

The Value and the Risk of Aggregate Human Capital Implications From a General Equilibrium Model*

JOB MARKET PAPER

Miguel Palacios[†]

November 19, 2008

Abstract

This paper derives the value and the risk of aggregate human capital in a dynamic equilibrium production model. The model shows that, under parsimonious assumptions, the relationship between consumption growth and the ratio of wages to consumption is counter-cyclical. As a result, human capital is less risky than equity, implying that the risk premium of human capital is lower than that of equity. With lower expected returns than equity, human capital has a weight in the aggregate wealth portfolio that exceeds the ratio of wages to consumption. Calibrating the model, I find three key results. First, the weight of human capital in the aggregate wealth portfolio is 85%. Second, a portfolio invested 35% in equity and 65% in risk-free bonds approximately replicates aggregate human capital returns. Third, the covariance between equity returns and human capital returns is twice as large as the covariance between equity returns and wage growth. Based on the results of the model, I estimate human capital returns between 1959 and 2007 and conclude that this estimate of returns to human capital better explains the cross-section of asset returns than an estimate based on aggregate wage growth.

*The financial support of the Batten Institute, the Dean Witter Foundation, and the White Foundation is gratefully appreciated. I thank Nigel Barradale, Jonathan Berk, Sebastien Betermier, Mario Capizzani, Jaime Cassasus, Pierre Collin-Dufresne, Stefano Corradin, Andrés Donangelo, Greg Duffee, Nicolae Gârleanu, Thomas Gilbert, Barney Hartman-Glaser, Sara Holland, Christine Parlour, Nishanth Rajan, Adam Szeidl, Carles Vergara-Alert, Johan Walden and participants at U.C. Berkeley's finance seminar for helpful comments and suggestions. Any errors remain my own.

[†]Haas School of Business, University of California, Berkeley. Email: palacios@haas.berkeley.edu. Phone: +1(510)717-6453. Address: S545, #1900, Berkeley, California, 94720-1900 (USA). <http://faculty.haas.berkeley.edu/palacios>

1 Introduction

Wages constitute more than 80% of consumption and about 60% of gross domestic product in the United States.¹ Indeed, the present value of future wages – human capital – represents the largest share of wealth in the economy and therefore must affect portfolio choice decisions and asset prices. Furthermore, wages are the main corporate expenditure. As a result, firms have a large exposure to shocks that affect the aggregate level of wages, which affects the riskiness of cash flows and financing decisions. In light of the significant role of human capital for both individuals and firms, I develop a production equilibrium model to analyze the drivers of human capital’s value and risk.

Human capital poses a challenge for researchers because even though it is the largest single asset class in the economy, we cannot observe its value or dynamics directly; we merely observe wages, human capital’s dividends. Thus, we need a framework to determine human capital’s value. I use a continuous-time version of the one-sector stochastic growth model to derive the endogenous dynamics of consumption, wages, dividends, and the stochastic discount factor. I then use these dynamics to determine the value and the risk characteristics of human capital and equity.

I use a general equilibrium production economy for two reasons. First, unlike endowment economies, a production-based model helps us understand the sources of the relative magnitudes of wages and dividends. In production models, wages are endogenously determined as the marginal product of labor, rather than assumed exogenously. Dividends are also determined endogenously as agents use their knowledge of the production function to make decisions about consumption and investment. Because the production function determines both wages and dividends, the model enables us to determine their joint dynamics.

¹Using data from the National Income and Product Accounts (NIPA) tables between 1947 and 2007, the average ratio of total compensation to consumption was 85%, and the average ratio of total compensation to production was 58%.

The second reason is that standard production technologies deliver intuitive mean-reverting dynamics that explain why conditional moments change over time. The equilibrium in this model is an expanded version of a Cox-Ingersoll-Ross (1985) economy in which expected returns are stochastic and determined by the production function.

The main contribution of this paper is to show that under a plausible set of assumptions, we should expect human capital to be less risky than stocks. While empirical evidence confirms this finding (dividend growth exhibits a volatility of 10%, whereas aggregate wage growth volatility is about 2.2%),² the empirical literature does not provide an explanation for why wages are less volatile than dividends.³ Existing theoretical studies justify these values by pointing out that labor contracts insure workers against idiosyncratic risk.⁴ In contrast, the mechanism in my model does not rely on idiosyncratic labor shocks.

In my model human capital is less risky than equity because of the dual role of production shocks in the economy. A production shock simultaneously alters both the productive capital base and the production per unit of capital. Under decreasing returns to scale, positive production shocks lead to lower production per unit of capital and vice versa. After a positive production shock, consumption rises. Wages rise as well, but because the positive production shock decreases labor's marginal product per unit of capital, wages do not rise as much as consumption. As a result, dividends that equity holders receive grow more than consumption, implying greater equity exposure to aggregate shocks.

The result that human capital is less risky than equity suggests that we should apply a lower discount rate to it. Thus, the weight of human capital in the aggregate wealth portfolio is higher than the fraction of wages to consumption observed in the data. This finding suggests that a weight for human capital closer to 85% seems more appropriate than a weight of around 70%, which is common in the literature.⁵ The result is relevant

²See Campbell and Cochrane (1999) who use 11.2%. Wage data comes from "total compensation", as reported in the NIPA tables, between 1947 and 2007.

³See Lustig, Van Nieuwerburgh and Verdelhan (2008).

⁴See Harris and Hölmstrom (1982).

⁵For example, see Baxter and Jermann (1997) and Chen et al. (2008).

for the portfolio choice literature because the weight and risk of human capital affect an agent's optimal portfolio.

The model also provides an alternative explanation for why human capital has an effect on asset-pricing. Previous work concludes that the impact of human capital on asset prices, if any, depends on the covariance of aggregate wage growth and equity returns.⁶ My model's results show that the covariance between human capital and equity returns can possibly be much higher than the covariance between aggregate wage growth and equity returns. To illustrate that human capital might indeed affect asset-prices, I use the model to construct a series of monthly human capital returns between 1959 and 2007. Using a Fama-Macbeth (1973) regression, I show that this measure of human capital better explains the cross-section of asset returns than one using aggregate wage growth.

The model's results follow from the dynamics of dividends and wages in financing consumption. Both the values of claims to human capital and claims to equity depend on the level of aggregate consumption and the share of aggregate consumption each claim holder receives. While aggregate consumption growth increases the value of both claims, changes to the share of consumption paid to each claim move their value in opposite directions. The share does not remain constant because the owner of equity receives dividends *net* of any investment, while the owner of human capital bears no responsibility for investment. If investment as a fraction of production changes over time, human capital and equity returns are not perfectly cointegrated. If these changes are systematic, the claim whose fraction increases when consumption increases will command a greater relative risk premium.

In this model, the value and riskiness of human capital are mean-reverting, as are the equity premium, risk-free rate, and dividend yield. This provides a convenient setting to explore how conditional moments of these variables change as the state of the economy changes. However, the model fails to reconcile asset pricing and macroeconomic dynamics, in particular the low volatility of capital in the presence of a significant equity risk premium.

⁶See Mayers (1972) and Fama and Schwert (1977).

Other equilibrium production-models share this shortcoming. I calibrate the model to match observed consumption growth, consumption volatility, and the long-term ratio of wages to consumption. I then draw implications about the value of and excess returns to human capital, while recognizing the unmet need to reconcile the dynamics of production based models with that of observed macroeconomic facts.

The structure of the rest of the article is as follows. Section 2 relates this paper to the existing literature. Section 3 presents the model with an infinitely-lived agent and derives the dynamics of the general equilibrium. Section 4 discusses the main implications from the model, and Section 5 analyzes the numerical calibration. Section 6 shows an empirical application from the model, and Section 7 concludes.

2 Relation to existing literature

The literature that originally tackled the impact of human capital on asset prices started from assumptions about the exogenous wage process (Mayers (1972)). Fama and Schwert (1977) tested the empirical predictions of Mayers' model and concluded that human capital, as proxied by wages, did not play a major role in determining asset prices.

More recent work, recognizing the weakness of treating wages as a proxy for human capital, tries to include human capital as relevant for determining other assets' prices. The two main characteristics of human capital that matter most for asset-pricing are its weight in the aggregate wealth portfolio and its riskiness.

The weight of human capital in the aggregate wealth portfolio is either assumed exogenously or derived endogenously under a restricted set of assumptions. Estimates of this weight typically range between 60% and 80%, all lower than the estimate presented here (Baxter and Jermann (1997), Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2006), Chen et al.(2008)). Two empirical papers that support higher values for the fraction of human capital in aggregate wealth are Jorgenson and Fraumeni (1989) and Lustig, Van Nieuwerburgh and Verdelhan (2008). This paper finds similar results, while highlighting

the mechanism that drives them.

The riskiness of human capital is comprised of idiosyncratic and systematic components. In my model I ignore idiosyncratic shocks, so systematic risk is the sole determinant of human capital's expected returns. Empirical studies require assumptions about human capital's expected returns. Shiller (1995) assumes that human capital's expected returns are constant. Campbell (1996) assumes that its expected returns are conditioned to be the same as those of stocks, while Jagannathan and Wang (1996) assume human capital's return is equal to labor income growth. Palacios-Huerta (2003a) uses the increase in labor income attributed to an extra year of education as a proxy for human capital returns and Lustig and Van Nieuwerburgh (2006) assume it to be the return that minimizes pricing errors in their model.⁷ The model presented here provides theoretical foundations to evaluate these assumptions. In particular, human capital's expected returns appear to be smaller than those of stocks, and innovations to expected returns of human capital and equity are negatively correlated.

Human capital is not only important on an aggregate level for asset-prices, but also important for portfolio choice. The portfolio choice literature assumes exogenous wage dynamics (for example, Merton (1971), Svensson (1988), Koo (1998), Campbell and Viceira (1999), and Viceira (2001)), but then wages and asset prices can diverge unrealistically. Benzoni, Collin-Dufresne and Goldstein (2007) provide a solution to this problem by assuming cointegration between wages and financial markets. However, the process driving the cointegration is also exogenous. In contrast, my model delivers cointegration endogenously, providing an intuitive reason for why wages and dividends follow a trend over time.

The model is grounded in the traditional one-sector stochastic growth literature.⁸ This literature goes back at least to Ramsey (1928) and the stochastic versions of Mirrlees

⁷Palacios-Huerta (2003b) uses the same method to compare the returns to investing in education of different demographic groups.

⁸A general closed-form solution to the growth model is not known, though solutions do exist restricting the parameters (Smith (2007)). The restrictions are not useful to obtain reasonable results, so I proceed with a numerical solution.

(1967), Brock and Birman (1972) and Merton (1975). Subsequent work tried to bridge macroeconomic observations with asset prices in an attempt to address the shortcomings pointed out by Mehra and Prescott (1985) on the predictions of Rubinstein (1976), Lucas (1978) and Breeden's (1979) consumption-based model.⁹ Importantly, none of these papers explored the implications for the value and expected return of human capital.

The paper also links to more recent work that explores the asset pricing implications of having multiple sources of income that add up to aggregate consumption, as in Cochrane, Longstaff and Santa Clara (2006), Başak (1999), Gomes et al.(2007), Gârleanu and Panageas (2008), and Santos and Veronesi (2006). In contrast to this work, I allow for capital accumulation, addressing the link between capital growth, production shocks, and human capital returns.

The implications for asset prices derived from the model follow empirical observations linking price-dividend ratios, forecasted economic growth, and asset returns (Fama and French (1989), Fama (1990), Lamont (1998), Lettau and Ludvigson (2001a and 2001b) and Santos and Veronesi (2006)). The theoretical model I present here is a step towards understanding the dynamics observed in the data.

3 A general equilibrium model of capital returns with decreasing returns to scale

This section describes and solves the equilibrium model, characterizes the representative agent's optimal consumption decision, and derives the joint dynamics of wages and dividends.

⁹See Brock (1982), Cochrane (1991, 1996), Belo (2007), Rouwenhorst (1995), Jermann (1998), Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (1992 and 2002).

3.1 Economic environment

Consider a competitive economy with a continuum of identical agents, whose mass is normalized to 1. Each agent works and chooses to invest her wealth W in a productive technology and financial claims. Agents can choose to enter or exit the productive technology at any time. I also assume agents can trade claims that span all possible outcomes in the economy, and therefore markets are complete. This assumption implies that a unique stochastic discount factor exists in the economy. Agents cannot enter into Ponzi games, so the value of the claims they trade cannot exceed the value of their wealth. We are interested in three of these claims.¹⁰ The first claim is “human capital,” and it is defined as a security that pays aggregate wages. The second claim is “equity,” and it consists of the dividend left after wages are paid and investments added to capital. The last claim is an instantaneously risk-free bond.

3.2 Production and profits

The single productive technology produces a single consumption good. At any point in time, the agent can consume the good or use it as capital to produce more of the same consumption good. Capital investment is perfectly reversible, so changes in its stock are costless. Production at each point in time depends on the amount of aggregate capital, labor, the technology level and production shocks. Denote by $(\Omega, \mathfrak{F}, \mathbb{P})$ a fixed complete probability state, and the stochastic process $(B_t)_{t \geq 0}$, a standard 1-dimensional Brownian motion with respect to the filtration (\mathcal{F}_t) . The brownian motion is a shock to production as described below.

I denote the technological level by A_t . A_t evolves deterministically over time and follows exponential growth.¹¹

¹⁰Note that because there is only one shock in the economy, only two claims are needed to span the whole economy.

¹¹Technology growth does not have to be deterministic and technology shocks can easily be added to the model.

$$dA_t = A_t \eta dt, \tag{1}$$

where η is the growth rate of the technology level.

Instantaneous production, dP_t , depends on a deterministic production function $F(K_t, L_t, A_t)$, and a shock $K \sigma_K dB_t$,¹²

$$dP_t = F(K_t, L_t, A_t)dt + K_t \sigma_K dB_t, \tag{2}$$

where K_t and L_t are the capital and labor inputs used for production, and σ_K is the instantaneous volatility of the production shocks. In what follows, I drop the time subscript t when not needed. The shock captures temporal idiosyncratic shocks that affect production. For example, one such shock would be the impact that rain or sun have on a crop's production. Another example is a depreciation shock that suddenly destroys capital. If $\sigma_K = 0$, then the technology is instantaneously riskless.

The literature concentrates on production functions $F(K, L, A)$ that are continuous, differentiable, and homogeneous of degree one in K and AL . Here, I will study the widely used labor-augmenting Cobb-Douglas production function,

$$F(K, L, A) = K^\alpha (AL)^{1-\alpha}. \tag{3}$$

The choice of making the technology labor-augmenting instead of Hicks-neutral or capital-augmenting does not affect the main results, but it affects the interpretation of the coefficients of A_t . The particular assumption for the production technology does impact wage and dividend dynamics, as different production functions result in different shares of pro-

¹²It is perhaps more natural to use shocks proportional to the production level, or proportional to the impact that A_t has on the production function, instead of shocks proportional to capital. The choice presented here is a simplification that keeps the instantaneous variance of capital constant. The alternative specification leads to stochastic volatility which adds one more layer of complexity to the results and makes the numerical results less stable, without necessarily adding interesting insights.

duction going to labor and capital. The specification presented here should be interpreted as a benchmark case, with richer structures possible when the shares of output that go to labor and capital change over time.

Profits are given by the amount produced net of wages paid to employees and depreciation:

$$d\pi = (F(K, L, A) - YL - \delta K)dt + K\sigma_K dB, \quad (4)$$

where Y is the market wide competitive wage and δ is capital's depreciation rate. I assume individuals have no disutility from working, so they do not face a work-leisure tradeoff.

3.3 Firms

Firms are run by managers who maximize the expected present value of profits. Managers choose how much labor to hire at every point in time, whereas shareholders decide how much dividends to pay themselves. Managers know the shareholders' dividend policy, so they know the dynamics of the firm's capital. I also assume that they know the dynamics of wages.

This specification of the problem is equivalent to the more frequent assumption of managers choosing how much capital to rent and how much labor to hire. In a setting in which managers choose how much capital to rent, competition drives profits to zero and agents are paid interest for their capital. However, a setting in which firms pay for rented capital and have no profits leads to a world without dividends. By defining the problem as above, I provide a way to calculate dividends within the standard setting.

Following the previous discussion, the manager's problem is:

$$\max_{\{L\}_t^\infty} E_t \left[\int_t^\infty M_s (F(K_s, L_s, A_s) - Y_s L_s - \delta K_s) ds \right] \quad (5)$$

$$s.t. \quad dK_t = \mu_k K_t dt + \sigma_K K_t dB_t \quad (6)$$

$$dY_t = \mu_{y,t} dt + \sigma_{y,t} dB_t \quad (7)$$

$$dA_t = \eta A_t dt, \quad (8)$$

where M_s is the stochastic discount factor, μ_K and μ_y are the drifts of capital and wages, respectively, and σ_y is the volatility of wages.

3.4 Agents

The agent-investor optimally invests so that she consumes all that she receives from wages and dividends. In equilibrium, the marginal product of labor determines wages, and the amount of capital invested in the production technology determines its expected return and volatility, R_t and V_t , respectively. The main driver of the dynamics that I derive below is the relationship between aggregate investment and expected returns.

At this point, following the approach of Cox, Ingersoll and Ross (1985) and without loss of generality, we can analyze the equilibrium when the only decision agents make is how much to consume and how much to invest in productive capital. In particular, we can abstract from the problem of how much the agent allocates to the riskless asset or to any other financial claim. To derive the prices of these claims we only need to define a claim and study the behavior of its dividends in relation to the stochastic discount factor.

I assume the agent chooses how much to consume at every instant (C_t) so that her capital investment W maximizes her expected time-separable utility function over an infinite horizon. The agent knows the expected instantaneous return R_t and volatility σ_K of investing in the technology at time t and assumes that her actions do not affect them.

R_t is derived in equilibrium as a function of the capital stock and the technology level. Therefore, the agent's problem can be defined as:

$$\max_{\{C\}_t^\infty} E_t \left[\int_t^\infty U(C_s, s) ds \right] \quad (9)$$

$$s.t. \quad dW_t = (W_t R_t + Y_t - C_t) dt + W_t V_{Kt} dB_t \quad (10)$$

$$0 = \lim_{t \rightarrow \infty} E \left[e^{-\rho t} J(W_t, Y_t, R_t, V_t) \right]. \quad (11)$$

where $J(W_t, Y_t, R_t, V_t, t)$ is the agent's indirect utility from future consumption. Equation (11) is the transversality condition.

The agent's utility function is of the traditional time-separable, constant relative risk aversion form:

$$U(C_t, t) = e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma}. \quad (12)$$

For technical reasons, I constrain the coefficient of relative risk aversion so that $\gamma > 1$.

3.5 Equilibrium

Because markets are complete, the solution to this economy is equivalent to solving the central planner's problem. This problem is easier to solve than the competitive economy problem and yields the same results. I start with the equilibrium's definition.

3.5.1 Definition

Capital and labor markets must clear in equilibrium. The definition that follows is standard:

Definition 1 *In this economy, an equilibrium is defined as a stochastic path for*

$$\{K_t, L_t, A_t, R_t, V_t, \{C_t\}_1^m, Y_t, \{W_t\}_1^m\}_t^\infty \quad \text{such that, for every } t,$$

1. Given the processes $\{Y_t, A_t, K_t\}$, each firm chooses L_t to maximize the present value of profits (Equation (5)).
2. Given the processes $\{Y_t, W_t, R_t, V_t\}$, the agent chooses C_t to maximize his expected lifelong utility (Equation (9)), subject to his budget constraint (Equation (10)) and the transversality condition (Equation (11)).
3. Capital supplied by agents equals the aggregate capital used to produce: $\sum_{s=1}^m W_{s,t} = K_t$.
4. Labor markets clear: $L_t = 1$.

3.5.2 Capital Dynamics

From the central planner's point of view, capital dynamics will be those of a classical optimal consumption choice model in which returns are time varying. Define Z_t as production per unit of capital:

$$Z_t = \left(\frac{A_t}{K_t} \right)^{1-\alpha}. \quad (13)$$

Then the solution to the equilibrium problem is that of a central planner who solves the following optimal control problem:¹³

$$\max_{\{C\}_t^\infty} E_t \left[\int_t^\infty U(C_s, s) ds \right] \quad (14)$$

$$\begin{aligned} s.t. \quad dK_t &= (K_t Z_t - \delta - C_t) dt + K_t \sigma_K dB_t \\ 0 &= \lim_{t \rightarrow \infty} E [e^{-\rho t} J(K_t, A_t, t)] \\ Z_t &= Z(K_t, A_t), \end{aligned}$$

where $J(K_t, A_t, t)$ is the indirect utility in the central planner's problem.

¹³In the limit when $K \rightarrow 0$, the drift term in the dynamics of capital is not well defined, posing well known problems for the existence of the solution. I assume that the system mean-reverts at a point far enough from $K = 0$ so that this is not a problem.

Z_t is the only state variable in the model. However, it is more convenient to define $z_t \equiv \frac{1}{1-\alpha} \log Z_t$ and use z_t throughout. Applying Itô-Doeblin's Lemma to z_t , its dynamics are given by:

$$dz = \left(c - e^{(1-\alpha)z} + \eta + \frac{\sigma_K^2}{2} \right) dt - \sigma_K dB, \quad (15)$$

where

$$c = \frac{C}{K}. \quad (16)$$

The dynamics of z depend on the joint dynamics of c and Z . When Z is very small, the agent behaves as if she had to choose how much to consume optimally when capital is not productive. In this case, consumption is larger than any amount produced (0 when $z \rightarrow 0$), and the drift of z is positive. On the other hand, as z grows consumption does not grow as much, since the agent faces a higher opportunity cost of consuming. Thus, for large values of z , its drift is negative. As a result, z is mean-reverting. Since z is the only state variable, all the relevant quantities in the model – the risk-free rate, the equity premium, the ratio of wages to consumption, and the dividend price ratio – are mean reverting as well.

3.5.3 Optimization solution

The central planner's optimization problem can be solved using the standard dynamic programming procedure as in Merton (1973). The planner's value function will depend only on her wealth, time, and the vector of technology efficiency A . Let $J(W, t, A)$ be the solution of the Bellman equation. The following proposition characterizes the solution of the agent's optimization problem:¹⁴

¹⁴ Note that neither the amount of capital invested in each technology nor wages enter into the Jacobian. These depend completely on changes to technology, and thus we only need to include A . If technology was irreversible, or if there was friction on the movement of wages, then the level of these variables would be needed here.

Proposition 1 *Assume $\rho > 0$ and $\gamma > 1$. If the value function $J(K_t, z_t, t)$ exists and is twice continuously differentiable, then the agent's value function can be expressed as:*

- $J(K_t, z_t, t) = e^{-\rho t} \frac{K_t^{1-\gamma}}{1-\gamma} Q(z_t)$
- $Q(z_t)$ solves the following ODE:

$$\begin{aligned}
0 = & Q(z) \left(\frac{\gamma}{1-\gamma} g(z)^{-\frac{1}{\gamma}} - \frac{\rho}{1-\gamma} + e^{(1-\alpha)z} - \frac{1}{2} \gamma \sigma_K^2 \right) \\
& + Q'(z) \frac{1}{1-\gamma} \left(g(z)^{-\frac{1}{\gamma}} - e^{(1-\alpha)z} + \eta + \left(\gamma - \frac{1}{2} \right) \sigma_K^2 \right) \\
& + Q''(z) \frac{1}{2(1-\gamma)} \sigma_K^2
\end{aligned} \tag{17}$$

with $Q(\infty) = 0$ and $Q'(\infty) = 0$ as boundary conditions.

The function $g(z)$ is defined as $g(z) \equiv \left(Q(z) - \frac{Q'(z)}{1-\gamma} \right)$.

Optimal consumption will be given by

$$c_t^* \equiv \frac{C_t^*}{K_t} = g(z)^{-1/\gamma}. \tag{18}$$

Proof: See Appendix.

The boundary conditions follow from the agent choosing to consume more when production per unit of capital increases. As production per unit of capital becomes arbitrarily large, the marginal utility approaches zero, implying that $Q(z)$ approaches zero for large values of z .

3.6 Dynamics of the economy

Having determined the optimal rule for choosing between consumption and saving, we now analyze the dynamics of the economy. The optimal consumption rule will determine the dynamics of consumption and the stochastic discount factor, which determines the risk-free rate. The optimal consumption rule in conjunction with the marginal product of

labor determines the dynamics of wages and dividends, which in turn determine the value of equity and human capital. Once we determine the value of equity and human capital, we can study the equity premium and the co-movement between asset returns and human capital. I derive all these results in this section.

3.6.1 Consumption, the stochastic discount factor, and the risk-free rate

Given the optimal policy and the dynamics of the state variable z , the dynamics of consumption will be:

$$\frac{dC}{C} = \mu_C(z)dt + \sigma_C(z)dB, \quad (19)$$

where the drift and volatility of consumption growth are equal to:

$$\begin{aligned} \mu_c(z) = & e^{(1-\alpha)z} - g(z)^{-1/\gamma} \\ & + \left(\frac{g'(z)}{g(z)}\right)^2 \frac{(1+\gamma)\sigma_K^2}{\gamma^2 2} \\ & + \left(\frac{g'(z)}{g(z)}\right) \frac{1}{\gamma} \left(e^{(1-\alpha)z} - g(z)^{-1/\gamma} - \eta + \frac{\sigma_K^2}{2} \right) \\ & - \left(\frac{g''(z)}{g(z)}\right) \frac{1}{\gamma} \frac{\sigma_K^2}{2}, \end{aligned} \quad (20)$$

$$\sigma_C(z) = \sigma_K \left(1 + \frac{1}{\gamma} \frac{g'(z)}{g(z)} \right). \quad (21)$$

The first two terms of μ_c are close to capital's growth and the other terms mean-revert around 0. Thus, consumption growth will mean-revert around capital's growth, as it should for the system to be stable. The term $\frac{1}{\gamma} \frac{g'(z)}{g(z)}$ is less than zero, so the fraction of consumption volatility coming from σ_K is lower than the volatility of capital. In the numerical calculations presented below, $\frac{1}{\gamma} \frac{g'(z)}{g(z)}$ increases as γ increases, implying that increasing levels of risk aversion also increase the smoothing of consumption over time. Rouwenhorst (1995) identified this result, noting that it creates a problem for matching production models with

macroeconomic variables because consumption becomes too smooth relative to plausible values for the volatility of capital. This model has the same problem, but its solution lies outside the scope of this paper. In particular, the calibration requires high volatilities of capital to deliver excess returns and equity volatilities of the same order of magnitude as the ones we observe in the data.

The dynamics of consumption determine the dynamics of the stochastic discount factor, which will allow us to price any claim in this economy. The stochastic discount factor will be given by the evolution of marginal utility:

$$\begin{aligned} M_t &= e^{-\rho t} C_t^{-\gamma} \\ &= e^{-\rho t} K_t^{-\gamma} g(z_t). \end{aligned} \tag{22}$$

Given the dynamics of capital and $g(z)$, applying Itô-Doebelin's lemma to Equation (22) we find that its drift is:

$$\begin{aligned} \frac{E[dM_t]}{M_t} &= -(e^{(1-\alpha)z} - g(z)^{-1/\gamma} - \delta)\gamma - \rho + \gamma(1 + \gamma)\frac{\sigma_K^2}{2} \\ &\quad + \frac{g'(z)}{g(z)} \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z}\delta + \eta + (1 + 2\gamma)\frac{\sigma_K^2}{2} \right) \\ &\quad + \frac{g''(z)}{g(z)} \frac{\sigma_K^2}{2}. \end{aligned} \tag{23}$$

The risk-free rate is the negative of the stochastic discount factor's drift. Thus, r_f will be given by:

$$\begin{aligned} r_f &= \gamma(e^{(1-\alpha)z} - g(z)^{-1/\gamma} - \delta) + \rho - \gamma(1 + \gamma)\frac{\sigma_K^2}{2} \\ &\quad - \frac{g'(z)}{g(z)} \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta + (1 + 2\gamma)\frac{\sigma_K^2}{2} \right) \\ &\quad - \frac{g''(z)}{g(z)} \frac{\sigma_K^2}{2}. \end{aligned} \tag{24}$$

3.6.2 Wages, human capital, and equity

Given the dynamics of the stochastic discount factor, we can now turn our attention to wages and the value of equity. In equilibrium, the competitive wage is the marginal product of labor. For the Cobb-Douglas case this implies that wages will be:

$$Y_t = (1 - \alpha)e^{(1-\alpha)z_t} K_t. \quad (25)$$

The value of equity equals the present value of dividends minus the present value of any additional capital that the representative firm needs to raise in order to follow the optimal investment plan. Capital needs to be raised when the difference of production and the sum of investment and wages is negative. This is equivalent to stating that equity equals the present value of dividends, where capital raised on any given period is considered a “negative” dividend.

Following the previous paragraph, dividends will be:

$$D_t = K_t g(z)^{-1/\gamma} - K_t e^{(1-\alpha)z_t} (1 - \alpha), \quad (26)$$

and equity’s value will be:

$$E_t = \int_t^\infty (K_s g(z_s)^{-1/\gamma} - K_s e^{(1-\alpha)z_s} (1 - \alpha)) M_s ds. \quad (27)$$

Given this information, we can derive the dynamics of human capital and equity using the stochastic differential equations implied by their definition as the present value of wages and dividends, and by the fact that the state of the economy is summarized by the level of capital and z_t . The following proposition summarizes this result:¹⁵

¹⁵A more frequent method is using the condition that under the risk-neutral measure, the expected return of a claim to equity *cum* dividend must equal the risk-free rate. This is, of course, equivalent to what I do above.

Proposition 2 *Let $H(K_t, z_t)$ denote the value of a claim to human capital. Then the value of human capital will be characterized by*

$$H(K_t, z_t) = K_t h(z_t). \quad (28)$$

$h(z_t)$ is given by the solution to the following ODE:

$$\begin{aligned} 0 = & (1 - \alpha)e^{(1-\alpha)z} - h(z)g(z)^{-1/\gamma} \\ & + h'(z) \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta - (1 - 2\gamma)\frac{\sigma_K^2}{2} + \frac{g'(z)}{g(z)}\sigma_K^2 \right) \\ & + h''(z)\sigma_K^2, \end{aligned} \quad (29)$$

subject to: $h(-\infty) = 0$ and $h'(-\infty) = 0$.

The value of equity per unit of capital will be given by $e(z) = 1 - h(z)$.

Proof: See Appendix.

The boundary conditions in Proposition 2 follow from wages approaching zero as production per unit of capital approaches zero. Dividends do not approach zero, since the agent still chooses to consume some of his capital. Therefore, a claim to equity will have some value when $z \rightarrow \infty$, but a claim to human capital will not.

$h(z)$ is the weight of human capital in the aggregate wealth portfolio. The results discussed below relate $h(z)$ to observable variables, in particular the ratio of wages to consumption.

4 Human capital and equity risk premium

Having derived the dynamics of the economy, and in particular the stochastic discount factor and the claims of human capital and equity, I explore the implications of those dynamics for the expected returns for both claims. Several papers have made assumptions about the value and expected returns to human capital with the purpose of estimating

the return to the aggregate, human capital-inclusive, wealth portfolio. For that purpose Jaganathan and Wang (1996) assume that the realized human capital return is equal to wage growth while Campbell (1996) assumes that expected human capital returns are identical to expected asset returns. We can compare these assumptions to what the model predicts.

In equilibrium, expected excess returns to human capital and equity are equal to the negative of the covariance between the stochastic discount factor and a claim's returns. Using Proposition 2 and applying Itô-Doebelin's Lemma, the diffusion term of equity's dynamics is:

$$\sigma_E = \sigma_K \left(1 + \frac{h'(z)}{1-h(z)} \right). \quad (30)$$

Using Equation (30) and Equation (22), the equity risk premium is:

$$r_e - r_f = \gamma \sigma_K^2 \left(1 + \frac{1}{\gamma} \frac{g'(z)}{g(z)} \right) \left(1 + \frac{h'(z)}{1-h(z)} \right). \quad (31)$$

The drivers of the equity premium are the volatility of the shocks, a factor that affects the sensitivity of consumption per unit of capital $\frac{1}{\gamma} \frac{g'(z)}{g(z)}$, and the sensitivity of changes in the weight of equity in the aggregate portfolio $\frac{h'(z)}{1-h(z)}$. In the numerical results that follow, $\frac{h'(z)}{1-h(z)} > 0$, implying that equity absorbs production shocks, which implies that their volatility *increases* the equity premium.

Equation (31) can be rewritten in terms of the volatility of consumption so as to separate the equity premium in this model from the one in the standard model:

$$r_e - r_f = \underbrace{\gamma \sigma_C^2}_{\text{Benchmark}} \underbrace{\frac{1}{\left(1 + \frac{1}{\gamma} \frac{g'(z)}{g(z)} \right)}}_{\text{Consumption smoothing}} \underbrace{\left(1 + \frac{h'(z)}{1-h(z)} \right)}_{\text{Weight changing}}. \quad (32)$$

Equation (32) shows the sources of equity premium. The first term corresponds to the benchmark result in which dividends equal consumption. The second term comes from

the difference between consumption volatility and capital volatility. In the production model, capital, which equals aggregate wealth, is more volatile than consumption because $g'(z) < 0$. In the special case when consumption and wealth are perfectly cointegrated $g'(z) = 0$, and the second term in Equation (32) disappears. The third term corresponds to the effect of changes to the fraction of aggregate wealth taken by equity. Movements in the state-variable impact the premium through $\frac{h'(z)}{1-h(z)}$, a “leverage” term which is positive because of the pro-cyclical movement between dividends and consumption. Given that $h(z)$ is relatively large (i.e., close to 1), the “leverage” effect can be quite large. If $h'(z) = 0$, then the third term in Equation (32) becomes 1. If the agent does not smooth consumption when production shocks arrive (as in Merton (1975)), or if the weight of total wealth for a given claim does not change, then Equation (32) becomes the well known benchmark case.

The expression for human capital’s risk-premium is different:

$$r_{hc} - r_f = \gamma\sigma_K^2 \left(1 + \frac{1}{\gamma} \frac{g'(z)}{g(z)}\right) \left(1 - \frac{h'(z)}{h(z)}\right). \quad (33)$$

If $h'(z) > 0$, then human capital’s risk-premium will be smaller than equity’s risk premium. This result comes from the sign of $h'(z)$ and from the larger value of $h(z)$, since any “leverage” effect will be smaller. Thus, human capital is weakly exposed to shocks, which lower its expected return.

Equations (31) and (33) illuminate the differences between previous researcher’s conclusions about the returns to human capital. Baxter and Jermann (1997) did not define the claims to equity and human capital as I have here, and implicitly assume that $h'(z) = 0$. With this assumption, the returns to human capital and to equity are identical, and perfectly cointegrated. Lustig and Van Nieuwerburgh’s (2006) results, however, imply that $h'(z) > 0$, so that innovations on returns have a negative covariance.

We can use the results in Equations (31) and (33) to derive the portfolio of equity and the risk-free asset that replicates human capital. The fraction $x(z)$ of the replicating

portfolio invested in stocks is:

$$\begin{aligned}
 x(z) &= \frac{1 - \frac{h'(z)}{h(z)}}{1 + \frac{h'(z)}{1-h(z)}} \\
 &= \frac{(1 - h(z))}{h(z)} \frac{h(z) - h'(z)}{1 - h(z) + h'(z)}.
 \end{aligned}
 \tag{34}$$

Equation (34) allows us to answer the question: is human capital a bond or a stock? The calibrated model provides plausible answers. Now we can analyze these results using plausible parameters to evaluate what the model predicts about the variables described above. The results are discussed below.

5 Model calibration and results

We now study the behavior of the variables of interest in the model. In the analysis that follows, I will study the risk-free rate, the equity premium, the volatility of consumption and equity, the weight of human capital in the aggregate wealth portfolio and human capital's expected excess return.

5.1 Parameter choice

Table 1 shows the parameters I used to calibrate the model. I chose the parameters so that the model's long-term expected values match consumption growth, consumption volatility and the ratio of wages to consumption observed in the data. Given the previous criteria, I chose parameters to minimize the risk-free rate, maximize the Sharpe ratio, and minimize the error in equity volatility against its historical average of 18%. This model is parsimonious, so matching all the moments with reasonable parameter values is arguably impossible. Yet, we can still use the output of the model to characterize human capital returns as a function of equity returns.

The technology's growth rate η determines consumption's long term growth. Thus I set η at 1.7% to match long-term consumption growth. The subjective discount rate ρ

plays a major role only in determining the risk-free rate, so to minimize the risk-free rate I set ρ equal to 1%. Typical assumptions for capital intensity are between 30% and 40%, so I constrain α to remain in this range.

After fixing these two parameters, the remaining parameters are the coefficient of relative risk aversion γ , the depreciation rate δ , capital intensity's α , and the volatility of production shocks σ_K .

Table 3 shows the sensitivity of the moments of interest to changes in these parameters. As Rouwenhorst (1995) discusses in his work, increasing the coefficient of risk aversion does not improve Sharpe ratios because a higher γ also implies a higher desire to smooth consumption over time. Thus, to obtain consumption growth volatility of 3.8% the only alternative is to increase the production shock's volatility σ_K at the same time that γ is increased. However, raising σ_K raises the volatility of equity beyond its historic value.

The depreciation rate δ and capital's intensity α are the main drivers affecting the ratio of wages to consumption. Lower capital intensities are associated with higher ratios of wages to consumption and higher equity volatilities. Higher depreciation rates have the same effect, increasing wages over consumption and equity volatilities. Thus, there is a tradeoff between matching the ratio of wages to consumption and matching equity volatility. However, these variables do not significantly affect the other moments of the model. I choose α to match the common value in the literature of 34%, and adjust δ to match the ratio of wages to consumption.

5.2 Results

The calibration provides several key insights. First, the behavior of consumption and the product per unit of capital shows the mechanism through which human capital is less risky than equity. Second, the size and risk of human capital are a function of the ratio of wages to consumption. Third, asset-prices change depending on the state of the economy. These results are discussed in more detail below.

Using the parameters discussed in the previous section, Table 2 summarizes the values

around which each of the variables mean-reverts. The risk-free rate of 6.1% is much higher than the historical average. Equity excess returns are about 2.5% and instantaneous equity volatility, which is endogenous in the model, is 26%. Equity’s volatility is an order of magnitude higher than the volatility of consumption growth, which is 3.8%.

5.2.1 Consumption and production

First, we look at the ratio of consumption to capital, which in this economy is the same as the ratio of consumption to aggregate wealth. In the simple constant expected-returns model (for example, see Merton (1971)) consumption is a constant fraction of capital. In the economy analyzed here, consumption is adjusting as shocks change expected returns, resulting in a time-varying consumption to wealth ratio. Figure 1 shows that the ratio of consumption to aggregate wealth changes significantly as a function of production per unit of capital. In particular, the ratio of consumption to aggregate wealth increases as production per unit of capital increases. Since production per unit of capital increases after negative production shocks, this result implies that consumption as a fraction of aggregate wealth increases in “bad times.”

Next, we study the ratio of consumption to production. Figure 2 shows that this ratio decreases as production per unit of capital increases. In other words, the agent invests more when the output for a given level of capital increases. This result is not surprising, since the agent’s opportunity cost of consuming today increases when production per unit of capital is relatively high. Recalling that wages are a constant fraction of output, this result implies that consumption falls relative to wages when production per unit of capital increases. Production per unit of capital increases after *negative* shocks, so consumption falls more than wages after a negative shock. As a result, wages are less exposed to the shock than consumption. This is the key result explaining why human capital is less risky than equity in this model.

Another way to visualize that wages are less exposed to shocks than consumption is to study the ratio of wages to consumption as a function of production per unit of capital.

Figure 3 shows that the ratio of wages to consumption increases with production per unit of capital. Because production per unit of capital is high after negative shocks, this figure stresses the counter-cyclical movement of wages and consumption.

5.2.2 Human capital and equity returns

Figures 4 and 5 show how large and how risky human capital is. Unconditionally, human capital is 84.6% of aggregate wealth, and, at .89%, its expected excess return is about a third of equity's. The weight of human capital in aggregate wealth is larger than the fraction of wages to consumption, which has an unconditional value of 83.5%. Furthermore, the weight of human capital in aggregate wealth follows closely, and remains above, the ratio of wages to consumption. Based on this result I construct an empirical measure of human capital returns in Section 6.

Figure 5 shows the relationship between the ratio of wages to consumption and excess returns. Higher-than-average ratios of wages to consumption will be associated with higher-than-average contemporaneous excess returns. Equity's expected excess return is 2.52% and its volatility is 26%, as shown in figure 6. These values are driven by the "leverage" effect induced by wages. When the ratio of wages to consumption is low the volatility of dividends is low, which in turn translates into low equity volatility.¹⁶ On the other hand, when productivity is extremely high, dividends can become negative, corresponding to states in which firms need to raise capital. In these states equity's volatility increases, even though consumption's volatility remains low.

Besides human capital's size and expected return, Figure 7 shows the relationship between the covariance of aggregate wages growth and equity, and the covariance between human capital and equity. The covariance of aggregate wages growth and equity returns is roughly two times as large as the covariance between wages and equity. This result shows that, even though the observed covariance between wage growth and equity returns is small, the covariance between human capital and equity returns can be much larger.

¹⁶In the limit, when the productivity of capital is 0, the volatility of dividends approaches that of consumption.

This result is relevant for asset-pricing tests and portfolio selection problems. After Mayer’s (1972) result linking optimal portfolios and asset prices to aggregate wages growth, several papers (Fama and Schwert (1977), Heaton and Lucas (1996)), documented that since the observed covariance between wages growth and equity returns was small, the effect of human capital on asset-prices was small. But the model presented here shows that the covariance of human capital and equity returns can be much larger than the covariance between wages growth and equity returns.

Finally, the portfolio that replicates a claim to human capital consists, on average, of 35% of a risk-free bond and 65% of equity. Figure 8 shows how this fraction changes conditionally on the state of the economy. When the ratio of wages to consumption is higher, human capital becomes less risky relative to equity, implying human capital becomes more like a “bond” in these states of the economy. On the other hand, when the ratio of wages to consumption is lower, the “leverage” effect driving equity’s volatility decreases, making human capital relatively more risky.

6 An empirical application

The motivation for this paper is a better understanding of the drivers of human capital and its riskiness. However, the model can also help us improve the specification of empirical tests that need the value and return of human capital as an input. One example is the classic test of the CAPM. In this section, I use the model to estimate the return to human capital, and then run a Fama-Macbeth (1973) regression on Fama and French’s (1992) 100 portfolios and 49 industry portfolios (1997). The result is that the measure of the return on human capital I use here has a positive loading for explaining the cross-section of portfolio returns. I contrast this result with that of using wage growth as a proxy for the return on human capital, like Jagannathan and Wang (1996) do. As is the case with almost all the literature, the empirical test rejects the CAPM.

To build a measure for the return of human capital, I use the model’s result that the

fraction of human capital in the aggregate wealth portfolio is closely linked to the fraction of wages over consumption. Assuming that the fraction of wages to consumption is a good proxy for the weight of human capital in the aggregate portfolio, and that market returns are a good proxy for the returns of all non-human capital returns, one can calculate the return to human capital from the return to financial assets and the change in the fraction of wages to consumption.

Let x_t denote the weight of human capital in the aggregate wealth portfolio at time t . Let V_t be the value of aggregate wealth at time t . The gross return of non-human capital wealth at time $t + 1$ can be expressed as:

$$R_m = \frac{V_{t+1}(1 - x_{t+1})}{V_t(1 - x_t)}. \quad (35)$$

The value of total wealth at time $t + 1$ is:

$$V_{t+1} = V_t(1 - x_t)R_m + V_t x_t R_{hc}, \quad (36)$$

where R_{hc} is the return to human capital. Substituting Equation (35) in Equation (36) and solving for R_{hc} we obtain the following expression for the return to human capital:

$$R_{hc} = R_m \frac{x_{t+1}}{x_t} \frac{(1 - x_t)}{(1 - x_{t+1})}. \quad (37)$$

Equation (37) is an accounting identity, though we still have the problem that we do not observe either x_t or x_{t+1} . However, if we assume that the ratio of wages to consumption is a good proxy for x_t , as suggested by the model, then we can use Equation (37) to calculate the return to human capital.

After estimating the return to human capital, we can test the CAPM using a simplified version (unconditional) of the test used by Jaganathan and Wang (1996). The CAPM

can be expressed as

$$\begin{aligned}
r_{i,t}^e &= B_i r_{w,t}^e + \epsilon_t \\
&= B_i (x_t r_{m,t}^e + (1 - x_t) r_{hc,t}^e) + \epsilon_t \\
&= B_{m,i} r_{m,t}^e + B_{hc,i} r_{hc,t}^e + \epsilon_t
\end{aligned} \tag{38}$$

where $r_{w,t}^e = R_{m,t} - R_{f,t}$ and $r_{hc,t}^e = R_{hc,t} - R_{f,t}$.

Equation (38) is a two factor model for explaining the cross-section of asset returns. I proceed to test this model using US monthly data between 1959 and 2007. The value for wages is “Total compensation” as reported in the National Income and Product Accounts (NIPA) tables. Consumption is the sum of nondurables and services, scaled by a factor,¹⁷ also reported in the NIPA tables. Market excess returns are monthly value-weighted returns for all NYSE, AMEX and NASDAQ stocks as reported in CRSP. Portfolio returns are those reported by Kenneth French on his website. To avoid problems of colinearity, I use market returns and the component of human capital return orthogonal to the market.

Table 4 shows the results of the second pass regression of the empirical specification described above and compares it with the results using wage growth. Using the 49 industry portfolios, the regressions support human capital as a risk factor regardless of the method used to calculate it. With either method the loading on the return to human capital is significant at the .001 level. The R^2 using labor income growth as a proxy for human capital returns is 55%, while the R^2 using the human capital return constructed as explained above is 78%. However, in a test of the 100 portfolios, the return to human capital calculated using the method above has a positive coefficient significant at the .01 level. The same test calculating human capital as labor income growth is not statistically significant. In either case the CAPM is rejected, with a significant negative loading associated with the return to the market.

This result suggests that a different specification for the returns to human capital

¹⁷See Santos and Veronesi (2006).

could improve our asset pricing tests. Future research can extend this test by applying the new specification for the returns to human capital to a wider range of tests, both in the cross-section of asset returns, and in the time series of market returns.

7 Conclusion

This paper explores the implications of a general equilibrium production model for the value and dynamics of human capital. Decreasing returns to scale and the sharing of consumption between labor and capital drive the results. The calibrated model predicts mean-reverting risk premia, dividend yields, and interest rates, with labor income growth and capital returns cointegrated over time.

The model suggests that human capital is less exposed to production shocks than equity. This result does not rely on frictions of labor markets such as labor contracts that protect workers from idiosyncratic productivity shocks. Aggregate human capital is analogous to holding a portfolio of 35% in risk-free bonds and 65% in stocks, though the actual portfolio changes with economic conditions.

An empirical construction of the returns to human capital based on this model is better at explaining the cross-section of asset returns than labor income growth. This result opens the possibility for other tests that can be performed with this measure of the returns to human capital.

The model can be extended in multiple directions. An obvious one is allowing the fraction of output received by workers to change over time. Acemoglu (2002) presents a model grounded on new growth models from which the fraction of output received by capital is mean-reverting. Alternatively, even though capital intensity remains constant, the realized fraction could change due to “sticky” wages, for example due to the existence of adjustment costs. A richer model in which the fraction of output that goes to workers changes over time can magnify the “leverage” effect that wages have on dividends, as long as the fraction increases when consumption decreases. While in the present model

uncertainty aligns the interests of investors and workers, translating into larger payoffs for both as technology changes, they are confronted by variations in the share of output they each receive. A larger pie benefits both stake holders, but the way the pie is divided clearly benefits one stakeholder at the expense of the other. Finding a way to separate these two effects is an extension that could help enhance our understanding of variations in labor income and asset returns.

Another extension is to consider heterogeneity in worker's skills and the inclusion of the dynamics of human capital accumulation. Distinguishing labor by its skill, and letting agents choose their level of skill could provide insights into our understanding of investments in human capital. Likewise, the model can be generalized to include demographic changes that would provide a link from intergenerational change to asset prices.

Finally, the role played by a competitive wage across different industries potentially affects the cross-section of asset returns. A positive technology shock in one industry will increase the demand for labor in that industry, which in turn increases economy-wide wages, affecting the returns in all other industries. Thus, labor markets play a significant role in the riskiness of firms' cash flows, and understanding that role better can improve our specification of the cross-section of asset returns.

References

- Acemoglu, D. (2003). Labor- and capital-augmenting technological change. *Journal of European Economic Association* 1, 1–37.
- Baxter, M. and U. Jermann (1997). The international diversification puzzle is worse than you think. *American Economic Review* 87(1), 170–180.
- Belo, F. (2006). A pure production-based asset pricing model. *Working paper, University of Minnesota*.
- Benzoni, L., P. Collin-Dufresne, and R. Goldstein (2007). Portfolio choice over the life cycle when the stock and labor markets are cointegrated. *The Journal of Finance* 62(5), 2123–2167.
- Boldrin, M., L. Christiano, and J. Fisher (2001). Habit persistence, asset returns, and the business cycle. *American Economic Review* 91(1), 149–166.
- Bourguignon, F. (1974). A particular class of continuous-time stochastic growth models. *Journal of Economic Theory* 9, 141–158.
- Brock, W. (1982). Asset prices in a production economy, in *Economics of Information and Uncertainty*, J.J. McCall, ed., University of Chicago Press, 1-43.
- Brock, W. and L. Mirman (1972). Optimal economic growth and uncertainty: The discounted case. *Journal of Economic Theory* 4(3), 479–513.
- Campbell, J. (1996). Understanding risk and return. *The Journal of Political Economy* 104(2), 298–345.
- Campbell, J. and J. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107(2), 205–251.
- Campbell, J. and L. Viceira (1999). Consumption and portfolio decisions when expected returns are time varying. *Quarterly Journal of Economics* 114(2), 433–495.
- Chen, X., J. Favilukis, and S. Ludvigson (2008). An Estimation of Economic Models with Recursive Preferences. *Working paper, New York University*.
- Cochrane, J. (1991). Production-based asset pricing and the link between stock returns and economic fluctuations. *Journal of Finance* 46(1), 209–237.
- Cochrane, J. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy* 104(3), 572–621.
- Cochrane, J., F. Longstaff, and P. Santa-Clara (2006). Two trees. *Review of Financial Studies* 21(1), 347–385.
- Cox, J., J. Ingersoll, and S. Ross (1985). An intertemporal general equilibrium model of asset prices. *Econometrica* 53(2), 363–384.
- Danthine, J. and J. Donaldson (2002). Labour relations and asset returns. *Review of Economic Studies* 69(1), 41–64.
- Danthine, J., J. Donaldson, and R. Mehra (1992). The equity premium and the allocation of income risk. *Journal of Economic Dynamics and Control* 16(3), 509–532.

- Duffee, G. (2005). Time variation in the covariance between stock returns and consumption growth. *The Journal of Finance* 60(4), 1673–1712.
- Fama, E. (1990). Stock returns, expected returns, and real activity. *Journal of Finance* 45(4), 1089–1108.
- Fama, E. and K. French (1989). Business conditions and expected returns on stocks and bonds. *Journal of Financial Economics* 25(1), 23–49.
- Fama, E. and K. French (1997). Industry costs of equity. *Journal of Financial Economics* 43(2), 153–193.
- Fama, E. and J. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy* 81(3), 607–636.
- Fama, E. and W. Schwert (1977). Human capital and capital market equilibrium. *Journal of Financial Economics* 4(1), 95–125.
- Gomes, J., L. Kogan, and M. Yogo (2007). Durability of output and expected stock returns. *NBER working paper W12986*.
- Harris, M. and B. Holmstrom (1982). A theory of wage dynamics. *The Review of Economic Studies* 49(3), 443–487.
- Heaton, J. and D. Lucas (2000). Portfolio choice and asset prices: the importance of entrepreneurial risk. *The Journal of Finance* 55(3), 1163–1198.
- Jagannathan, R. and Z. Wang (1996). The conditional CAPM and the cross-section of expected returns. *The Journal of Finance* 51(1), 3–53.
- Jermann, U. (1998). Asset pricing in production economies. *Journal of Monetary Economics* 41(2), 257–275.
- Jorgenson, D. and B. Fraumeni (1989). The accumulation of human and nonhuman capital, 1948-84, in *The Measurement of Saving, Investment, and Wealth*, R.E. Lipsey and H.S. Tice, eds., University of Chicago Press, 227-282.
- Koo, H. K. (1998). Consumption and portfolio selection with labor income: A continuous-time approach. *Mathematical Finance* 8(1), 49–65.
- Lamont, O. (1998). Earnings and expected returns. *The Journal of Finance* 53(5), 1563–1587.
- Lettau, M. and S. Ludvigson (2001). Resurrecting the (c)capm: A cross-sectional test when risk premia are time-varying. *The Journal of Political Economy* 109(6), 1238–1286.
- Lustig, H. and S. Van Nieuwerburgh (2008). The returns on human capital: Good news on wall street is bad news on main street. *Review of Financial Studies* 21(5), 2097–2137.
- Lustig, H., S. Van Nieuwerburgh, and A. Verdelhan (2008). The wealth-consumption ratio. *Working Paper, UCLA*.
- Marimon, R. and A. Scott (1999). *Computational Methods for the Study of Dynamic Economies*. Oxford University Press.

- Mayers, D. (1972). Non-marketable assets and capital market equilibrium under uncertainty, in *Studies in the Theory of Capital Markets*, M.C. Jensen, ed., Praeger, 223-248.
- Mehra, R. and E. Prescott (1985). The equity premium. *Journal of Monetary Economics* 15(2), 145–61.
- Merton, R. (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory* 3(4), 373–413.
- Merton, R. (1973). An intertemporal capital asset pricing model. *The Journal of Financial Econometrics* 1(2), 272–289.
- Merton, R. (1975). An asymptotic theory of growth under uncertainty. *Review of Economic Studies* 42(3), 375–393.
- Mirrlees, J. (1967). Optimum growth when technology is changing. *Review of Economic Studies* 34(1), 95–124.
- Palacios-Huerta, I. (2003a). An empirical analysis of the risk properties of human capital returns. *American Economic Review* 93(3), 948–964.
- Palacios-Huerta, I. (2003b). The robustness of the conditional CAPM with human capital. *Journal of Financial Econometrics* 1(2), 272–289.
- Ramsey, F. (1928). A mathematical theory of saving. *Economic Journal* 38(152), 543–559.
- Rouwenhorst, K. (1995). Asset pricing implications of equilibrium business cycle models. *Frontiers of Business Cycle Research*, 294–330.
- Santos, T. and P. Veronesi (2006). Labor income and predictable stock returns. *Review of Financial Studies* 19(1), 1–44.
- Shiller, R. (1995). Aggregate income risks and hedging mechanisms. *The Quarterly Review of Economics and Finance* 35(2), 119–152.
- Smith, W. (2007). Inspecting the mechanism exactly: A closed-form solution to a stochastic growth model. *The BE Journal of Macroeconomics* 7(1).
- Svensson, L. (1988). Portfolio choice and asset pricing with nontraded assets. *NBER Working Paper 2774*.
- Tallarini, T. (2000). Risk-sensitive real business cycles. *Journal of Monetary Economics* 45(3), 507–532.
- Viceira, L. (2001). Optimal portfolio choice for long-horizon investors with nontradable labor income. *The Journal of Finance* 56(2), 433–470.

8 Appendix A: Symbols and variables

Variable	Description
α	Capital intensity.
A	Efficiency parameter in technology function
C	Consumption
c	Consumption per unit of wealth ($\frac{C}{W}$)
δ	Depreciation rate
γ	Coefficient of risk aversion
J	Agent's value function
K	Aggregate capital
L	Aggregate Labor
m	Labor supply, number of agents in the economy
E	Value of equity
e	Value of equity per unit of capital
H	Value of human capital
h	Value of human capital per unit of capital
ρ	Subjective discount rate
R	Expected returns of investing in a technology
r_e	Expected return of equity
r_f	Risk free rate
σ_K	Volatility of output production shocks
y	Labor income
Z	State variable ($\frac{A^{1-\alpha}}{K^{1-\alpha}}$)
z	$\frac{1}{1-\alpha} \log Z$

9 Appendix B: Proofs

9.1 Proof of proposition 1

The solution follows that of a central planner who maximizes utility for the representative investor-worker-agent. The central planner's problem is:

$$\begin{aligned} \max_{\{C_t\}_t^\infty} E_t \left[\int_t^\infty e^{-\rho s} \frac{C_s^{1-\gamma}}{1-\gamma} ds \right] & \quad (39) \\ \text{s.t. } dK_t &= (K_t Z_t - C_t) dt + \sigma_K dB_t \\ Z_t &= \left(\frac{A_t}{K_t} \right)^{1-\alpha}. \end{aligned}$$

Assuming that a solution exists, the solution to the problem requires solving for the Hamilton-Jacobi-Bellman equation:

$$0 = \max_c E_t \left[\frac{C_t^{1-\gamma}}{1-\gamma} + J_t + J_K dK_t + \frac{1}{2} J_{KK} (dK_t)^2 \right]. \quad (40)$$

Now assume that the function $J(K_t, A_t, t)$ is of the form:¹⁸

$$J(K_t, A_t, t) = e^{-\rho t} \frac{K_t^{1-\gamma}}{1-\gamma} Q(z_t), \quad (41)$$

where $z_t = \log \left(\frac{A_t}{K_t} \right)$.

Defining $c \equiv \frac{C}{W}$, substituting the dynamics of K and A , and taking expectations, the HJB equation becomes:

$$\rho J = \max_c \frac{K^{1-\gamma} c^{1-\gamma}}{1-\gamma} + J_K K (Z - c) + \frac{1}{2} J_{KK} K^2 \sigma_K^2. \quad (42)$$

The first order condition simplifies to:

$$J_K = K^{-\gamma} c^{-\gamma}. \quad (43)$$

From the definition in Equation (41) we can calculate every element of Equation (40). The partial derivatives are:

$$J_K = K^{-\gamma} \left(Q(z) - Q'(z) \frac{1}{1-\gamma} \right) \quad (44)$$

$$J_{KK} = K^{-\gamma-1} \left(-\gamma(z) - \frac{\gamma}{1-\gamma} Q'(z) + \frac{1}{1-\gamma} Q''(z) \right). \quad (45)$$

Replacing the optimal value of consumption into Equation (42), and simplifying, we obtain the following differential equation:

$$\rho J = \frac{\gamma J_K^{\frac{\gamma-1}{\gamma}}}{1-\gamma} + J_K K Z + \frac{1}{2} J_{KK} (K \sigma_K)^2. \quad (46)$$

¹⁸That this is indeed a solution to the problem will be verified after finding a candidate solution.

Substituting Equation (44) and Equation (45) into Equation (46) the PDE becomes:

$$\begin{aligned} \rho \frac{K^{1-\gamma}}{1-\gamma} Q(z) &= \frac{\gamma K^{1-\gamma} Q(z)^{\frac{\gamma-1}{\gamma}}}{1-\gamma} + K^{1-\gamma} Q(z) Z - \frac{1}{2} \gamma K^{1-\gamma} Q(z) \sigma_K^2 \\ &\quad + K^{1-\gamma} \frac{Q'(z)}{1-\gamma} \eta. \end{aligned} \quad (47)$$

Defining $g(z) = Q(z) - \frac{1}{1-\gamma} Q'(z)$, and noticing that the term $K^{1-\gamma}$ cancels out of Equation (47), we are left with the following ordinary differential equation:

$$\begin{aligned} 0 &= Q(z) \left(\frac{\gamma}{1-\gamma} g(z)^{-\frac{1}{\gamma}} - \frac{\rho}{1-\gamma} + Z - \frac{1}{2} \gamma \sigma_K^2 \right) \\ &\quad + Q'(z) \frac{1}{1-\gamma} \left(g(z)^{-\frac{1}{\gamma}} - Z + \eta + \left(\gamma - \frac{1}{2} \right) \sigma_K^2 \right) \\ &\quad + Q''(z) \frac{1}{2(1-\gamma)} \sigma_K^2. \end{aligned} \quad (48)$$

If the function Q exists, then it solves the planner's optimization problem. To verify that our candidate value function is indeed an optimal solution, we need to verify that the transversality condition holds. Numerical solutions suggest the condition is satisfied, but I omit a formal proof here.

9.2 Proof of proposition 2

I drop the subscript t where it is not needed for clarity. The discounted process for the value of human capital is:

$$M_t H_t = E_t \left[\int_0^\infty M_{\tau+t} Y_{\tau+t} d\tau \right] \quad (49)$$

$$= E_t \left[\int_0^\infty M_{\tau+t} (1-\alpha) K_{\tau+t} e^{z\tau+t} d\tau \right]. \quad (50)$$

The state variable z_t and the stock of productive capital K_t describe the economy, and so the value of human capital will only be a function of these two variables. Taking advantage of the homogeneity of the model, we guess (and then verify) that $H[K_t, z_t] = K_t h(z_t)$. Using this definition, and recalling that $M_t = e^{-\rho t} K_t^{1-\gamma} g(z_t)$, we can express 9.2 as:

$$K_t^{1-\gamma} g(z_t) h(z_t) = E_t \left[\int_0^\infty e^{-\rho\tau} K_{t+\tau}^{1-\gamma} g(z_{t+\tau}) (1-\alpha) e^{z\tau+t} d\tau \right]. \quad (51)$$

The drift of the discounted process is:

$$\frac{E[dK_t^{1-\gamma} g(z) h(z)]}{K^{1-\gamma} g(z) h(z)} = \left(\rho - \frac{(1-\alpha)e^{(1-\alpha)z}}{h(z)} \right) dt. \quad (52)$$

Applying Itô-Doebelin's Lemma and taking expectations to the expression on the left hand side we obtain the following expression for the discounted process' drift:

$$\frac{E[dK^{1-\gamma} g(z) h(z)]}{K^{1-\gamma} g(z) h(z)} = ((1-\gamma)(e^{(1-\alpha)z} - \delta - g(z)^{-1/\gamma}) - \gamma(1-\gamma) \frac{\sigma_K^2}{2}) dt \quad (53)$$

$$\begin{aligned}
& + \frac{g'(z)}{g(z)} \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta + (2\gamma - 1) \frac{\sigma_K^2}{2} \right) \\
& + \frac{g''(z)}{g(z)} \frac{\sigma_K^2}{2} \\
& + \frac{h'(z)}{h(z)} \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta - (1 - 2\gamma) \frac{\sigma_K^2}{2} + \frac{g'(z)}{g(z)} \sigma_K^2 \right) \\
& + \frac{h''(z)}{h(z)} \sigma_K^2 dt.
\end{aligned}$$

The first terms of Equation (53):

$$\begin{aligned}
& (1 - \gamma)(e^{(1-\alpha)z} - \delta - g(z)^{-1/\gamma}) - \gamma(1 - \gamma) \frac{\sigma_K^2}{2} \\
& + \frac{g'(z)}{g(z)} \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta + (2\gamma - 1) \frac{\sigma_K^2}{2} \right) \\
& + \frac{g''(z)}{g(z)} \frac{\sigma_K^2}{2},
\end{aligned}$$

are the same as in a differential equation of a discounted claim to consumption without the adjustment for time $-\rho$. Adding the dividend paid by this claim and adjusting for time, the differential equation must equal 0, so the first few terms can be replaced by $-g(z)^{-1/\gamma} + \rho$. Thus, the differential equation becomes:

$$\begin{aligned}
\frac{E[dK^{1-\gamma}g(z)h(z)]}{K^{1-\gamma}g(z)h(z)} & = -g(z)^{-1/\gamma} + \rho \tag{54} \\
& + \frac{h'(z)}{h(z)} \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta - (1 - 2\gamma) \frac{\sigma_K^2}{2} + \frac{g'(z)}{g(z)} \sigma_K^2 \right) \\
& + \frac{h''(z)}{h(z)} \sigma_K^2.
\end{aligned}$$

Equations (54) and (53) must be equal, so combining we obtain the differential equation that h must satisfy:

$$\begin{aligned}
0 & = (1 - \alpha)e^{(1-\alpha)z} - h(z)g(z)^{-1/\gamma} \tag{55} \\
& + h'(z) \left(g(z)^{-1/\gamma} - e^{(1-\alpha)z} + \delta + \eta - (1 - 2\gamma) \frac{\sigma_K^2}{2} + \frac{g'(z)}{g(z)} \sigma_K^2 \right) \\
& + h''(z) \sigma_K^2.
\end{aligned}$$

If the function $h(z)$ exists, then it will satisfy Equations (54) and (53), and $H(K, z) = Kh(z)$ is the arbitrage-free price of a claim to human capital.

Finally, since K_t is the value of a claim to consumption, and consumption is the sum of dividends and wages, then $h(z) + e(z) = 1$, so that $e(z) = 1 - h(z)$.

10 Appendix C: Numerical solution method

The two ordinary differential equations (the first one solves the value function and the second one solves for the value of equity) are found using the same method. I use the finite-difference Crank-Nicholson method as described in Marimon and Scott (1999) (chapter 8).

I first solve the ODE for the value function, using a grid of 1001 points for z ranging from -4 to -2.5, a range that captures the area of interest. The initial value is $g(z) = \frac{\rho}{\gamma} + e^{(1-\alpha)z} + \frac{(1-\gamma)}{2}(\sigma_K^2)$, corresponding to the function's value if the agent didn't consider that the state variable z changes over time. The initial guess is very important for the algorithm's convergence. I iterate until the largest error in the iteration is smaller than 1E-6.

Solving the ODE for the value of equity requires the value of $g(z)$ as an input. The initial guess is with $e(z) = 1$, corresponding to the limit of $e(z)$ as $z \rightarrow -\infty$. I iterate until the root of the sum of squared errors between iterations is less than 1E-6.

Table I
Parameters used in calibration

This table shows the parameters used to calibrate the model. Section 5 describes the choice of each parameter in more detail.

Variable	Symbol	Value
Coefficient of relative risk aversion	γ	2.5
Subjective discount rate	ρ	.01
Production shocks volatility	σ_K	12%
Productivity growth	η	1.7%
Capital intensity	α	.34
Depreciation rate	δ	.01

Table II
Long-term trend values

This table records the values around which variables mean-revert in the calibration. Data on the risk-free rate, the risk-free premium, consumption growth volatility, equity volatility, and the Sharpe ratio is as reported in Campbell and Cochrane's (1999) long sample. The ratio of wages to consumption is the average for the period 1947 - 2007. Wages are equal to total compensation as reported in the National Income and Product Accounts (NIPA) tables. Consumption is the scaled sum of nondurables and services, also reported in the NIPA tables.

Variable	Model	Data
Risk-free rate	6.1%	2.9%
Equity risk premium	2.5%	3.9%
Consumption growth volatility	3.8%	3.8%
Equity volatility	26%	18%
Sharpe ratio	9.5%	22%
Wages over consumption	83.5%	85%
Human capital risk premium	0.9%	–
Human capital weight in aggregate wealth	84.6%	–

Table III
Sensitivity analysis

This table shows the sensitivity of the long-term values to changes in the parameters. The first three columns are parameters: α is capital's intensity, γ is the coefficient of relative risk aversion and σ_K is the volatility of production shocks. The middle columns show the results for the volatility of consumption growth and the ratio of wages to consumption w/c . The last columns show the results for the risk-free rate r_f , the equity risk premium r_e^e , the volatility of equity σ_e , equity's Sharpe ratio, and the excess return to human capital r_{hc}^e . Other parameters in the calibration are $\delta = 0.01$, $\eta = 0.017$ and $\rho = 0.01$.

Parameters			Macro variables		Asset-prices				
γ	σ_K	α	σ_c	w/c	r_f	r_e^e	σ_e	Sharpe	r_{hc}^e
2.5	10%	0.34	3.16%	81.1%	5.87%	1.46%	18%	0.079	0.64%
		0.36	3.27%	78.9%	5.83%	1.25%	15%	0.082	0.70%
		0.38	3.41%	76.9%	5.79%	1.15%	13%	0.085	0.77%
	12%	0.34	3.81%	83.5%	6.13%	2.52%	26%	0.095	0.89%
		0.36	3.90%	80.8%	6.08%	2.09%	21%	0.098	0.97%
		0.38	4.03%	78.5%	6.04%	1.82%	18%	0.101	1.05%
2.7	10%	0.34	2.83%	80.8%	6.37%	1.44%	19%	0.076	0.61%
		0.36	2.93%	78.4%	6.33%	1.24%	16%	0.079	0.67%
		0.38	3.07%	76.2%	6.28%	1.12%	14%	0.083	0.73%
	12%	0.34	3.45%	83.1%	6.67%	2.48%	27%	0.093	0.87%
		0.36	3.53%	80.4%	6.64%	2.07%	22%	0.095	0.93%
		0.38	3.64%	77.9%	6.58%	1.80%	18%	0.098	1.02%

Table IV
Fama Macbeth regressions

This table gives the estimates for the cross-sectional regression model

$$E[R_i] = c_0 + c_{vw}\beta_i^{vw} + c_{labor}\beta_i^{labor}$$

R_i is the return of portfolio i . β_i^{vw} is the coefficient in a regression of portfolio i 's returns on the market's returns, human capital's returns, and a constant. β_i^{labor} is the corresponding coefficient for human capital's returns. c_{hc} is the coefficient when human capital's return is calculated using the ratio of wages to consumption as a proxy for the weight of human capital in aggregate wealth. $c_{\Delta w}$ is the coefficient when human capital's return is calculated using labor income growth as a proxy for the return to human capital. Panel A shows the results of the regression using Fama and French's (1996) 49 industry portfolios. Panel B reports the results using Fama and French's 100 size and book-to-market sorted portfolios. Coefficients reported in bold are significant at the 95 percent level. The values presented here were multiplied by a factor of 100. The sample period is 1959-2007.

Cross-sectional regressions: 1959-2007					
PANEL A: 49 Industry Portfolios					
Coefficient	c_0	c_{vw}	c_{hc}	$c_{\Delta w}$	R^2
Estimate	-14.14	10.53	9.78		78.49
Standard error	(7.96)	(7.15)	(1.35)		
Estimate	-8.06	8.45		9.98	55.67
Standard error	(6.55)	(4.52)		(2.36)	
PANEL B: 100 Size and Book-to-market Portfolios					
Coefficient	c_0	c_{vw}	c_{hc}	$c_{\Delta w}$	R^2
Estimate	6.37	-6.50	1.86		15.09
Standard error	(3.27)	(3.06)	(0.82)		
Estimate	9.92	-9.96		2.11	20.60
Standard error	(3.40)	(3.39)		(1.70)	

Figure 1

Consumption per unit of capital as a function of production per unit of capital

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

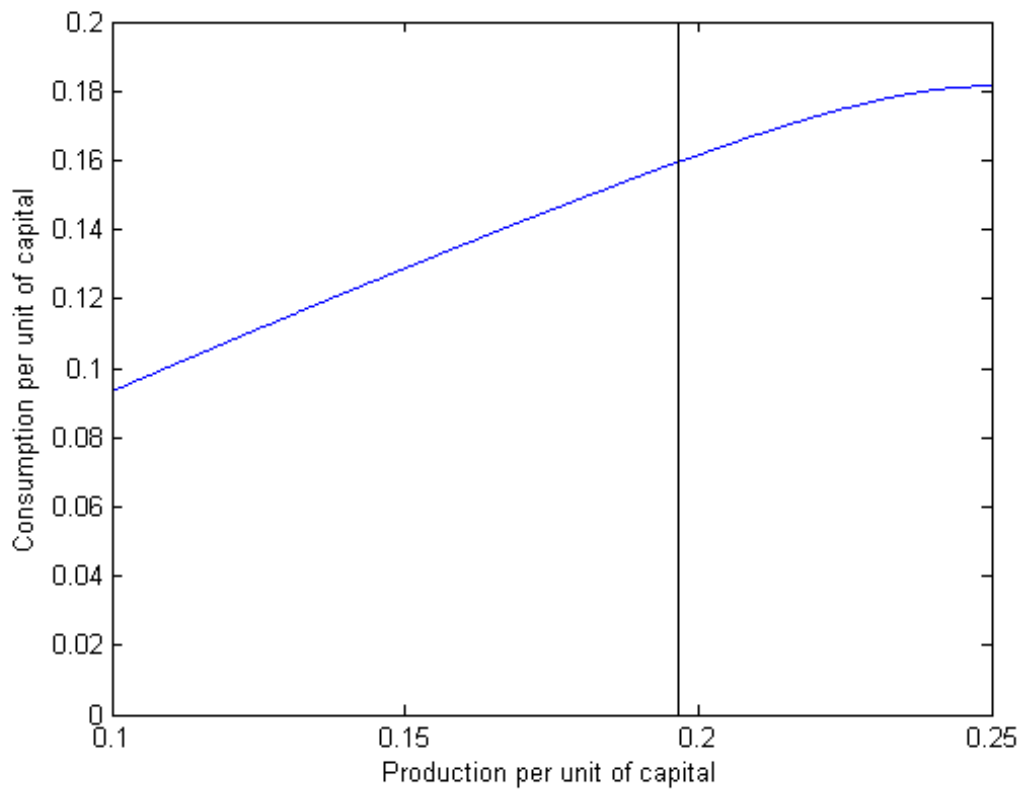


Figure 2

Consumption over production as a function of production per unit of capital

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

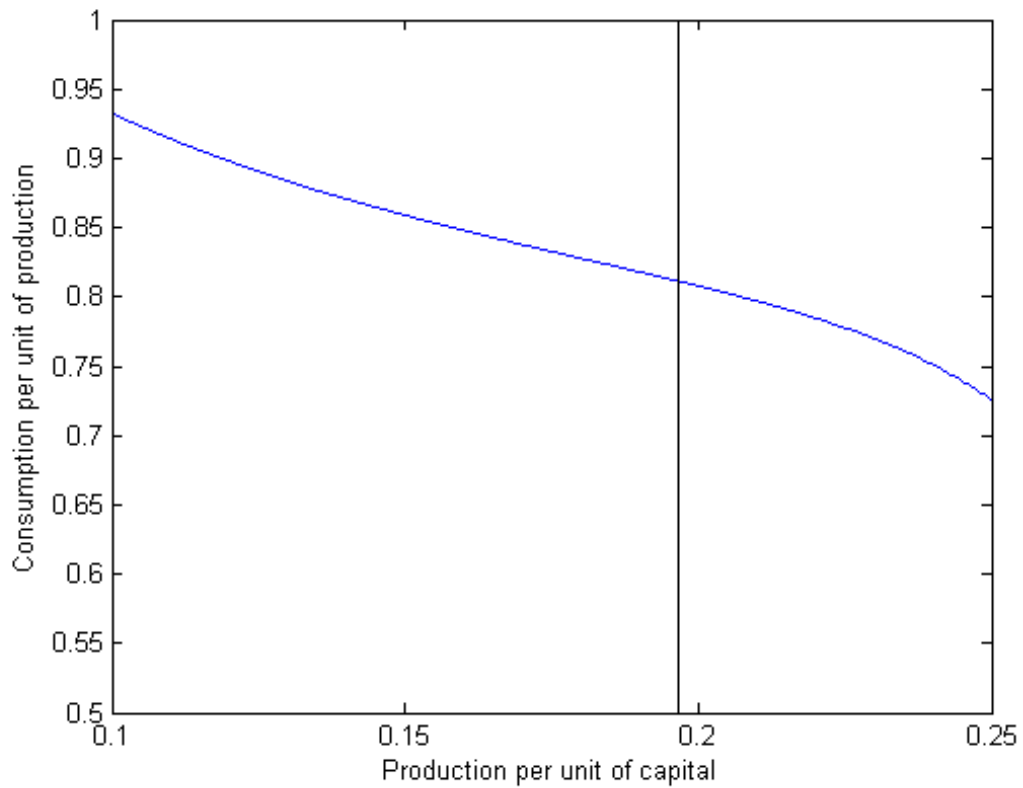


Figure 3

Wages over consumption as a function of production per unit of capital

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

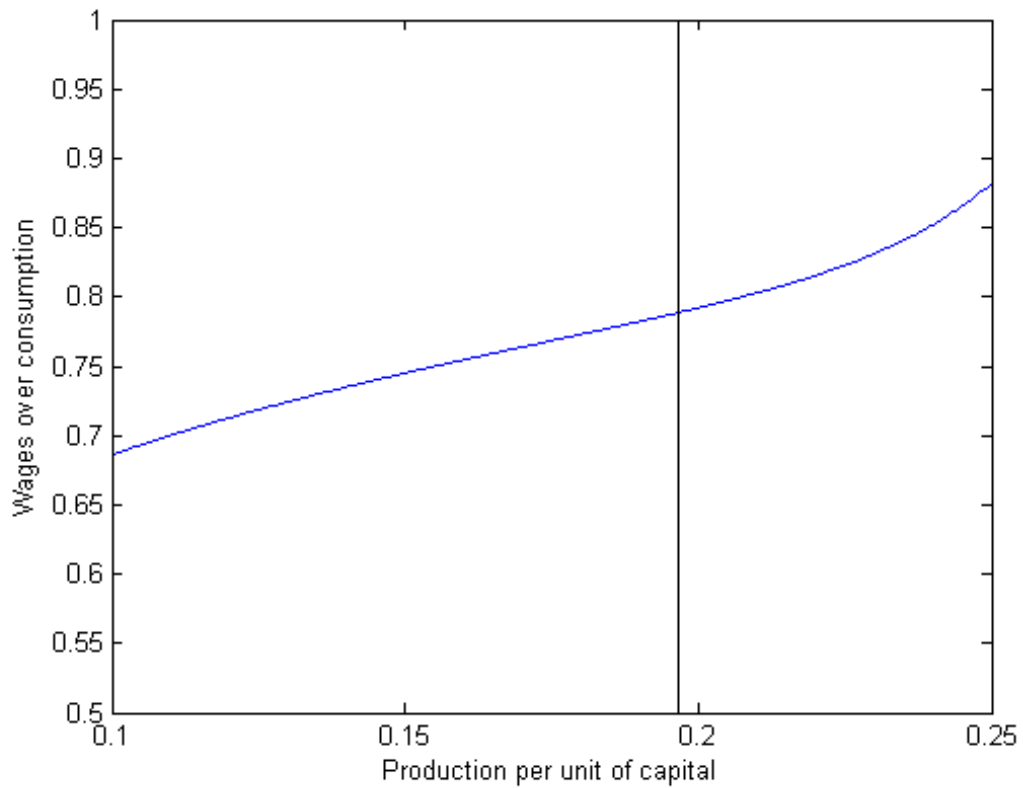


Figure 4
Human capital as a fraction of the wealth portfolio

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

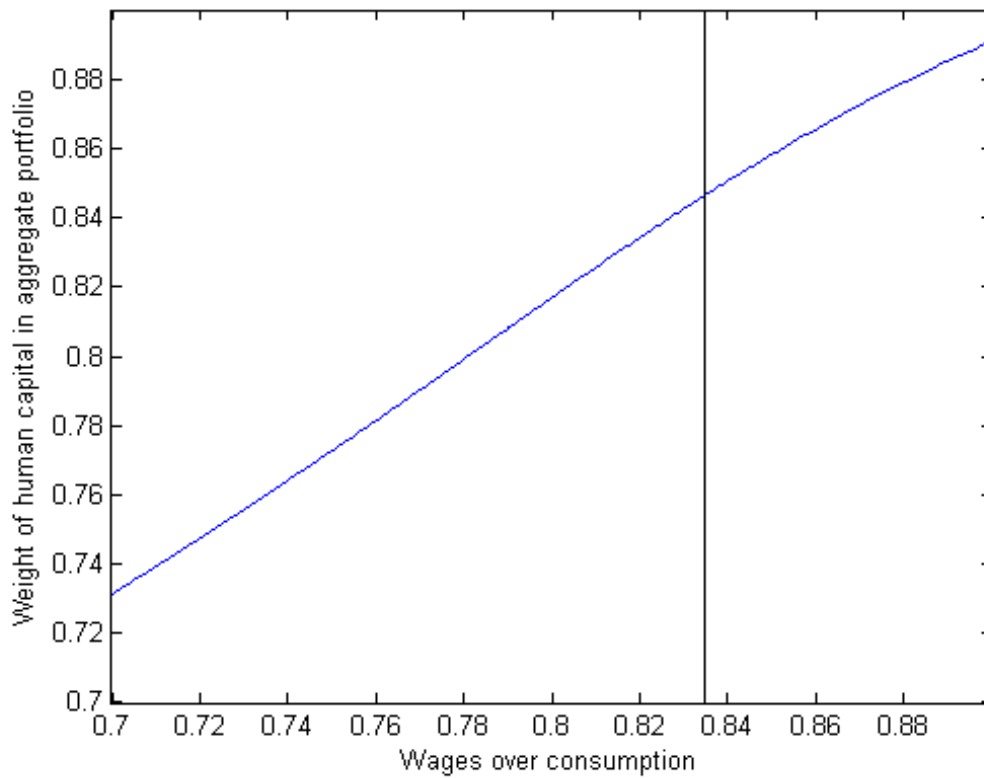


Figure 5
Expected equity risk premium and human capital risk premium as a function of wages over consumption

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

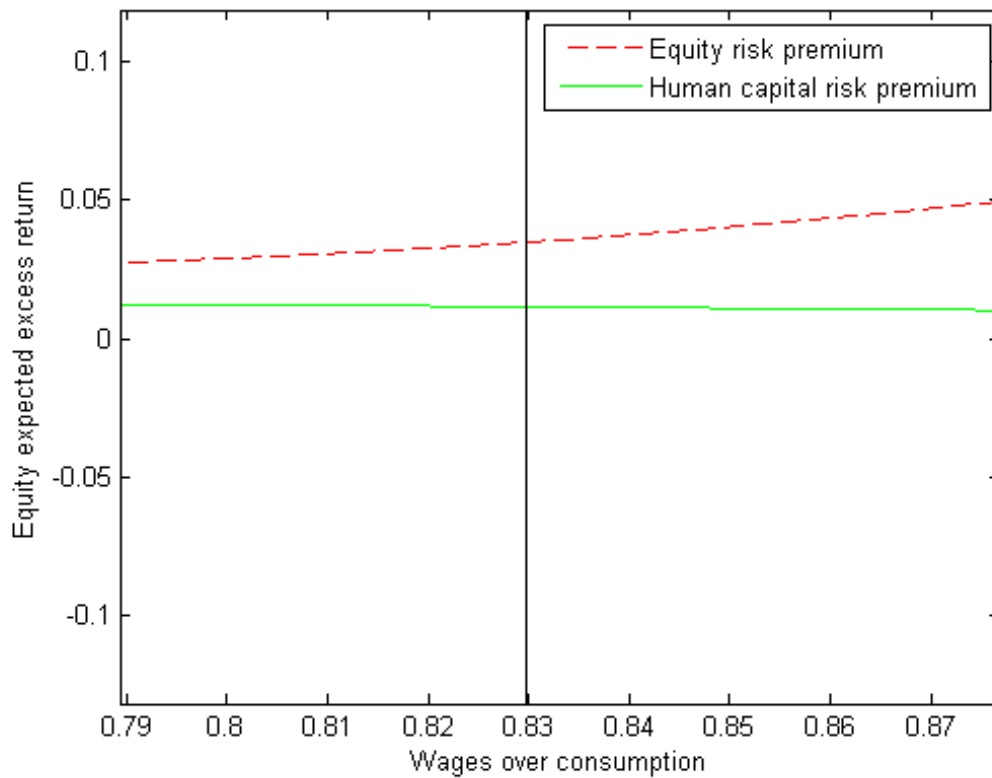


Figure 6

Consumption and equity volatility as a function of wages over consumption

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

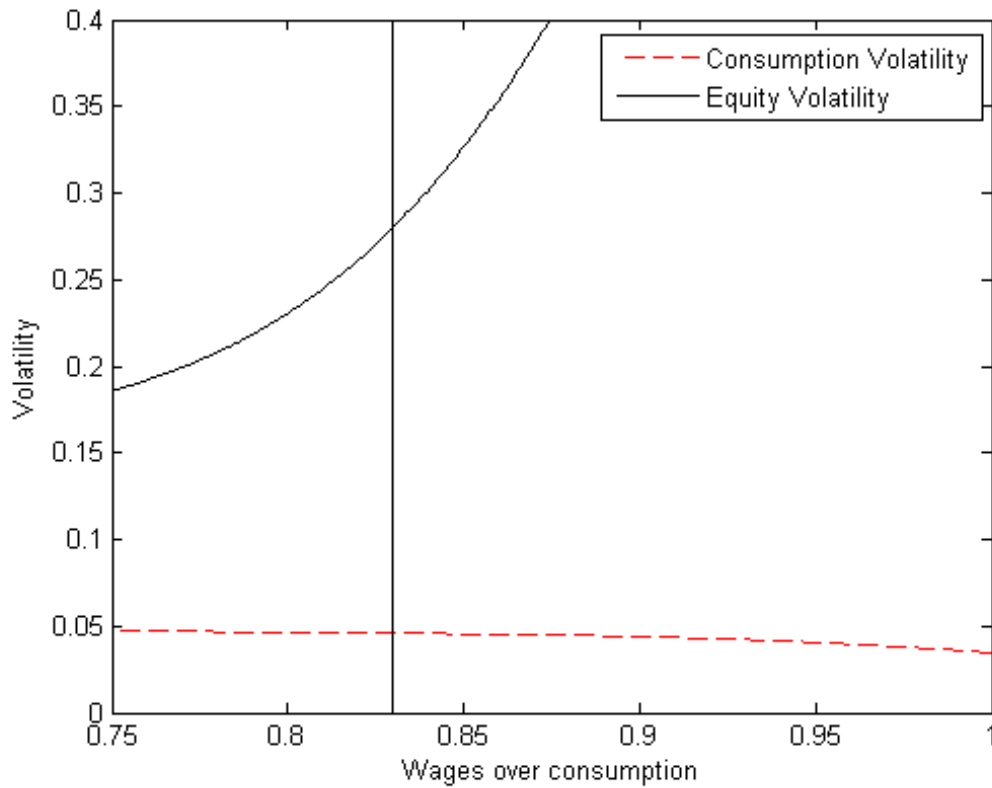


Figure 7

Covariance between wages, human capital, and equity returns

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

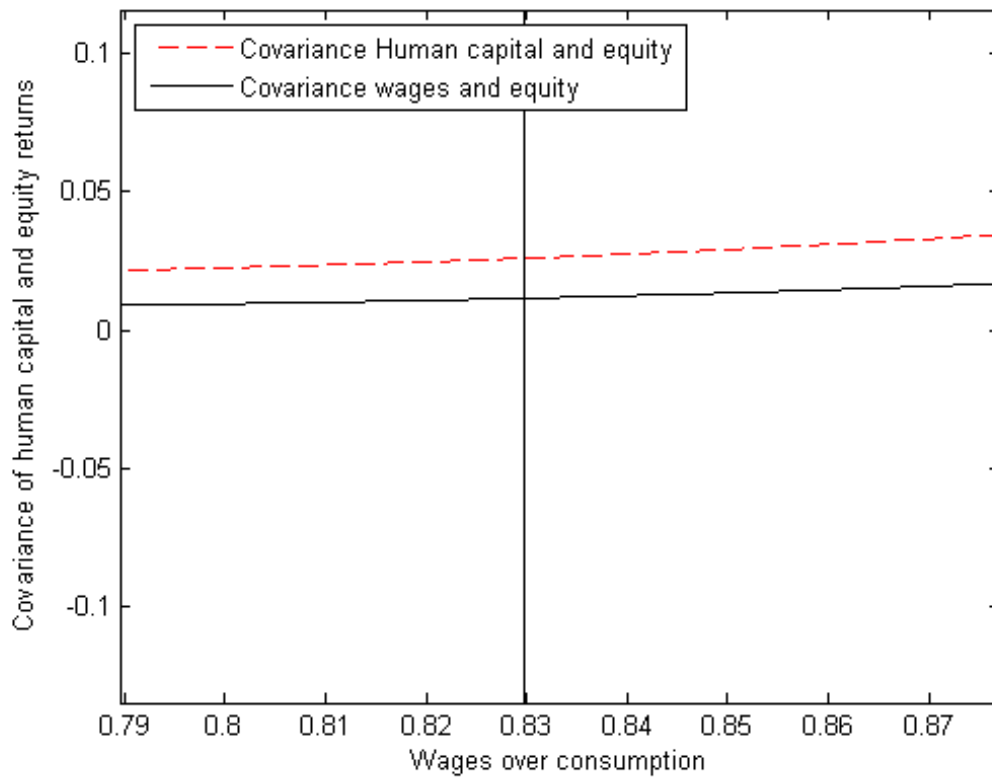


Figure 8

Replicating human capital: weight of equity in the replicating portfolio

Results with the following parameters: $\alpha = 0.34$, $\rho = 0.01$, $\eta = 0.017$, $\gamma = 2.5$, $\sigma_K = 0.12$.
The vertical black line denotes the long-term value around which the economy oscillates.

