td>16.7</td>
</tr>
<tr>
<td>A</td>
<td>24.9</td>
<td>5.99</td>
<td>1.55</td>
<td>22.6</td>
<td>4.57</td>
<td>25.1</td>
<td>123</td>
<td>20.4</td>
</tr>
<tr>
<td>Baa</td>
<td>33.8</td>
<td>6.55</td>
<td>4.39</td>
<td>22.2</td>
<td>4.56</td>
<td>60.3</td>
<td>194</td>
<td>31.1</td>
</tr>
<tr>
<td>Ba</td>
<td>41.8</td>
<td>7.30</td>
<td>20.63</td>
<td>27.4</td>
<td>4.99</td>
<td>203.5</td>
<td>299</td>
<td>68.1</td>
</tr>
<tr>
<td>B</td>
<td>51.2</td>
<td>8.76</td>
<td>43.91</td>
<td>32.8</td>
<td>5.86</td>
<td>410.5</td>
<td>408</td>
<td>100.6</td>
</tr>
</tbody>
</table>

Table 10 reports results for leverage ratios that are higher and lower than the base case assumption by 30%. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio δ = 6%. The base parameters for the interest rate process are r0 = 8%, κr = 0, θ = 0.113, σr = 0.%, and λ0 = -0.248. Recovery rate is fixed at 51.31%.
Table 11: Sensitivity Analysis: Different Asset Payout Ratios

Panel A: Payout ratio $\delta = 0$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob (%)</th>
<th>Cum. Asset (%)</th>
<th>Asset Risk (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Spread (bps)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>0.77</td>
<td>36.6</td>
<td>4.96</td>
<td>8.8</td>
<td>63</td>
<td>14.0</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
<td>0.99</td>
<td>33.1</td>
<td>4.91</td>
<td>12.2</td>
<td>91</td>
<td>13.4</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
<td>1.55</td>
<td>30.5</td>
<td>4.88</td>
<td>19.6</td>
<td>123</td>
<td>16.0</td>
</tr>
<tr>
<td>Baa</td>
<td>43.3</td>
<td>6.55</td>
<td>4.39</td>
<td>31.1</td>
<td>4.98</td>
<td>48.8</td>
<td>194</td>
<td>25.1</td>
</tr>
<tr>
<td>Ba</td>
<td>53.5</td>
<td>7.30</td>
<td>20.63</td>
<td>38.4</td>
<td>5.42</td>
<td>180.5</td>
<td>299</td>
<td>60.4</td>
</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
<td>43.91</td>
<td>46.1</td>
<td>6.38</td>
<td>377.9</td>
<td>408</td>
<td>92.6</td>
</tr>
</tbody>
</table>

Panel B: Payout ratio $\delta = 0.08$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob (%)</th>
<th>Cum. Asset (%)</th>
<th>Asset Risk (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Spread (bps)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>0.77</td>
<td>30.5</td>
<td>4.96</td>
<td>10.5</td>
<td>63</td>
<td>16.7</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
<td>0.99</td>
<td>26.7</td>
<td>4.91</td>
<td>15.1</td>
<td>91</td>
<td>16.6</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
<td>1.55</td>
<td>23.9</td>
<td>4.89</td>
<td>25.3</td>
<td>123</td>
<td>20.5</td>
</tr>
<tr>
<td>Baa</td>
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<td>6.55</td>
<td>4.39</td>
<td>23.9</td>
<td>5.02</td>
<td>60.7</td>
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<td>31.3</td>
</tr>
<tr>
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<td>7.30</td>
<td>20.63</td>
<td>30.2</td>
<td>5.52</td>
<td>198.4</td>
<td>299</td>
<td>66.4</td>
</tr>
<tr>
<td>B</td>
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<td>8.76</td>
<td>43.91</td>
<td>37.1</td>
<td>6.50</td>
<td>392.8</td>
<td>408</td>
<td>96.3</td>
</tr>
</tbody>
</table>

Table 11 reports results for two other different asset payout ratios: $\delta = 0$, and $\delta = 8\%$. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The base parameters for the interest rate process are $r_0 = 8\%$, $\kappa_r = 0$, $\theta = 0.113$, $\sigma_r = 0$, and $\lambda_0 = -0.248$. Recovery rate is fixed at 51.31\%.
Table 12: Sensitivity Analysis: Target Default Probability Higher By Half

Panel A: Default boundary $V^* = F$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Target Default Prob. (%)</th>
<th>Cum. Asset Risk Vol. (%)</th>
<th>Implied Asset Risk Prem (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Yield Spread (bps)</th>
<th>% of Default due to Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>1.16</td>
<td>28.4</td>
<td>4.69</td>
<td>15.3</td>
<td>63</td>
<td>24.3</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
<td>1.49</td>
<td>24.1</td>
<td>4.47</td>
<td>21.7</td>
<td>91</td>
<td>23.9</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
<td>2.33</td>
<td>20.6</td>
<td>4.20</td>
<td>35.5</td>
<td>123</td>
<td>28.9</td>
</tr>
<tr>
<td>Baa</td>
<td>43.3</td>
<td>6.55</td>
<td>6.59</td>
<td>19.8</td>
<td>4.10</td>
<td>85.1</td>
<td>194</td>
<td>43.9</td>
</tr>
<tr>
<td>Ba</td>
<td>53.5</td>
<td>7.30</td>
<td>30.95</td>
<td>25.6</td>
<td>4.60</td>
<td>296.0</td>
<td>299</td>
<td>99.0</td>
</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
<td>65.86</td>
<td>34.2</td>
<td>5.41</td>
<td>620.8</td>
<td>408</td>
<td>152.2</td>
</tr>
</tbody>
</table>

Panel B: Default boundary $V^* = 0.6F$

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Target Default Prob. (%)</th>
<th>Cum. Asset Risk Vol. (%)</th>
<th>Implied Asset Risk Prem (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Yield Spread (bps)</th>
<th>% of Default due to Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>1.16</td>
<td>33.4</td>
<td>4.96</td>
<td>13.6</td>
<td>63</td>
<td>21.6</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
<td>1.49</td>
<td>29.6</td>
<td>4.92</td>
<td>19.1</td>
<td>91</td>
<td>21.0</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
<td>2.33</td>
<td>26.9</td>
<td>4.90</td>
<td>31.1</td>
<td>123</td>
<td>25.3</td>
</tr>
<tr>
<td>Baa</td>
<td>43.3</td>
<td>6.55</td>
<td>6.59</td>
<td>27.6</td>
<td>5.05</td>
<td>75.2</td>
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<td>38.7</td>
</tr>
<tr>
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<td>7.30</td>
<td>30.95</td>
<td>36.9</td>
<td>5.59</td>
<td>263.6</td>
<td>299</td>
<td>88.2</td>
</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
<td>65.86</td>
<td>51.5</td>
<td>6.63</td>
<td>558.0</td>
<td>408</td>
<td>136.8</td>
</tr>
</tbody>
</table>

Table 12 reports results for the case where the target default probability for each credit rating is higher than the base case by 50% (of the base case amount). The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio $\delta = 6\%$. The base parameters for the interest rate process are $r_0 = 8\%, \kappa_r = 0, \theta = 0.113, \sigma_r = 0$. Recovery rate is fixed at 51.31%.
Table 13: Sensitivity Analysis: A Lower Recovery Rate at 45%

Panel A: Default boundary \( V^* = F \)

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob (%)</th>
<th>Cum. Asset Risk (%)</th>
<th>Implied Asset Risk (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Spread (bps)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>0.77</td>
<td>27.2</td>
<td>4.69</td>
<td>12.9</td>
<td>63</td>
<td>20.4</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
<td>0.99</td>
<td>23.1</td>
<td>4.46</td>
<td>18.5</td>
<td>91</td>
<td>20.3</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
<td>1.55</td>
<td>19.6</td>
<td>4.18</td>
<td>30.6</td>
<td>123</td>
<td>24.8</td>
</tr>
<tr>
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<td>6.55</td>
<td>4.39</td>
<td>18.5</td>
<td>4.06</td>
<td>73.8</td>
<td>194</td>
<td>38.0</td>
</tr>
<tr>
<td>Ba</td>
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<td>7.30</td>
<td>20.63</td>
<td>22.3</td>
<td>4.57</td>
<td>254.8</td>
<td>299</td>
<td>85.2</td>
</tr>
<tr>
<td>B</td>
<td>65.7</td>
<td>8.76</td>
<td>43.91</td>
<td>25.6</td>
<td>5.56</td>
<td>530.5</td>
<td>408</td>
<td>130.0</td>
</tr>
</tbody>
</table>

Panel B: Default boundary \( V^* = 0.6F \)

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Leverage Ratio (%)</th>
<th>Equity Prem (%)</th>
<th>Default Prob (%)</th>
<th>Cum. Asset Risk (%)</th>
<th>Implied Asset Risk (%)</th>
<th>Calc Credit Spread (bps)</th>
<th>Avg Spread (bps)</th>
<th>% of Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>13.1</td>
<td>5.38</td>
<td>0.77</td>
<td>32.1</td>
<td>4.96</td>
<td>11.3</td>
<td>63</td>
<td>17.9</td>
</tr>
<tr>
<td>Aa</td>
<td>21.2</td>
<td>5.60</td>
<td>0.99</td>
<td>28.4</td>
<td>4.91</td>
<td>16.0</td>
<td>91</td>
<td>17.6</td>
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<tr>
<td>A</td>
<td>32.0</td>
<td>5.99</td>
<td>1.55</td>
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<td>4.90</td>
<td>26.4</td>
<td>123</td>
<td>21.5</td>
</tr>
<tr>
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<td>6.55</td>
<td>4.39</td>
<td>25.8</td>
<td>5.03</td>
<td>64.3</td>
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<td>20.63</td>
<td>32.4</td>
<td>5.56</td>
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<td>43.91</td>
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<td>408</td>
<td>110.9</td>
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</table>

Table 13 reports results for a lower target default recovery rate of 45%. The setup is otherwise identical to that in the base case. Column 2 represents the firm leverage ratio for six rating categories. Columns 3 through 5 show respectively the target equity risk-premium, the total yield spreads, and the target cumulative real default probability over a given horizon. Columns 6 through 9 report respectively implied asset volatility, implied asset risk-premium, the resulted credit spreads, and the percentage of total yield spreads (shown in column 4) due to credit risk. Coupon rate is 8.162%. The payout ratio \( \delta = 6\% \). The base parameters for the interest rate process are \( r_0 = 8\% \), \( \kappa_r = 0 \), \( \theta = 0.113 \), \( \sigma_r = 0 \), and \( \lambda^D_0 = -0.248 \). Recovery rate is fixed at 51.31\%. 

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