

Optimal Debt Mix and Priority Structure: The Role of Bargaining Power*

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April 25, 2002

ABSTRACT

This paper examines the optimal priority structure and mix of bank and bond market debt within a continuous-time asset pricing framework. *Closed-form* expressions are derived for the values of renegotiable bank debt, non-renegotiable bond market debt, equity, and levered firm values. We show that the optimal debt structure hinges upon the *ex post* division of bargaining power between the firm and bank. The optimal debt structure for firms that are weak *vis-à-vis* the bank entails financing exclusively with bank debt. Strong firms find it optimal to issue a mix of bank and bond market debt, with the bank senior in the priority structure. The model explains: (i) why it is optimal for small firms to avoid public debt markets, (ii) why large firms employ mixed debt financing (iii) why banks are senior in the priority structure, and (iv) why firms shift from bank debt into a mixture of bond market and bank debt over their life-cycle. These predictions are generated within a tax shield-bankruptcy cost tradeoff model in which the only unique feature of banks is their ability to renegotiate.

JEL Codes: G13, G32, G33.

Keywords: Banking, Capital Structure, Priority Structure, and Contingent Claims Pricing.

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I. Introduction

In the years following the publication of the Modigliani and Miller (MM, 1958) irrelevancy result, debt tax shields and the costs of default or bankruptcy have been cited as important determinants of a firm's optimal capital structure. A common shortcoming of this literature is that it typically ignores lender heterogeneity. However, debt contracts are shaped in critical ways by the type of lender and the division of bargaining power between borrower and lender in the event of nonpayment of initially-promised amounts (a "renegotiation"). For instance, bank debt is typically *soft* in the sense that its terms can be renegotiated without necessarily incurring formal default or bankruptcy, while bond market debt is typically *hard*, in that renegotiation with dispersed creditors is costly or impossible and may well lead to formal default and/or bankruptcy.¹ Costs, both direct and indirect, are generally incurred in bankruptcy.² In addition, the outcome of the debt renegotiation process itself is influenced by the division of bargaining power between lenders and the firm whenever the firm chooses not to make the promised debt service. Renegotiation between the firm and lenders is also affected by the priority structure of debt, which affects the threat points of heterogeneous lenders.

This paper examines the effect of debt heterogeneity and priority structure on the value of the corporate tax shield, bankruptcy costs, and renegotiation costs within the context of a continuous-time asset pricing framework. We solve for the optimal priority structure and mix of renegotiable bank debt and non-renegotiable bond market debt. It is shown that *the optimal debt structure depends critically upon the ex post division of bargaining power between the firm and bank*. Throughout the paper, the term *ex ante* refers to times prior to the issuance of debt contracts, while *ex post* refers to times when the firm's debt structure is in place. By *optimal* financial structure, we mean the debt mix and priority structure that maximizes the *ex ante* value of all marketable claims on the firm.

We consider two polar cases regarding the distribution of bargaining power: when the firm is *strong*, in which case the firm can offer reduced debt service that just avoids the bank's demand

¹Bondholders acting atomistically may hold out and preclude desirable debt restructuring; cf. Gertner and Scharfstein (1991), Brown, James, and Mooradian (1993), Kahan and Tuckman (1993), and Morris and Shin (2001). Further, informational asymmetries can make Chapter 11 more attractive for individual bondholders than private debt reorganizations; cf. Giammarino (1991).

²See Warner (1977), Altman (1984), Summers and Cutler (1988), Weiss (1990), Franks and Torous (1989, 1994), and Andrade and Kaplan (1998).

for liquidation; or when the firm is *weak*, in which case the bank can demand payments which just avoid the firm's refusing to pay and voluntarily declaring bankruptcy. In particular, we address four related questions: (i) If the choice between bond market and bank debt is mutually exclusive, which should be chosen? (ii) If the firm may issue both bond market and bank debt, what is the optimal mix of the two? (iii) What is the optimal priority structure when multiple debt classes are issued? (iv) How does the *ex post* division of bargaining power between the firm and lending bank(s) affect the answer to each of these questions?³

These questions are addressed using a unified continuous-time asset pricing framework that bridges the gap between the Leland (1994) model, which treats non-renegotiable debt, and the model of Mella-Barral and Perraudin (MBP, 1997) which features renegotiable debt. Analytical solutions are derived for bank debt, bond market debt, equity, and levered firm values as functions of the endogenously determined variables (promised coupon payments and priority structure) and exogenous parameters (*ex post* bargaining power, underlying volatility, corporate tax rates, negotiation costs, and bankruptcy costs).

The model generates several predictions that are supported by empirical observation. First, *weak firms*, those ceding all bargaining power to banks in the event of renegotiation, find it optimal to finance exclusively with bank debt. That is, for weak firms, bank debt dominates any mix of bond market and bank debt, regardless of the priority structure under the mixed debt policy. This result holds in the absence of any notion of monitoring or certification by banks, transaction costs, economies of scale, and other common rationales for why small firms fail to tap public debt markets. Second, for *strong firms*, those with full bargaining power in renegotiations with banks, the optimal financial structure entails a mixture of bank and bond market debt. In particular, the analysis indicates that there is a complementarity effect between bank debt and bond market debt for strong firms. Thus, the model provides a rationale for the coexistence of bank debt (indirect or intermediated finance) and bond market debt (direct finance). Third, for reasonable parameter values, it is optimal for strong firms that optimally choose mixed debt to place bank debt senior. Intuitively, placing the bank senior in the priority structure raises the bank's reservation value, which thereby commits equity to higher debt service in renegotiations. The increase in debt

³Ex post bargaining power in our model is not ex ante contractible and therefore exogenous. Note that when renegotiations commence, both sides will want maximal bargaining power. Thus, a non-contracted "agreement" to be weak is incentive incompatible ex post.

service raises the value of both the tax shield and bankruptcy costs. We find that the former effect dominates for reasonable parameter values.

To the extent that one views young firms as having a weak *ex post* bargaining position *vis-à-vis* banks, and gaining bargaining strength as they mature, the model generates a *life-cycle hypothesis for debt structure*. Young firms begin by relying exclusively on bank debt. As they grow and gain bargaining power, they shift away from bank debt, placing more reliance on bond market debt. This prediction is consistent with observed financing patterns, and is not dependent on the notion of bank certification. In fact, the model predicts that even if a weak firm could tap public debt markets with fair pricing, it would be sub-optimal to do so.

Finally, when the choice between bank and bond market debt is mutually exclusive, we find that the optimal debt structure of strong firms depends on their level of bankruptcy costs and tax shield values. Interestingly, strong firms are *ceteris paribus* more likely to choose bond market over bank debt when bankruptcy costs are high. Taken together, these results suggest the more general conclusion that bank debt is relatively more desirable to firms *ex ante* when banks have a stronger bargaining position in renegotiations. For instance, pure bank debt financing is strictly preferred to any mixture of bond market and bank debt when banks are strong and firms are weak. When firms are strong and optimally seek a mixture of bank and bond market financing, bank debt seniority—which improves the threat point of the bank—is optimal *ex ante* for the firm. When firms are strong and financing is mutually exclusive, bank debt will be preferred when bankruptcy costs are low, since low bankruptcy costs raise the threat point of the bank in renegotiations.

The remainder of the paper is organized as follows. Section II contains a review of the related empirical and theoretical literature. Section III presents the model. The contingent claims on the levered firm are valued in Section IV. Section V analyzes the case of mutually exclusive financing choices and Section VI contains the analysis of the mixed debt firm. Section VII concludes.

II. Literature Review

A number of papers have incorporated strategic interactions between debt and equity as well as debt renegotiation in the context of contingent claims valuation models. Anderson and Sundaresan (1996) examine strategic debt service in a Cox-Ross-Rubinstein binomial-tree setting, while MBP

(1997) studies a continuous-time model of debt renegotiation. More recently, Fan and Sundaresan (2000) characterize two Nash bargaining formulations in which equity and debt bargain over the value of the assets of the firm (debt-equity swap) or over the value of the firm (strategic debt service).

Our model may be viewed as bridging the gap between Leland (1994) and MBP (1997). Leland (1994) represents a unified theory of optimal leverage when the firm may only issue non-renegotiable debt. Optimal leverage is chosen to maximize the value of tax shield benefits less bankruptcy costs. MBP (1997) analyze a firm that may only issue renegotiable debt, with bargaining power treated as a choice variable. In their continuous-time model, they show that strategic debt service eliminates direct bankruptcy costs as well as inefficient liquidation. This result restores the Modigliani-Miller irrelevance result of financial policy, since there are no tax benefits in their basic model. In the presence of tax shields, they argue that distributing all bargaining power to the bank is optimal.

Since the Leland (1994) and MBP (1997) papers analyze situations in which the firm issues only a single class of debt, the question of the optimal *mix* between bank and bond market debt is beyond the scope of their papers. Similarly, the question of the optimal priority structure is left unaddressed.⁴ The present paper stresses the notion that in many settings the distribution of *ex post* bargaining power must be taken as a *given* by the firm, and cannot be treated as a choice variable. Therefore, even in settings where a particular distribution of bargaining power may be said to dominate in the sense of resulting in higher firm valuations, the firm may be unable to commit to such a division of bargaining power, and must optimize subject to this constraint.

It has been well documented empirically that banks are generally senior in the priority structure. For example, Carey (1995) finds that in the 18,000 loans made between 1986 and 1993 and recorded in the Dealscan database, over 99 percent of all bank loans contain a senior priority clause, regardless of whether or not the borrower has bond market debt outstanding. Mann (1997) and Schwartz (1997) find that in addition to taking senior positions, banks also collateralize as much of their debt as possible and incorporate protective covenants limiting the ability of the firm to issue additional debt.

⁴Hackbarth and Leland (2001) analyze the role of priority structure of a firm issuing multiple classes of non-renegotiable bond market debt.

One of the contributions of this paper to the banking literature is that it suggests an alternative theory regarding the optimal debt mix and priority structure that rests upon a standard tax shield-bankruptcy cost trade-off story. The model does not invoke any notion of monitoring by the bank, costly state verification, or asset substitution problems. Yet, it generates many of the stylized facts that the banking literature has attempted to explain. This banking literature is discussed below.

Much of the theoretical banking literature attempts to explain why banks are senior. Diamond (1993a, 1993b) develops a model of both adverse selection and moral hazard, in which he obtains an optimal debt structure resulting from a trade-off between informational sensitivity and preservation of control rent. In his model, because of the non-assignable control rent, short-term lenders have excessive incentives to liquidate the firm at the intermediate date, which imposes a distortion on long-term lenders. Seniority permits short-term creditors to dilute the other claims and thereby moderate their incentives to liquidate the firm early. By invoking the assumption that renegotiation of bond market debt is very difficult, he finds that making short-term (bank) debt senior relative to long-term (bond market) debt improves the bank's incentive to monitor.

Rajan (1992) arrives at the opposite conclusion regarding priority structure. The main benefit of bank debt in his model is flexibility in times of distress. However, banks develop informational monopolies and can distort investment incentives by demanding a share of the rents from profitable projects as a condition for rolling over short-term loans. To weaken the bargaining power of informed lenders, the firm grants uninformed lenders higher priority. Welch (1997) analyzes the role of influence costs on the optimal priority structure of debt. Placing bank debt senior is optimal in his model since influence costs are lower when the strongest creditor, *ex post*, is given more power *ex ante*.

Repullo and Suarez (1998) abstract from Rajan's informational monopoly problem and conclude that the optimal priority rule entails placing informed lenders senior. Dewatripont and Tirole (1994) demonstrate that in order to prevent borrower moral hazard, it is best for control rights in bad states to be given to lenders rather than to equity holders since lenders have a more credible threat to liquidate the firm. Since equity in their model is indistinguishable from junior debt, one can draw many conclusions about the optimal debt and priority structure from their model. Rajan and Winton (1995) and Chemmanur and Fulghieri (1994) propose models of banks as delegated monitors, but neither analyzes the optimal debt mix or priority structure.

In a first effort to reconcile these two conflicting theories, Park (2000) develops a theory of optimal debt structure by explicitly modeling a costly monitoring technology in a model with asset substitution. Because the firm's contracting costs are reduced when lenders have stronger incentives to monitor, firms structure debt securities to maximize the lenders' incentives to monitor. This is achieved only if the delegated monitor can recover all of his monitoring costs from the lending relationship, which is feasible only if there are no lenders above him in the priority structure.

Winton (1995) analyzes the priority structure problem building on Townsend's (1979) costly state verification model. As in the original work, lenders are assumed to be able to commit themselves to verification when reported income is low, so that inspections occur even when banks know that the firm has truthfully reported low cash flow truthfully. Winton's model implies that junior lenders verify more often than senior lenders, which is consistent with Fama's (1990) intuition.

Diamond's (1993a, 1993b) theory does not provide a role for multiple lenders holding different debt securities of the firm. Extending the model of Hart and Moore (1998), Berglöf and von Thadden (1994) develop a rationale for multiple creditors and seniority structure by illustrating the role of bargaining power in a renegotiation game of symmetric information. The main insight is that the optimal debt contract drives a wedge between short-term and long-term lenders. Bolton and Scharfstein (1996) put forth a related model of the optimal number of creditors. In their paper, multiple lenders provides a commitment mechanism for the borrower due to the difficulty of renegotiating with multiple lenders of the same class.

Our model also generates a life-cycle hypothesis regarding optimal debt financing policy, which rests upon a rather different line of causation than that invoked in the existing banking literature. Diamond (1991) considers a setting in which banks ignore rents accruing to borrowers in late periods, resulting in suboptimal liquidation. On the other hand, young firms are able to establish a good reputation by borrowing from banks with monitoring skills. The Diamond model thus predicts that firms will use bond market debt only after a favorable track record is established via bank loans. Another closely related theory on the coexistence of financial intermediaries and bond market finance has been developed by Bolton and Freixas (2000) within an asymmetric information framework, absent the debt tax shield. Similar to our approach, these authors emphasize that banks provide value to firms since they are good at helping firms through times of financial distress.

However, the cost of intermediation creates a trade-off between bond market and bank finance. They also obtain the result that younger firms may be constrained to bank debt, while larger and more mature firms access both markets.

III. The Model

The underlying state variable X is the firm's EBIT, which is assumed to follow a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 > 0 \quad (1)$$

where μ and σ are known positive constants and W is a Wiener process. All agents are risk neutral, and there is a risk-free asset yielding a constant rate of return $r > \mu$. Within this setting, consider an arbitrary claim paying the state contingent flow $mX + k$. The value function (G) for that claim must satisfy the following ODE:

$$\frac{1}{2}\sigma^2 X^2 G''(X) + \mu X G'(X) - rG(X) + mX + k = 0. \quad (2)$$

The general solution to this ODE is:

$$G(X) = K_1 X^a + K_2 X^z + \frac{mX}{r - \mu} + \frac{k}{r}, \quad (3)$$

where $a < 0$ and $z > 1$ denote the roots of the quadratic:

$$Q(a) \equiv \frac{1}{2}\sigma^2 a^2 + (\mu - \frac{1}{2}\sigma^2)a - r = 0. \quad (4)$$

All of the contingent claims priced below have solutions of this form, with suitable boundary conditions pinning down unknown constants.

There is a linear tax at rate τ levied on corporate income, which is computed as EBIT less instantaneous debt service.⁵ It is never optimal to shut-down an unlevered firm, and the implied value of the unlevered corporation at any instant is given by V :

$$V(X) \equiv (1 - \tau)X/(r - \mu). \quad (5)$$

The firm may issue two classes of consol bond debt: bank debt, which carries a promised coupon flow b , and bond market debt which carries a promised coupon flow c . *Bond Market Debt*

⁵FUTURE: Simulate effect of non-linearities in tax schedule.

(BMD) may not be renegotiated, with failure to pay the promised coupon leading to immediate reorganization of the firm. In contrast, bank debt may be renegotiated, with the character of the debt service depending on the bargaining power of equity *vis-à-vis* the bank, as well as the parties' respective threat points. We distinguish between two types of firms. *Strong Firms* have full bargaining power in renegotiation and we label their bank debt commitments *Equity Bargaining Power Debt* (EBPD). *Weak Firms* cede all bargaining power to the bank in renegotiation and we label their bank debt commitments *Bank Bargaining Power Debt* (BBPD).

A central argument in this paper is that the optimal debt mix and priority structure depends critically on the *ex post* distribution of bargaining power between the firm (equity) and the bank. We do not formally derive the relationship between bargaining power and underlying firm/bank characteristics. Rather, the model treats the distribution of bargaining power as a “fact of life” for the business. In our view, it is most natural to think of weak firms as being small corporations that are possibly reliant on the bank for a portfolio of services. Strong firms are best viewed as those that are large (perhaps too large to fail from the bank’s perspective), sophisticated, and aggressive in negotiations due to reputational considerations.

For the remainder of the paper, the term *default states* refers to situations in which either BMD is not serviced or in which the firm and the bank fail to reach agreement on debt service, bringing about immediate *reorganization*, with the proceeds from sale of the firm being paid to creditors in accordance with the Absolute Priority Rule (APR).⁶ States in which equity fails to make the *promised* payment (*b*) and enters into successful renegotiations with the bank, while any BMD is still serviced, are referred to as *renegotiation states*.

Following Leland (1994), there is a deadweight loss in the event of default, with the parameter $\alpha > 0$ representing *net default costs* as a percentage of unlevered firm value. The value of the reorganized (liquidated) firm is denoted L , which is equal to:

$$L(X) \equiv (1 - \alpha)V(X) \tag{6}$$

It should also be noted that in using the term “default costs,” we have in mind the broad concept of the term which includes more than the fees paid to the attorneys as they carve up the

⁶The model is general enough to encompass deviations from APR, although discussion of the optimal contractual priority structure would then necessitate some assumption regarding the behavior of the courts.

carcass. Rather, default costs include: superior ability of existing management in running the firm; distraction of management; impaired ability to contract; and suboptimal investments in firm-specific human capital. It should be noted that it is not assumed that the reorganized firm does not issue debt. Rather, the creditors taking over the firm are assumed to revalue optimally, implying that α is expressed as net of the tax shield benefit associated with the reorganized firm. While the model is general enough to allow for a negative α parameter, empirical evidence on recovery rates on defaulted debt as a percentage of par supports the assumption that $\alpha > 0$. See Ross (1997) for a discussion.

To capture the fact that debt renegotiation is costly, there is a proportional deadweight cost from renegotiation:

$$\text{Flow Renegotiation Cost} = \delta[b - s(X)]^+, \quad (7)$$

where s represents the state contingent debt service to the bank, and $\delta \geq 0$.

A. Bank Debt Renegotiation

A.1. Equity Bargaining Power Debt

Consider first the dynamics of renegotiation for EBPD. In renegotiation states, equity makes take-it-or-leave-it offers to the bank, pushing the bank down to its reservation value.⁷ The bank's EBIT contingent reservation value is denoted R , and is equal to the value the bank would receive in the event of reorganization. We model debt priority with the parameter $\eta \in [0, 1]$, which represents the percentage of reorganization value paid to the bank. There are a number of advantages to this parametric approach to modeling debt priority: *(i)* It subsumes both Senior and Equal Priority bank debt; *(ii)* It covers a broad range of sharing rules, some of which may entail court imposed deviations from APR, which are then capitalized into asset prices in a rational expectations equilibrium; *(iii)* It yields closed-form solutions for asset prices.

Assumption 1: $R(X) \equiv \eta L(X)$.

⁷The analysis in this section is similar to that of MBP (1997), although we employ a somewhat different optimal control approach.

The priority structure for many debt contracts when APR is followed entails non-linearity in the reservation value function:

$$\begin{aligned}
\text{Bank Senior} &\Rightarrow R(X) = \min\{b/r, L(X)\} \\
\text{Equal Priority} &\Rightarrow R(X) = \min\{b/r, (b/(b+c))L(X)\} \\
\text{Bank Junior} &\Rightarrow R(X) = \min\{b/r, \max\{0, L(X) - c/r\}\}.
\end{aligned} \tag{8}$$

However, for EBPD, the assumed linear sharing rule in default subsumes both the Bank Senior and Equal Priority cases shown above.⁸ The reasoning is as follows:

- For EBPD, if there is no renegotiation, then the analysis of the mixed debt firm issuing the pair (b, c) is subsumed in the case of the firm financing exclusively with BMD carrying the coupon $b + c$.
- For Senior EBPD, if there is renegotiation, it must be the case that $L(X) < b/r$ on the renegotiation region, otherwise the bank rejects any offer less than b . Therefore, setting $\eta = 1$ produces the correct debt service and correct valuations of all contingent claims on the firm.
- For Equal Priority EBPD, if there is renegotiation, it must be the case that $(b/(b+c))L(X) < b/r$ on the renegotiation region, otherwise the bank rejects any offer less than b . Therefore, setting $\eta = b/(b+c)$ produces the correct debt service and correct valuations of all contingent claims on the firm.
- Junior bank debt (To Be Added). Results thus far suggest this is suboptimal.

The *incentive compatible* (IC) strategic debt service function for EBPD is denoted \hat{s} . It is inappropriate to think of \hat{s} as being optimal from an *ex ante* perspective. Rather, \hat{s} is optimal for equity, *ex post*, given that in order for the bank to accept an offer, the value of its claim cannot be below $R(X)$. The essence of the problem is that even if \hat{s} is not optimal *ex ante*, equity cannot commit to an alternative stream of promised payments. This points out the key difference between EBPD and BMD in the model. EBPD offers financial flexibility, which preserves value in terms of reducing expected bankruptcy costs and maintaining the ability of the firm to exploit existing

⁸The priority structure for BBPD is shown to be moot, as mixed debt finance is dominated.

tax shields. BMD has superior commitment value, keeping the flow tax shield benefit high in all non-default states.

Below, the term $X_d(b, c)$ refers to the endogenous default-triggering EBIT level that is chosen by equity for an arbitrary mix of EBPD and BMD, with the shorthand X_d often being used for the sake of brevity. Letting E denote the value of equity, the Bellman equation in non-default states ($X > X_d$) is:⁹

$$rE(X) = \max_{s(X) \in S(X)} (1 - \tau) [X - c - s(X)] - \delta[b - s(X)]^+ + \mu X E'(X) + \frac{1}{2} \sigma^2 X^2 E''(X). \quad (9)$$

where:

$$S(X) \equiv \{s(X) : s(X) < b \Rightarrow B(X) \geq R(X)\}. \quad (10)$$

Note that the state-contingent feasible set S simply represents the set of all strategic debt payments such that if the promised coupon is not paid, the value of bank debt is weakly above the reservation value. Dropping terms on the right-side of the Bellman that do not involve the strategic debt service term, equity's problem reduces to:

$$\hat{s}(X) \equiv \arg \min_{s(X) \in S(X)} s(X)(1 - \tau) + \delta[b - s(X)]^+. \quad (11)$$

This expression (11) indicates that the IC debt service under EBPD is that which minimizes the sum of the after-tax cost of debt service and renegotiation costs, subject to the constraint that debt service offers below b are accepted by the bank.

For the remainder of the paper, it is assumed that δ is sufficiently low so that the total cost of debt service, inclusive of renegotiation costs, is increasing in s .¹⁰

Assumption 2: $\delta < (1 - \tau)$

We now characterize the IC strategic debt service that obtains when the parameters of the problem are such that renegotiation occurs. The function B denotes the value of bank debt. Reference to (11) indicates that when equity has maximum bargaining power, the IC debt service entails

⁹As in MBP (1997), attention is confined to debt service functions that are piecewise right-continuous functions of the state variable. This avoids problems associated with deviations on sets of measure zero.

¹⁰If $\delta \geq (1 - \tau)$, then $\hat{s}(X) = b$ in all non-default states, which is subsumed in the analysis of the Bond Market Debt financed firm.

$B(X) = R(X)$ in the renegotiation region. Based on (2), the function B must satisfy the following ODE on the region (X_d, ∞) :

$$\frac{1}{2}\sigma^2 X^2 B''(X) + \mu X B'(X) - rB(X) + s(X) = 0. \quad (12)$$

Substituting in R and its derivatives for B implies that in the renegotiation region, strategic debt service for EBPD is:

$$\widehat{s}(X) = \eta(1 - \alpha)(1 - \tau)X. \quad (13)$$

The equation (13) is informative about the role that priority plays in the context of EBPD. When the bank is placed higher in the priority structure (η is increased) the flow debt service under EBPD is higher. Bargaining power is not changed, but the *threat point* of the bank in instantaneous renegotiations is higher, implying that while equity retains full bargaining power, it cannot push the bank as hard. Similarly, if bankruptcy costs (α) are lower, the IC debt service is higher due to the higher threat point of the bank.

The IC switch point for entering into renegotiation is denoted X_s , with arguments (b, c) . Letting the subscripts L and H denote claim values for $X < X_s$ and $X \geq X_s$, respectively, it follows that for EBPD:¹¹

$$B_L(X) = \eta(1 - \alpha)V(X) \quad \forall X \in [X_d, X_s] \quad (14)$$

$$B_H(X) = A_1 X^a + A_2 X^z + b/r \quad \forall X \in [X_s, \infty). \quad (15)$$

The unknowns for this problem are (A_1, A_2, X_s) , which are derived using the following boundary conditions, which represent value matching, smooth pasting and asymptotic conditions, respectively:

$$B_H(X_s) = \eta(1 - \alpha)V(X_s), \quad (16)$$

$$\frac{\partial B_H(X_s)}{\partial X} = \frac{\eta(1 - \alpha)(1 - \tau)}{r - \mu}, \quad (17)$$

$$\lim_{X \rightarrow \infty} B_H(X) = b/r. \quad (18)$$

The solution yields the following bank debt values and renegotiation threshold for EBPD:

$$B_L(X) = \eta(1 - \alpha)V(X) \quad (19)$$

¹¹The valuation formulas assume the existence of an endogenous default threshold. In the case of no default, the term X_d is simply replaced with a zero.

$$B_H(X) = \frac{b}{r} \left[1 - \frac{1}{1-a} \left(\frac{X}{X_s} \right)^a \right] \quad (20)$$

$$X_s = \left(\frac{r-\mu}{r} \right) \left(\frac{a}{a-1} \right) \left[\frac{b}{\eta(1-\alpha)(1-\tau)} \right]. \quad (21)$$

Finally, it is worth noting that the IC debt service function *must* exhibit a jump at the point X_s since:

$$\lim_{X \uparrow X_s} \widehat{s}(X_s) = \left(\frac{r-\mu}{r} \right) \left(\frac{a}{a-1} \right) b < b. \quad (22)$$

The existence of a default trigger for equity ($X_d > 0$) under EBPD depends on the existence of negotiation costs ($\delta > 0$) and/or a BMD coupon commitment ($c > 0$). Intuitively, for EBPD default is triggered when equity is unwilling to pay the flow costs associated with servicing BMD and renegotiation costs that are necessary to maintain its claim on positive cash-flows in good states. Absent fixed costs associated with BMD and negotiation costs, default never occurs because equity incurs no flow cost in keeping alive its option. Of course, if renegotiation costs are sufficiently high, renegotiation does not occur. Additionally, *for strong firms* financing with a mixture of EBPD and BMD, renegotiation does not occur if c is sufficiently high relative to b . This is because for EBPD, the optimal renegotiation threshold X_s as given in (21) does not depend on c , whereas X_d increases monotonically in c .

The following cases characterize the range of possible equilibria under EBPD:

- For the firm financed exclusively with EBPD, if $\delta = 0$, default never occurs. There exists an EBIT level X_s such that $\widehat{s}(X) = \eta(1-\alpha)(1-\tau)X$ on the interval $(0, X_s)$ and $\widehat{s}(X) = b$ for all $X \geq X_s$. This equilibrium is depicted in Figure 1.
- If δ or c is positive, $\exists X_d$ such that equity declares default.
- There is no renegotiation of EBPD for (δ, c) sufficiently high relative to b .
- For positive δ or c , and (δ, c) sufficiently low relative to b , $\exists X_d$ such that default occurs endogenously; $\exists X_s > X_d$ such that $\widehat{s}(X) = \eta(1-\alpha)(1-\tau)X$ on (X_d, X_s) and $\widehat{s}(X) = b$ for all $X \geq X_s$. This equilibrium is depicted in Figure 2.

A.2. Bank Bargaining Power Debt

With BBPD, in the event that equity fails to pay b and enters into renegotiation, it is the bank that makes take-it-or-leave-it offers. In renegotiation, equity is pushed down to its reservation value,

which is zero under limited liability. Since equity value is everywhere zero on the renegotiation region, satisfaction of (2) implies the cash-flow to equity is zero in renegotiation states. That is, equity pays all free cash-flow, $X - c$, to the bank. So long as the bank is willing to service the BMD coupon commitment (c) as well as pay any negotiation costs, reorganization is avoided. Therefore, under BBPD, it is the bank that chooses when to enter into reorganization, whereas under EBPD it is equity that chooses when to enter into reorganization. The bank's endogenous reorganization threshold under BBPD is denoted $X_b(b, c)$.

Lemma 1 proves that equity's endogenous *renegotiation* threshold under BBPD for the mixed debt policy (b, c) is identical to the *default* threshold chosen by equity facing the BMD coupon commitment $b + c$. Intuitively, from the perspective of equity, there is no difference between default and entering into renegotiations with a bank that has full bargaining power, since both outcomes generate a claim worth zero.

Lemma 1 *For Bank Bargaining Power Debt, equity's optimal switch point for entering into renegotiation satisfies $X_s(b, c) = X_d(0, b + c)$.*

Proof. The cash-flow to equity in non-renegotiation states under BBPD for the pair (b, c) is identical to that under exclusive BMD finance with coupon $\tilde{c} = b + c$. Default on BMD and renegotiation of the BBPD entail $E[X_s(b, c)] = E[X_d(0, \tilde{c})] = 0$. Result follows from the value matching and smooth pasting conditions being identical. ■

To characterize the optimal financial structure of the firm in light of the distribution of bargaining power, the constituent pieces of the firm must be priced. The value of the marketable claims of the levered firm (v) is equal to the sum of the values of equity (E), bond market debt (C), and bank debt (B):

$$v(X) = E(X) + C(X) + B(X). \quad (23)$$

Lemma 2 proves that the optimal financial policy for a weak firm that is able to issue BBPD entails financing with bank debt *exclusively*. *This result holds regardless of the assumed values of the parameters $(\tau, \alpha, \delta, \sigma)$ and regardless of the assumed priority structure on any proposed mixed debt financing arrangement.*

Lemma 2 *For the Weak Firm, financing exclusively with Bank Bargaining Power Debt dominates any policy involving nonzero Bond Market Debt, regardless of priority structure.*

Proof. The optimal debt policy maximizes v . Consider an arbitrary mixed debt policy (b, c) with an arbitrary priority rule f that maps L to payments to the bank. Now consider the pure BBPD policy setting $\tilde{b} = b + c$. From Lemma 2 it follows that the value of equity in the two firms is identical in all states. Let (\tilde{s}, \tilde{B}) denote the bank debt service and bank debt value under pure BBPD finance. The mixed debt firm is represented by (s, B, C) . Note that in non-reorganization states, $\tilde{s}(X) = s(X) + c$. On this same region, letting Φ be an indicator for renegotiation, flow renegotiation costs are identical for both firms and equal to $\Phi(X)\delta[\tilde{b} - \tilde{s}(X)]$. The stopping time problem for the bank under pure BBPD is:

$$\tilde{B}(X_0) \equiv \max_T \mathbf{E}_0 \left[\int_0^T e^{-rt} [\tilde{s}(X_t) - \Phi(X_t)\delta(\tilde{b} - \tilde{s}(X_t))] dt + e^{-rT} L(X_T) \right].$$

The stopping time problem for the bank under mixed debt is:

$$B(X_0) \equiv \max_T \mathbf{E}_0 \left[\int_0^T e^{-rt} [s(X_t) - \Phi(X_t)\delta(b - s(X_t))] dt + e^{-rT} f(L(X_T)) \right].$$

Let $T_m^* \equiv \inf\{t \geq 0 : X_t = X_b(b, c)\}$ represent the optimal stopping time for the bank under mixed debt. Using the adding up constraint for recoveries in default, the equality of renegotiation costs, and the fact that $\tilde{s}(X) = s(X) + c$ in non-reorganization states we have:

$$B(X_0) + C(X_0) = \mathbf{E}_0 \left[\int_0^{T_m^*} e^{-rt} [\tilde{s}(X_t) - \Phi(X_t)\delta(\tilde{b} - \tilde{s}(X_t))] dt + e^{-rT_m^*} L(X_{T_m^*}) \right] \leq \tilde{B}(X_0).$$

■

Lemma 2 suggests the following intuition for the dominance of pure BBPD finance over any mixed debt policy for weak firms. For any mixed debt policy (b, c) , the firm can achieve the same equity valuation by issuing BBPD with coupon $\tilde{b} = b + c$. However, since it is the bank that decides when to terminate renegotiations, through its choice of X_b , it is optimal to avoid *ex post* inefficient bank decision rules that emerge when the choice of X_b generates an externality to other classes of debt. Similarly, BBPD dominates pure BMD finance since the option to remain open under BBPD has weakly positive value *ex post*.

There is an alternative graphical interpretation regarding why pure BBPD dominates any mixed debt or pure BMD policy for weak firms. Assume that negotiation costs are zero. Consider

first the choice between issuing pure BMD versus pure BBPD when there are no negotiation costs. Assuming that the BMD policy entails coupon c , consider issuing BBPD with promised debt service $b = c$. Figure 3 depicts the debt service under BBPD. Note that the debt service under the BMD contract is represented by the horizontal segment. Clearly, the BBPD contract dominates since it entails higher tax shield benefits as well as lower bankruptcy costs. Figure 4 illustrates why BBPD dominates an arbitrary mixed debt policy entailing (b, c) . The schedule beginning at $X_b(b, c) > 0$ depicts total debt service under the mixed policy. If the firm had financed with pure BBPD for $\tilde{b} = b + c$ no default would occur, implying bankruptcy costs are reduced. Further, additional tax shield benefits are realized, as the firm is able to shield income on the interval $(0, X_b)$.

Pricing of Bank Bargaining Power Debt Due to Lemma 2, in the remainder of the paper discussion of weak firms is confined to those financing exclusively with BBPD. We now solve for the price of BBPD and the bank's optimal reorganization threshold X_b . The cash-flow to the bank in renegotiation states is equal to debt service, denoted by $\bar{s}(X)$, less renegotiation costs, if any:

$$\begin{aligned} \text{Cash-Flow to Bank} &= \bar{s}(X) - \delta[b - \bar{s}(X)]^+ \\ &= \begin{cases} b & \text{for non-renegotiation states} \\ X - \delta[b - X] & \text{for renegotiation states} \end{cases} \end{aligned} \quad (24)$$

Below, we prove that an endogenous reorganization threshold for the bank does not exist if $\delta = 0$. For $\delta > 0$, the general solution for the value of BBPD is as follows:

$$B_L(X) = A_3X^a + A_4X^z + \frac{X(1+\delta)}{r-\mu} - \frac{\delta b}{r} \quad \forall X \in [X_b, X_s] \quad (25)$$

$$B_H(X) = A_5X^a + A_6X^z + b/r \quad \forall X \in [X_s, \infty). \quad (26)$$

Based on Lemma 1, the optimal renegotiation point for equity (X_s) is derived below in (51), where the BMD model is solved. Taking X_s as given, the unknowns for this problem are $(A_3, A_4, A_5, A_6, X_b)$ which are derived using the following boundary conditions:

$$B_L(X_b) = (1 - \alpha)V(X_b), \quad (27)$$

$$\frac{\partial B_L(X_b)}{\partial X} = (1 - \alpha)V'(X_b), \quad (28)$$

$$B_L(X_s) = B_H(X_s) \quad (29)$$

$$\frac{\partial B_L(X_s)}{\partial X} = \frac{\partial B_H(X_s)}{\partial X} \quad (30)$$

$$\lim_{X \rightarrow \infty} B_H(X) = b/r. \quad (31)$$

It is interesting to note that since it is the *bank* that chooses when to enter into reorganization, we have imposed value matching and smooth pasting conditions for the bank value function, with B pasting up smoothly to the reservation value function (R) at the point X_b . The solution is as follows:¹²

$$B_L(X) = \frac{X(1+\delta)}{r-\mu} - \frac{\delta b}{r} \left[1 - \frac{1}{(1-a)} \left(\frac{X}{X_b} \right)^a \right] \quad \forall X \in [X_b, X_s) \quad (32)$$

$$B_H(X) = \left[\frac{\delta b}{r(1-a)} \right] \left[\frac{X}{X_b} \right]^a - \left[\frac{(1+\delta)b}{r(1-a)} \right] \left[\frac{X}{X_s} \right]^a + b/r \quad \forall X \in [X_s, \infty). \quad (33)$$

$$X_b = \left[\frac{\delta}{\delta + \tau + \alpha(1-\tau)} \right] X_s \quad (34)$$

$$= \left(\frac{a}{1-a} \right) \left(\frac{\delta b}{r} \right) \left[\frac{\delta + \tau + \alpha(1-\tau)}{r-\mu} \right]^{-1} \quad (35)$$

Lemma 3 follows from inspection of the equation for X_b :

Lemma 3 *For Bank Bargaining Power Debt, $\exists X_b > 0$ iff $\delta > 0$.*

In terms of the optimal reorganization point, one can think of the bank as solving an optimal stopping problem which involves an irreversible switch from the stream of cash-flows associated with the BBPD to that associated with the reorganized firm. The reorganized firm generates the perpetual cash-flow $(1-\alpha)(1-\tau)X$. The difference in instantaneous cash-flows is:

$$X[\delta + \tau + \alpha(1-\tau)] - \delta b. \quad (36)$$

The second expression for X_b embeds this cash-flow differential.

In summary, the following cases characterize the range of possible equilibria under BBPD:

- For the firm financed with BBPD, if $\delta = 0$, default never occurs and $\exists X_s$ such that $\bar{s}(X) = X$ on the interval $(0, X_s)$ and $\bar{s}(X) = b$ for all $X \geq X_s$.
- If $\delta > 0$, $\exists X_b$ such that the bank chooses to enter reorganization; $\exists X_s > X_b$ such that $\bar{s}(X) = X$ on (X_b, X_s) and $\bar{s}(X) = b$ for all $X \geq X_s$.

¹²The expression for X_s is derived below in (51).

IV. Valuation of the Levered Firm

Having characterized debt service functions for both EBPB and BBPD we can now solve for the constituent pieces of levered firm value. It is easily verified that the sum of instantaneous cash flows to equity, bond market debt, and bank debt is:

$$\text{Firm Cash Flow} = (1 - \tau)X + \tau[c + s(X)] - \delta[b - s(X)]^+. \quad (37)$$

This implies that the value of the levered firm can be expressed as the sum of the unlevered firm value, plus the value of the tax shield (TB), less negotiation costs (N), less bankruptcy costs (BC):

$$v(X) = V(X) + TB(X) - N(X) - BC(X) \quad (38)$$

Before proceeding, the price of a particular primary claim will be of use for valuing a number of claims on the firm. Consider a primary claim paying \$1 at the first-passage time from above of the EBIT process X to some threshold X^* . Based on (2), it may be verified that for $X \geq X^*$, this claim's value is:

$$\text{Hitting Claim} = \left(\frac{X}{X^*}\right)^a \quad (39)$$

A. Tax Shield

The value of the tax shield generated by bank and bond market debt is readily computed once one recognizes that the tax shield value is equal to the tax rate τ multiplied by the expected present value of future debt service. For the case of EBPB this latter value is equal to EBPB plus BMD less the creditors' claims on recoveries in the event of default. To determine the value of the recovery claim held by the firm's creditors, one multiplies the value of a hitting claim that pays off at X_d times the value of payoffs in default, $L(X_d)$. Therefore, the tax shield value is:

$$TB(X) = \tau \left[C(X) + B(X) - L(X_d) \left(\frac{X}{X_d}\right)^a \right] \text{ for EBPB.} \quad (40)$$

The value of the tax shield in the case of BBPD is computed similarly, with negotiations costs, $N(X)$, being added back to the value of the BBPD in order to compute the value of debt service:

$$TB(X) = \tau \left[B(X) + N(X) - L(X_b) \left(\frac{X}{X_b}\right)^a \right] \text{ for BBPD.}$$

B. Bankruptcy Costs

For EBPD the deadweight loss in the event of default is given by $\alpha V(X_d)$. Therefore, we have the following expression for bankruptcy costs:

$$BC(X) = \alpha V(X_d) \left(\frac{X}{X_d} \right)^a \text{ for EBPD.}$$

Similarly:

$$BC(X) = \alpha V(X_b) \left(\frac{X}{X_b} \right)^a \text{ for BBPD.}$$

C. Negotiation Costs

The negotiation cost function satisfies (2), with $\delta[b - s(X)]$ representing the flow term in the renegotiation region, while there is zero flow cost for $X \geq X_s$. The following boundary conditions pin down the solution for EBPD. For BBPD, one simply replaces X_d with X_b in the first boundary condition.

$$N_L(X_d) = 0, \quad (41)$$

$$N_L(X_s) = N_H(X_s), \quad (42)$$

$$\frac{\partial N_L(X_s)}{\partial X} = \frac{\partial N_H(X_s)}{\partial X}, \quad (43)$$

$$\lim_{X \rightarrow \infty} N(X) = 0. \quad (44)$$

For EBPD the solution for the negotiation cost function is:

$$N_L(X) = \delta \frac{b}{r} \left[1 - \left(1 - \left(\frac{a}{a-z} \right) \left(\frac{X_d}{X_s} \right)^z \right) \left(\frac{X}{X_d} \right)^a - \left(\frac{a}{a-z} \right) \left(\frac{X}{X_s} \right)^z \right] \quad (45)$$

$$- \delta \left(\frac{\eta(1-\alpha)(1-\tau)}{r-\mu} \right) \left[X - \left(X_d - \left(\frac{a-1}{a-z} \right) X_s \left(\frac{X_d}{X_s} \right)^z \right) \left(\frac{X}{X_d} \right)^a - \left(\frac{a-1}{a-z} \right) X_s \left(\frac{X}{X_s} \right)^z \right]$$

$$N_H(X) = \delta \frac{b}{r} \left[\left(\left(\frac{a}{a-z} \right) \left(\frac{X_d}{X_s} \right)^z - 1 \right) \left(\frac{X}{X_d} \right)^a - \left(\frac{z}{a-z} \right) \left(\frac{X}{X_s} \right)^a \right] \quad (46)$$

$$+ \delta \left(\frac{\eta(1-\alpha)(1-\tau)}{r-\mu} \right) \left[\left(X_d - \left(\frac{a-1}{a-z} \right) X_s \left(\frac{X_d}{X_s} \right)^z \right) \left(\frac{X}{X_d} \right)^a - \left(\frac{1-z}{a-z} \right) X_s \left(\frac{X}{X_s} \right)^a \right]$$

For BBPD the solution for the negotiation cost function is:

$$N_L(X) = \delta \frac{b}{r} \left[1 - \left(1 - \left(\frac{a}{a-z} \right) \left(\frac{X_b}{X_s} \right)^z \right) \left(\frac{X}{X_b} \right)^a - \left(\frac{a}{a-z} \right) \left(\frac{X}{X_s} \right)^z \right] \quad (47)$$

$$\begin{aligned}
& -\delta \left(\frac{1}{r - \mu} \right) \left[X - \left(X_b - \left(\frac{a-1}{a-z} \right) X_s \left(\frac{X_b}{X_s} \right)^z \right) \left(\frac{X}{X_b} \right)^a - \left(\frac{a-1}{a-z} \right) X_s \left(\frac{X}{X_s} \right)^z \right] \\
& N_H(X) = \delta \frac{b}{r} \left[\left(\left(\frac{a}{a-z} \right) \left(\frac{X_b}{X_s} \right)^z - 1 \right) \left(\frac{X}{X_b} \right)^a - \left(\frac{z}{a-z} \right) \left(\frac{X}{X_s} \right)^a \right] \\
& + \delta \left(\frac{1}{r - \mu} \right) \left[\left(X_b - \left(\frac{a-1}{a-z} \right) X_s \left(\frac{X_b}{X_s} \right)^z \right) \left(\frac{X}{X_b} \right)^a - \left(\frac{1-z}{a-z} \right) X_s \left(\frac{X}{X_s} \right)^a \right]
\end{aligned} \tag{48}$$

D. Bond Market Debt Value

The value function for bond market debt satisfies (2) with flow payment equal to c on the non-default region. The boundary conditions are value matching at X_d , and an asymptotic condition:

$$C(X_d) = (1 - \eta)(1 - \alpha)V(X_d), \tag{49}$$

$$\lim_{X \rightarrow \infty} C(X) = c/r. \tag{50}$$

The solution for the value of BMD is given by:

$$C(X) = \frac{c}{r} \left[1 - \left(\frac{X}{X_d} \right)^a \right] + (1 - \eta)(1 - \alpha)V(X_d) \left(\frac{X}{X_d} \right)^a \tag{51}$$

E. Equity Value

Finally, having derived each of the terms in the levered firm value expression (38), the value of equity can be expressed as a residual based on (23). In the case of EBPD, the following valuation equation holds on the interval $[X_d, \infty)$:

$$E(X) = V(X) - V(X_d) \left(\frac{X}{X_d} \right)^a - (1 - \tau) \left[B(X) + C(X) - L(X_d) \left(\frac{X}{X_d} \right)^a \right] - N(X) \text{ for EBPD.} \tag{52}$$

In the case of BBPD, equity is worth zero for $X \leq X_s$, since it is pushed down to its reservation value in renegotiations. The following pricing equation holds on the interval (X_s, ∞) :

$$E(X) = V(X) - V(X_b) \left(\frac{X}{X_b} \right)^a - (1 - \tau) \left[B(X) + N(X) - L(X_b) \left(\frac{X}{X_b} \right)^a \right] \text{ for BBPD.}$$

The two expressions for the value of equity under EBPD and BBPD have intuitive interpretations. The first two terms represent the value of holding an unlevered firm that will be worthless at the first-passage of the EBIT process to the relevant reorganization threshold. The third term represents the expected present value of the after-tax cost of total debt service. The fourth term

is renegotiation costs, which is paid by equity in the case of EBPD but is paid by the bank in the case of BBPD.

The remaining unknown variable for the case of EBPD is the optimal default threshold, $X_d(b, c)$. The brute force method of solving for equity value and the endogenous default threshold is to solve for the flow payments to equity and then utilize the following boundary conditions:

$$E_L(X_d) = 0 \quad (53)$$

$$E_L(X_s) = E_H(X_s) \quad (54)$$

$$\frac{\partial E_L(X_s)}{\partial X} = \frac{\partial E_H(X_s)}{\partial X} \quad (55)$$

$$\lim_{X \rightarrow \infty} E(X) = \frac{(1 - \tau) X}{r - \mu} - \frac{(1 - \tau)(b + c)}{r} \quad (56)$$

$$\frac{\partial E_L(X_d)}{\partial X} = 0 \quad (57)$$

However, it can be verified that the solution (52) necessarily satisfies the first four boundary conditions. The smooth pasting condition at default must therefore be exploited in solving for X_d . Using (21), the endogenous default-triggering EBIT level under an arbitrary mixture of EBPD and BMD is:

$$X_d(b, c) = \frac{a(r - \mu) [\delta b + (1 - \tau)c]}{(a - 1)r(1 - \tau) [1 - \eta(1 - \alpha)(1 - \tau - \delta)]} \quad (58)$$

V. Optimal Debt Structure under Mutually Exclusive Financing

A. Analysis of Strong Firm

A.1. Bond Market Debt Finance

We begin with an analysis of the bond market debt financed firm, confirming that the qualitative features of the Leland (1994) framework carry over to our model. The discussion will be brief, since the main objective of this section is to contrast the bond market and bank debt firms. Dropping terms in (38) that involve bank debt, the expression for the value of the marketable claims of the corporation is given by:

$$v(X) = V(X) + \frac{\tau c}{r} \left(1 - \left(\frac{X}{X_d} \right)^a \right) - \alpha V(X_d) \left(\frac{X}{X_d} \right)^a. \quad (59)$$

Value matching and smooth pasting conditions for equity yield the following solution for the endogenous default-triggering EBIT level as a function of the promised coupon payment:

$$X_d(0, c) = \left(\frac{r - \mu}{r} \right) \left(\frac{a}{a - 1} \right) c. \quad (60)$$

Differentiating the previously derived expression for the value of bond market debt given in (51), with respect to c and setting it equal to zero yields the *bond market debt coupon capacity*, denoted c^{\max} :

$$c^{\max}(X_0) = \frac{r(a-1)X_0}{a(r-\mu)} [1 - a(\tau + \alpha(1-\tau))]^{1/a}. \quad (61)$$

Substituting this expression back into the pricing equation yields the *bond market debt value capacity*:

$$C^{\max}(X_0) = \frac{X_0 [1 - a(\alpha + (1-\alpha)\tau)]^{1/a}}{(r-\mu)}. \quad (62)$$

Figure 5 parallels Figure 1 in Leland (1994) presenting comparative statics results for the value of bond market debt as a function of leverage for varying degrees of underlying cash flow risk. It is easy to see that a debt capacity c^{\max} obtains. Perhaps surprisingly, it can be seen that the firm's *bond market debt coupon capacity* is increasing in EBIT risk, but the *bond market debt value capacity* decreases in the riskiness of the underlying cash flows.

We can compute the optimal coupon by solving the first-order condition $\partial v(X_0; c^*)/\partial c = 0$:

$$c^*(X_0) = \frac{r(a-1)X_0}{a(r-\mu)} \left[\frac{\tau - a(\tau + \alpha(1-\tau))}{\tau} \right]^{1/a} \quad (63)$$

Substituting this expression back into the total firm value expression yields the *ex ante* value of an optimally levered bond market debt firm:

$$v^*(X_0; 0, c^*) = \frac{X_0}{r-\mu} \left\{ 1 - \tau + \tau \left[\frac{\tau - a(\tau + \alpha(1-\tau))}{\tau} \right]^{1/a} \right\} \quad (64)$$

Figure 6 depicts the optimal coupon level c^* . Although high volatility firms have a higher *debt coupon capacity* than low volatility firms, the *optimal choice of leverage* goes in the opposite direction in that it is greater for low volatility firms than it is for high volatility firms. Altogether, less risky firms attain a higher total levered firm value by issuing a higher promised coupon level relative to riskier firms.

A.2. Equity Bargaining Power Debt Finance

We begin this subsection by verifying that some of the key results in MBP (1997) relating to the case in which equity has full bargaining power are also valid in our modeling framework.

Lemma 4 *If $c = \delta = 0$, the Strong Firm financing with EBPD never defaults.*

Proof. Follows from the fact that default is not optimal when the cash-flow to equity is everywhere positive. ■

Lemma 5 *For the Strong Firm, if $\tau = 0$, $\delta = 0$, and $\alpha > 0$, then the optimal financial structure entails no Bond Market Debt, and the firm is indifferent regarding the level of b .*

Proof. Issuing BMD entails default costs, whereas EBPD entails no such costs. ■

Lemma 6 *For the Strong Firm, if $\delta = 0$ and $\tau > 0$, it is optimal to increase b up to the bank debt capacity, b^{\max} , which satisfies $B[X_0; b^{\max}] = L(X_0)$. The ex ante firm value for any $b \geq b^{\max}$ is $V(X_0)[1 + \tau(1 - \alpha)]$.*

Proof. Since there are no negotiation costs and default never occurs, the firm solves the problem $\max_b \tau B[X_0; b]$. ■

The intuition for bank debt capacity in this setting is as follows. From the equation for the endogenous renegotiation threshold given in (21), it can be seen that X_s is increasing in b . For a fixed initial EBIT level, low levels of b are consistent with $X_0 > X_s(b)$. However, for b sufficiently high, the bank recognizes that equity will push it down to the reservation value $L(X_0)$ immediately, assuming equity has maximum bargaining power. This implies that increasing the promised coupon will not raise the value of debt, since the reservation value is invariant to b . More formally, the bank debt capacity, b^{\max} , satisfies:

$$X_s(b^{\max}) = X_0 \quad \Leftrightarrow \quad b^{\max} = \left(\frac{r}{r - \mu} \right) \left(\frac{a - 1}{a} \right) (1 - \alpha) (1 - \tau) X_0. \quad (65)$$

We now proceed to a discussion of some numerical results. Figure 7 depicts the bank's reservation value R and the value of bank debt B as functions of EBIT for varying levels of bankruptcy costs,

assuming that equity has full bargaining power. In the model, the strategic debt service to the bank in the renegotiation region depends crucially on the level of bankruptcy costs (and corporate taxes, for which the effect is essentially the same). The dashed rays from the origin represent the bank's threat point or, more formally, reservation value function $R(\alpha, \cdot)$. The construction of the *incentive compatible* switch point $X_s(\alpha, \cdot)$ implies that the bank debt value B is tangent to $R(\alpha, \cdot)$ up to $X_s(\alpha, \cdot)$. Note that *ceteris paribus*, firms with higher bankruptcy costs enter negotiations with their bank sooner. The figure clearly depicts the fact that the bank is pushed down to its reservation value in the renegotiation region, where $B(X) = R(X)$, with value matching and smooth pasting being exhibited at X_s .

Figure 8 depicts the value of EBPD (B) and BMD (C) as functions of EBIT (X) evaluated at the same promised coupon, $b = c = 20$ in the presence of negotiation costs. Note that default occurs earlier with BMD than with EBPD. However, for points to the right of X_d , the value of BMD exceeds that of EBPD. Figure 9 reports comparative static results for the price of bank debt when the firm's EBIT volatility varies. EBPD is worth weakly more when the firm's operations are less risky. This is not surprising given that in Figure 7 it was shown that EBPD is concave in EBIT. It is also interesting to note that the higher the underlying volatility, the longer the firm waits before entering into renegotiation.

Figures 10-12 explore some comparative statics for the effects of key parameters on the total leveraged firm value v . Lemma 6 demonstrated that with $\delta = 0$, it is optimal for the firm to lever up to its debt capacity. Figure 10 illustrates the effect variations in negotiation costs. It can be seen that the introduction of negotiation costs generates an interior optimal coupon, with higher negotiation costs implying a lower firm value and a lower optimal coupon b^* .

Figure 11 depicts the effect of variations in bankruptcy costs. It can be seen that the firm's *debt capacity and optimal coupon are inversely related to bankruptcy costs*. In other words, the model predicts that firms with lower bankruptcy cost have more *access* to bank debt and will optimally choose to borrow more. Figure 12 indicates that firm value is decreasing in underlying cash flows' volatility.

Finally, the effect of an increase in perpetual coupon payments from $b = 10$ to $\tilde{b} = 15$ on strategic debt service is illustrated in Figure 13. Provided negotiation costs are strictly positive, an increase in the promised coupon: (i) shifts the default-triggering EBIT level up $\partial X_d(b, 0) / \partial b > 0$; (ii)

results in a loss in tax shield benefits in the region between the previous switch point X_s and the new switch point $\tilde{X}_s > X_s$; and (iii) brings about an increase in tax shield benefits in the non-renegotiation region, i.e., for all $X \in [\tilde{X}_s, \infty)$. This provides the underlying basis for the interior optimum b^* .

A.3. Equity Bargaining Power and Bond Market Debt Comparison

The purpose of this section is to compare EBPD and BMD, when the choice between the two is *mutually exclusive*.¹³ We begin by comparing EBPD and BMD when the firm may choose the optimal coupon commitment for each, respectively. Under the base case parameter values there exist threshold values of bankruptcy costs α^* and corporate taxes τ^* at which the firm is indifferent between EBPD and BMD. Figure 14 reports valuations applicable to optimally levered EBPD and BMD firms as α is varied. For the optimal promised coupon payments $b^*(\alpha, \cdot)$ and $c^*(\alpha, \cdot)$, firms with bankruptcy costs of 31% or less prefer EBPD. At first the result may seem counter-intuitive. After all, casual intuition suggests that issuing a hard claim like BMD is less attractive when bankruptcy costs increase. There is nothing wrong with this intuition, as far as it goes. That is, an increase in α reduces the optimal coupon commitment under BMD and the total firm value attained. However, increases in bankruptcy costs also make EBPD less attractive and lower the value attained. As Lemma 6 demonstrates, this effect is present *even when the issuance of EBPD entails no default* (when $\delta = 0$). This is because increases in α reduce the value of the tax shield created by EBPD. Recall that under EBPD, the bank is pushed down to its reservation value in renegotiations. Increases in α rotate the function R clockwise downwards, implying lower debt service under EBPD.

Figure 15 reports valuations applicable to optimally levered EBPD and BMD firms as τ is varied. At the promised coupon levels $b^*(\tau, \cdot)$ and $c^*(\tau, \cdot)$ firms with corporate taxes of less than 26% achieve a higher value with bank debt. Finally, Figure 16 traces out the locus of points at which the firm is indifferent between the two sources of capital so long as they are mutually

¹³This case would be of particular interest when there are substantial fixed costs associated with tapping either source of debt capital.

exclusive. If $\delta = 0$, the following condition identifies parameters such that EBPD dominates BMD, when both are optimally levered:

$$(1 - \alpha)(1 - \tau) > \{1 - a[\tau + \alpha(1 - \tau)]/\tau\}^{1/a}. \quad (66)$$

Finally, by parametrically varying the firm's EBIT volatility another empirical prediction emerges. In line with previous theoretical and empirical studies, we obtain the result that safer firms find it optimal to tap public debt markets, while riskier firms are more likely to be financed by banks.

Turning now to the analysis of *equal promised coupon payments* to the bank and the bond market, Figures 17–19 confirm in principle all of the previous findings for optimally levered firms. Assuming that $\delta = 0$, we can derive an analytic expression for the cut-off contour under equal promised payments to EBPD and BMD. The following condition defines parameter values such that the EBPD financed firm with the same perpetual coupon promise as the BMD firm achieves higher value (when both are evaluated at $X > X_s$):

$$[(1 - \alpha)(1 - \tau)]^{-a} > (1 - a)^{1/a}. \quad (67)$$

Figure 19 confirms the intuition that *ceteris paribus* bank debt is more desirable for lower negotiation costs δ ; that is, a decrease in the negotiation costs δ implies that more (α, τ) pairs lead to the optimality of EBPD for mutually exclusive financing decisions.

B. Analysis of Weak Firm

This section presents numerical results for the weak firm. We confine analysis to issuance of BBPD, since for a weak firm the issuance of BMD is dominated, as was shown in Lemma 2. To begin, we parallel the analysis of EBPD by plotting BBPD as a function of EBIT when the cost of negotiation with the bank changes (Figure 20) and when the firm's cash flow riskiness varies (Figure 21). It is important to note that under BBPD, the bank becomes the residual claimant on the firm's cash flows in the renegotiation region. The value function for BBPD therefore pastes smoothly to the reservation function R at the endogenous reorganization threshold X_b . Figure 21 illustrates that for any positive negotiation costs ($\delta > 0$) both the default-triggering EBIT level X_b and the *incentive compatible* switch point X_s are decreasing in the firm's riskiness σ . Also, the value function for BBPD exhibits a convex-concave shape, illustrating that the bank also becomes risk-loving in some states.

As was already noted, under BBPD the bank becomes the residual claimant in the renegotiation region. This has an important implication for the equity value function, which is depicted in Figure 22. In particular, the value of equity is equal to zero in the renegotiation region since the bank pushes equity down to its reservation value. At equity's *incentive compatible* switch point for entering into renegotiations (X_s), the equity value function exhibits value-matching and smooth-pasting.

Figure 23 illustrates that lower negotiation costs generate a higher optimal coupon under BBPD. Figure 24 indicates that the value attained by the optimally levered firm is higher when the volatility of underlying EBIT is reduced. The effect of volatility on firm value under BBPD seems qualitatively similar to the results reported in Figure 5 for firm value under BMD. However, it is interesting to note that the increase in firm value stemming from lower volatility is achieved via different means, depending on the type of debt issued. For BMD, the optimal coupon commitment increases when volatility is reduced. For BBPD, on the other hand, the optimal coupon commitment decreases when volatility is reduced.

VI. Optimal Debt Structure of the Mixed Debt Firm

This section confines discussion to the case of the *strong firm* since Lemma 2 demonstrates that the optimal debt structure for the *weak firm* entails exclusive BBPD financing. We begin with an analysis of the optimal mix of EBPD and BMD when the bank is placed senior in the priority structure. Analysis of *pari passu* treatment follows. Comparison of *ex ante* firm valuations achieved under the optimal debt mix for each priority rule, respectively, pins down the optimal priority structure.

A. Senior Bank Debt

The following result is useful before proceeding to the numerical results for senior bank debt.

Lemma 7 *For a Strong Firm issuing the mixed debt pair (b, c) such that there is no renegotiation region, the ex ante firm value is equivalent to that under BMD finance with $\tilde{c} = c + b$.*

Proof. If there is no renegotiation, then bank debt represents a hard claim from the perspective of equity, implying that the optimal default threshold is based upon facing a fixed flow cost $c + b$ at each instant. This is the same situation facing a firm that has promised $\tilde{c} = c + b$ to the bond markets. ■

Intuitively, no renegotiation region exists when the bank is senior and the debt mix is weighted heavily towards BMD. In such cases:

$$R[X_d(b, c)] = \frac{(1 - \alpha)(1 - \tau) X_d(b, c)}{r - \mu} > \frac{b}{r}, \quad (68)$$

implying that no renegotiation occurs, since the bank rejects any debt service payment less than b . Therefore, full analysis of levered firm valuations for the strong firm is conducted as follows. Senior bank debt is examined by setting $\eta = 1$ and solving the model for parameters (b, c) are such that there exists a renegotiation region. Other (b, c) pairs are subsumed under the exclusive BMD financing case.

The analysis of the model in this subsection has been accompanied by several consistency checks of the numerical scheme. First, the theoretical results for the pricing of claims for the mixed-debt firm are predicated on the assumption that there exists a non-trivial renegotiation region. Therefore, we need to impose the following regularity condition in our numerical analysis of the mixed debt firm:

$$\text{Condition 1: } X_d(b, c, \cdot) < X_s(b, \cdot) \quad \forall (b, c) \in \mathbb{R}_+^2 \quad (69)$$

Secondly, it was stated that for certain (b, c) pairs the firm may be in the position to issue safe, senior bank debt. These cases are subsumed in the analysis of the BMD firm. To avoid any pathological cases, the numerical scheme consequently needs to contain a second regularity condition for the mixed debt firm:

$$\text{Condition 2: } L[X_d(b, c, \cdot)] < b/r \quad \forall (b, c) \in \mathbb{R}_+^2 \quad (70)$$

Figure 25 indicates that for the strong firm there exists an interior optimum debt mix $(b^*, c^*) = (24.09, 23.63)$, when the bank is senior in the priority structure. Although the promised coupon to the bank is higher, the initial value of BMD (334.94) exceeds that on EBPD (315.31). Regardless of the measure, the optimal debt mix when the bank is senior is relatively even. Substantial increases in firm value are achieved when the firm can exploit a mix of EBPD and BMD, rather than relying

on one source exclusively. The first line below reports the *ex ante* firm value attained when the firm may mix EBPD and BMD, assuming the bank is senior. The second and third lines report valuations under exclusive BMD and EBPD finance, respectively:

$$\begin{aligned}
 v(b^*, c^*) &= 844.09 \\
 v(c^*) &= 809.30 \\
 v(b^*) &= 752.01.
 \end{aligned}
 \tag{71}$$

An interesting feature of the model is that it may be used to generate estimates regarding the magnitude of the benefit the firm enjoys when it gains access to a previously untapped source of funds. The results above indicate that firms without access to the bond markets reap large gains when this constraint is removed. Note that this result holds in the absence of any notion that bond markets are “cheaper.” In this model all claims are priced fairly, with all claimants discounting flows at the rate r . All of the benefit from mixed debt finance comes from the complementarity effect. Firm value increases by 4.3% when mixed debt is employed, rather than financing exclusively with BMD. The increase in firm value is 12.2% when mixed debt is employed, rather than financing exclusively with EBPD.

The *complementarity effect* is due the relationship between bank and bond market debt coupon payments and the default-triggering EBIT level $X_d(b, c, \cdot)$. Therefore, we fix the default-triggering EBIT level at $X_d(b, c, \cdot) = 5$ in Figure 33 and solve for the (b, c) pairs such that our root equation is satisfied. This analysis reveals that bank and bond market debt have an uneven contribution to the endogenous bankruptcy boundary and hence a *complementarity effect* of the nature discussed above obtains. Secondly, in comparing these numerical results, it can be seen that placing bank debt senior in the priority structure leads to higher total firm value. Lastly, at a broader level these findings show that our model yields a product differentiation result for financial markets in the following sense. In order to maximize firm value, the role of banks is to issue genuinely soft debt such that they offer a financial product, which is as “distant” as possible from hard bond market debt. Thus we provide a rationale for the coexistence of bank debt (indirect/intermediated finance) and bond market debt (direct finance).¹⁴

¹⁴See for instance Bolton and Freixas (2000) for a similar result. These authors obtain coexistence of bank and bond market debt from an asymmetric information model of financial intermediation.

The individual value components of the total leveraged firm are reported in Figures 27–32. Valuations for bank B and bond market debt C are depicted as functions of coupon payments to the bank b and to the bond market c in Figures 31 and 32, respectively. Our earlier discussion on comparative statics results concerning debt capacity of the all soft (hard) debt firm are confirmed by these graphs.

B. Equal Priority Bank Debt

The following result will be useful before proceeding to the numerical results for the case of equal priority bank debt.

Lemma 8 *Under equal priority with endogenous default, it is impossible for the firm to issue safe EBPD or BMD.*

Proof. Suppose to the contrary that either class is safe. Then it must be the case that $L[X_d(b, c)] \geq \frac{b+c}{r}$ and there is no renegotiation. With no renegotiation the endogenous default threshold is the same as that pertaining to a firm issuing bond market debt exclusively, with promised payment $\tilde{c} = c + b$. But $L[X_d(0, \tilde{c})] < \frac{\tilde{c}}{r}$. This is a contradiction. ■

Corollary 9 *A renegotiation region always exists for the Strong Firm issuing equal priority EBPD.*

Figure 26 indicates that for the strong firm there exists an interior optimum debt mix $(b^*, c^*) = (5.89, 32.53)$, when the bank is *pari passu* in the priority structure. However, the debt mix is slanted towards BMD under *pari passu*, whereas it was roughly equal for senior bank debt. More modest increases in firm value are achieved when the firm can exploit a mix of EBPD and BMD, rather than relying on one source exclusively. The first line below reports the *ex ante* firm value attained when the firm may mix EBPD and BMD, when the bank is *pari passu*. The second and third lines report valuations under exclusive BMD and EBPD finance, respectively:

$$\begin{aligned} v(b^*, c^*) &= 816.15 & (72) \\ v(c^*) &= 809.30 \\ v(b^*) &= 752.01 \end{aligned}$$

Under equal priority, firm value increases by 0.8% when mixed debt is employed, rather than financing exclusively with BMD. The increase in firm value is 8.5% when mixed debt is employed, rather than financing exclusively with EBPD. For our baseline parameter values, placing the bank senior in the priority structure dominates *pari passu*, with the *ex ante* firm values equal to 844.09 and 816.15, respectively. The increase in firm value associated with moving away from *pari passu* is 3.4%.

Why does placing the bank senior generate superior *ex ante* firm valuations? Table 1 suggests an answer. Placing the bank senior in the priority structure offers costs and benefits relative to *pari passu* treatment. On the one hand, placing the bank senior leads to an increase in bankruptcy costs equal to 4.78. The increase in bankruptcy costs is generated by the increase in the optimal default threshold from 8.11 to 8.76. The optimal default threshold increases because placing the bank senior raises the bank's threat point in instantaneous negotiations, leading to higher requisite debt service. Outweighing this cost is the benefit of high *ex post* debt service in terms of combined tax shield increases and renegotiation cost reductions. Most of the benefit is generated by the increase in the value of the tax shield, which increases in value by 32.27 when the bank is placed senior in the priority structure.

VII. Conclusions/Extensions

- Non-linear tax schedule.
- Agency problems, e.g. endogenous volatility as in Leland (1998).
- Costly monitoring by banks.
- Monitoring of bank debt.
- Maturity structure, as in Leland and Toft (1996).

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Table I

Comparison of Optimal Debt and Priority Structures under EBPD and BBPD

This table reports our numerical results for the mixed debt firm for different priority structures, i.e., bank debt senior ($\eta = 1$) and equal priority ($\eta = b/(b + c)$) when the *strong firm* issues Equity Bargaining Power Debt (EBPD) and the *weak firm* issues Bank Bargaining Power Debt. The firm's optimal promised coupon payments for EBPD/BBPD b^* and BMD c^* are studied. In particular, we evaluate the *closed-form* expressions for leveraged firm value $v(X)$, endogenous default-triggering EBIT level $X_d(b, c)/X_b(b, c)$, *incentive compatible* switch point from the renegotiation to the non-renegotiation region $X_s(b)$, unleveraged firm value $V(X)$, tax shield values $TB(X)$, bankruptcy cost values $BC(X)$, negotiation cost values $N(X)$, equity values $E(X)$, bank debt values $B(X)$, and bond market debt values $C(X)$ once we solved numerically for the optimal leverage of the mixed debt firm, where X denotes the initial EBIT level $X \equiv X_0 = 20$, and the base case parameter values $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, $\delta = 10\%$ are assumed.

Panel A. Equity Bargaining Power Debt (EBPD)												
Priority Structure	$v(X)$	b^*	c^*	$X_d(b, c)$	$X_s(b)$	$V(X)$	$TB(X)$	$BC(X)$	$N(X)$	$E(X)$	$B(X)$	$C(X)$
$\eta = 1$	844.09	24.09	23.63	8.76	17.23	650	220.13	21.29	4.75	193.85	315.31	334.94
$\eta = b/(b + c)$	816.15	5.89	32.53	8.11	27.47	650	187.86	16.51	5.2	281.48	46.42	488.26
Panel B. Bank Bargaining Power Debt (BBPD)												
Priority Structure	$v(X)$	b^*	c^*	$X_b(b, c)$	$X_s(b)$	$V(X)$	$TB(X)$	$BC(X)$	$N(X)$	$E(X)$	$B(X)$	$C(X)$
	964.09	77.22	0.00	2.32	17.95	650	343.90	0.26	29.55	10.81	953.28	0.00

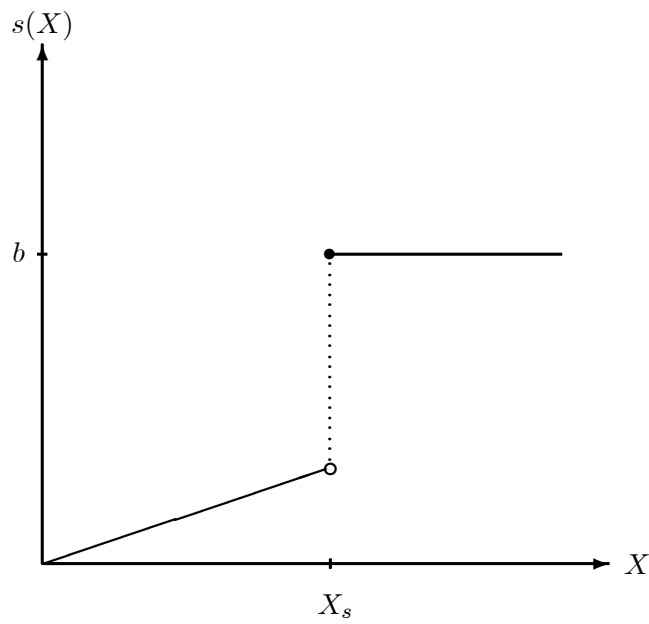


Figure 1: Strategic debt service $s(X)$ as a function of EBIT X in case of Equity Bargaining Power Debt (EBPD). It is assumed that there are no negotiation costs ($\delta = 0$).

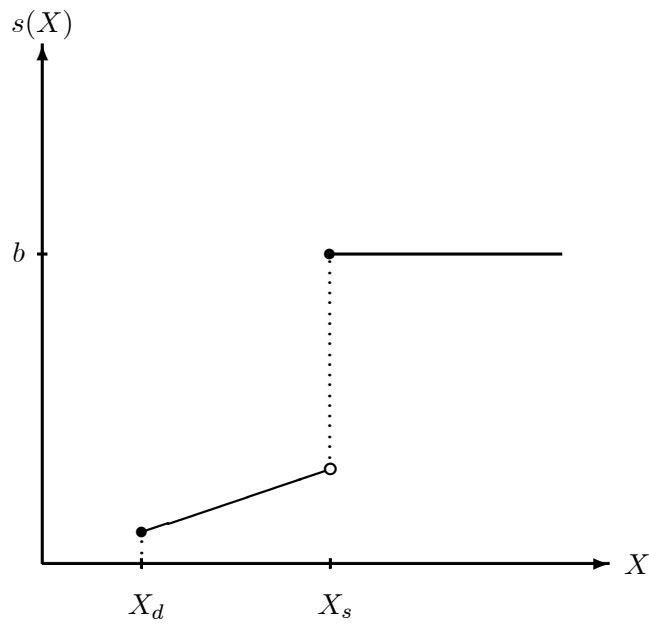


Figure 2: Strategic debt service $s(X)$ as a function of EBIT X in case of Equity Bargaining Power Debt (EBPD). It is assumed that either there are negotiation costs ($\delta > 0$) or the firm issued Bond Market Debt ($c > 0$). Note that for positive δ or c , and (δ, c) sufficiently low relative to b , $\exists X_d(b, c; \delta)$ such that default occurs endogenously and a nontrivial renegotiation region exists: $X_d(b, c; \delta) < X_s$.

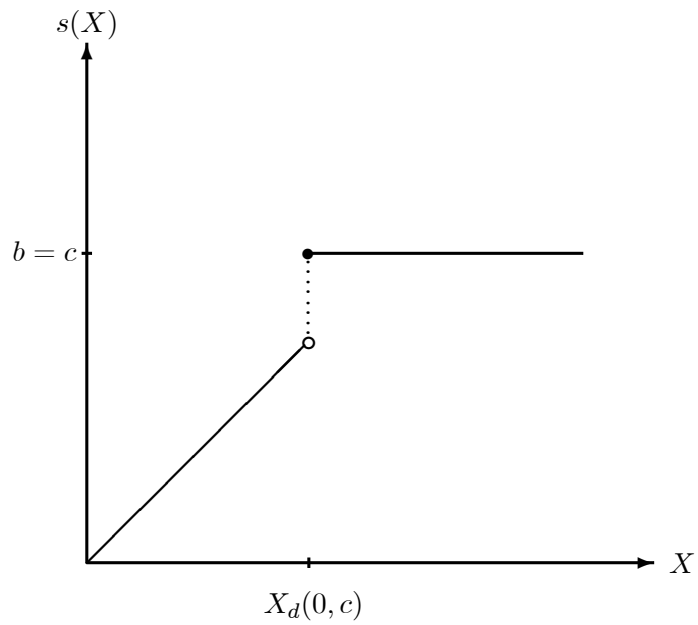


Figure 3: Strategic debt service $s(X)$ as a function of EBIT X in case of Bank Bargaining Power Debt (BBPD). It is assumed that there are no negotiation costs ($\delta = 0$) and that the promised coupon payments under Bank Bargaining Power Debt and Bond Market Debt are identical: $b = c$. Note that the default-triggering EBIT level under BMD is equal to the switch point under BBPD, i.e., $X_d(0, c) = X_s(b)$.

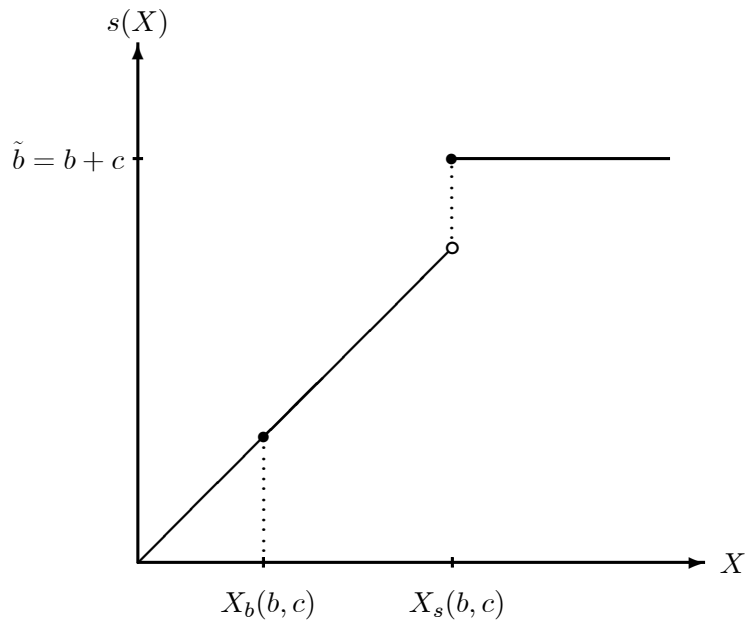


Figure 4: Strategic debt service $s(X)$ as a function of EBIT X in case of Bank Bargaining Power Debt $\tilde{b} = b + c$ versus strategic debt service of a mixed debt firm financed by Bank Bargaining Power Debt b and Bond Market Debt c . Note that the *incentive compatible* switch point X_s is the same under both debt structures, but the latter is associated with a strictly positive default point $X_b(b, c)$. It is assumed that there are no negotiation costs ($\delta = 0$).

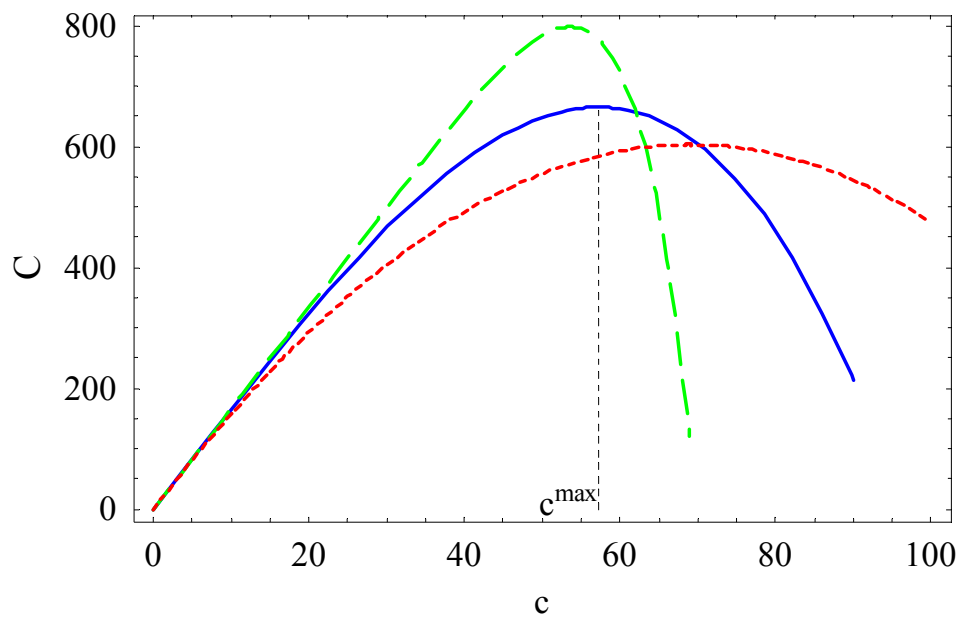


Figure 5: Bond market debt value C as a function of promised coupon payments c when the firm's EBIT volatility varies: $\sigma = 10\%$ (*long-dashed line*), $\sigma = 20\%$ (*solid line*), and $\sigma = 30\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, $\eta = 0$. Note that the hard debt capacity c^{\max} depends on firm's riskiness σ . Cf. Figure 1 in Leland (1994).

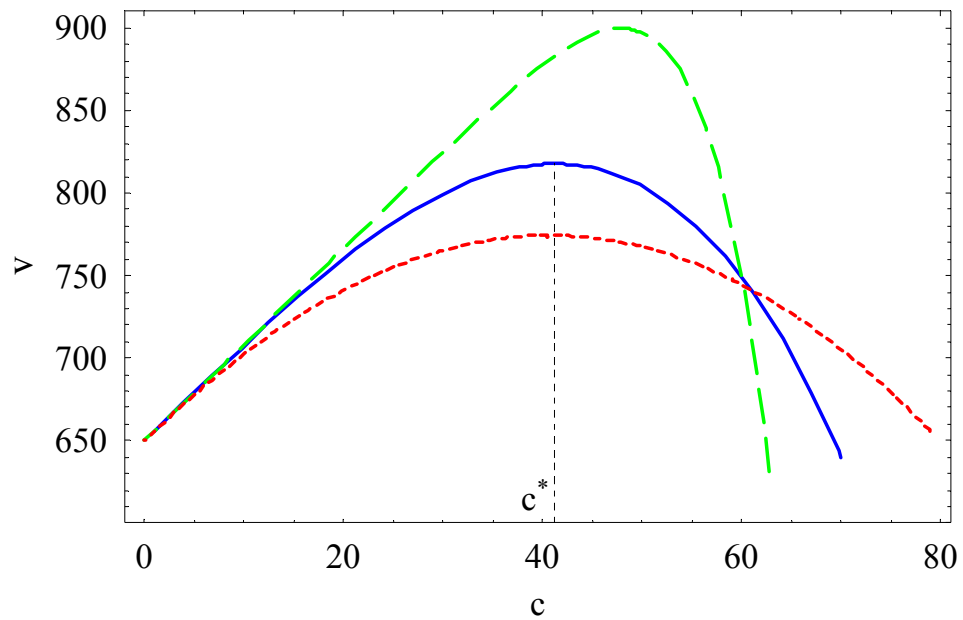


Figure 6: Total firm value v of a Bond Market Debt firm as a function of promised coupon payments c when EBIT volatility varies: $\sigma = 10\%$ (*long-dashed line*), $\sigma = 20\%$ (*solid line*), and $\sigma = 30\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, $\eta = 0$. Note that the optimal leverage c^* depends on firm's riskiness σ . Cf. Figure 6 in Leland (1994).

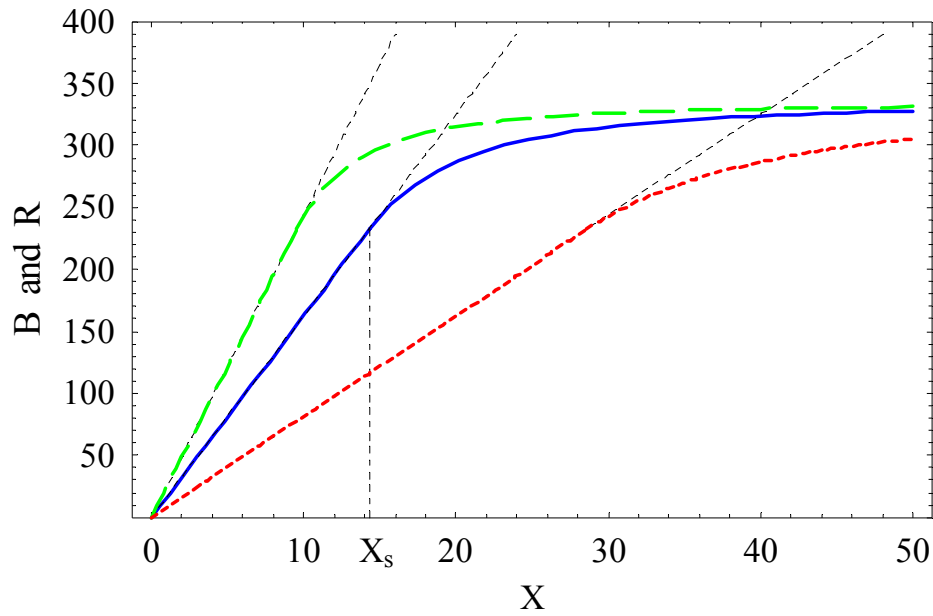


Figure 7: Equity Bargaining Power Debt B and bank's reservation value R as a function of EBIT X for a given coupon level b as the firm's bankruptcy costs vary: $\alpha = 25\%$ (*long dashed line*), $\alpha = 50\%$ (*solid line*), and $\alpha = 75\%$ (*short-dashed line*). It is assumed that $X_0 = 20$, $b = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\tau = 35\%$, $\delta = 0$, $\eta = 1$. Note that both the bank's reservation value R (*thin short-dashed lines*) and the *incentive compatible* switch point X_s depend on the firm's bankruptcy costs α .

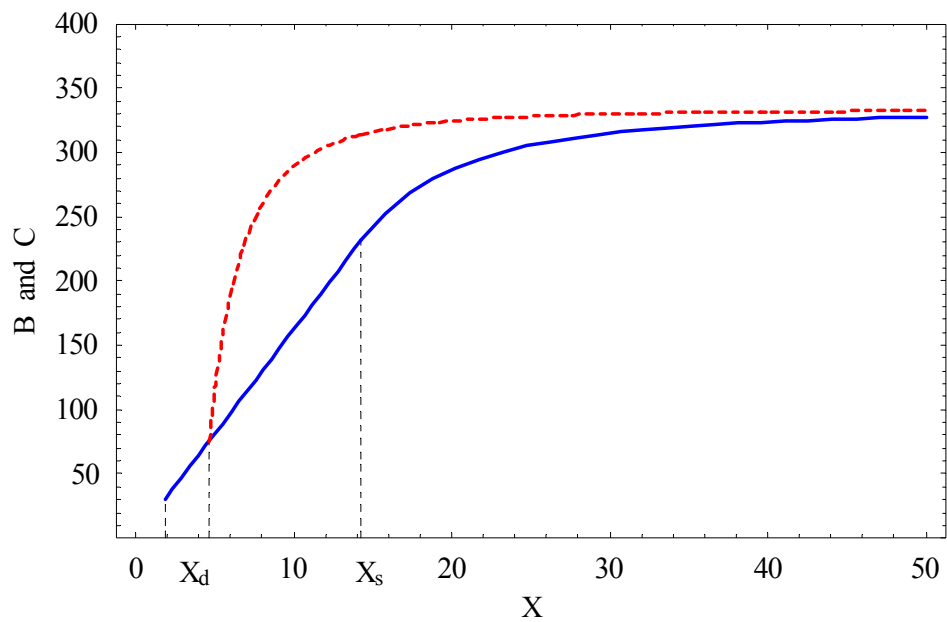


Figure 8: Equity Bargaining Power Debt B as a function of EBIT X for a given coupon level b . It is assumed that $X_0 = 20$, $b = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, $\delta = 10\%$, $\eta = 1$. The Bond Market Debt C as a function of EBIT X for the same coupon level $b = c$ is depicted by the dashed line. Note that *ceteris paribus* default occurs earlier with Bond Market Debt than with Equity Bargaining Power Debt.

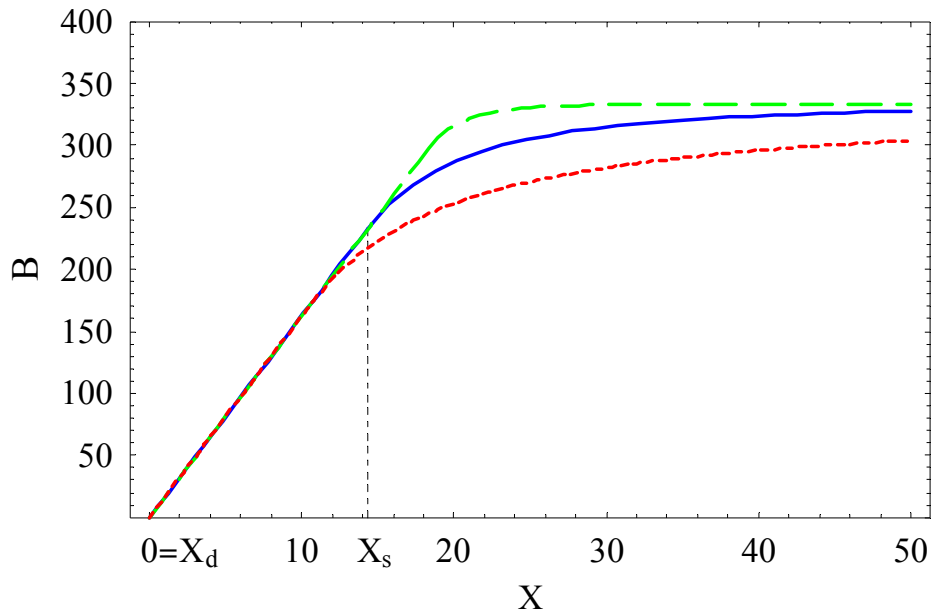


Figure 9: Equity Bargaining Power Debt B as a function of EBIT X for a given coupon level b as EBIT volatility varies: $\sigma = 10\%$ (*long-dashed line*), $\sigma = 20\%$ (*solid line*), and $\sigma = 30\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $b = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, $\delta = 0\%$, $\eta = 1$. Note that the default-triggering EBIT level X_d is equal to zero in all cases because the negotiation cost δ is equal to zero, but the *incentive compatible* switch point X_s still varies with the firm's riskiness σ .

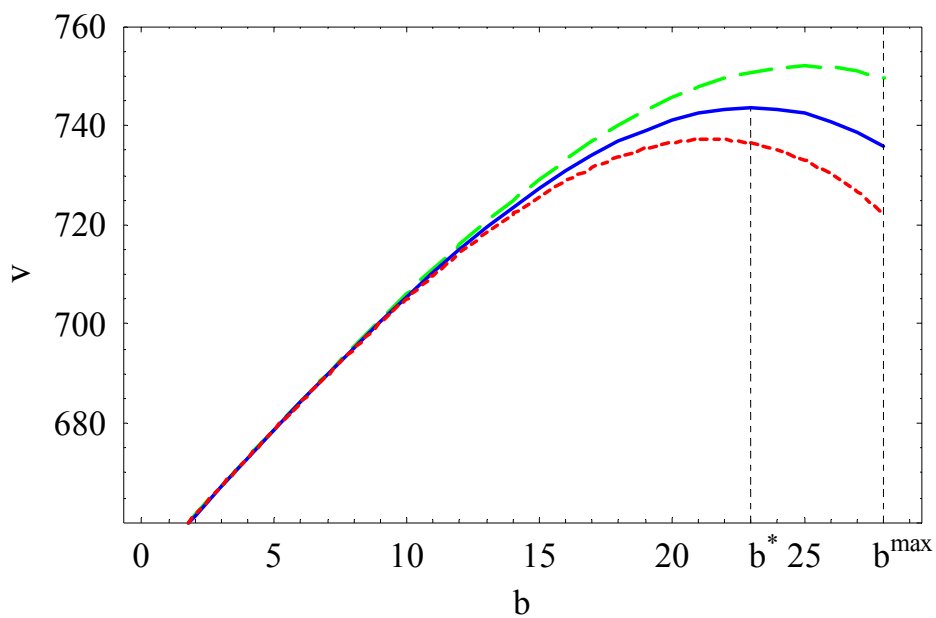


Figure 10: Total firm value v of an Equity Bargaining Power Debt firm as a function of promised coupon b when the firm's negotiation cost varies: $\delta = 10\%$ (*long-dashed line*), $\delta = 20\%$ (*solid line*), and $\delta = 30\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, $\eta = 1$. Note that optimal leverage b^* depends on the size of negotiation costs δ , but not the firm's debt capacity b^{\max} .

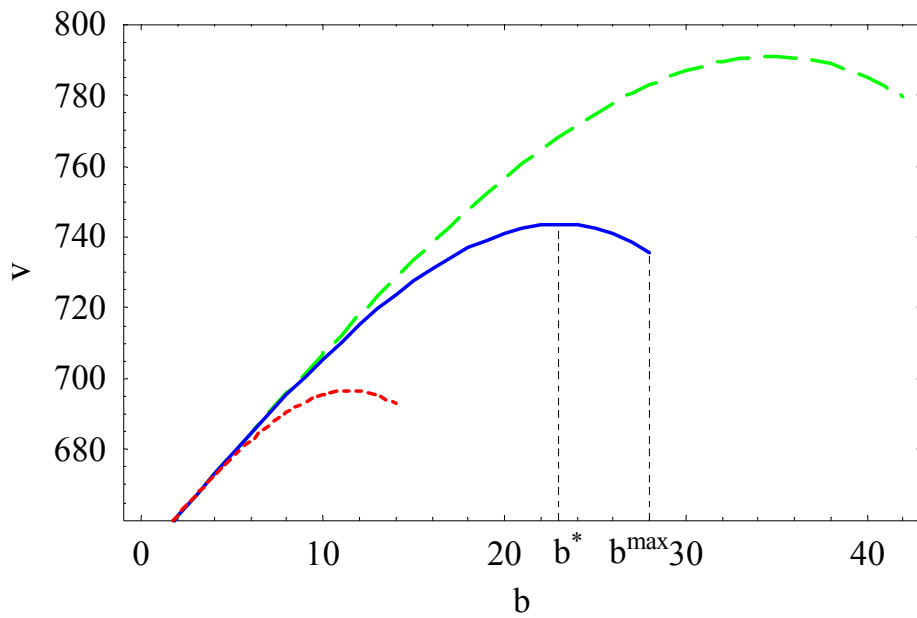


Figure 11: Total firm value v of an Equity Bargaining Power Debt firm as a function of promised coupon b when the firm's bankruptcy costs vary: $\alpha = 25\%$ (*long dashed line*), $\alpha = 50\%$ (*solid line*), and $\alpha = 75\%$ (*short-dashed line*). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\tau = 35\%$, $\delta = 10\%$, and $\eta = 1$. Note that both optimal leverage b^* and debt coupon capacity b^{\max} depend on the firm's bankruptcy costs α .

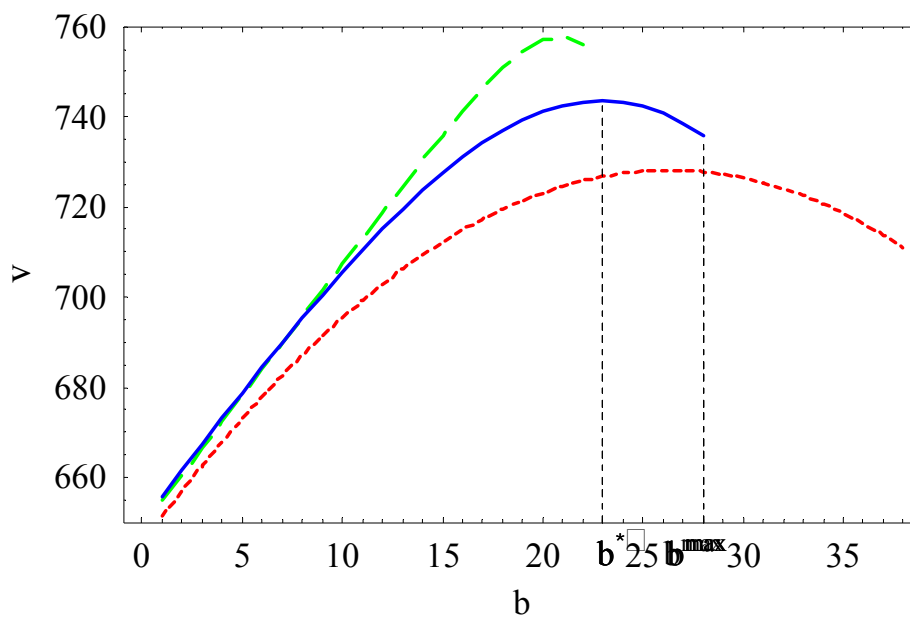


Figure 12: Total firm value v of an Equity Bargaining Power Debt firm as a function of promised coupon b when the firm's EBIT volatility varies: $\sigma = 10\%$ (*long-dashed line*), $\sigma = 20\%$ (*solid line*), and $\sigma = 30\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, $\delta = 10\%$, $\eta = 1$. Note that both optimal leverage b^* and debt capacity b^{\max} depend on the firm's riskiness σ .

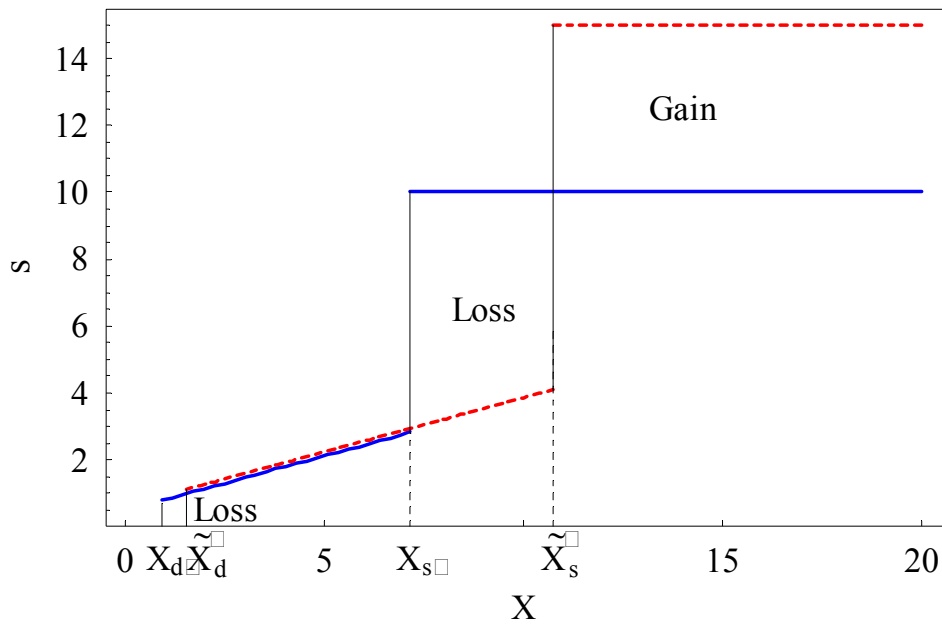


Figure 13: “Loss” versus “Gain” of bank debt’s strategic service flow s with equity making take-it-or-leave-it offers as a function of EBIT X when leverage increases from $b = 10$ to $\tilde{b} = 15$. It is assumed $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, $\delta = 10\%$, $\eta = 1$. Note that both the default-triggering EBIT level X_d and the *incentive compatible* switch point X_s are (strictly) increasing in the firm’s leverage choice: $X_d < \tilde{X}_d$ and $X_s < \tilde{X}_s$.

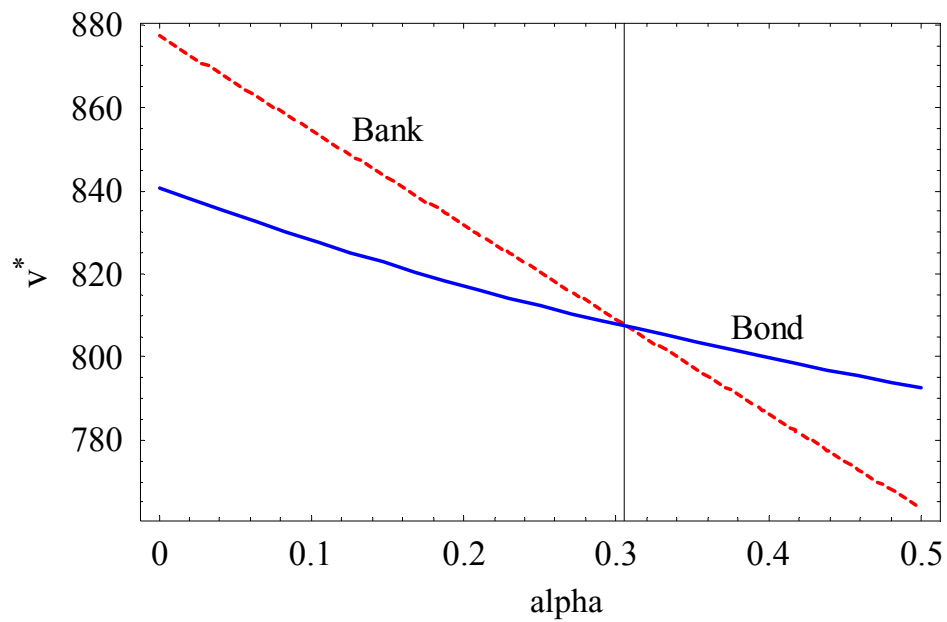


Figure 14: Trade-off in terms of total (*optimally levered*) firm value when choosing – as an exclusive source of financing – between Equity Bargaining Power Debt (*dashed line*) and Bond Market Debt (*solid line*) as a function of bankruptcy costs α . An *optimal* degree of leverage, i.e., promised coupon payments $b^*(\alpha, \cdot)$ and $c^*(\alpha, \cdot)$, $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 30\%$, $\tau = 35\%$, and $\delta = 10\%$ are assumed.

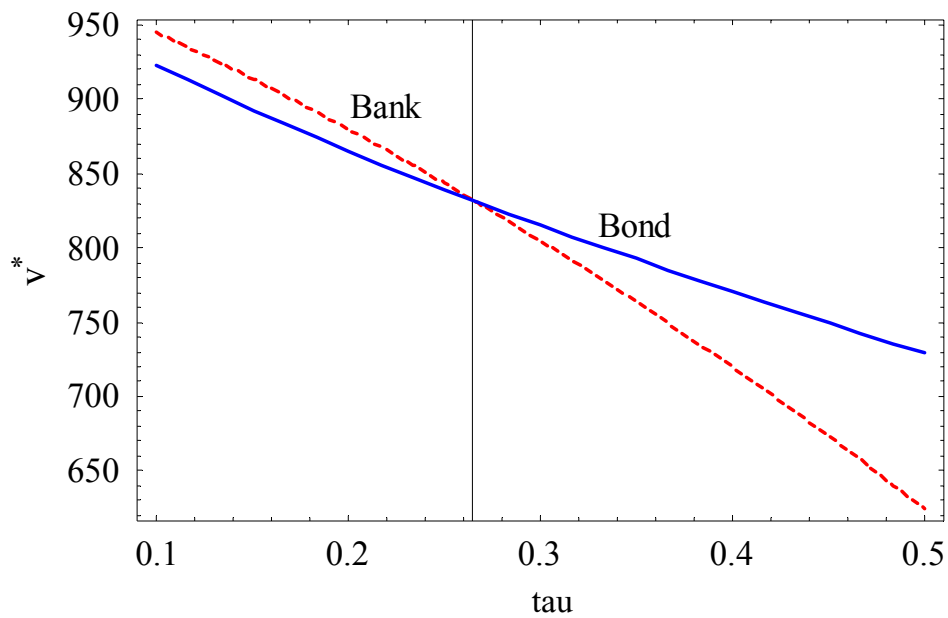


Figure 15: Trade-off in terms of total (*optimally levered*) firm value when choosing – as an exclusive source of financing – between Equity Bargaining Power Debt (*dashed line*) and Bond Market Debt (*solid line*) as a function of corporate taxes τ . An *optimal* degree of leverage, i.e., promised coupon payments $b^*(\tau, \cdot)$ and $c^*(\tau, \cdot)$, $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 30\%$, $\alpha = 50\%$, and $\delta = 10\%$ are assumed.

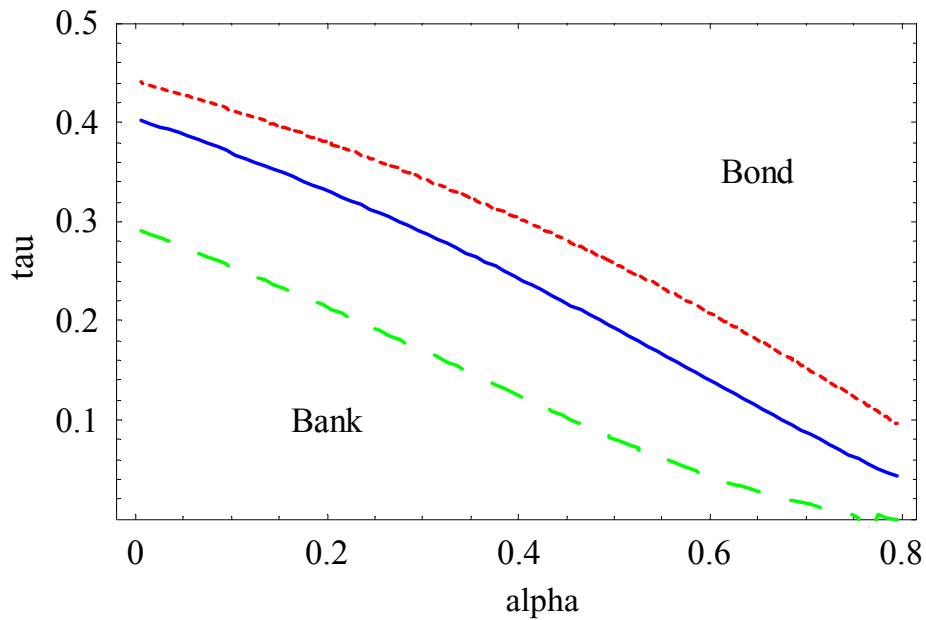


Figure 16: Implicit trade-off based on total (*optimally levered*) firm value when choosing between Equity Bargaining Power Debt and Bond Market Debt as an exclusive source of financing in terms of bankruptcy costs α and corporate taxes τ when EBIT riskiness varies: $\sigma = 15\%$ (*long-dashed line*), $\sigma = 20\%$ (*solid line*), $\sigma = 25\%$ (*short-dashed line*). An *optimal* degree of leverage, i.e., promised coupon payments $b^*(\alpha, \tau, \cdot)$ and $c^*(\alpha, \tau, \cdot)$, $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, and $\delta = 10\%$ are assumed. Note that *ceteris paribus* it is more likely to be optimal for riskier firms to choose Equity Bargaining Power Debt over Bond Market Debt and *vice versa*.

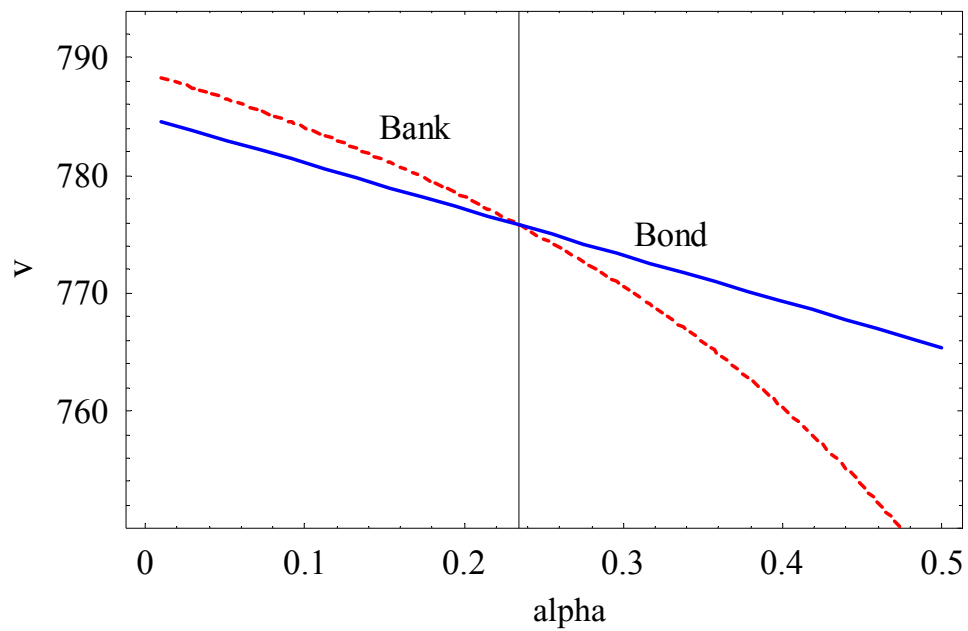


Figure 17: Trade-off in terms of total (*equally levered*) firm value when choosing – as an exclusive source of financing – between Equity Bargaining Power Debt (*dashed line*) and Bond Market Debt (*solid line*) as a function of bankruptcy costs α . An *equal* degree of leverage, i.e., promised coupon payments $b = c = 30$, and $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 30\%$, $\tau = 35\%$, $\delta = 10\%$ are assumed.

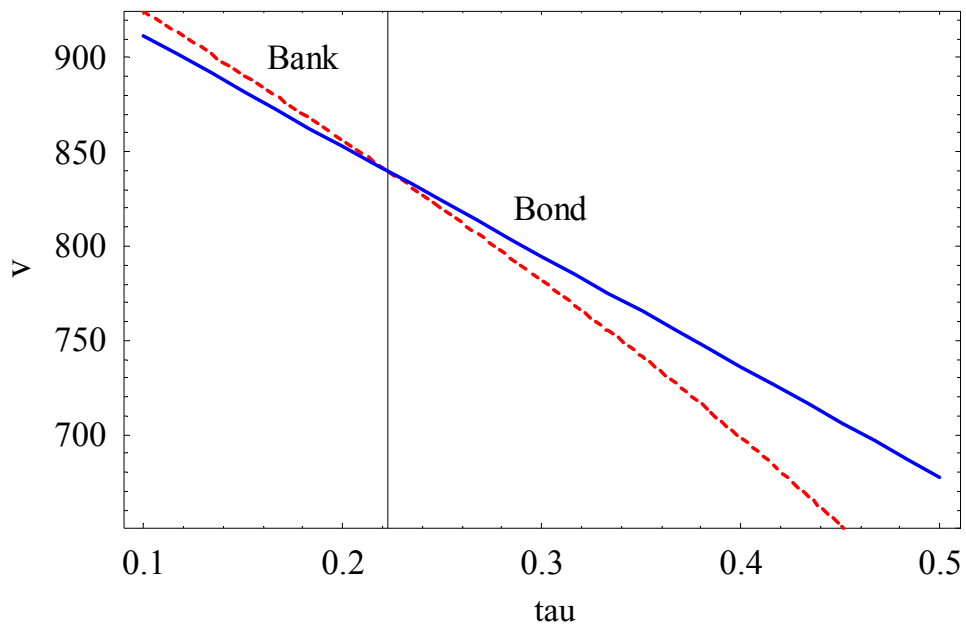


Figure 18: Trade-off in terms of total (*equally levered*) firm value when choosing – as an exclusive source of financing – between Equity Bargaining Power Debt (*dashed line*) and Bond Market Debt (*solid line*) as a function of corporate taxes τ . An *equal* degree of leverage, i.e., promised coupon payments $b = c = 30$, and $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 30\%$, $\alpha = 50\%$, $\delta = 10\%$ are assumed.

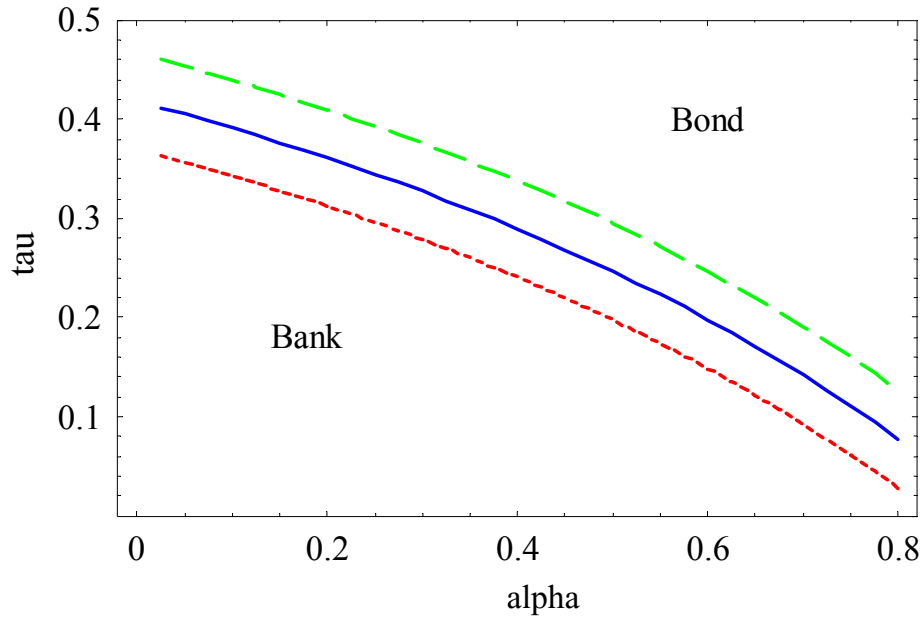


Figure 19: Implicit trade-off based on total (*equally levered*) firm value when choosing between equity bargaining power bank and Bond Market Debt as an exclusive source of financing in terms of bankruptcy costs α and corporate taxes τ when bank debt's negotiation costs vary: $\delta = 5\%$ (*long-dashed line*), $\delta = 10\%$ (*solid line*), $\delta = 15\%$ (*short-dashed line*). An equal degree of leverage, i.e., promised coupon payments $b = c = 30$, and $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 30\%$ are assumed. Note that *ceteris paribus* it is more likely to be optimal for firms with lower negotiation costs to choose Equity Bargaining Power Debt over Bond Market Debt and *vice versa*.

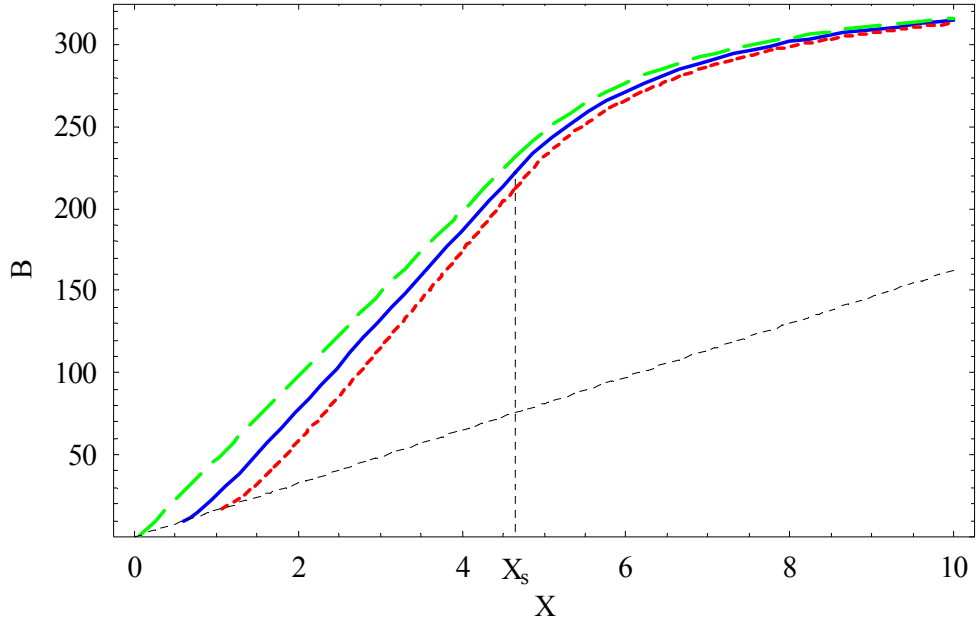


Figure 20: Bank Bargaining Power Debt B as a function of EBIT X for a given coupon level b when the firm's negotiation cost varies: $\delta = 5\%$ (*long-dashed line*), $\delta = 10\%$ (*solid line*), and $\delta = 20\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $b = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, and $\tau = 35\%$. Note that in the renegotiation region the bank is the residual claimant on the firm's cash flows and hence at the default-triggering EBIT level X_b the value and the slope of bank bargaining power debt are equal to the corresponding value and slope of the firm's restructuring function $(1 - \alpha)(1 - \tau)X / (r - \mu)$ (*thin short-dashed line*).

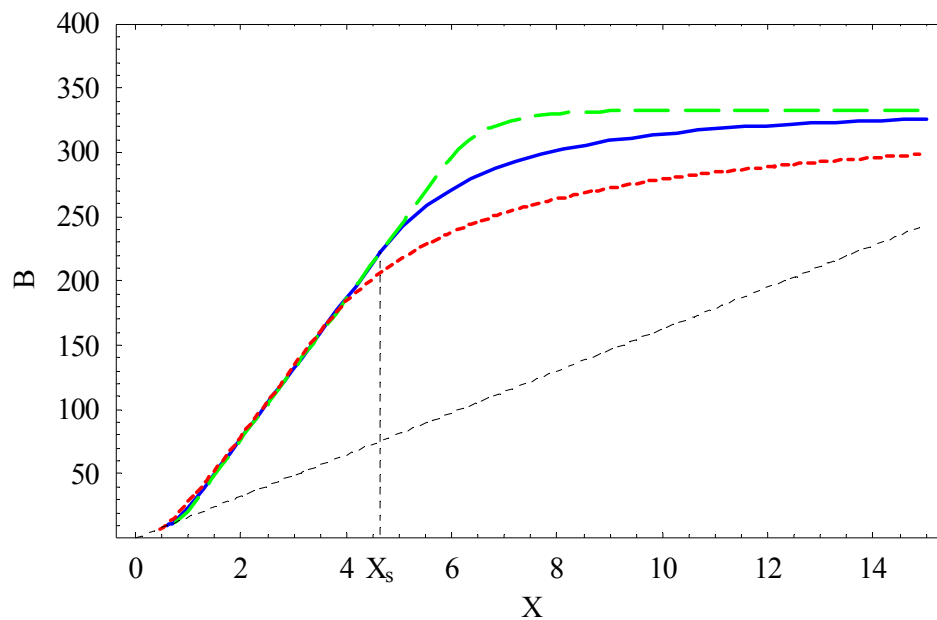


Figure 21: Bank Bargaining Power Debt B as a function of EBIT X for a given coupon level b as EBIT volatility varies: $\sigma = 10\%$ (*long-dashed line*), $\sigma = 20\%$ (*solid line*), and $\sigma = 30\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $b = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$. Note that for any nontrivial negotiation costs ($\delta > 0$) both the default-triggering EBIT level X_b and the *incentive compatible* switch point X_s are decreasing in the firm's riskiness σ .

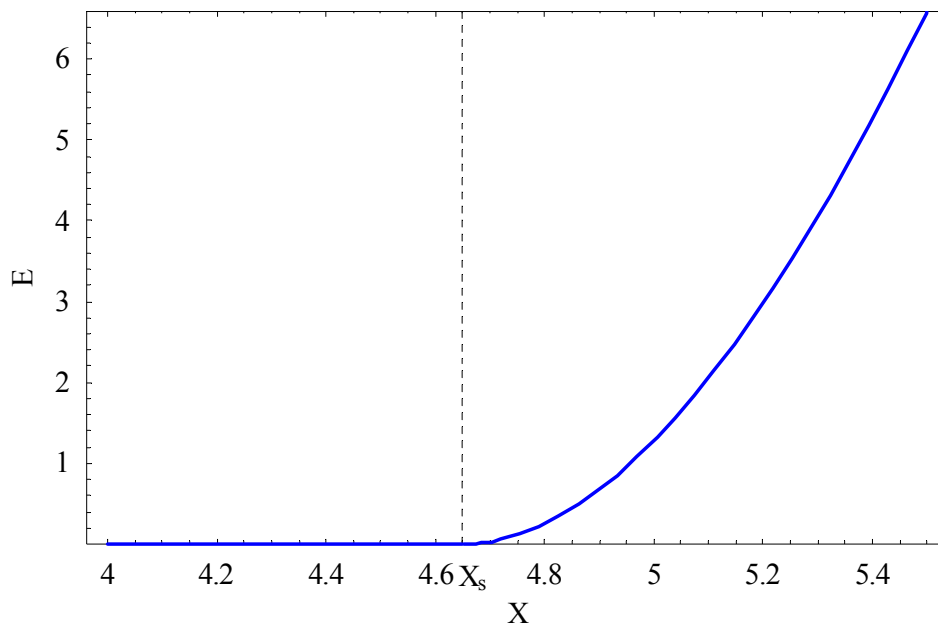


Figure 22: Equity E under Bank Bargaining Power Debt as a function of EBIT X . It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$. Note *value-matching* and *smooth-pasting* obtain at the switch point X_s .

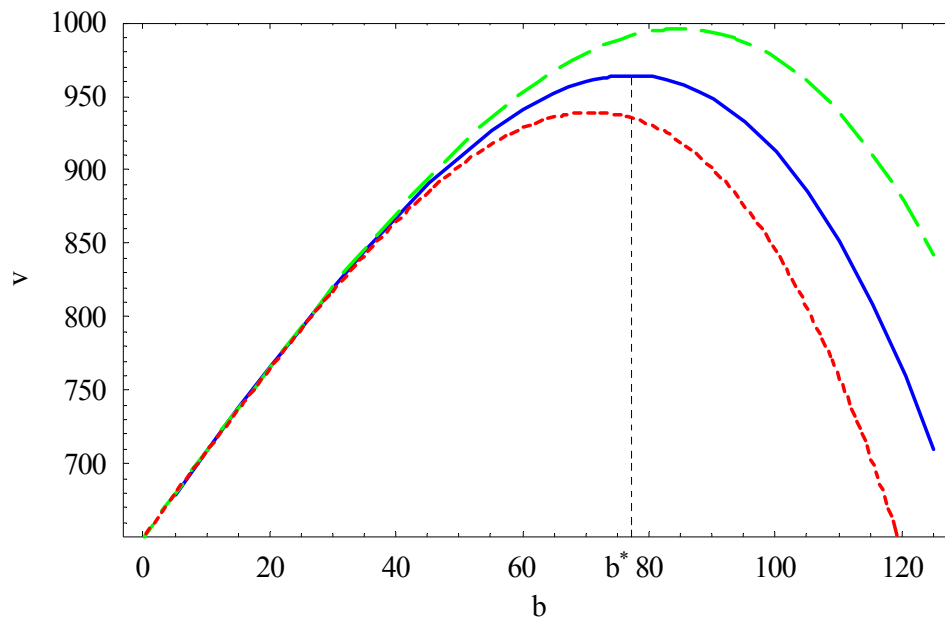


Figure 23: Total firm value v of a Bank Bargaining Power Debt firm as a function of promised coupon b when the firm's negotiation cost varies: $\delta = 5\%$ (*long-dashed line*), $\delta = 10\%$ (*solid line*), and $\delta = 20\%$ (*short-dashed line*). It is assumed $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, and $\tau = 35\%$. Note that optimal leverage b^* depends on the size of negotiation costs δ .

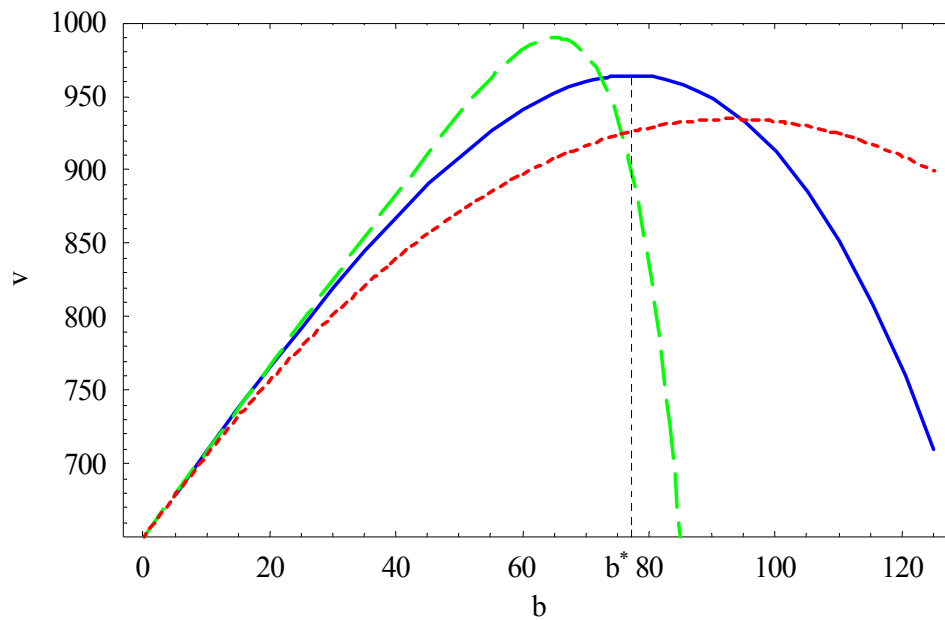


Figure 24: Total firm value v of a Bank Bargaining Power Debt firm as a function of promised coupon b when the firm's EBIT volatility varies: $\sigma = 10\%$ (*long dashed line*), $\sigma = 20\%$ (*solid line*), and $\sigma = 30\%$ (*short-dashed line*). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$. Note that optimal leverage b^* depends on the firm's riskiness σ .

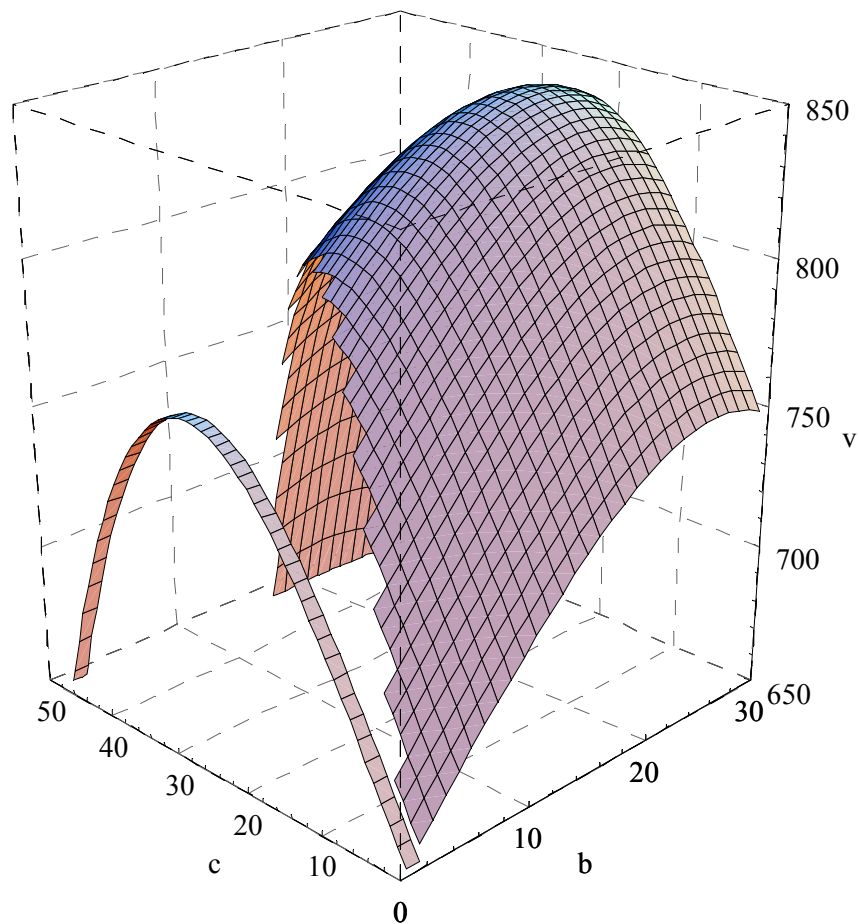


Figure 25: Total firm value v as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$, which implies that the unlevered firm is worth $V(X_0) = (1 - \tau) X_0 / (r - \mu) = 650$. Note that there exists an interior firm value optimum for both classes of debt $v(b^*, c^*)$.

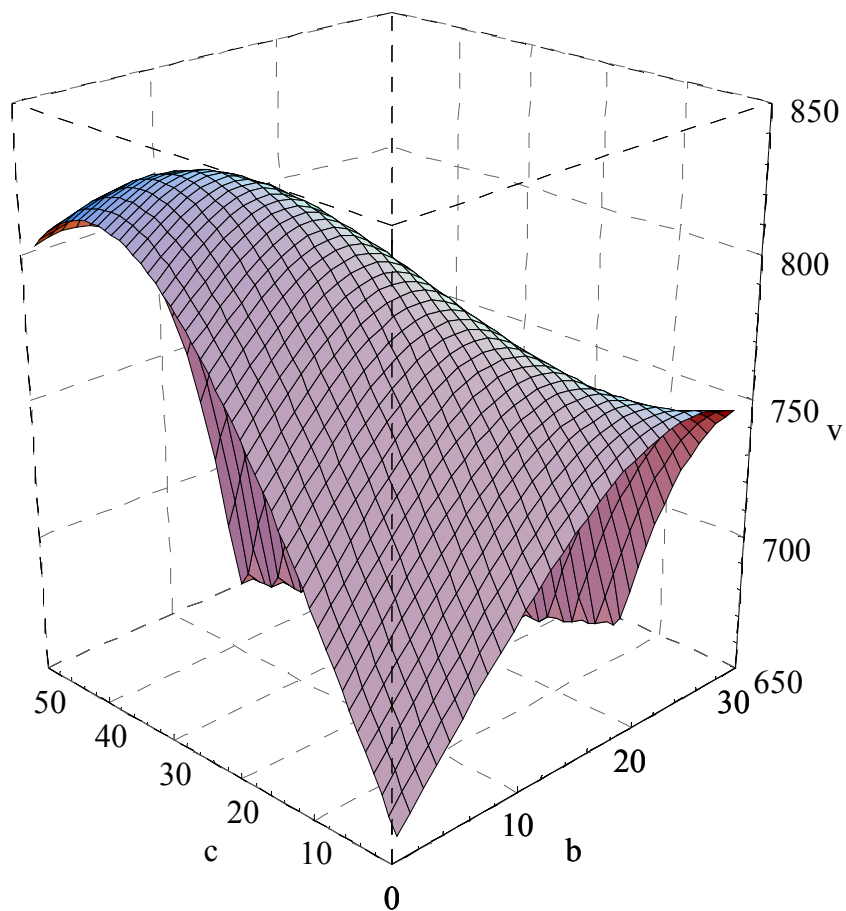


Figure 26: Total firm value v as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power and soft and hard debt are *pari passu*, i.e., equal priority: $\eta = b/(b + c)$. It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, $\delta = 0.1$, which implies that the unlevered firm is worth $V(X_0) = (1 - \tau) X_0 / (r - \mu) = 650$. Note that there exists an interior firm value optimum for both classes of debt $v(b^*, c^*)$.

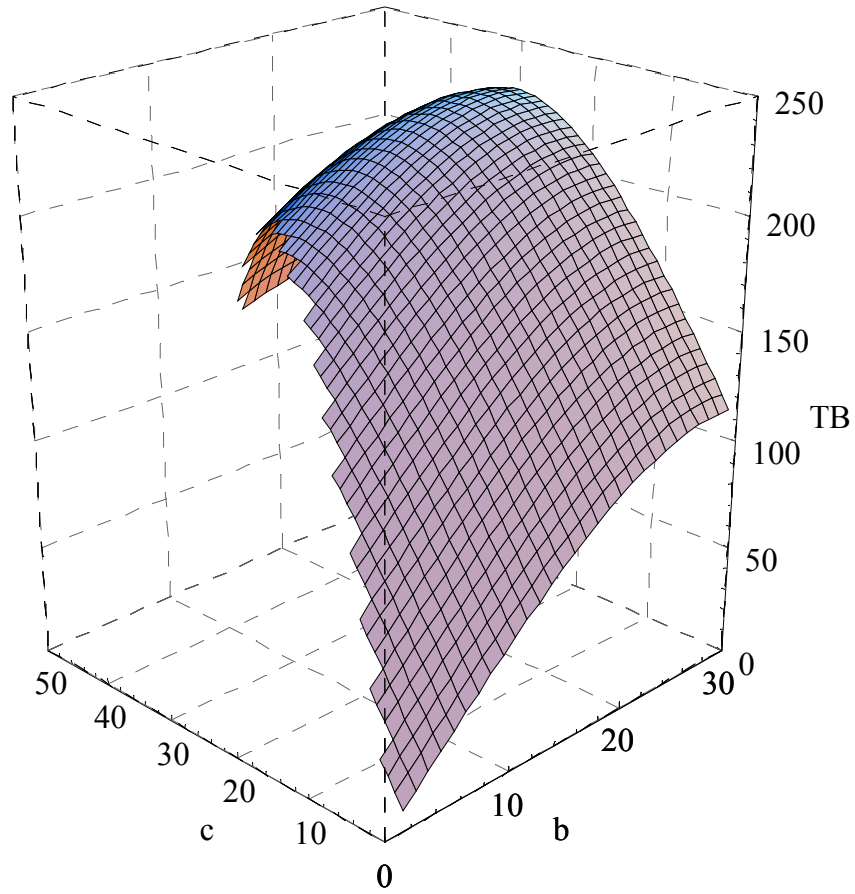


Figure 27: Tax benefits TB as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$.

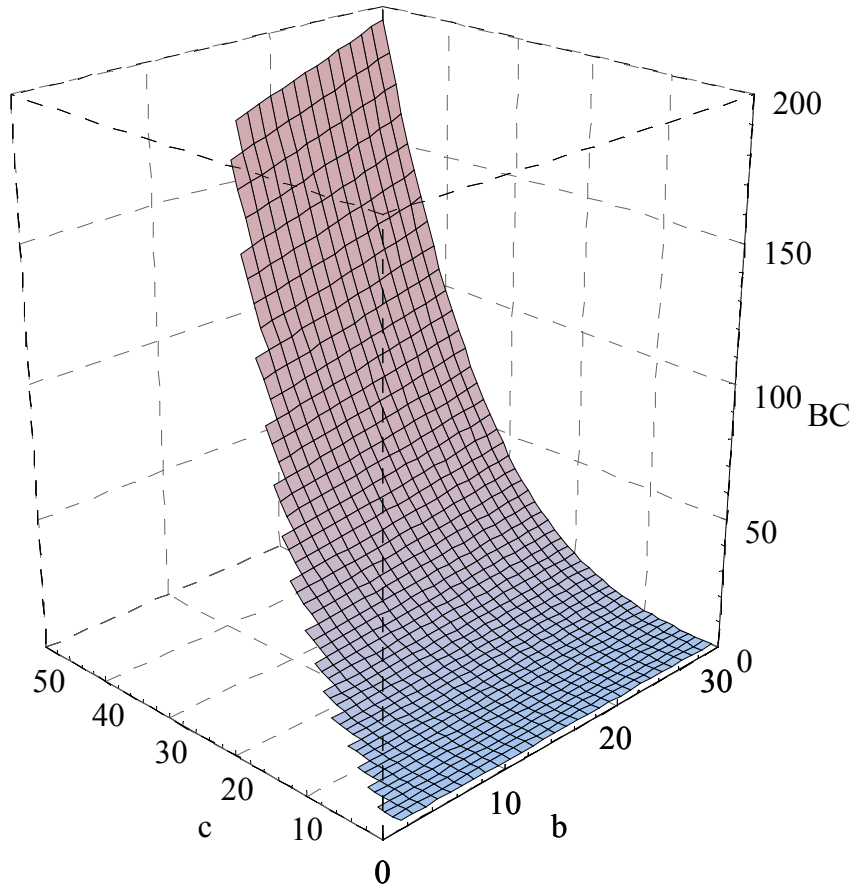


Figure 28: Bankruptcy costs BC as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$.

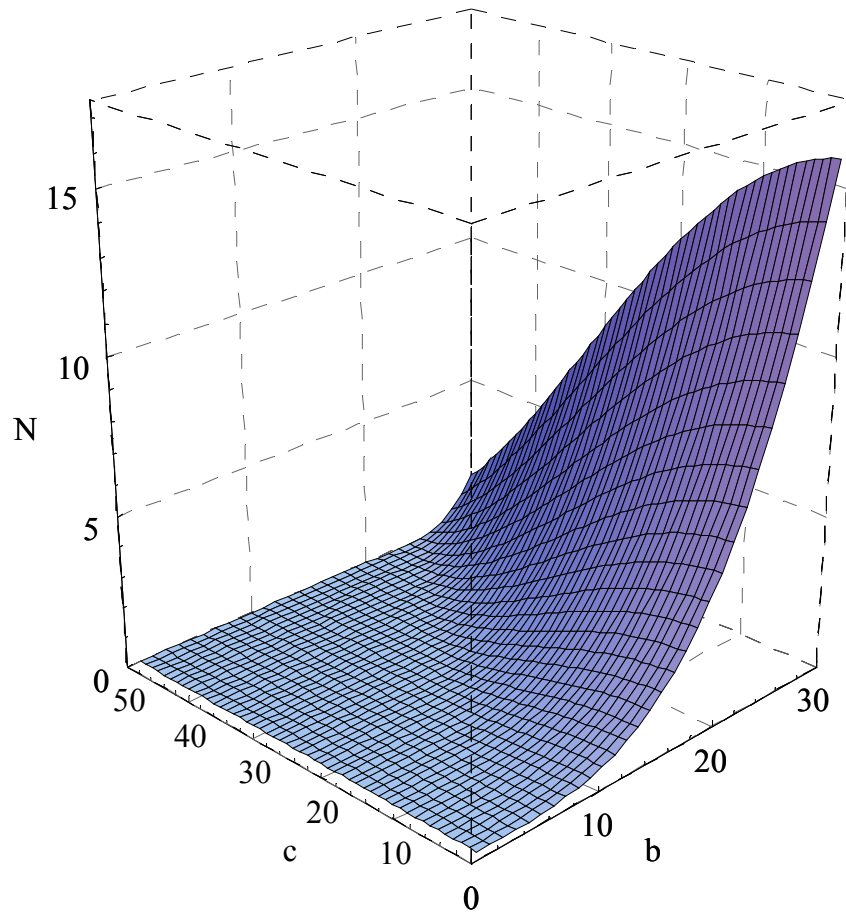


Figure 29: Negotiation costs N as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$.

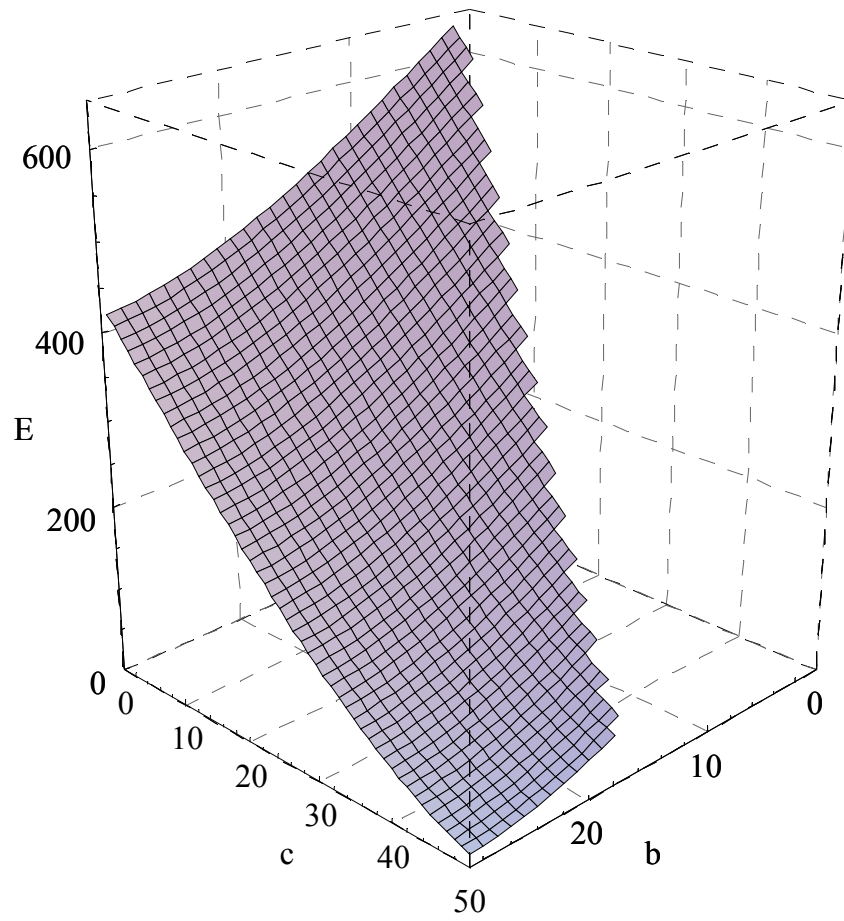


Figure 30: Equity value E as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$.

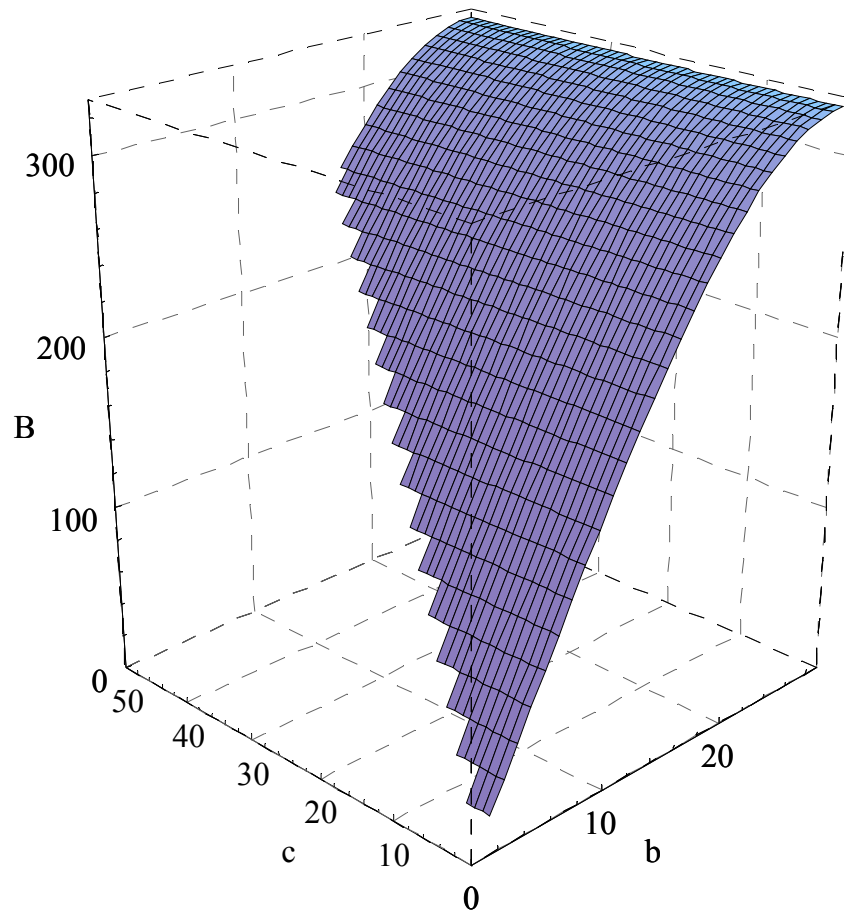


Figure 31: Bank debt value B as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$.

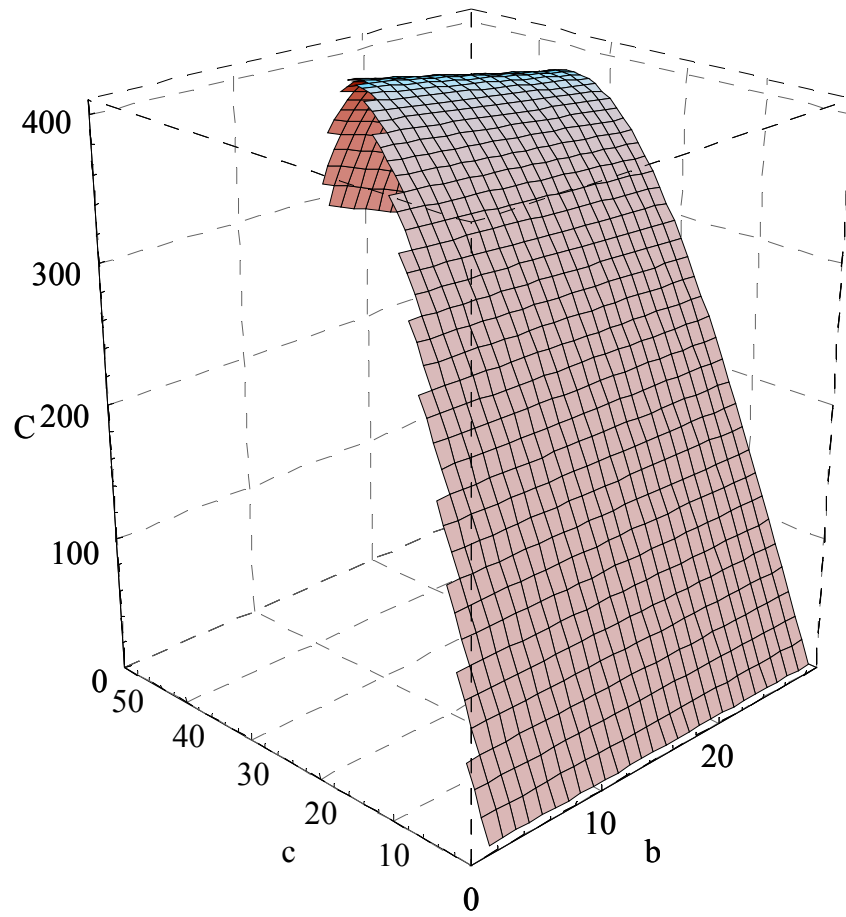


Figure 32: Hard debt value C as a function promised coupon payments to the bank b and to the bond market c when equity holders have all the bargaining power *vis-à-vis* the bank and the bank debt is *senior* ($\eta = 1$). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\sigma = 20\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\delta = 10\%$.

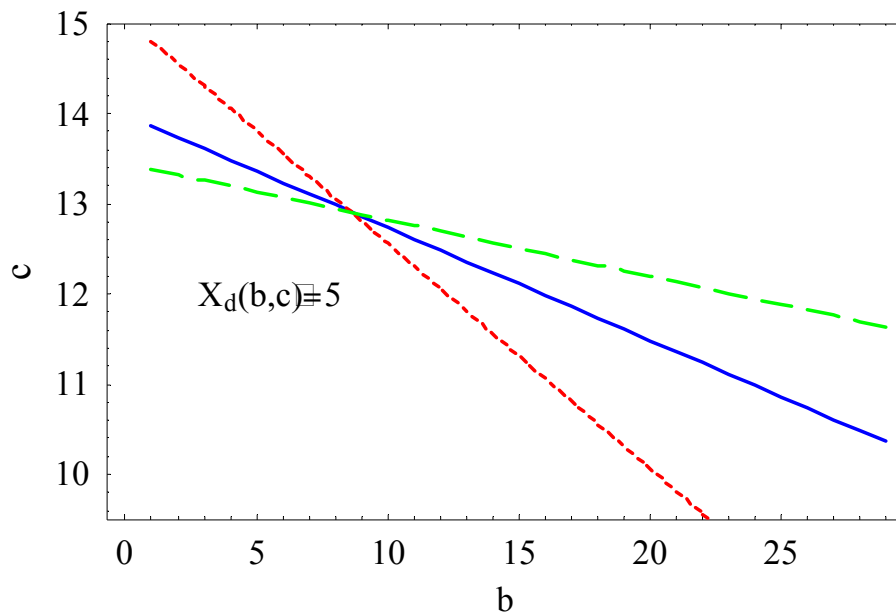


Figure 33: Complementarity effect between Equity Bargaining Power Debt and Bond Market Debt. Feasible pairs of Equity Bargaining Power Debt and Bond Market Debt (b, c) when the equity's *endogenous* default-triggering EBIT level is fixed at $X_d(b, c; \delta) = 5$ and the firm's negotiation costs vary: $\delta = 5\%$ (*long-dashed line*), $\delta = 10\%$ (*solid line*), $\delta = 15\%$ (*short-dashed line*). It is assumed that $X_0 = 20$, $r = 6\%$, $\mu = 4\%$, $\alpha = 50\%$, $\tau = 35\%$, and $\sigma = 20\%$.