Financial Synergies and the Optimal Scope of the Firm: Implications for Mergers, Spinoffs, and Structured Finance

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ABSTRACT

Multiple activities may be separated financially, allowing each to optimize its financial structure, or combined in a firm with a single optimal financial structure. We consider activities with nonsynergistic operational cash flows, and examine the purely financial benefits of separation versus merger. The magnitude of financial synergies depends upon tax rates, default costs, relative size, and the riskiness and correlation of cash flows. Contrary to accepted wisdom, financial synergies from mergers can be negative if firms have quite different risks or default costs. The results provide a rationale for structured finance techniques such as asset securitization and project finance.

DECISIONS THAT ALTER THE SCOPE of the firm are among the most important faced by management, and among the most studied by academics. Mergers and spinoffs are classic examples of such decisions. More recently, structured finance has seen explosive growth: Asset securitization exceeded $6.8 trillion in 2004, and Esty and Christov (2002) report that in 2001, more than half of capital investments with costs exceeding $500 million were financed on a separate project basis.[1] Yet financial theory has made little headway in explaining structured finance.

Positive or negative *operational* synergies are often cited as a prime motivation for decisions that change the scope of the firm. A rich literature addresses the roles of economies of scope and scale, market power, incomplete contracting, property rights, and agency costs in determining the optimal boundaries of the firm.[2] But operational synergies are difficult to identify in the case of asset securitization and structured finance.

[1] "Structured finance" typically refers to the transfer of a subset of a company's assets (an "activity") into a bankruptcy-remote corporation or other special purpose vehicle or entity (SPV/SPE). These entities then offer a single class of securities (a "pass-through" structure) or multiple classes of securities (a "pay-through" structure). Structured finance techniques include asset securitization and project finance and are discussed in more detail in Section VI. See Esty and Christov (2002), Esty (2003, 2004), Gorton and Souleles (2005), Kleimeier and Megginson (1999), Oldfield (1997, 2000), and Skarabot (2002).

[2] Classic studies include Coase (1937), Williamson (1975), and Grossman and Hart (1986). See also Holmstrom and Roberts (1998) and the references cited therein.

This paper examines the existence and extent of purely *financial* synergies. To facilitate this objective, we assume that the operational cash flow of the combined activities is nonsynergistic.[3] If operational synergies exist, their effect will be incremental to the financial synergies examined here.

In a Modigliani-Miller (1958) world without taxes, bankruptcy costs, informational asymmetries, or agency costs, there are no purely financial synergies, and capital structure is irrelevant to total firm value. In a world with taxes and default costs, however, capital structure matters. Therefore, changes in the scope of the firm that affect optimal capital structure typically create financial synergies.

Financial synergies can be positive (favoring mergers) or negative (favoring separation). When activities' cash flows are imperfectly correlated, risk can be lowered via a merger or initial consolidation. Lower risk reduces expected default costs. Leverage can potentially be increased, with greater tax benefits, as first suggested by Lewellen (1971). However, Lewellen asserts that the financial benefits of mergers *always* are positive. If his assertion is correct, then purely financial benefits cannot explain structured finance. But we show below that Lewellen's argument is incomplete. Financial separation of activities—whether through separate incorporation or a special purpose entity (SPE)—allows each activity to have its appropriate capital structure, with an optimal amount of debt and equity. Separate capital structures and separate limited liabilities may allow for greater leverage and financial benefits than when activities are merged with the resultant single capital structure.[4] We show that this is likely to be the case when activities differ markedly in risk or in default costs. Further, as Scott (1977) and Sarig (1985) observe, separation bestows the advantage of multiple limited liability shelters.

This paper develops a simple trade-off model of optimal capital structure to address three questions:

1. What are the characteristics of activities that benefit from merger versus separation?
2. How important are the *magnitudes* of potential financial synergies?
3. How do synergies depend upon the volatility and correlation of cash flows, and on tax rates, default costs, and relative size?

While operational synergies may exceed financial synergies in many mergers or spinoffs, financial synergies can be sizable in the specific situations that we identify. Indeed, financial synergies are often cited as the *principal* reason for structured finance, and our model shows potentially significant financial benefits to using these techniques. The results have implications for empirical work

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[3] The assumption that operational cash flows are additive, and therefore invariant to firm scope, parallels the Modigliani-Miller (M-M, 1958) assumption that cash flows are invariant to changes in capital structure. While much of capital structure theory deals with relaxing this M-M assumption, it still stands as the base from which extensions are made.

[4] The analysis considers a single class of debt for each separate firm. Multiple seniorities of debt within a firm will not affect the results as long as all classes have potential recourse to the firm's total assets.

attempting to explain the sources of merger gains or to predict merger activity. Aspects of firms' cash flows that create substantial financial synergies, such as differences in volatility and differences in default costs, should be included as possible explanatory variables.

This paper is organized as follows. Section I summarizes previous work on financial synergies. Section II introduces a simple two-period valuation model. Closed-form valuation formulas when cash flows are normally distributed are derived in Section III, and properties of optimal capital structure are considered. Section IV introduces measures of financial synergies from mergers. Section V examines the nature and extent of synergies when cash flows are jointly normal. This section contains our core results, including a counterexample to Lewellen's (1971) conjecture that financial synergies are always positive. Section VI considers spinoffs and structured finance, providing examples that illustrate the benefits of asset securitization and project finance. Section VII considers the distribution of benefits between stockholders and bondholders. Section VIII concludes, and identifies several testable hypotheses.

I. Previous Work

Numerous theoretical and empirical studies consider the effects of conglomerate mergers and diversification. Lewellen (1971) correctly argues that combining imperfectly correlated nonsynergistic activities, while not value-enhancing per se, has a coinsurance effect: Mergers reduce the risk of default, and thereby increase debt capacity. He then conjectures that higher debt capacity leads to greater optimal leverage, tax savings, and value for the merged firm. Stapleton (1982) uses a slightly different definition of debt capacity but also argues that mergers have a positive effect on total firm value. In Section V below we quantify the coinsurance effect, and show that it does *not* always overcome the disadvantage of forcing a single financial structure onto multiple activities. When the latter dominates, separation rather than merger creates greater total value.

Higgins and Schall (1975), Kim and McConnell (1977), Scott (1977), Stapleton (1982), and Shastri (1990) consider the distribution of merger gains between extant bondholders and stockholders. They argue that while total firm value may increase with a merger due to lower risk, bondholders may gain at the expense of shareholders. Similar to Lewellen's work, these papers do not have an explicit model of optimal capital structure before and after merger.[5] Nonetheless, our results support many of their conclusions.

Examples in the papers above assume that activities' future cash flows are always positive. In this case, limited firm liability has no value. However, Scott (1977) and Sarig (1985) independently note that, if activities' future cash flows can be negative, limited firm liability provides a valuable option to walk away

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[5] Scott (1977) presents a simple two-state example in which capital structure is optimized, and shows that a profitable merger could result in lower total debt.

from future activity losses.[6] Mergers may incur a value loss in this case: The sum of separate nonsynergistic cash flows, with limited liability on the sum, can be less (but never more) than the sum of cash flows each with separate limited liability. Thus, although activity cash flows are additive, firm cash flows are subadditive. We term the loss in value that results from the loss of separate firm limited liability the "*LL* effect." Its magnitude depends upon the distribution of activities' future cash flows, and is independent of capital structure. The *LL* effect is negligible in some of the cases that we examine. Yet it can be substantial under realistic circumstances.

Numerous papers consider the potential impact of firm scope on *operational* synergies. Flannery, Houston, and Venkataraman (1993) consider investors who issue external debt and equity to invest in risky projects and must decide upon separate or joint incorporation of the projects. They find that joint incorporation is more valuable when project returns have similar volatility and lower correlation, results that are consistent with our conclusions. However, operational rather than financial synergies drive their results: Investment and therefore the cash flows of the merged firm will be different from the sum of investments and cash flows of the separate firms. John (1993) uses a related approach to analyze spinoffs, while Chemmanur and John (1996) use managerial ability and control issues to explain project finance and the scope of the firm.

Several recent papers use an incomplete contracting approach to determining firm scope. Inderst and Müller (2003) assume nonverifiable cash flows and examine investment decisions. Separation of activities may be financially desirable, but only if there are increasing returns to scale for second-period investment. While cash flows are additive in the first period, investment levels and therefore operational cash flows in the second period depend upon whether activities are merged (centralized) or separated (decentralized). Faure-Grimaud and Inderst (2004) also consider nonverifiable cash flows in the context of mergers. In contrast with our model, firm access to external finance is restricted because of nonverifiability. Mergers affect these financing constraints and in turn future cash flows. Chemla (2005) introduces an incomplete contracting environment where ex post takeovers can affect ex ante effort by stakeholders, and therefore future operational cash flows are functions of the likelihood of takeover. Rhodes-Kropf and Robinson (2004) focus on incomplete contracting and asset complementarity (implying operational synergies) in explaining merger benefits. They find empirical evidence that similar firms merge. Our results (e.g., Proposition 1 in Section V.C) suggest that financial synergies could also explain the merger of similar firms.

Morellec and Zhdanov (2005) use a continuous time model to consider mergers as exchange options. Positive operational synergies are assumed but are not explicitly examined; their focus is on the dynamic evolution of firm values and the timing of mergers.

Finally, numerous papers consider the effect of mergers on managerial decision making. Agency costs may give rise to negative operational synergies and

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[6] Sarig (1985) cites the potential liabilities of tobacco and asbestos companies as examples of cases in which activity cash flows can be negative.

a "conglomerate discount."[7] A rich but still inconclusive empirical literature tests whether such a discount exists.[8]

The purely financial synergies we examine are in most cases supplemental to, rather than competitive with, the existence of operational synergies. The works cited above generally ignore optimal capital structure and the resulting tax benefits and default costs that are the key sources of synergies in this paper. Our approach is simple: Information is symmetric, cash flows are verifiable, and there are no agency costs. Despite its lack of complexity, our model shows that financial synergies can be of significant magnitude, and it provides a clear rationale for asset securitization and project finance.

II. A Two-Period Model of Capital Structure

The analysis of financial synergies requires a model of optimal capital structure. This section develops a simple two-period model to value debt and equity. The approach is related to the two-period models of DeAngelo and Masulis (1980) and Kale, Noe, and Ramirez (1991). In contrast with these authors, we distinguish between activity cash flow and corporate cash flow, since the latter reflects limited liability and is affected by the boundaries of the firm. Also in contrast with these authors, our analysis makes the more realistic assumption that only interest expenses are tax deductible. This, however, creates an endogeneity problem. When interest only is deductible, the fraction of debt service attributed to interest payments depends on the value of the debt, which in turn (when the tax rate is positive) depends on the fraction of debt service attributed to interest payments. We use numerical techniques to find debt values and optimal leverage. But the lack of closed-form solutions limits the comparative static results that can be obtained analytically.

A. Operational Cash Flows, Taxes, and Limited Liability of the Firm

Consider a risk-neutral environment with two periods $t = \{0, T\}$, where T is the length of time spanned by the two periods. The (nonannualized) risk-free interest rate over the entire time period T is r_T. An activity generates a random future operational cash flow X at time $t = T$. Following Scott (1977) and Sarig (1985), future operational cash flows may be negative.

Risk neutrality implies that the value X_0 of the operational cash flow at $t = 0$ is its discounted expected value; that is

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[7] See, for example, Jensen (1986), Aron (1988), Rotemberg and Saloner (1994), Harris, Kriebel, and Raviv (1982), Shah and Thakor (1987), John and John (1991), John (1993), Li and Li (1996), Stein (1997), Rajan, Servaes, and Zingales (2000), and Scharfstein and Stein (2000). Maksimovic and Philips (2002) focus on inefficiencies (that generate negative cash flow synergies) of conglomerates in a model that does not include agency costs.

[8] Martin and Sayrak (2003) provide a useful summary of this research. Berger and Ofek (1995) document conglomerate discounts on the order of 15%. This and related results have been challenged on the basis that diversified firms trade at a discount prior to diversifying: see, for example, Lang and Stulz (1994), Campa and Kedia (2002), and Graham, Lemmon, and Wolf (2002). Mansi and Reeb (2002) conclude that the conglomerate discount of *total* firm value is insignificantly different from zero when debt is priced at market rather than book value.

$$X_0 = \frac{1}{(1+r_T)} \int_{-\infty}^{\infty} X \, dF(X), \tag{1}$$

where $F(X)$ is the cumulative probability distribution of X at $t = T$. Claims to operational cash flows need not be traded. With limited liability, the firm's owners can "walk away" from negative cash flows through the bankruptcy process. Thus, the (pre-tax) value of the activity with limited liability is

$$H_0 = \frac{1}{(1+r_T)} \int_{0}^{\infty} X \, dF(X), \tag{2}$$

and the pre-tax value of limited liability is

$$L_0 = H_0 - X_0$$
$$= -\frac{1}{(1+r_T)} \int_{-\infty}^{0} X \, dF(X) \geq 0. \tag{3}$$

Note that $L_0 = 0$ if the probability of negative future cash flows value is zero.

Now consider an unlevered firm with limited liability when future cash flows are taxed at the rate τ.[9] The after-tax value of the unlevered firm is

$$V_0 = \frac{1}{(1+r_T)} \int_{0}^{\infty} (1-\tau)X \, dF(X)$$
$$= (1-\tau)H_0, \tag{4}$$

and the present value of taxes paid by the firm (with no debt) is

$$T_0(0) = \tau H_0. \tag{5}$$

B. Debt, Tax Shelter, and Default

Similar to Merton (1974), firms can issue zero-coupon bonds at time $t = 0$ with principal value P due at $t = T$. Let $D_0(P)$ denote the $t = 0$ market value of the debt. Then the promised interest payment at T is

$$I(P) = P - D_0(P). \tag{6}$$

Hereafter we often suppress the argument P of D_0 and I.[10]

Interest is a deductible expense at time $t = T$. Thus, taxable income is $X - I$ and the zero-tax or "break-even" level of cash flow, X^Z, is

$$X^Z = I = P - D_0. \tag{7}$$

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[9] After personal taxes, τ is typically less than the corporate income tax rate (currently 35%). See footnote 16.

[10] Our analysis can also be interpreted in terms of a coupon-paying bond, with D_0 representing the market (and principal) value of the debt at $t = 0$, and an amount $P = D_0 + I$ due at maturity, where I is the promised coupon. In a two-period model, there is little need to distinguish zero-coupon from coupon-paying debt. This distinction is more important in multiple-period models, since default can occur prior to debt maturity if the bond defaults on a coupon payment.

We assume that taxes have zero loss offset: If $X < X^Z$, no tax refunds are paid. The present value of future tax payments of the levered firm with zero-coupon debt principal P is given by the discounted expected value

$$T_0(P) = \frac{\tau}{(1+r_T)} \int_{X^Z}^{\infty} (X - X^Z) \, dF(X). \tag{8}$$

The future random equity cash flow E is equal to operational cash flows less taxes and the repayment of principal, bounded below by zero to reflect limited liability. Thus, E can be written as

$$E = \text{Max}[X - \tau \text{Max}[X - X^Z, 0] - P, 0]. \tag{9}$$

Default occurs if operational cash flow X results in a negative equity cash flow E but for limited liability. Default occurs whenever $X < X^d$, where from equation (9) the default-triggering level of cash flow, X^d, is determined by

$$X^d = P + \tau \text{Max}[X^d - X^Z, 0]. \tag{10}$$

We now show that $X^d \geq X^Z$. Assume the contrary, that $X^Z > X^d$. Then from equation (10), $X^d = P$. But from equation (7), $X^Z = P - D_0 < P = X^d$, a contradiction. It therefore follows from (10) that

$$X^d = P + \tau(X^d - X^Z),$$

which in turn implies

$$X^d = P + \frac{\tau}{(1 - \tau)} D_0, \tag{11}$$

where (11) uses equation (7).

With X^Z and X^d from (7) and (11), we can now determine $D_0(P)$, the value of zero-coupon debt given the principal P. The cash flows to bondholders at time $t = T$ are equal to P when $X \geq X^d$ and the firm is solvent. In the event of default, we assume that bondholders receive a fraction $(1 - \alpha)$ of nonnegative pre-tax operational cash flows $X \geq 0$, where α is the fraction of cash flows lost due to default costs.[11] Limited liability allows bondholders to avoid payments when $X < 0$. Finally, recalling that the government has seniority over bondholders in default, bondholders must absorb the tax liability $\tau(X - X^Z)$ whenever $X^Z \leq X \leq X^d$.[12] The present value of debt is therefore given by

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[11] Note that at $X = X^d - \varepsilon$ (implying default), the value received by bondholders must not exceed their promised value P. Thus, α must be sufficiently large that $((1 - \alpha)X^d - \tau(X^d - X^Z)) \leq P$. Our examples below satisfy this constraint when α is chosen to match observed recovery rates.

[12] In principle, limited liability of debt introduces an additional requirement that $((1 - \alpha)X - \text{Max}[\tau(X - X^Z), 0]) \geq 0$ for $X_Z < X < X^d$. A sufficient condition is that $(1 - \alpha - \tau) \geq 0$. The examples we consider always satisfy this constraint.

$$D_0(P) = \frac{P \int_{X^d}^{\infty} dF(X) + (1-\alpha) \int_0^{X^d} X \, dF(X) - \tau \int_{X^Z}^{X^d} (X - X^Z) \, dF(X)}{1 + r_T}. \quad (12)$$

Note that (12) is an implicit equation for D_0, since X^Z and X^d are themselves functions of D_0 through (7) and (11). Thus, numerical methods are typically required to obtain a solution.

Our treatment of taxes in bankruptcy is consistent with the "interest first" repayment regime analyzed by Baron (1975), where bondholders retain the full interest rate deduction when determining taxes owed in default. An alternative considered by Turnbull (1979), the "principal first" repayment regime, is more complex: Partial payments to debt are treated first as principal, with the remainder (if any) as interest. Talmor, Haugen, and Barnea (1985) suggest that optimal leverage can be quite sensitive to the repayment regime. They find that corner solutions often prevail for optimal leverage under either regime. However, we limit our analysis here to the simpler interest first case. With realistic levels of default costs, we find interior solutions for optimal leverage (see Section III).

The expected recovery rate (after taxes) on debt, conditional on the event of default, is

$$R(P) = \frac{\left((1-\alpha) \int_0^{X^d} X \, dF(X) - \tau \int_{X^Z}^{X^d} (X - X^Z) \, dF(X) \right) \Big/ \int_{-\infty}^{X^d} dF(X)}{P}. \quad (13)$$

While the parameter α is difficult to observe directly, in subsequent examples we choose it such that equation (13) matches observed recovery rates.[13]

Equity cash flows are given by equation (9) when $X \geq X^d$, and zero otherwise. Recalling that $X^d \geq X^Z$, the value of equity is

$$E_0(P) = \frac{1}{1 + r_T} \left(\int_{X^d}^{\infty} (X - P) \, dF(X) - \tau \int_{X^d}^{\infty} (X - X^Z) \, dF(X) \right). \quad (14)$$

C. Optimal Capital Structure

The initial value of the leveraged firm, $v_0(P)$, is the sum of debt and equity values,

$$v_0(P) = D_0(P) + E_0(P), \quad (15)$$

where $D_0(P)$ satisfies the implicit equation (12) and $E_0(P)$ is given by equation (14). The optimal capital structure is the debt P that maximizes total firm value $v_0(P)$. Given the distribution of X and the parameters r_T, α, and τ, we can

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[13] With zero-coupon debt, there is a question as to whether recovery rates should be scaled relative to P or $D_0(P)$. We choose the former for our analysis. If we had chosen the latter, a higher α would be required to match observed recovery rates.

determine numerically the optimal amount of debt $P = P^*$ and therefore optimal leverage $D_0(P^*)/v_0(P^*)$. In Section III, we derive optimal leverage assuming normally distributed future cash flows.

D. Sources of Gains to Leverage

The increase in value from leverage, $v_0(P) - V_0$, reflects the present value of tax savings from the interest deduction less default costs. It is straightforward to show that the value of the levered firm (15) can also be expressed as

$$v_0(P) = V_0 + TS_0(P) - DC_0(P), \tag{16}$$

where $TS_0(P)$ is the present value of tax savings, which is equal to the difference in taxes between the levered and unlevered firm,

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\begin{aligned}
TS_0(P) &= T_0(0) - T_0(P) \\
&= \tau H_0 - \frac{\tau}{(1 + r_T)} \int_{X^Z}^{\infty} (X - X^Z) dF(X),
\end{aligned}
\tag{17}
$$

recalling (5) and (8), and that $DC_0(P)$ is the present value of the default costs,

$$DC_0(P) = \frac{\alpha}{(1 + r_T)} \left(\int_0^{X^d} X \, dF(X) \right), \tag{18}$$

recalling (11). Because V_0 in equation (16) is independent of P, the optimal leverage problem can also be posed as choosing the debt level P to maximize tax savings less default costs.

III. Optimal Capital Structure with Normally Distributed Cash Flows

In Appendix A, we derive closed-form expressions for debt, equity, and firm value using the formulas in Section II, assuming that future operational cash flow is normally distributed with mean Mu and standard deviation SD. The normal distribution is particularly suited to our purpose of exploring firm scope since the sum of normally distributed cash flows is also normally distributed, and hence the formulas in Appendix A can be used for merged as well as separate firms. Note that the equations in Appendix A for debt, equity, and firm values are homogeneous of degree one in the variables Mu, SD, and P.

A. A Base Case Example

Table I gives parameters for a base case consistent with a typical firm that issues BBB-rated unsecured debt. The annual risk-free interest rate $r = 5\%$ approximates recent intermediate-term Treasury note rates. The length of the time period T is assumed to be 5 years, consistent with estimates of the average

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Table I

Base-Case Parameters

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This table shows the parameter values chosen for the base case.

Variable	Symbols	Values
Annual risk-free rate	r	5.00%
Time period/debt maturity (yrs)	T	5.00
T-period risk-free rate	$r_T = (1 + r)^T - 1$	27.63%
Capitalization factor	$Z = (1 + r_T)/r_T$	4.62
Unlevered Firm Variables		
Expected future operational cash flow at T	Mu	127.63
Expected operational cash flow value (PV)	$X_0 = Mu/(1 + r)^T$	100.00
Cash flow volatility at T	Std	49.19
Annualized operational cash flow volatility	$Std/(X_0 \, T^{0.5})$	22%
Tax rate	τ	20%
Value of unlevered firm w/limited liability	V_0	80.05
Value of limited liability (after tax)	$(1 - \tau)L_0$	0.05

maturity of debt.[14] The resulting capitalization factor for 5-year cash flows is $Z = (1 + r)^T/((1 + r)^T - 1) = 4.62$.[15]

Expected operational cash flow $Mu = 127.6$ is chosen such that its present value is $X_0 = 100$. More generally, the expected cash flow $Mu = X_0 (1 + r)^T$. Operational cash flow at the end of 5 years has a *SD* of 49.2, consistent with an annual standard deviation of cash flows equal to 22.0 ($= 49.2/\sqrt{5}$) if annual cash flows are additive and identically and independently distributed (i.i.d.).[16] Henceforth we express volatility σ as an annual percent of initial activity value X_0, for example, $\sigma = 22\%$ in the base case. More generally, the standard deviation of cash flows *SD* is related to σ by $SD = X_0 \, \sigma \sqrt{T}$.

The tax rate $\tau = 20\%$ is selected in conjunction with other parameters to generate a capitalized value of optimal leverage ($Z(v_0^* - V_0)/V_0$) of 8.2%.[17] The

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[14] This lies between Stohs and Maurer's (1996) estimate for average debt maturity (4.60 years for BBB-rated firms, based on data 1980–1989) and the Lehman Brothers Credit Investment Grade Index average duration (5.75 years as of September 30, 2004).

[15] For a discussion of capitalizing T-period flows and the factor Z, see Section IV.D below.

[16] Annualized operational cash flow volatility $\sigma = 22\%$ is based on Schaefer and Strebulaev (2004), who estimate asset volatility from equity volatility for firms with investment grade debt over the period 1996 to 2002. This volatility also approximates the 23% asset volatility that Leland (2004) finds, using a structural model of debt, to match Moody's observed default rates on long-term investment grade debt over the period 1980 to 2000.

[17] This premium for optimal leverage is consistent with estimates by Graham (2000) and Goldstein, Ju, and Leland (2001). If the corporate tax rate is τ_C and the marginal investor is taxable at rates τ_E on equity income and τ_P on interest income, then Miller (1977) derives an effective tax rate $\tau = 1 - (1 - \tau_C)(1 - \tau_E)/(1 - \tau_P)$. For post-1986 average personal and corporate tax rates, Graham (2003) shows that this formula would imply $\tau = 10\%$. However, several authors find empirical evidence that τ may be considerably larger. Kemsley and Nissim (2002) estimate τ to be almost 40%, and Engel, Erickson, and Maydew (1999) estimate $\tau = 31\%$.

Table II
Optimal Capital Structure

This table shows the optimal leverage for the firm and the resulting gains to leverage given the base-case parameters and a default cost $\alpha = 23\%$ (consistent with a recovery rate on debt of 49.3%). The annual volatility of the firm is $\sigma = 22\%$, the time horizon is $T = 5$ years, the risk-free interest rate is $r = 5\%$, and the tax rate is $\tau = 20\%$.

Variable	Symbols	Values
Default costs	α	23%
Optimal zero-coupon bond principal	P^*	57.1
Default value	X^d	67.7
Breakeven profit level	X^z	14.9
Value of optimal debt	D_0^*	42.2
Value of optimal equity	E_0^*	39.2
Optimal levered firm value	$v_0^* = D_0^* + E_0^*$	81.47
Optimal leverage ratio	D_0^*/v_0^*	51.8%
Annual yield spread of debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.23%
Recovery rate	R	49.3%
Tax savings of leverage (PV)	TS_0	2.32
Expected default costs (PV)	DC_0	0.89
Value of optimal leveraging	$v_0^* - V_0$ (or $TS_0 - DC_0$)	1.42
Capitalized value of optimal leverage	$Z(v_0^* - V_0)/V_0$	8.21%

default cost parameter $\alpha = 23\%$ is chosen to give an expected recovery rate of 49.3%, which is close to empirical estimates of recovery rates.[18]

Table II shows the optimal capital structure for a firm with base-case parameters. We derive the optimal debt level P^* numerically. Given P^*, other variables are computed using the formulas in Appendix A. The model predicts a base-case optimal leverage of 51.8%. This figure is somewhat greater than the leverage of an average BBB-rated firm, but is consistent with Graham's (2000) conclusion that firms are less leveraged than the optimal level.[19] The model also predicts a yield spread of 123 basis points on debt, which is also close to empirical estimates.[20] Thus, the appropriately calibrated two-period model generates both yield spreads and an optimal capital structure that are highly plausible. In Section V, we use this capital structure model to explore the optimal scope of the firm.

[18] The recovery rate depends upon the level of debt as well as other parameters including the default cost fraction α. We assume debt principal is equal to its optimal level, P^*. Elton et al. (2001) report an average recovery rate on BBB-rated debt of 49.4% for the period 1987 to 1996. Acharya, Bharath, and Srinivasan (2005) estimate median recovery of 49.1% for their 1982 to 1999 sample of defaulted debt. Direct evidence on default costs, α, is mixed. Andrade and Kaplan (1998) suggest a range of default costs, from 10% to 23% of firm value at default, based on studies of firms undergoing highly leveraged takeovers (HLTs). However, firms subject to HLTs are likely to have lower-than-average default costs, since high leverage is more likely to be optimal for firms with this characteristic.

[19] Schaefer and Strebulaev (2004) estimate that the average leverage of a large sample of BBB-rated firms is 38% over the period 1996 to 2002.

[20] Elton et al. (2001) report 5-year maturity BBB yield spreads of 120 bps for the period 1987 to 1996.

Recalling that debt, equity, and firm values are homogeneous of degree one in *Mu, SD,* and *P,* and that *Mu* and *SD* are proportional to operational activity value X_0, it follows directly that both the optimal debt $P^*(X_0, \sigma, \alpha)$ and the optimal value of the firm $v_0^*(X_0, \sigma, \alpha)$ are proportional to X_0, given fixed annual percent volatility σ and default cost fraction α. Thus, the optimal leverage is invariant to the size of the firm as reflected by X_0, when other parameters remain fixed.

B. Volatility and Capital Structure

Figure 1 plots optimal leverage as a function of volatility for two levels of default costs: $\alpha = 23\%$ (the base case), and $\alpha = 75\%$ (high default costs). Other parameters remain as in the base case. Note that when $\alpha = 23\%$, optimal leverage initially declines with volatility. For annual volatility exceeding 25%, optimal leverage increases. Thus, with moderate levels of default costs, both a low and a high volatility can generate the same optimal leverage ratio. When $\alpha = 75\%$, optimal leverage is lower. Note that the optimal leverage monotonically decreases until extreme levels of volatility are reached. This suggests that volatile firms with high default costs—for example, firms with substantial risky growth options—should avoid high leverage, consistent with the conclusions of Smith and Watts (1992) and others.
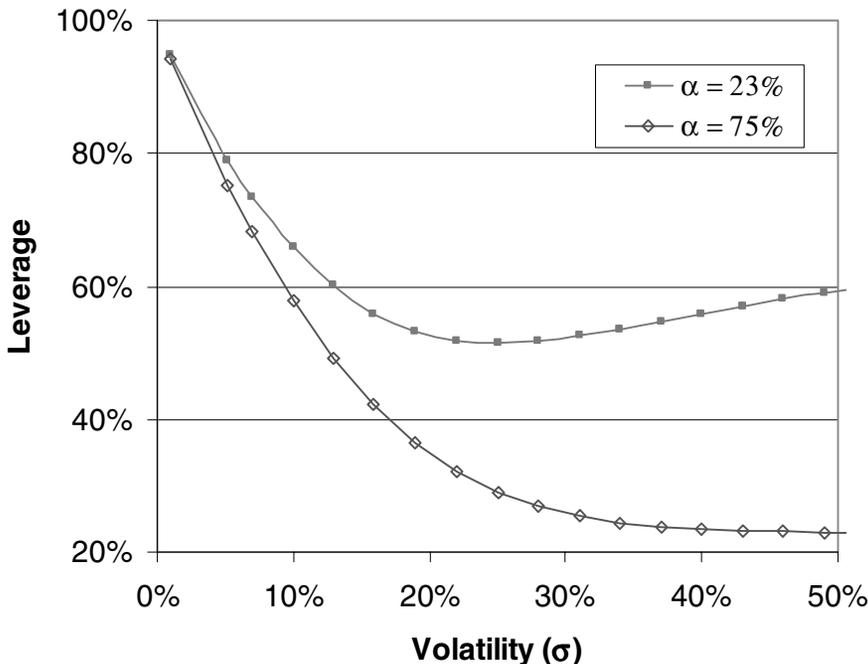
Figure 1. Optimal firm leverage. The lines plot the optimal leverage ratio for a firm as a function of the annualized volatility of cash flows σ, when default costs are $\alpha = 23\%$ (the base case) and $\alpha = 75\%$. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, and the effective corporate tax rate is $\tau = 20\%$.
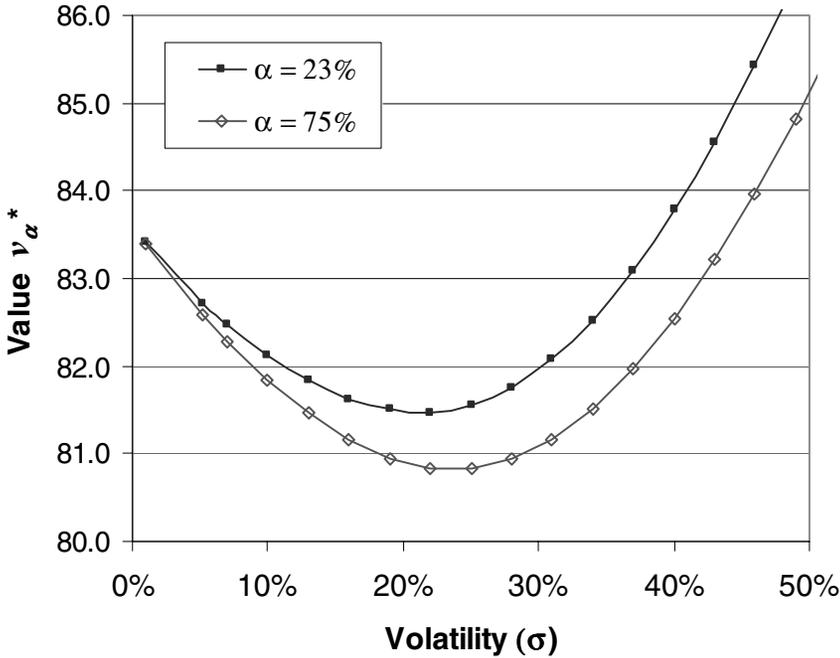
Figure 2. Optimal firm value. The lines plot the value of the optimally leveraged firm $v_\alpha(\sigma)$ as a function of the annualized volatility of cash flows σ, when unlevered operational value $X_0 = 100$ and default costs are $\alpha = 23\%$ (the base case) and $\alpha = 75\%$. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, and the effective corporate tax rate is $\tau = 20\%$. The minimum optimal firm value is reached at $\sigma_L = 21.5\%$ when $\alpha = 23\%$ and $\sigma_L = 23.7\%$ when $\alpha = 75\%$.

Define $v_\alpha^*(\sigma) \equiv v_0^*(100, \sigma, \alpha)$, that is, the value of the optimally levered firm as a function of volatility when X_0 is normalized to 100. Figure 2 plots $v_\alpha^*(\sigma)$ for default costs $\alpha = 23\%$ and $\alpha = 75\%$. Value declines with risk at low volatility levels. As volatility increases, however, the value of the firm's limited liability shelter increases and eventually value increases with volatility. Although we do not prove a general result, the optimal value function $v_\alpha^*(\sigma)$ is strictly convex and U-shaped for all combinations of the parameters we examine. We denote σ_L as the volatility at which $v_\alpha^*(\sigma)$ reaches a minimum value.[21]

IV. Measures of Financial Synergies for Merged Activities

A. *Measuring Financial Synergies*

In this section, we consider optimal firm scope. The decision is whether to incorporate and then leverage two activities $i = \{1, 2\}$ separately, or to combine ("merge") the activities into a single firm $i = M$ and leverage the merged firm.[22]

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[21] Observe that σ_L depends upon other parameters, including α.

[22] When firms are separately incorporated initially and later merged, or vice versa, how previously issued debt is retired (or assumed) becomes important. We discuss this question in Section VII below.

Nonsynergistic operational cash flows imply additivity, $X_M = X_1 + X_2$, which in turn implies from equation (1) that

$$X_{0M} = X_{01} + X_{02}. \tag{19}$$

The financial benefit of merger Δ is defined as the difference between the value of the optimally levered merged firm, and the sum of the values of the optimally levered separate firms:

$$\Delta \equiv v_{0M}^* - v_{01}^* - v_{02}^*, \tag{20}$$

recalling that $v_{0i}^* \equiv v_{0i}(P_i^*)$, where $v_{0i}(P_i)$ is given by equation (15) or (16), and P_i^* is the debt principal that maximizes firm value $v_{0i}(P_i)$, $i = \{1, 2, M\}$. Positive Δ implies that a merger increases total firm value, while negative Δ implies that separation increases value.

B. Identifying the Sources of Financial Synergies

From (20) and (16), financial synergies Δ can be decomposed into three components:

$$\Delta = \Delta V_0 + \Delta TS - \Delta DC, \tag{21}$$

where $\quad \Delta V_0 \equiv V_{0M} - V_{01} - V_{02}, \Delta TS \equiv TS_{0M} - TS_{01} - TS_{02}, \quad$ and $\quad \Delta DC \equiv DC_{0M} - DC_{01} - DC_{02}$.

The first component of financial synergies, ΔV_0, denotes the change in *unlevered* firm value that results from a merger. The other two components are directly related to changes in financial structure, with ΔTS denoting the change in the value of tax savings from optimal leveraging of the merged versus separate firms and ΔDC denoting the change in the value of default costs.

Despite operational cash flow additivity, mergers can create value changes ΔV_0. When tax rates are identical across firms ($\tau_i = \tau$), from equation (4) the change in unlevered firm values ΔV_0 can be written as

$$
\begin{aligned}
\Delta V_0 &= (1 - \tau)(H_{0M} - H_{01} - H_{02}) \\
&= (1 - \tau)((X_{0M} - X_{01} - X_{02}) + (L_{0M} - L_{01} - L_{02})) \qquad \text{using (3)} \\
&= (1 - \tau)(L_{0M} - L_{01} - L_{02}) \qquad\qquad\qquad\qquad \text{using (19)} \\
&= LL,
\end{aligned} \tag{22}
$$

where

$$LL \equiv (1 - t)(L_{0M} - L_{01} - L_{02}). \tag{23}$$

Thus, LL reflects the difference between the after-tax value of limited liability to the merged firm and the total value of limited liability to the separate firms. As noted by Scott (1977) and Sarig (1985), the LL effect is never positive, and is strictly negative if operational cash flows have a positive probability of being

negative and are less than perfectly correlated. Given (22), we shall occasionally refer to ΔV_0 directly as the *LL* effect.[23]

The second component of financial synergies from mergers, ΔTS, is the gain (or loss) in tax savings solely related to the effects of optimal merged leverage versus optimal separate leverage. The examples in Section V show that ΔTS can have either sign. Even when debt principal increases after a merger, a significantly lower debt credit spread may result in lower total interest deductions, with a subsequent loss of expected tax savings. The final component of financial synergies is the change in the value of default costs at the optimal leverage levels, ΔDC. This term is negative in all examples considered, indicating that although leverage may increase after a merger, the expected losses from default are nonetheless reduced by the lower operational risk of the merged firm.

The tax and default cost benefits may be combined into a single net "leverage effect" term, $LE \equiv \Delta TS - \Delta DC$. Thus, an alternative decomposition of merger benefits is

$$\Delta = LL + LE. \tag{24}$$

In the cases we study below, the leverage effect can be positive or negative.

C. Scaled Measures of Synergies

We consider three ways in which financial synergies Δ may be scaled. We adopt the convention that Firm 1 is the acquiring firm, and Firm 2 is the acquired or target firm.

Measure 1. $\Delta/(V_{01} + V_{02})$.

Measure 2. Δ/v_{02}^*.

Measure 3. Δ/E_{02}^*.

Measure 1 expresses synergies as a percentage of the sum of the separate firms' unlevered pre-merger values.[24] When $\Delta < 0$, Measure 1 is negative, reflecting the benefits of separation.

Competition may induce an acquiring firm to bid an amount that reflects total synergies, including financial synergies.[25] If the target receives all the

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[23] The presence of nondebt tax deductions complicates associating ΔV_0 with the after-tax loss of separate limited liability shelters alone. With additional nondebt tax deductions and no (or limited) loss offset, a tax schedule convexity effect arises, favoring mergers and partially or even fully offsetting the loss of limited liability. We do not pursue the impact of nondebt tax shelters in this paper.

[24] An alternative to Measure 1 is to express Δ as a percentage of the sum of the optimally levered separate firm values, that is, $\Delta/(v_1^* + v_2^*)$. This measure (in contrast to Measure 1) will not necessarily be monotone in absolute benefits, Δ. Note that Measures 2 and 3 below are potentially nonmonotonic in Δ, but are comparable to available statistics on the percentage merger gains for target firms.

[25] Numerous studies (e.g., Andrade, Mitchell, and Stafford (2001)) suggest that acquiring firms realize little or no increases in market value. Target firms, however, realize substantial value premiums.

potential merger benefits, Measure 2 reflects the percentage value premium on its pre-merger value, v_{02}^*. Measure 2 is also informative in the case of spinoffs or asset securitization. The negative of Measure 2, $-\Delta/v_{02}^*$, reflects the benefits of separation as a percent of the value of the assets spun off.

Measure 3 is relevant when all financial benefits accrue to the *stockholders* of the target firm. It reflects the percentage premium that the acquiring firm could pay for the optimally levered target firm's equity based on financial synergies alone. Note that generally, |Measure 1| < |Measure 2| < |Measure 3|.

D. Adjusting Benefit Measures for an Infinite Horizon

The benefit measures introduced above reflect the length T of the single time period assumed. Shorter time periods will generate smaller tax benefits (and usually lower expected default costs). Since firms do not have finite maturity, however, they can realize additional benefits in subsequent time periods with positive probability.

A complete solution to the multi-period problem is difficult in the normally distributed future cash flow case, and we do not attempt a fully dynamic modeling.[26] Rather, we follow Modigliani and Miller (1958) and many others by capitalizing the value of cash flows that occur over the single T-year period. Recall that the present value of a perpetual stream of expected payments Δ received at the end of each period of length T is $PV(\Delta) = \Delta/r_T$, where r_T is the (constant) interest paid over a period of length T years. Risk neutrality implies that the appropriate interest rate is the risk-free rate. The present value of these future payments, plus Δ at $t = 0$, is $\Delta/r_T + \Delta = Z\Delta$, where $Z = (1 + r_T)/r_T$.[27] Equivalently, $Z = (1 + r)^T/((1 + r)^T - 1)$, where r is the annual interest rate. Subsequent examples scale Measures 1–3 by the factor Z.

Note that $Z\Delta$ preserves the ordering of Δ. Therefore, the nature of our results is independent of Z, which serves only as a reasonable means for scaling the net benefits derived for a period of T years to a long-term horizon.

V. How Large Are Financial Synergies?

This section assumes that activities' future cash flows are normally distributed. When the separate cash flows X_i are jointly normally distributed

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[26] In contrast, the multiperiod case is quite straightforward to model when cash flows of firms follow a logarithmic random walk, resulting in lognormally distributed future values (e.g., Leland (1994)). However, a problem arises in studying mergers, as the sum of lognormally distributed cash flows is not lognormally distributed.

[27] A stylized environment that justifies capitalizing the gain is as follows. An entrepreneur initially owns two activities, each with a life of T years. If the entrepreneur merges the activities, optimally leverages them, and immediately sells at a fair price to outside investors, she will realize a value Δ greater than if the activities were separately incorporated, optimally leveraged, and then sold. The entrepreneur has a subsequent set of activities with identical characteristics available at time T, again, each with a life until time $2T$, and so on. In addition to the gain Δ at time $t = 0$, gains of Δ can therefore be realized at times $t = T$, $t = 2T$, etc. The present value of this infinitely repeated set of incremental cash flows is $Z\Delta$, where $Z = (1 + r_T)/r_T$, and represents the infinite-horizon value of merging activities versus separation.

$(i = \{1,2\})$ with means Mu_i, standard deviations SD_i, and correlation ρ, the merged activity cash flow $X_M = X_1 + X_2$ is normally distributed with

$$Mu_M = Mu_1 + Mu_2; \quad Std_M = \left(Std_1^2 + Std_2^2 + 2\rho Std_1 Std_2\right)^{0.5}. \quad (25)$$

Whenever $\rho < 1$, the merger creates risk reduction due to diversification: $SD_M < SD_1 + SD_2$. Recalling that the annualized standard deviation of cash flow i, expressed as a percent of initial activity value X_{0i}, is $\sigma_i = SD_i/(X_{0i}T^{0.5})$, it follows that the annualized percent standard deviation of the merged firm is

$$\sigma_M(\rho) = \left(\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2\rho\sigma_1\sigma_2 w_1 w_2\right)^{0.5}, \quad (26)$$

where $w_i = X_{0i}/X_{0M}$ is the relative value weight of activity i. Note that $\sigma_M(\rho)$ is an increasing function of ρ for given σ_1 and σ_2. When correlation takes on extreme values $\rho = \pm 1$, then $\sigma_M(1) = (w_1\sigma_1 + w_2\sigma_2)$ and $\sigma_M(-1) = |w_1\sigma_1 - w_2\sigma_2|$.

We now apply the valuation and capital structure results from Section III and Appendix A to determine the sign and magnitude of the measures of financial synergies developed in Section IV.

A. *Mergers of Identical Base-Case Firms*

We first consider the case in which the activities are identical, with parameters as listed in Table I. The correlation between the activities' cash flows is assumed to be $\rho = 0.20$. The merged firm has annualized percent volatility $\sigma_M = 17.0\%$. This compares with $\sigma_1 = \sigma_2 = 22.0\%$ for the separate firms. Diversification therefore provides a substantial percentage risk reduction.

Table III presents the optimal capital structure of the merged activities and compares it with the capital structure of the two activities when separately incorporated. Optimal debt usage rises, with optimal leverage increasing from 52% to 55%. Nonetheless, the yield spread falls from 123 basis points to 60 basis points, reflecting lower risk of default and a higher expected recovery rate.

By Measure 1, the merger provides only a 0.60% increase in value. As a fraction of the value of the target (Firm 2), the financial benefits are 1.18% (Measure 2). Measure 3 indicates that financial synergies would allow the acquirer to bid a premium of 2.45% for the target's equity.

As in Section IV.B, Measure 1 synergies can be decomposed into three components. The unlevered value, ΔV_0, falls by 0.25%, reflecting the *LL* effect. This is more than offset by the leverage effect *LE*, which yields a gain of 0.85%. The leverage effect itself consists of diminished tax savings from leverage, ΔTS, of -0.72%, which is offset by the change in expected default costs, ΔDC, of -1.57%. Given the greater use of leverage after a merger, it may seem strange that tax savings fall. However, the increased amount of post-merger debt is more than offset by the lower coupon rate paid. Thus, the interest deduction and resultant tax savings are reduced.

Financial synergies to the merger of identical base-case firms are positive but very modest. Transactions fees associated with a merger would likely outweigh the benefits gained. This result is reassuring, as it would be surprising if our analysis suggested that identical "average" firms should merge solely to

Table III
Financial Effects of Merging Identical Firms

This table shows the financial effects of merging two identical firms with base-case parameters when the correlation of cash flows $\rho = 0.20$. It is assumed that the firms are optimally leveraged both before and after merging. The separate firms have annual standard deviation $\sigma_1 = \sigma_2 = 22\%$. The annualized standard deviation of the merged firm is $\sigma_M = 17.04\%$. For all firms, the tax rate $\tau = 20\%$, the default cost fraction $\alpha = 23\%$, and time horizon $T = 5$ years. The annual risk-free rate is $r = 5\%$, resulting in a capitalization factor of $Z = 4.62$.

Variables	Symbols	Sum of Separate Firm Values	Merged Firm Value	Change
Value of unlevered firm	V_0	160.09	160.01	−0.09
Optimal zero-coupon bond principal	P^*	114.27	117.42	3.15
Value of optimal debt	D_0^*	84.47	89.40	4.94
Value of optimal equity	E_0^*	78.47	73.74	−4.73
Optimal levered firm value	$v_0^* = D_0^* + E_0^*$	162.94	163.15	0.21
Tax savings of leverage (PV)	TS	4.63	4.39	−0.25 ΔTS
Expected default costs (PV)	DC	1.79	1.25	−0.54 ΔDC
Net leverage benefit	$TS - DC$	2.85	3.14	0.30

Ratios	Symbols	Separate Firms	Merged
Optimal leverage ratio	D_0^*/v_0^*	51.84%	54.80%
Annual yield spread of debt	$(P^*/D_0^*)^{1/T} - 1 - r$	1.23%	0.60%
Recovery rate	R	49.29%	56.48%

| Summary of Benefits			
Source	Symbols	Values	Effect
Change in unlevered firm value	ΔV_0	−0.09	*LL* Effect
Benefit to leverage	$\Delta TS - \Delta DC$	0.30	Leverage Effect
Net benefit of merger Δ	$\Delta V_0 + \Delta TS - \Delta DC$	0.21	Total Benefits
	$Z\Delta/(V_{01} + V_{02})$	0.60%	Measure 1
	$Z\Delta/v_2^*$	1.18%	Measure 2
	$Z\Delta/E_2^*$	2.45%	Measure 3

realize financial synergies. However, as we diverge from the base-case scenario and symmetric activities, situations arise in which financial synergies can be substantial, especially when synergies are negative. This provides a rationale for many aspects of structured finance, as we see in Section VI below.

B. Comparative Statics: The Symmetric Case

Figure 3A shows how financial synergies change in the base case, as the correlation of cash flows varies from zero to one. Higher correlation reduces merger benefits as diversification is less pronounced, and for correlations between 0.80 and 1 become slightly negative.[28] Financial synergies are zero when cash flows are perfectly correlated.

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[28] Benefits are negative for high correlations because $\sigma_1 = \sigma_2 = 22\% > 21.5\% = \sigma_L$ in the base case. For a more complete analysis, see Proposition 2 in Section V.C.

Figure 3B decomposes the financial synergies of Measure 1 in Figure 3A into two components: the loss of separate limited liability (the *LL* effect), and the net benefits of financial structure (the leverage effect). As expected, the *LL* effect is negative for all $\rho < 1$. The leverage effect is positive for $\rho < 1$ and outweighs the *LL* effect for $\rho < 0.80$.

(A)
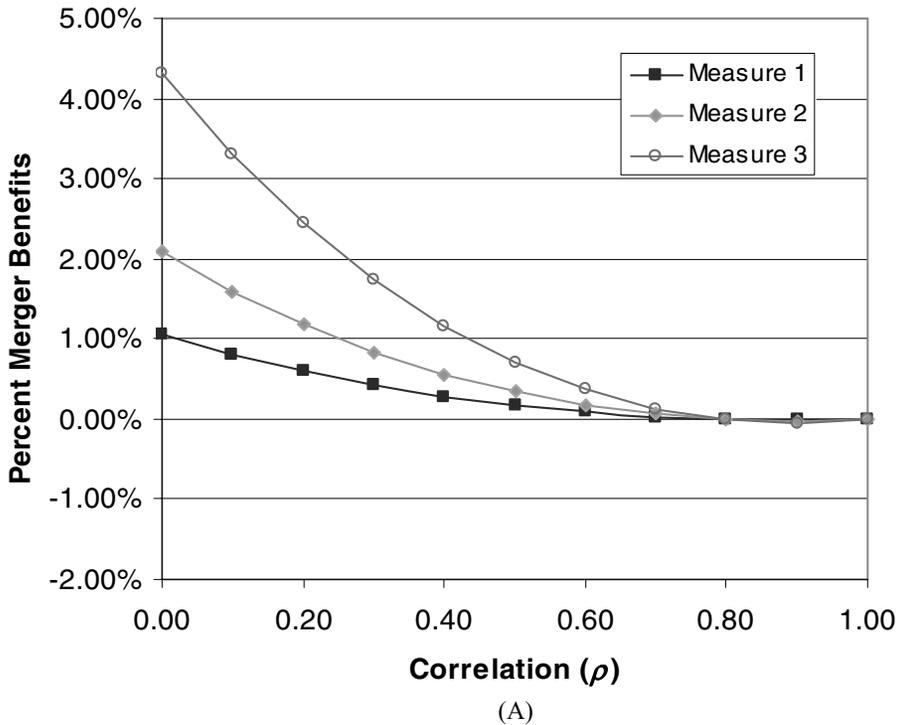
Figure 3. (A) Merger benefits as a function of correlation. The lines plot different measures of the value of merging two identical base-case firms as a function of the correlation between their cash flows. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, default costs are $\alpha = 23\%$, and the annualized volatility of both firms is 22%. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Measure 2 is capitalized merger benefits divided by the optimally levered target firm's total value. Measure 3 is capitalized merger benefits divided by the optimally levered target firm's equity value. **(B) Decomposition of merger benefits in the base case.** The lines plot the leverage effect, the loss of separate limited liability (*LL*) effect, and their combined total effect (Measure 1) from merging identical base-case firms with volatility $\sigma = 22\%$ as a function of the correlation between their cash flows. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, and default costs are $\alpha = 23\%$. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. **(C) Decomposition of merger benefits when volatility $\sigma = 24\%$.** The lines plot the leverage effect, the loss of separate limited liability (*LL*) effect, and their combined total effect (Measure 1) from merging identical firms with annualized volatility $\sigma = 24\%$ as a function of the correlation between their cash flows. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, and default costs are $\alpha = 23\%$. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values.
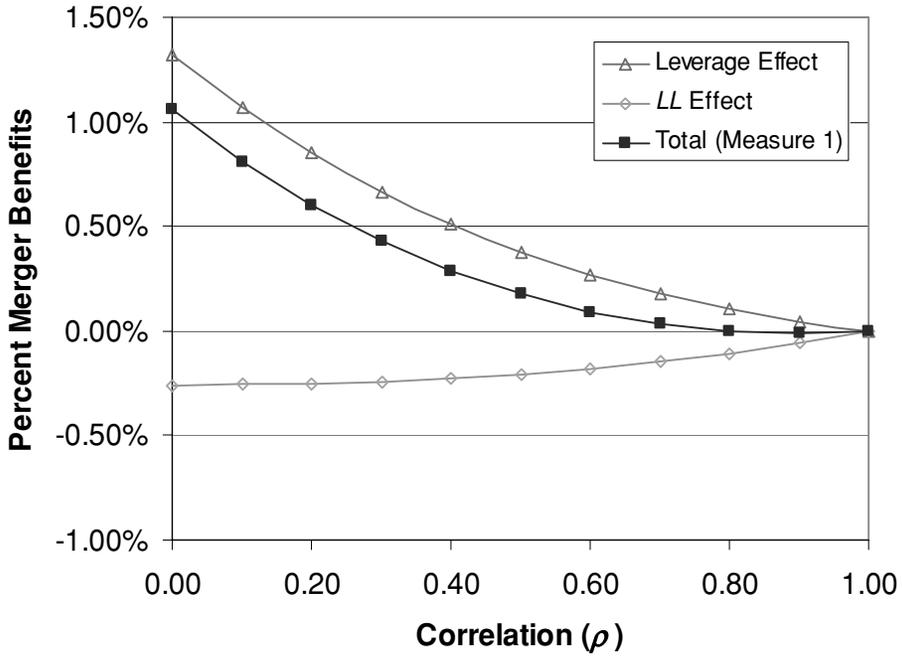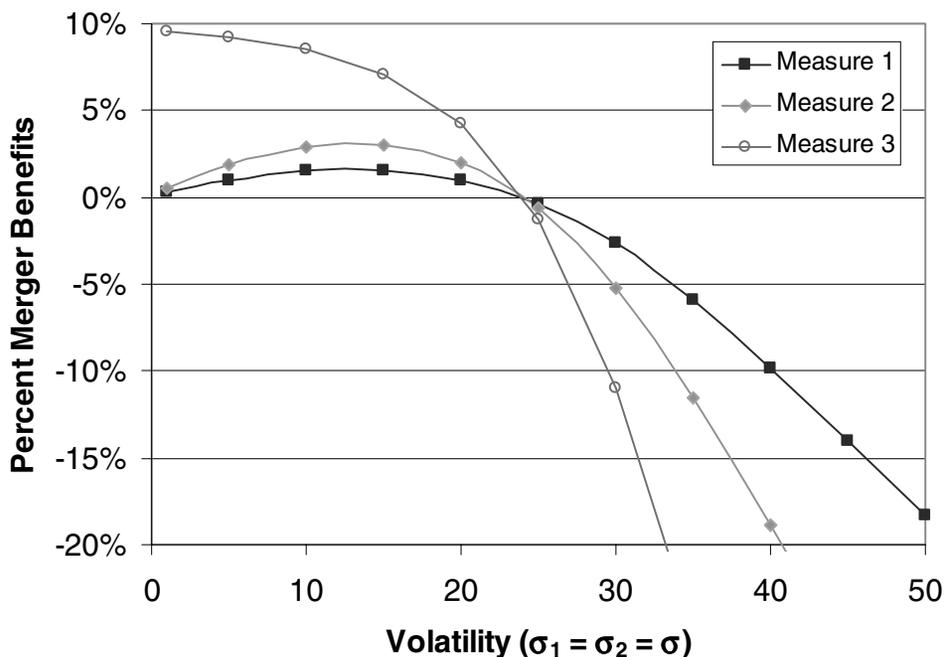
(B)
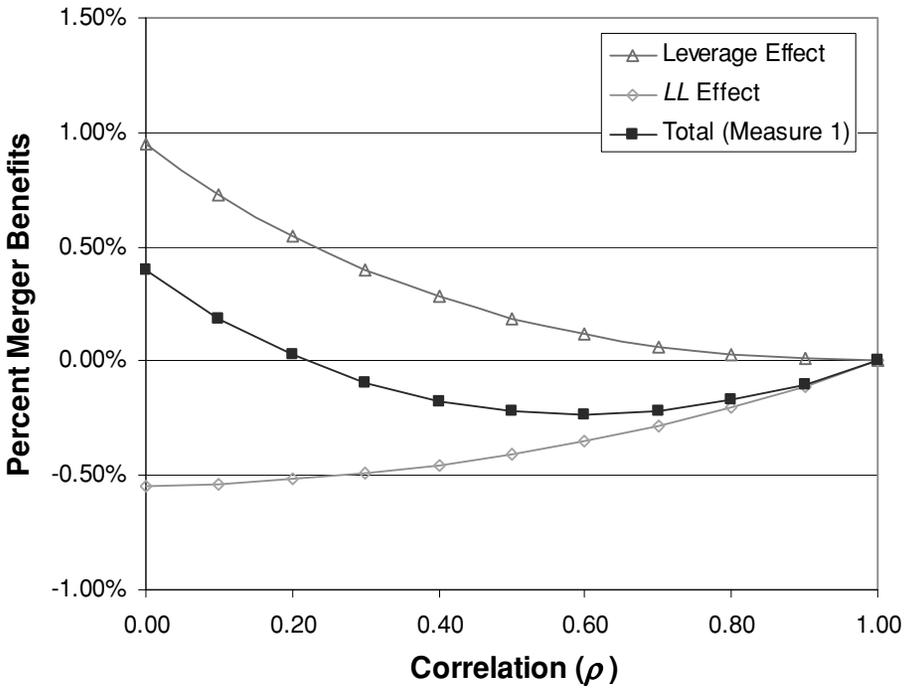
(C)

Figure 3—*Continued*

Figure 4. Merger benefits as a function of volatility. The lines plot three different measures of merger benefits as a function of the annualized volatility of identical firms. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, default costs are $\alpha = 23\%$, and the correlation between cash flows is 0.20. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Measure 2 is capitalized merger benefits divided by the optimally levered target firm's total value. Measure 3 is capitalized merger benefits divided by the optimally levered target firm's equity value.

Figure 3C modifies the parameters of the base case in one way: Volatilities of the (identical) firms are now 24% rather than 22%. Because the likelihood of negative cash flows is larger, the *LL* effect is more pronounced. The leverage effect is somewhat smaller but remains positive. Merger benefits are positive only for low levels of correlation.

Figure 4 demonstrates the effect of changing (joint) volatility on merger benefits. With the correlation fixed at $\rho = 0.20$, benefits decline and become negative at high levels of volatility. While quite low volatilities lead to greater Measure 1 and Measure 2 benefits compared to the base case, these benefits decline as volatilities approach zero (and optimal leverage for both separate and merged firms approaches 100%). Measure 3 benefits continue to increase as volatilities approach zero because equity values also approach zero as leverage increases toward 100%.

In sum, mergers of similar firms tend to have greater financial synergies when the correlation of cash flows is low and volatilities are somewhat lower than the base case. Financial synergies of a merger also increase when default

costs α rise above the base-case level.[29] When $\alpha = 75\%$, merger synergies are 2.5 times as large as when $\alpha = 23\%$. Thus, ceteris paribus, firms with high default costs realize greater financial synergies from a merger, as diversification reduces the risks of incurring such costs.

Does target size matter? Figure 5A plots the three measures of financial synergies as a function of the size of the target firm for base-case parameters. The value of the target's operations (X_{02}) varies from 1% to 100% of the size of the acquirer's operations (X_{01}). Measure 1 benefits are monotonically increasing in target firm size, with the benefits of the merger shifting from negative to positive.[30] However, as a proportion of the value of the target firm (Measure 2) or the value of the target firm's equity (Measure 3), there can be an optimal-sized target. An "ideal" target, that which yields the highest financial benefits as a percent of its value (or equity value), would range between about 40% and 80% of the acquirer's size. Figure 5B examines the size effect for identical (but for size) firms with different parameters. The firms $i = \{1, 2\}$ each have volatility $\sigma_i = 15\%$ and $\alpha_i = 75\%$, and their returns are uncorrelated. A merger is now beneficial for any target size. An "ideal" target, as reflected by Measure 3, is less than one-fifth of the acquirer's size.

How large can positive synergies realistically become when firms are symmetric? The parameters for Figure 5B are chosen with this question in mind. When the target firm is 10% of the acquirer's size, financial synergies represent an 8.2% value premium on the target firm's value (Measure 2), and 14.6% of the target firm's equity value (Measure 3). Andrade, Mitchell, and Stafford (2001) estimate the median target firm is about 11.7% the size of the acquiring firm, based on a sample of mergers over the period 1973 to 1998. They find that the 3-day abnormal return to acquired firms is 16% of equity value when the merger is announced (24% over a longer window that includes closing of the merger). Their estimate of the return to acquiring firms is slightly negative but insignificantly different from zero, consistent with the assumption underlying Measures 2 and 3 that all benefits accrue to the target firm. The Measure 3 return here of 14.4% suggests that financial synergies have the potential to explain a substantial proportion of realized merger gains in specialized situations.

Negative synergies can be even larger. If annual volatility is 40% for both firms, Measure 2 is -18.8%, indicating that the parent firm (Firm 1) could spin off Firm 2 and realize a premium of almost 20% of the value of the assets spun off. This advantage to separation primarily reflects the negative LL effect, but the leverage effect is negative as well.[31]

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[29] For low levels of α, the optimum capital structure may require 100% leverage. Except as noted, the (interior) leverage optimum is the global optimum in all examples below.

[30] The fact that the benefits can become negative when the target is very small also reflects the fact that with base-case parameters, $\sigma_1 = \sigma_2 = 22\% > 21.5\% = \sigma_L$. With the parameters in Figure 5B, $\sigma_1 = \sigma_2 = 15\% < 23.7\% = \sigma_L$ and benefits are positive for all target firm sizes: See Proposition 1 of Section V.C.

[31] In very high-risk cases, courts may disallow spinoffs if found to be undertaken principally to avoid future liabilities. When Firm 2 volatility is 40%, the risk-neutral probability of negative cash flow is about 7.7% at the end of 5 years. Assuming a 6% annual risk premium for operational cash flow value (implying an expected annual return on value X_0 of 11%), the actual probability of negative terminal cash flow is 3.0%.
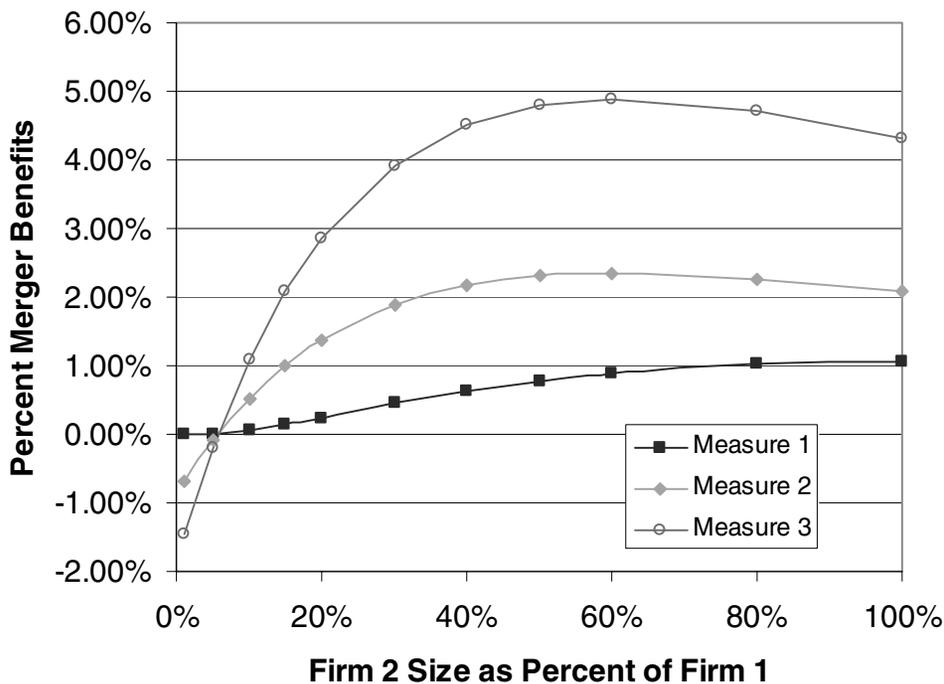
(A)

Figure 5. (A) The effect of relative size on merger benefits: The base case. The lines plot three different measures of the value of merging two firms of different asset value, as a function of the size of Firm 2 relative to Firm 1. The annualized volatility of each firm is 22%. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, default costs are $\alpha = 23\%$, and the correlation between cash flows is 0.20. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Measure 2 is capitalized merger benefits divided by the optimally levered target firm's total value. Measure 3 is capitalized merger benefits divided by the optimally levered target firm's equity value. **(B) The effect of relative size on merger benefits: An alternative case.** The lines plot three different measures of the value of merging two firms of different asset value as a function of the size of Firm 2 relative to Firm 1. The annualized volatility of each firm is 15%. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, default costs are $\alpha = 75\%$, and the correlation between cash flows is 0. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Measure 2 is capitalized merger benefits divided by the optimally levered target firm's total value. Measure 3 is capitalized merger benefits divided by the optimally levered target firm's equity value.

C. Comparative Statics: Asymmetric Volatility

We now vary the parameters of the target firm, but let Firm 1 retain the base-case parameters. Figure 6 examines the effect of changing the target firm's volatility σ_2, keeping the acquirer's annualized volatility fixed at $\sigma_1 = 22\%$. The measures of merger benefits are humped. Maximum financial synergies occur when Firm 2 has slightly lower volatility. Financial synergies are negative when the target's volatility is very different from the acquirer's volatility.
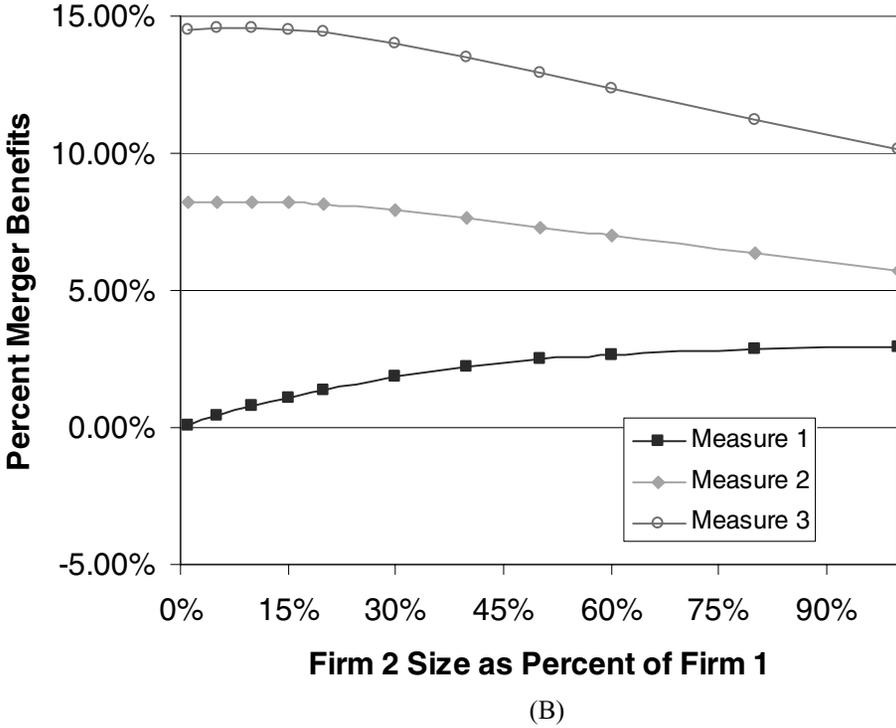
(B)

Figure 5—*Continued*

Figure 7 decomposes the financial synergies of Measure 1 in Figure 6 into the *LL* effect and the leverage effect. When Firm 2 volatility is high, mergers become costly, largely because of the negative *LL* effect. A spinoff is desirable if the activities are already merged. When Firm 2 volatility is very low, the negative leverage effect dominates. Spinoffs are desirable here because Firm 2 can benefit from high leverage, whereas the optimal leverage and tax savings of the combined firm are considerably less. Section VI.A below explores this rationale further in the context of asset securitization.

Figure 8 provides a framework for conceptualizing and extending the above results.[32] From Section III.B, recall that $v_\alpha^*(\sigma) \equiv v_0^*(100, \sigma, \alpha)$, and because the optimal firm value function $v_0^*(X_0, \sigma, \alpha)$ is proportional to X_0, it follows that

$$v_0^*(X_0, \sigma, \alpha) = (X_0/100)v_\alpha^*(\sigma). \tag{27}$$

With default costs α of 23% and other parameters as in Table I, the $v_\alpha^*(\sigma)$ curve in Figure 8 is identical to the upper $v_\alpha^*(\sigma)$ curve in Figure 2. Note that $v_\alpha^*(\sigma)$ is a strictly convex function of σ, and reaches a minimum at $\sigma = \sigma_L = 21.5\%$.

Consider two firms with identical default costs $\alpha_i = 23\%$ and cash flow values $X_{0i} = 100, i = \{1, 2\}$. Firm 1 has volatility $\sigma_1 = 16\%$, which results in an optimal

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[32] The author thanks Josef Zechner for suggesting this visualization.
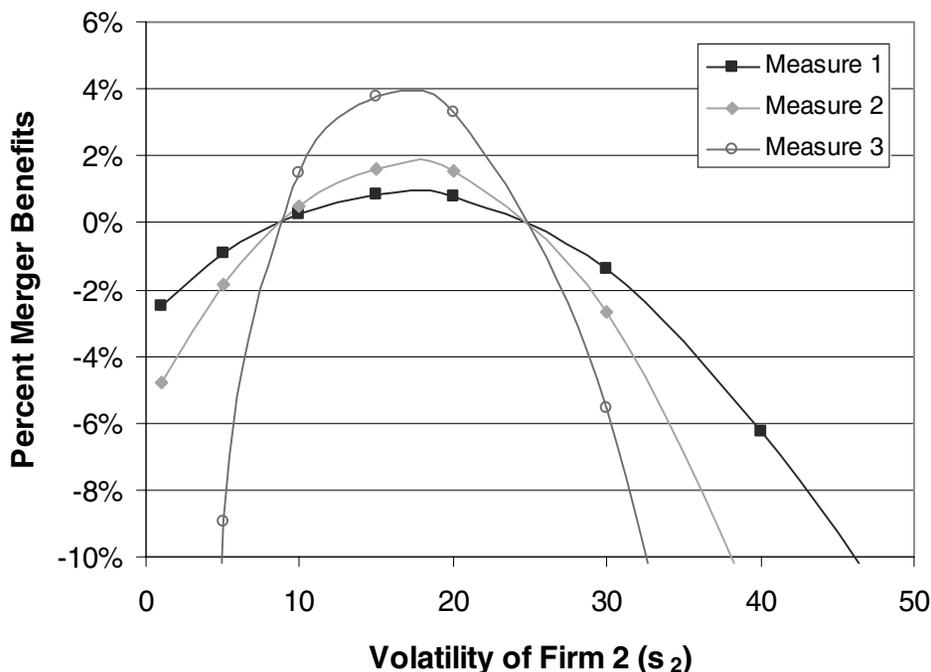
Figure 6. Merger benefits with asymmetric volatility. The lines plot three different measures of the value of merging two firms of equal asset value, as a function of the annualized volatility of Firm 2. The annualized volatility of Firm 1 is 22%. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, default costs are $\alpha = 75\%$, and the correlation between cash flows is 0. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Measure 2 is capitalized merger benefits divided by the optimally levered target firm's total value. Measure 3 is capitalized merger benefits divided by the optimally levered target firm's equity value.

value $v_\alpha^*(16) = 81.6$ (point X in Figure 8). Firm 2 has volatility $\sigma_2 = 40\%$ and value $v_\alpha^*(40) = 83.8$ (point Y). Let S denote the midpoint of the straight line (chord) joining X and Y.[33] The vertical coordinate of S is $v_S^* \equiv 0.5v_\alpha^*(16) + 0.5v_\alpha^*(40) = 82.7$. Observe that $2v_S^* = 165.4$ is the value of separation, that is, the sum of the values $v_\alpha^*(16)$ and $v_\alpha^*(40)$ of the separate optimally levered firms. The horizontal coordinate of S, $\sigma_S \equiv (\sigma_1 + \sigma_2)/2 = 28\%$, is the average of the two separate firms' volatilities. From point W in Figure 8 it can be seen that $v_\alpha^*(\sigma_S) = v_\alpha^*(28) = 81.6 < 82.7 = v_S^*$. This inequality follows from the strict convexity of $v_\alpha^*(\sigma)$ in σ.

We now show that a merger of the two firms is undesirable. This is first shown for the case in which the cash flows are perfectly correlated, and then for arbitrary correlation. The merged firm has cash flow value $X_{0M} = X_{01} + X_{02} = 200$ and volatility $\sigma_M(\rho)$ given by equation (26). From (27), the

[33] It is straightforward to extend the results to the case in which the two firms are of different sizes.
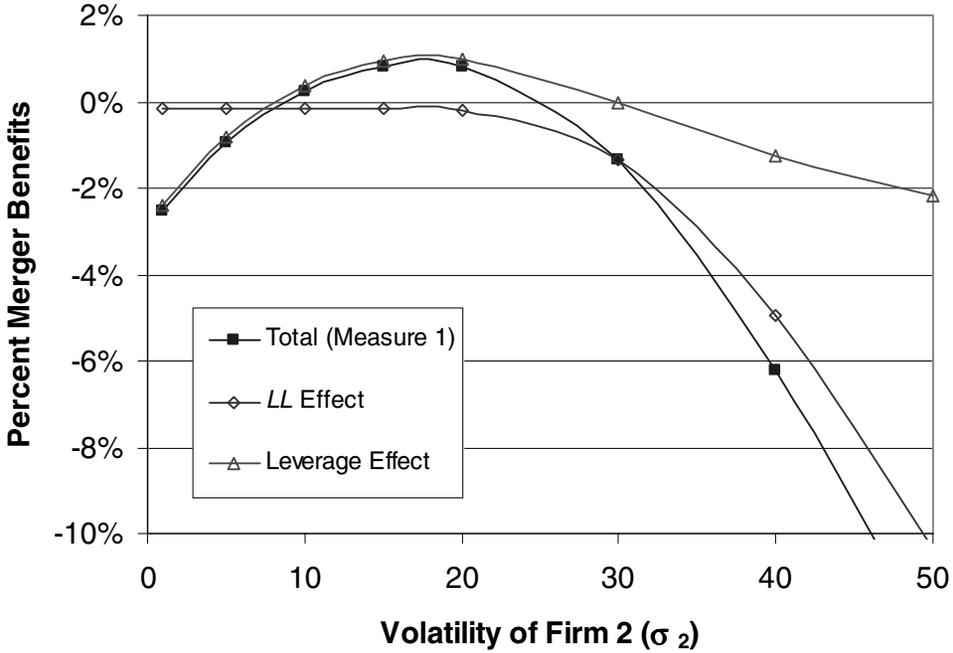
Figure 7. Decomposition of merger benefits with asymmetric volatility. The lines plot the loss of separate limited liability (*LL*) effect, the leverage effect, and their combined total effect (Measure 1) from merging two firms of equal asset value as a function of the annualized volatility of Firm 2. It is assumed that the debt maturity and time horizon is 5 years, the risk-free interest rate is 5%, the effective corporate tax rate is 20%, the default costs of both firms is 23%, the annualized volatility of Firm 1 is 22%, and the correlation of cash flows is 0.20. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, the effective corporate tax rate is $\tau = 20\%$, default costs are $\alpha = 23\%$, and the correlation between cash flows is 0.20. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Note that Measure 1 values here are identical to Measure 1 values in Figure 6.

optimal value of the merged firm is $v_0^*(X_{0M}, \sigma_M(\rho), \alpha) = (X_{0M}/100)v_\alpha^*(\sigma_M(\rho)) = 2v_\alpha^*(\sigma_M(\rho))$.

When $\rho = 1$, $\sigma_M(\rho) = (\sigma_1 + \sigma_2)/2 = \sigma_S$. The value of the merged firm is $2v_\alpha^*(\sigma_M(1)) = 2v_\alpha^*(\sigma_S) = 2(81.6) = 163.2$, or twice the height of the point W. But we showed above that the value of the separate firms was $2v_S^* = 2(82.7) = 165.4$, or twice the height of point S. Therefore, merger is undesirable if correlation $\rho = 1$.

What if the cash flows are less than perfectly correlated? As the correlation ρ decreases, $\sigma_M(\rho)$ falls and (half) the optimal value of the merged firm moves leftward from W along the $v_\alpha^*(\sigma)$ curve. A merger will be desirable only if $2v_\alpha^*(\sigma_M(\rho)) > 2v_S^*$, or $v_\alpha^*(\sigma_M(\rho)) > v_S^* = 82.7$. As can be seen from Point Z in Figure 8, $v_\alpha^*(\sigma_M(\rho)) > v_S^* = 82.7$ requires that $\sigma_M(\rho) < 5.2\%$. But the minimum
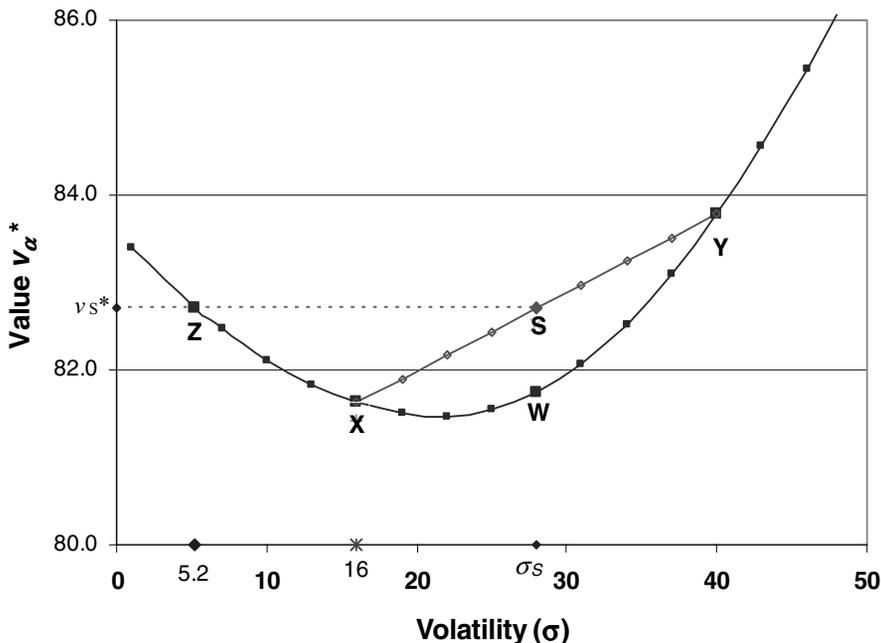
Figure 8. Analysis of merger benefits. The curved line plots the value of the optimally lever-aged firm as a function of the annualized volatility of cash flows, when default costs are $\alpha = 23\%$ and unlevered operational value $X_0 = 100$. The assumed debt maturity and time horizon are $T = 5$ years, the risk-free interest rate is $r = 5\%$, and the effective corporate tax rate is $\tau = 20\%$. The points $S, W, X, Y, Z,$ and v_S^* are defined in Section V.C of the paper.

possible volatility for $\sigma_M(\rho)$, which occurs when $\rho = -1$, is $|\sigma_1 - \sigma_2|/2 = |16 - 40|/2 = 12\%$. Thus, a merger is undesirable for any correlation in this particular case.[34]

We now derive more general conclusions about merger desirability. The separate firms may differ in size X_0 and volatility σ, but we assume that the separate and merged firms have identical default costs, tax rates, and horizons. Define the relative size of the firm i by $w_i \equiv X_{0i}/X_{0M}$. Recalling (19), it follows that $w_1 + w_2 = 1$. Without loss of generality, we can scale values so $X_{0M} = 100$. Thus, the optimally leveraged value of the merged firm is given by $v_\alpha^*(\sigma_M(\rho))$, with $\sigma_M(\rho)$ given by (26), and from (27), each separate firm has value $w_i v_\alpha^*(\sigma_i)$. The total value of the separate firms is $v_S^* \equiv w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2)$. A merger will be desirable if and only if $v_\alpha^*(\sigma_M(\rho)) > v_S^*$.

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[34] The example in Figure 8 has one other special property. The initial points X and Y are chosen to have equal leverage, as Figure 1 illustrates. Thus, mergers may be undesirable between firms whose initial leverage is the same when the firms are identical except for volatility. Presuming that mergers are always beneficial when separate firms have equal leverage is therefore incorrect. The author thanks the referee for clarifying this point.

The propositions below assume that cash flows are normally distributed, that the optimally levered firm value $v_\alpha^*(\sigma)$ is strictly convex in σ and reaches a minimum at $\sigma = \sigma_L$, where $0 < \sigma_L \le \infty$.[35] Proofs of all propositions are provided in Appendix B.

PROPOSITION 1: *A merger of firms with identical volatilities $\sigma_1 = \sigma_2 = \sigma_0 < \sigma_L$ will be desirable for all correlations $\rho < 1$.*

PROPOSITION 2: *A merger of firms with identical volatilities $\sigma_1 = \sigma_2 = \sigma_0 > \sigma_L$ will be undesirable for high correlations (in the case of perfect correlation, mergers will be weakly undesirable).*

More formally, Proposition 2 can be stated as follows: If $\sigma_1 = \sigma_2 = \sigma_0 > \sigma_L$, there exists an open interval $Q = (\rho^Q, 1)$ such that a merger will be undesirable when the correlation of cash flows, ρ, lies within Q.

Propositions 1 and 2 indicate that lower volatilities and a lower correlation favor mergers of firms with identical volatility. Proposition 2 explains that the negative merger values at high correlations observed in Figure 3A depend critically on whether σ_0 exceeds σ_L. The Propositions imply mergers are more likely to be desirable for all correlations when σ_L is large. From Figure 2, it can be observed that larger default costs lead to a higher σ_L, implying that high default costs favor mergers.

PROPOSITION 3: *A merger of firms with identical volatilities $\sigma_1 = \sigma_2 = \sigma_0$ will be undesirable for all correlations if and only if $v_\alpha^*(\sigma_0|w_1 - w_2|) < v_\alpha^*(\sigma_0)$.*

Since $v_\alpha^*(\sigma)$ is increasing in σ at σ_0 if and only if $\sigma_0 > \sigma_L$, and $\sigma_0|w_1 - w_2| < \sigma_0$, the inequality in Proposition 3 can hold only if $\sigma_0 > \sigma_L$. Thus, Proposition 3 does not contradict Proposition 1. The total value of the separate firms is $v_S^* \equiv w_1 v_\alpha^*(\sigma_0) + w_2 v_\alpha^*(\sigma_0) = v_\alpha^*(\sigma_0)$. Recall that $v_\alpha^*(\sigma_0|w_1 - w_2|)$ is the value of the merged firm when the separate firms' cash flows are perfectly negatively correlated ($\rho = -1$). Therefore, Proposition 3 can alternatively be stated: Mergers of firms will be undesirable for all correlations if and only if the value of the merged firm when $\rho = -1$ is less than the total value of the separate firms.

Mergers are unlikely to be desirable between high-volatility firms ($\sigma_0 > \sigma_L$) of unequal size (w_1 or w_2 close to one). This follows because, as $|w_1 - w_2| \to 1$, $\sigma_0 > \sigma_0 |w_1 - w_2| > \sigma_L$. Since $v_\alpha^*(\sigma)$ is increasing in σ when $\sigma > \sigma_L$, $v_\alpha^*(\sigma_0 |w_1 - w_2|) < v_\alpha^*(\sigma_0)$, and from Proposition 3 a merger will never be desirable. This result explains the negative merger benefits for low target firm size observed in Figure 5A, in which $\sigma_0 > \sigma_L$. Negative benefits are not observed in Figure 5B, where $\sigma_0 < \sigma_L$.

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[35] Tax rates and time horizons are also assumed equal across the separate and merged firms, although not necessarily at base-case levels. The lack of a closed-form expression for optimal leverage has precluded a proof of the convexity of $v_\alpha^*(\sigma)$. However, all examples considered over a wide range of parameters exhibit a strictly convex, U-shaped $v_\alpha^*(\sigma)$ schedule, with $0 < \sigma_L < \infty$ as in Figure 2.

PROPOSITION 4: *A merger of firms with differing volatilities will be undesirable for high correlations.*

More formally, Proposition 4 can be stated as follows: For any given volatilities $\sigma_1 \neq \sigma_2$, an interval $R = (\rho^R, 1)$ exists such that a merger will be undesirable when the correlation of cash flows, ρ, lies within R. In comparison with Proposition 1, Proposition 4 suggests that mergers of firms with different volatilities are less likely: Mergers will be undesirable for high correlations even though σ_1 and σ_2 may be lower than σ_L.

PROPOSITION 5: *A merger of firms with differing volatilities will be undesirable for all correlations ρ if and only if $v_\alpha^*(|w_1\sigma_1 - w_2\sigma_2|) < w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2)$.*

Proposition 5 is a generalization of Proposition 3, allowing for different firm volatilities. Again, the necessary and sufficient condition requires that the value of the merged firm when $\rho = -1$ is less than the total value of the separate firms. The example underlying Figure 8 satisfies the required inequality of Proposition 5 and a merger is undesirable for all correlations in that case.

Corollaries 1 and 2 below provide sufficient conditions for the inequality in Proposition 5 to hold or to be reversed, respectively.

COROLLARY 1: *If $|w_1\sigma_1 - w_2\sigma_2| > \sigma_L$, a merger of firms with differing volatilities is undesirable for all correlations.*

Thus, a firm with high volatility ($\sigma > \sigma_L$) is unlikely to desire a merger with a much smaller firm, particularly if that firm has low volatility.

COROLLARY 2: *If (i) $\sigma_1, \sigma_2 < \sigma_L$, and (ii) $|w_1\sigma_1 - w_2\sigma_2| < \mathrm{Min}[\sigma_1, \sigma_2]$, a merger of firms with differing volatilities is desirable if correlation is low.*

More formally, Corollary 2 states that if conditions (i) and (ii) hold, then there exists an interval $Z = [-1, \rho^Z)$ such that a merger will be desirable for firms when $\rho \; \varepsilon \; Z$. Accordingly, mergers of firms are more likely to be desirable if firms have low correlation, have low volatilities ($\sigma < \sigma_L$), and have similar size-weighted volatilities.

Collectively, the propositions confirm that mergers do not always provide positive financial synergies. The sole case in which mergers are beneficial for any correlation is very special: The volatilities of the merging firms must be identical and moderate ($< \sigma_L$). When volatilities differ, or are identical but large ($> \sigma_L$), separation is desirable at high correlations, and may be preferred at any correlation.

D. A Specific Counterexample to Lewellen

Lewellen's (1971) contention that financial synergies are always positive does not explicitly contemplate negative future cash flows and the resulting *LL*

effect. Absent negative future cash flows, might Lewellen's conjecture, that the leverage effect is always positive, be correct? While the previous results suggest not, here we provide a specific counterexample.

When cash flow volatilities are 15% or less, the probability of a negative cash flow given other base-case parameters is less than 0.07% and the *LL* effect is negligible. Consider an example with Firm 1 volatility equal to 15%, Firm 2 volatility equal to 5%, and a correlation of 0.70. Then the financial benefits to a merger are negative: $\Delta = -0.073$. Decomposition of benefits gives $\Delta V_0 = LL = -0.000$ (as expected), tax savings $\Delta TS = -0.153$, and default cost change $\Delta DC = -0.081$. Thus, the leverage effect ($\Delta TS - \Delta DC$) equals -0.073, and it is responsible for the significantly negative financial synergies.

Our examples indicate that, as a general rule of thumb (but not an exact guide), mergers are beneficial (costly) if total debt value increases (decreases) after a merger.[36] Thus, our predictions of situations in which mergers will be beneficial largely coincide with situations in which the optimal post-merger debt exceeds the total optimal debt of the separate firms. This is true in the counterexample above: Optimal total debt value when firms are separate is 112.4 versus 110.4 when merged.

E. Hedging and Mergers

If activities' cash flows can be hedged, both absolute and relative volatilities can be altered. Figure 2 indicates that firms can increase value by hedging if initial risk $\sigma_0 < \sigma_L$. However, merger benefits may be reduced if volatilities fall too far, as seen in Figure 4. Relative risk as well as absolute risk determines whether mergers are desirable. Consider the example in Figure 6. If Firm 2 can reduce its volatility from 30% to 20% by hedging, an undesirable merger becomes desirable.[37] If Firm 2 can reduce its volatility to 5% from 20% by further hedging, a merger no longer is desirable. These observations, coupled with the results in Proposition 3 and Corollary 2, suggest that a merger is more likely to be beneficial when size-weighted volatilities of the two firms are similar. But a full examination of the interaction between hedging and merger benefits is beyond the scope of this paper.

F. Comparative Statics: Asymmetric Default Costs

Figure 9 charts the value of a merger between two firms that are identical but for default costs. The default costs of Firm 1 remain as in the base case ($\alpha_1 = 23\%$), while default costs of Firm 2 (α_2) vary. The merged firm is presumed to have a default cost equal to the size-weighted default costs of the separate

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[36] Kim and McConnell (1977), Cook and Martin (1991), and Ghosh and Jain (2000) find that leverage increases after mergers.

[37] For the parallel between hedging and our analysis to be precise, it must be the case that hedges are fairly priced, that the distribution of cash flows will continue to be normally distributed (though with lower volatility), and that hedging will not affect the correlation between the activities.
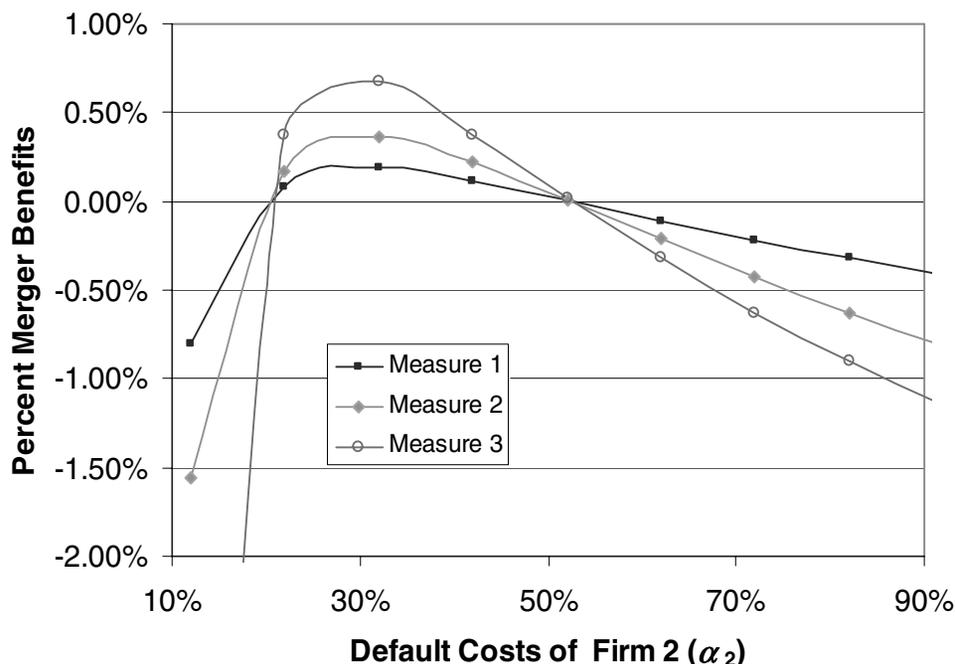
Figure 9. The lines plot three different measures of the value of merging two base-case firms as a function of the default costs of Firm 2. It is assumed that the debt maturity and time horizon are 5 years, the risk-free interest rate is 5%, the effective corporate tax rate is 20%, the default costs of Firm 1 are 23%, the annualized volatility of both firms is 22%, and the correlation of cash flows is 0.50. The default cost of the combined firm is the operational value-weighted average of separate firm default costs. Measure 1 is capitalized merger benefits divided by the sum of the separate firms' unlevered values. Measure 2 is capitalized merger benefits divided by the optimally levered target firm's total value. Measure 3 is capitalized merger benefits divided by the optimally levered target firm's equity value.

firms. The benefits function is humped and becomes negative when the default costs of Firm 2 are very high or very low.[38] Like differences in volatilities, large differences in default costs favor separation.

VI. Spinoffs and Structured Finance

Spinoffs are the reverse of mergers: Two previously combined activities are separated into distinct corporations. These corporations then leverage themselves individually. "Structured finance" is another means to separate an activity from the originating or sponsoring organization. Asset securitization and project finance are both examples of structured finance. Assets generating cash flows are placed in a bankruptcy-remote SPE formed specifically to hold those

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[38] We use correlation $\rho = 0.50$ in Figure 9 to illustrate that Figure 2's benefits can be negative both for low and for high default costs (α_2). With correlation $\rho = 0.20$, benefits shift upward; they are still negative for low α_2, but are positive as α_2 exceeds 0.23.

assets.[39] An SPE raises funds to compensate the sponsor by selling securities that are collateralized by the cash flows of the transferred assets. The SPE typically issues multiple tranches of debt with differing seniority, including a residual tranche (often termed the "equity" tranche). A bankruptcy-remote SPE with limited-recourse financing has the key features of a separate firm from our analytical perspective.[40]

Structured finance has boomed in recent years. Financial and industrial firms have transferred trillions of dollars of mortgages, commercial loans, accounts receivables, power plants, motorway rights, and other cash flow sources to special purpose entities. This raises the question: How does structured finance create value? Advocates claim that structured finance benefits activities both with very low-risk cash flows (e.g., mortgages) and with very high-risk cash flows (e.g., some major investment projects). It is sometimes vaguely argued that spinoffs and structured finance "unlock asset value." Little formal analysis has accompanied such claims.[41]

Securitization has also been justified by the assertion that separate, low-volatility assets can attract lower cost financing. This is not convincing a priori, as the assets remaining with the sponsor will have higher volatility and higher financing cost. Gorton and Souleles (2005) argue that SPEs exist to avoid bankruptcy costs. Other reasons cited for structured finance, which are not explored here, include the issuance of multiple debt classes (tranching) to specific clienteles, relaxation of capital constraints (for financial institutions), and reduced informational asymmetry and agency costs.[42]

In the subsections below, the trade-off model developed in previous sections provides a straightforward rationale for structured finance based on purely financial synergies. The sources of these synergies can be clearly identified,

[39] We focus on transfers of assets from sponsor to SPE that receive sale accounting treatment under FAS 140. To qualify for sale accounting treatment, it must be shown (i) that there has been a "true sale" of assets to the SPE, and (ii) that on the bankruptcy of the sponsor, its creditors have no recourse to the assets of the SPE. A true sale precludes explicit or implicit credit guarantees to the SPE, and therefore helps to assure that the sponsor is bankruptcy remote from the SPE. No recourse to SPE assets helps to assure that the SPE is bankruptcy remote from the sponsor. If the transfer receives sale accounting treatment and the SPE is a qualifying SPE (see Gorton and Souleles (2005)), then the SPE's assets and liabilities may be excluded from the sponsor's balance sheet.

[40] Our analysis presumes that the sponsoring firm does not guarantee the SPE debt. Explicit guarantees would abrogate a "true sale" of assets to the SPE, entailing the negative consequences noted in footnote 39. Gorton and Souleles (2005) argue that nonetheless there may be implicit guarantees between the SPE and its sponsor.

[41] An exception is that of Chemmanur and John (1996), who examine several of the issues we consider here. However, their focus is not on purely financial synergies, but rather on operational synergies resulting from differential managerial abilities across projects and different benefits of control. Flannery et al. (1993) consider operational synergies related to underinvestment.

[42] See, for example, DeMarzo (2005), Esty (2003), Greenbaum and Thakor (1987), Lockwood, Rutherford, and Herrera (1996), Oldfield (1997), and Rosenthal and Ocampo (1988). Oldfield (1997) and DeMarzo (2005) attribute tranching to price discrimination and information asymmetries, respectively. Note that the benefits discussed in these papers may be complementary to the benefits we examine.

giving meaning to the vague claims that structured finance can unlock asset value. The theory explains the use of these techniques for both low-risk and high-risk assets.

A. Asset Securitization

We now develop an example of asset securitization. Prior to securitization, an originating bank (or "sponsor") has two sources of cash flows: mortgages (or other types of loans) that have low cash flow risk, and residual banking activities that have greater risk. The low-risk assets are transferred to a bankruptcy-remote SPE that issues securities collateralized by the assets' cash flows. In return, the sponsor receives the proceeds from the SPE's issuance of securities. While the typical SPE may issue multiple tranches of debt, we assume a single (senior) class of debt, plus equity.[43] The originating bank is presumed to retain no equity in the SPE.[44]

Table IV outlines the parameters and results for our example. The pre-securitization firm is viewed as the merged firm, having parameters equal to those assumed in Table I. The securitized assets represent 25% of the firm's operational value prior to securitization, and are assumed to have low annual cash flow volatility (4% of cash flow value), similar default costs ($\alpha_2 = 23\%$, later reduced), and a correlation with the sponsor's other cash flows of 0.50. After securitization, Firm 2 represents the SPE that receives the cash flows of the securitized assets, and Firm 1 represents the sponsoring firm after securitization which receives the cash flows of the residual banking activities. The parameters above imply that these residual cash flows have an annual volatility of $\sigma_2 = 28.6\%$.

Securitization generates an almost 14% increase in value relative to the value of the assets secured (the negative of Measure 2). This is a substantial value increase from a purely financial change. The optimal leverage for the securitized assets rises to 83%. (This is equal to the proportion of the SPE's "senior tranche" debt). The debt has minimal risk of default and a very low credit spread, consistent with the observed high credit ratings on the senior tranche of securitized debt. Securitization leads to a substantial increase (15%) in the total amount of debt that is issued, compared with the pre-securitization firm.

These results justify some of the informal arguments for asset securitization cited above. Securitization permits the use of very high leverage on the subset of

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[43] Some asset securitizations offer a single-class participation only, and are termed "pass-through" structures. More complex SPEs issue multiple securities (we consider one debt tranche and one equity tranche) and are termed "pay-through" structures. See Oldfield (2000). It is not uncommon for senior debt tranches to include third-party credit enhancements, but we do not consider the effects of such enhancements here.

[44] Originating banks typically retain no equity in current asset securitizations. We assume that the funds received from an SPE financing are entirely distributed to the originating bank's debt holders and shareholders through repurchases and/or dividends. After distributions and restructuring, the bank after securitization is optimally levered. If funds are not paid out to security holders, the problem becomes one of optimal investment policy, which we do not address here.

Table IV

Asset Securitization Example

The table details an optimally leveraged firm before and after it has securitized a fraction (25%) of low-risk assets. Before securitization, the firm consists of two (merged) activities: the assets subsequently securitized, whose parameters are listed in the column "SPE," and other assets, whose parameters are listed in the column "Firm." The correlation of the activities' cash flows is assumed to be 0.50. Parameters associated with the combination of these activities are listed in the column "Firm Before Securitization." Financial synergies are given by Δ. Since these synergies are negative, separation ("securitization") is desirable. The benefits to securitization can be decomposed into the increase in value due to separate limited liability shelters $(-\Delta V_0)$, plus the increase in total tax savings due to optimal leverage $(-\Delta TS)$ less the increase in expected default costs $(-\Delta DC)$.

	Symbols	Firm Before Securitization	After Securitization SPE	After Securitization Firm	Change
Value of operational cash flows	X_0	100	25	75	0
Value of unlevered firm	V_0	80.05	20.00	60.26	$0.21 - \Delta V_0$
Pre-tax value of limited firm liability	L_0	0.06	0.000	0.33	0.27
Annual volatility (as % of X_0)	σ	22.0%	4.0%	28.6%	
Optimal zero-coupon bond principal	P^*	57.13	21.96	45.74	10.56
Value of optimal debt	D_0^*	42.23	17.18	31.85	6.79
Value of equity	E_0^*	39.23	3.54	29.51	−6.18
Optimal levered firm value	$v_0^* = D_0^* + E_0^*$	81.47	20.72	61.35	$0.61 - \Delta$
Optimal leverage ratio	D_0^*/v_0^*	51.8%	82.9%	51.9%	
Annual yield spread of debt (%)	$(P^*/D_0^*)^{1/T} - 1 - r$	1.23%	0.04%	2.51%	
Recovery rate	R	49.3%	70.6%	41.7%	
Tax savings of leverage (PV)	TS	2.32	0.75	2.11	$0.54 - \Delta TS$
Expected default costs (PV)	DC	0.89	0.03	1.01	$-0.14 - \Delta DC$
Summary of Benefits to Asset Securitization					
(Negative) merger benefits	$-\Delta$	0.61			
(Negative) measure 1	$-Z\Delta/(V_{01} + V_{02})$	3.51%			
(Negative) measure 2	$-Z\Delta/v_{02}^*$	13.57%			

low-risk assets. About two-thirds of the benefits come from the leverage effect. The remaining benefits come from the *LL* effect, reflecting the increased value of the separate limited liability shelters after securitization.

Securitization is even more desirable when the originating firm is riskier. If the volatility of the sponsor before securitization is 25% rather than 22%, benefits rise from 14% to 20% of secured asset value. Consistent with the model's prediction that riskier sponsors benefit more from securitization, Gorton and Souleles (2005) present preliminary empirical results suggesting that the

riskiest banks (with single B debt ratings) are the most likely to securitize credit card debt.

Gorton and Souleles (2005) further argue that the primary benefit of asset securitization is the low bankruptcy (default) costs associated with the SPE structure. This conjecture can be directly addressed with our model. For example, reducing the SPE's default costs to 5% from 23% raises its optimal leverage from 83% to 88%, but the advantage of securitization rises by only a modest amount, from 13.6% to 14.4%.[45] Thus, the importance of lower default costs seems relatively small in the example considered.

B. Separate Financing of High-Risk Projects

Large and risky investment projects can be internally financed by a firm, or financed separately as a spinoff or through project finance. The formation of a separate firm or SPE ensures that debt financing for the project has recourse only to the project's cash flows and assets.[46] Commonly cited justifications for separation include greater total financing ability, cheaper financing for assets that remain in the firm, and preserving core firm assets from bankruptcy risk. Berkovitch and Kim (1990) and Esty (2003) also mention the possible benefits of project finance in reducing the agency costs of underinvestment in large and risky projects. These benefits would be incremental to the purely financial synergies that we consider here.

Our model provides insight into the decision to use project finance. From Figure 6, it can be seen that separate financing benefits (the negative of merger benefits) increase as the annual volatility of Firm 2 ("the investment project") rises above the volatility of Firm 1 ("the parent"). If the new project is half the size of the parent, and has annual volatility of 40%, the benefits of separate financing to the parent firm represent 13% of project value. Approximately 80% of the value increase comes from the additional limited liability shelter provided by separate financing, with the remainder coming from the leverage effect. Compared with internal financing, the use of separate financing allows greater additional debt financing (56% of project value versus 52% if the project is financed within the parent firm).

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[45] The optimal leverage of 87.7% is a global optimum within leverage ranges of 0–99.7%. However, by issuing an enormous amount of highly risky debt (driving leverage to 100% and virtually guaranteeing default), the value can be made higher since the interest deduction will eliminate taxes for virtually any level of X (i.e., X^Z is very large), and default costs are modest. Since such risky debt is never observed, and the interest deduction would almost surely be disallowed, we ignore this corner solution.

[46] Esty and Christov (2002) and Esty (2003) provide details on project financing. Our approach assumes that the parent firm retains no equity risk in the project, which is often the case with spinoffs but unusual with project finance. The analysis is more complex if the originating firm retains equity in the SPE: The equity of the originating firm becomes an option (due to the nature of equity) on a value that includes an option on another asset (SPE equity). We are unaware of closed-form solutions to this problem, although preliminary numerical analysis suggests that the nature of our results will not be significantly changed.

As project volatility rises further, the gains to project finance accelerate. A project with 50% annualized volatility realizes benefits of 27% when separately financed. In cases in which project finance is used for low-risk activities (e.g., cash flows from toll highway revenues), the analysis of Section VI.A pertains.

VII. Who Realizes the Financial Benefits of Mergers and Spinoffs?

If equity holders are to capture the entire financial synergies derived above, bondholders must not participate in gains or losses. This poses no problem for startup firms, for which there is no initial debt. The entrepreneur decides whether to incorporate activities jointly or separately, and subsequently levers the firm(s) optimally by issuing debt at a price that fairly reflects the risks. If the separate firms have extant debt that is callable at par, or otherwise can be retired at a price that reflects pre-merger risks, again the separate firms' bondholders will not participate in windfall gains from a merger.

As Higgins and Schall (1975), Stapleton (1982), and Shastri (1990) suggest, potential problems arise when the extant debts of the separate firms are non-callable and are assumed by the post-merger firm.[47] Transfer of value to bondholders can lead to value-enhancing mergers being rejected by shareholders. This inefficiency is similar to the "debt overhang" problem discussed in Myers (1977), which also can prevent a value-improving investment decision by the firm. Although Higgins and Schall and others do not have explicit models of optimal capital structure, we can show that their concerns are indeed warranted.

In the base case of Table III, the value of the outstanding bonds at the time of merger would rise by 3.08 after the merger, reflecting their lower risk. This increase in value is far greater than the benefits of $\Delta = 0.21$ resulting from the merger itself. Shareholders will suffer a substantial loss of value from merger in this case. In the more favorable merger environment, when firms have 15% volatility, merger benefits of $\Delta = 0.93$ will be substantially reduced (but not eliminated) to shareholders by an increase in extant bond value of 0.60.[48]

The results underscore an important reason for firms to issue callable debt. Such debt enables corporations to reduce or eliminate windfall gains to bondholders. Windfall gains can also be reduced when the firm uses short-term debt. Subsequent debt rollovers will carry an interest rate that reflects post-merger risks and overpayment of interest to extant bondholders will occur only for a

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[47] These authors consider value transfers to bondholders when the merged firm assumes the debt of the separate firms but does not issue or retire additional debt. They do not explicitly consider optimal financial structure.

[48] Billett, King, and Maurer (2004) report significant excess returns to target bondholders and negative excess returns to acquirers' bonds at the time of merger announcements. Total returns to bondholders as a group are insignificantly different from zero. They also find positive correlations between stock and bond returns, suggesting either that there are minimal transfers between stockholders and bondholders, or that synergies in mergers overshadow any wealth transfers that might occur. Mansi and Reeb (2002), while not studying mergers explicitly, conclude that the "conglomerate discount" disappears when the increased market (relative to book) value of bonds, presumably resulting from lower risk due to diversification, is included in total firm value.

short time. In the base case with debt with maturity of 2 years (rather than 5 years), extant debt value increases only by 0.56 compared with 3.08.

Spinoffs or structured finance can cause an opposite problem. Extant bond values may fall after separation, with a consequent transfer of value to shareholders.[49] Short-term debt can alleviate the extent of value transfer. Covenants can potentially enable bondholders to extract compensation, for example, through an option to redeem (or "put") the bonds at face value in the event of asset divestment. When such protective covenants do not exist, value-diminishing spinoffs or structured finance may nonetheless be profitable for shareholders.

VIII. Conclusion

We analyze the financial synergies that result from combining versus separating multiple activities. To focus on financial effects alone, we assume that the operational cash flows of activities are additive and therefore nonsynergistic. A simple two-period model generates the optimal capital structure for separate or combined activities. When cash flows are jointly normally distributed, we derive closed-form expressions for debt and equity values. Numerical optimization determines optimal leverage.

Value when activities are separated and optimally levered is compared to value when activities are merged in a single optimally levered firm. The complex interaction of activities' cash flow risks, correlation, tax rates, default costs, and relative size determines the value-maximizing scope of the firm. Despite this complexity, some general principles can be ascertained.

There are two sources of purely financial benefits (or costs) from a merger. The first source is the loss of separate firm limited liability, which we term the *LL* effect. This effect is unrelated to leverage and it always favors separation if activity cash flows can be negative. The *LL* effect is more significant when the activities have high cash flow volatilities and low correlation.

The second source of financial synergies is the leverage effect. This effect reflects the difference in leverage benefits when activities are jointly versus separately incorporated. The leverage effect can be decomposed into the change in tax savings from leverage less the change in default costs, when comparing merger with separation. It can have either sign, contrary to Lewellen's (1971) contention that it always favors mergers. Combining activities into a single firm offers the advantage of risk-reduction from diversification. But keeping activities separate offers the advantage of optimizing the separate capital structures. As a general (but not exact) rule, the leverage effect is positive (negative) when the optimally levered merged firm has greater (lesser) debt value than the sum of the separate optimally levered debt values.

The theory generates several testable hypotheses. Ceteris paribus, financial synergies from mergers are more likely to be positive when correlations are low and volatilities are low and similar. Jointly high default costs also make mergers

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[49] The 1993 Marriott spinoff is a classic example. See Parrino (1997). In the example in Table IV, extant bondholders lose almost 4% of value (even assuming that the amount of principal retired is bought back at par value).

more desirable, reflecting an increased value from risk-reduction through diversification. Substantial differences in activities' volatilities or default costs favor separation.

Our results have implications for empirical work that examines the sources of merger gains or predicts merger activity. Those aspects of firms' cash flows noted above, which can create substantial financial synergies, should be included as possible explanatory variables. The results in Section VII also suggest that the mergers can have importantly different impacts on debt and equity values.

How large are purely financial synergies? In many cases, examples calibrated to empirical data suggest that financial synergies by themselves are insufficient to justify mergers. But they can become important in specialized circumstances, as noted in Section V.B. The argument that purely financial synergies can justify separation is much stronger. When the volatilities are jointly high or are quite different, the benefits to separation can be significant. The theory provides a strong rationale for the existence of structured finance, including asset securitization and separate large project financing.

Appendix A: Values with Normally Distributed Returns

We now develop formulas for asset values and recovery rates when the future cash flow X has a normal distribution with mean Mu and standard deviation SD. Recall that for a normally distributed random variable z,

$$G(x, y) \equiv \int_x^y z \, pr(z) \, dz = Mu(N(d(y)) - N(d(x))) - SD(n(d(y)) - n(d(x))), \tag{A1}$$

where $d(y) = (y - Mu)/SD$, $N(\bullet)$ is the standard normal cumulative distribution function, and $n(\bullet)$ is the standard normal density function. Applying (A1) to the formulas in Section III, key variables can now be expressed as follows:
Value of limited liability:

$$L_0 = \frac{G(-\infty, 0)}{(1 + r_T)}, \tag{A2}$$

Value of unlevered firm:

$$V_0 = \frac{(1 - \tau)G(0, \infty)}{(1 + r_T)}, \tag{A3}$$

Debt value:

$$D_0 = $$

$$\frac{P(1 - N(d(X^d))) + (1 - \alpha)G(0, X^d) - \tau(G(X^d, X^Z) - X^Z(N(d(X^d) - N(d(X^Z)))))}{(1 + r_T)}, \tag{A4}$$

Equity value:

$$E_0 = \frac{G(X^d, \infty) - \tau(G(X^d, \infty) - X^Z(1 - N(d(X^d)))) - P(1 - N(d(X^d)))}{(1 + r_T)},$$

(A5)

Recovery rate:

$$R = \frac{(1 - \alpha)G(0, X^d) - \tau(G(X^d, X^Z) - X^Z(N(d(X^d)) - N(d(X^Z))))}{P(N(d(X^d)))},$$ (A6)

Tax savings:

$$TS_0 = \frac{\tau(G(0, \infty)) - G(X^Z, \infty) + X^Z(1 - N(d(X^Z)))}{(1 + r_T)}, \quad \text{and}$$ (A7)

Default costs:

$$DC_0 = \frac{\alpha G(0, X^d)}{(1 + r_T)},$$

(A8)

where $X^Z = I = P - D_0$ and $X^d = P + \tau D_0/(1 - \tau)$.

Appendix B: Proofs of Propositions

Notes: As before, X_{0i} denotes the operational value of firm i, $i = \{1, 2, M\}$, with $X_{0M} = X_{01} + X_{02}$. Without loss of generality, we set $X_{0M} = 100$. Separate firm weights are given by $w_i = X_{0i}/X_{0M} = X_{0i}/100$, with $w_1 + w_2 = 1$. The optimally leveraged value of the merged firm is given by $v_\alpha^*(\sigma_M(\rho))$, with $\sigma_M(\rho)$ given by (26), and from (27), each separate firm has value $w_i v_\alpha^*(\sigma_I)$. Thus the total value of the separate firms is $v_S^* \equiv w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2)$. Merger will be desirable if and only if $v_\alpha^*(\sigma_M(\rho)) > v_S^*$. All propositions assume that $v_\alpha^*(\sigma)$ is continuous and strictly convex in σ and reaches a minimum at $0 < \sigma_L \leq \infty$ Thus, $v_\alpha^*(\sigma)$ is strictly decreasing (increasing) in σ for $\sigma <(>) \sigma_L$. It is further assumed that default costs α, tax rates τ, and horizon T are the same for the separate and merged firms.

PROPOSITION 1: *From (26), $\sigma_M(\rho) < \sigma_0$ for any $\{w_1, w_2\}$ when $\sigma_1 = \sigma_2 = \sigma_0$ and $\rho < 1$. Recall that $v_\alpha^*(\sigma)$ is decreasing in σ for $0 < \sigma < \sigma_0 < \sigma_L$. Thus, $v_\alpha^*(\sigma_M(\rho)) > v_\alpha^*(\sigma_0) = w_1 v_\alpha^*(\sigma_0) + w_2 v_\alpha^*(\sigma_0) = v_S^*$ for $\rho < 1$. A merger therefore is desirable for all $\rho < 1$.*

PROPOSITION 2: *Because $v_\alpha^*(\sigma)$ is continuous and strictly increasing in σ at $\sigma = \sigma_1 = \sigma_2 = \sigma_0 > \sigma_L$, and $\sigma_M(\rho)$ is continuous and strictly increasing in ρ, there exists a neighborhood $Q = (\rho^Q, 1)$ such that for ρ in this neighborhood, $\sigma_M(\rho) < \sigma_M(1)$ and therefore $v_\alpha^*(\sigma_M(\rho)) < v_\alpha^*(\sigma_M(1))$. From (26), $\sigma_M(1) = \sigma_0$. Thus, $v_\alpha^*(\sigma_M(\rho)) < v_\alpha^*(\sigma_0) = w_1 v_\alpha^*(\sigma_0) + w_2 v_\alpha^*(\sigma_0) = v_S^*$, and a merger is undesirable for $\rho \varepsilon Q$. When $\rho = 1$, $v_\alpha^*(\sigma_M(1)) = v_\alpha^*(\sigma_0) = v_S^*$, and merging is weakly undesirable.*

PROPOSITION 3: *Recall from (26) that for $\rho = -1$, $\sigma_M(-1) = \sigma_0|w_1 - w_2|$ and for $\rho = 1$, $\sigma_M(1) = \sigma_0 (w_1 - w_2)$.*

Necessity: Assume the contrary, that $v_\alpha^*(\sigma_0|w_1 - w_2|) = v_\alpha^*(\sigma_M(-1)) > v_S^*$. Because $v_\alpha^*(\sigma_M(\rho))$ is continuous in $\sigma_M(\rho)$, and $\sigma_M(\rho)$ is continuous in ρ, there exists an open neighborhood $Y = [-1, \rho^Y)$ such that $v_\alpha^*(\sigma_M(\rho)) > v_S^*$ and merger is desirable for $\rho \, \varepsilon \, Y$, a contradiction.

Sufficiency: By assumption, $v_\alpha^*(\sigma_M(-1)) = v_\alpha^*(\sigma_0|w_1 - w_2|) < v_S^*$. Further, $v_\alpha^* \times (\sigma_M(1)) = v_\alpha^*(\sigma_0(w_1 + w_2)) = v_\alpha^*(\sigma_0) = w_1 v_\alpha^*(\sigma_0) + w_2 v_\alpha^*(\sigma_0) = v_S^*$. Since $v_\alpha^*(\sigma_M) < v_S^*$ at $\sigma_M = \sigma_M(-1)$ and $v_\alpha^*(\sigma_M) = v_S^*$ at $\sigma_M = \sigma_M(1)$, $v_\alpha^*(\sigma_M) < v_S^*$ for all $\sigma_M \times \varepsilon[\sigma_M(-1), \sigma_M(1))$ by the strict convexity of v_α^*. Because σ_M is monotonic in ρ, this in turn implies $v_\alpha^*(\sigma_M(\rho)) < v_S^*$ for all $\rho \, \varepsilon \, [-1, 1)$. Thus, merger is never desirable for $\rho < 1$. When $\rho = 1$, $v_\alpha^*(\sigma_M(1)) = v_\alpha^*(\sigma_{0)}) = v_S^*$, and merging is weakly undesirable.

PROPOSITION 4: *From equation (26), the volatility of the merged firm, $\sigma_M(\rho)$, is continuous and increasing in ρ. Strict convexity of $v_\alpha^*(\sigma)$ implies that when $\rho = 1$, $v_\alpha^*(\sigma_M(1)) = v_\alpha^*(w_1\sigma_1 + w_2\sigma_2) < w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2) = v_S^*$. Continuity of v_α^* in σ_M and of σ_M in ρ implies that there exists an open neighborhood $R = (\rho^R, 1]$ for which $v_\alpha^*(\sigma_M(\rho)) < v_S^*$ for $\rho \, \varepsilon \, R$. Thus, a merger is undesirable for $\rho \, \varepsilon \, R$.*

PROPOSITION 5: *Recall from (26) that for $\rho = -1$, $\sigma_M(-1) = (|w_1\sigma_1 - w_2\sigma_2|)$ and for $\rho = 1$, $\sigma_M(1) = \sigma_0 (w_1 - w_2)$.*

Necessity: Assume the contrary, that $v_\alpha^*(|w_1\sigma_1 - w_2\sigma_2|) = v_\alpha^*(\sigma_M(-1)) > v_S^*$. Because $v_\alpha^*(\sigma_M(\rho))$ is continuous in $\sigma_M(\rho)$, and $\sigma_M(\rho)$ is continuous in ρ, there exists an open neighborhood $Z = [-1, \rho^Z)$ such that $v_\alpha^*(\sigma_M(\rho)) > v_S^*$ and merger is desirable for $\rho \, \varepsilon \, Z$, a contradiction.

Sufficiency: By assumption, $v_\alpha^*(\sigma_M(-1)) = v_\alpha^*(|w_1\sigma_1 - w_2\sigma_2|) < v_S^*$. From the strict convexity of $v_\alpha^*(\sigma_M)$ in σ_M, $v_\alpha^*(\sigma_M(1)) = v_\alpha^*(w_1\sigma_1 + w_2\sigma_2) < w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2) = v_S^*$. Since $v_\alpha^*(\sigma_M) < v_S^*$ at $\sigma_M = \sigma_M(-1)$ and $v_\alpha^*(\sigma_M) < v_S^*$ at $\sigma_M = \sigma_M(1)$, by convexity it follows that $v_\alpha^*(\sigma_M) < v_S^*$ for all $\sigma_M \, \varepsilon \, [\sigma_M(-1), \sigma_M(1)]$. Because σ_M is monotonic in ρ, this in turn implies $v_\alpha^*(\sigma_M(\rho)) < v_S^*$ for all $\rho \, \varepsilon \, [-1, 1]$. Thus, merger is never desirable.

COROLLARY 1: *By the strict convexity of v_α^* in σ, $v_S^* = w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2) > v_\alpha^*(w_1\sigma_1 + w_2\sigma_2)$. By assumption $|w_1\sigma_1 - w_2\sigma_2| = \sigma_M(-1) > \sigma_L$, and recall that $v_\alpha^*(\sigma_M)$ is increasing in σ_M for $\sigma_M > \sigma_L$. Therefore, $v_\alpha^*(w_1\sigma_1 + w_2\sigma_2) > v_\alpha^*(|w_1\sigma_1 - w_2\sigma_2|)$ since $w_1\sigma_1 + w_2\sigma_2 > |w_1\sigma_1 - w_2\sigma_2|$, and by transitivity it follows that $v_S^* = w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2) > v_\alpha^*(|w_1\sigma_1 - w_2\sigma_2|)$. From Proposition 5, a merger therefore is never desirable.*

COROLLARY 2: *Without loss of generality, let $\sigma_1 < {}_2 < \sigma_L$, where the latter inequality holds by assumption. Also by assumption, $|w_1\sigma_1 - w_2\sigma_2| < \text{Min}[\sigma_1, \sigma_2] < \sigma_L$. Recall that $v_\alpha^*(\sigma)$ is decreasing in σ when $\sigma < \sigma_L$. Thus $v_\alpha^*(|w_1\sigma_1 - w_2\sigma_2|) > v_\alpha^*(\sigma_1) > w_1 v_\alpha^*(\sigma_1) + w_2 v_\alpha^*(\sigma_2)$. This contradicts the necessary and sufficient condition in Proposition 5, and there will exist a $\rho^Z > -1$ such that mergers will be desirable for correlations $\rho \, \varepsilon \, [-1, \rho^Z)$.*

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