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Gauging the Risk Premium
For Bonds Subject to Default

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INTRODUCTION  

The yield on the Moody's Baa industrial bond average typically ranges from 50 to 125 basis points above that available on Aaa bonds. The question arises as to whether this premium adequately compensates for the additional risk on the lower quality securities. This is a very difficult question to answer unless the size of the premium can be related to the probability of default and the losses which may result. This paper presents a very simple method for doing so. The method is applicable to judging the adequacy of the risk premium on any type of fixed-income security, including term loans and mortgages. 

There are taken to be two basic parameters describing future expectations about default risk. One is the probability of default occurring and the other is the expected loss rate if default occurs. Suppose that default has not occurred up to a particular year. The probability of default in that year is taken to be \( \alpha \). Suppose that default does occur. Typically, this will arise from the firm's inability to renew its short-term credit or pay off maturing long-term debt. Upon petition, the courts appoint a trustee to manage the firm, and a reorganization takes place. In extreme cases, the firm may be liquidated.  

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Eventually, the bondholders will receive either new securities in the reorganized firm or liquidating payments. They may also receive some payments during the reorganization period.

The value of the bonds upon default is taken to be the present value at time of default of any payments or securities received in reorganization or liquidation. The present value of the bonds upon default, divided by the value of comparable default-free securities prior to default, is taken as the loss rate. Before the fact, this loss rate will not be known. It is assumed, however, that an estimate can be made of its mean expected value. This value will be denoted by $\lambda$. Junior and senior securities for a given firm may have the same probability of default, $\alpha$. The larger risk premium on the junior securities will be reflected by the larger value of $\lambda$ on the subordinated issues. In some cases—the equipment trust certificates of railroads, for instance—the probability of default may also be less on the more senior securities.

THE MODEL

Let $P_t$ be the price of the bond in year $t$, excluding the coupon received in that year, and assuming that default has not occurred up to year $t$. Suppose that default does not occur in the following year. The return earned by a bondholder will be equal to the coupon payment plus the capital gain or loss, all divided by the price in $t$, 

$$\frac{c + P_{t+1} - P_t}{P_t}.$$ 

If default does occur in the following year, a bondholder will suffer a negative return equal to the loss rate. The mean expected value of this negative return is $-\lambda$. The probability that
default does not occur is \(1 - \alpha\), while the probability of default is \(\alpha\). Multiplying the return for each of these two possibilities by the probability of occurrence and adding gives the mean anticipated return:

\[
\text{Mean Return} = (1-\alpha) \frac{(c+P_{t+1}-P_t)}{P_t} - \alpha \lambda.
\]

Any investor will presumably require a mean return on securities of lower quality at least as great as that available on governments or other securities which are essentially default free. Default experience on the bonds of different firms will tend to be correlated through their mutual dependence on general business conditions. Nevertheless, as with stocks, it will be possible for large investors, or small investors acting in concert through financial intermediaries, to diversify away part of the fluctuation about the mean by investing in a portfolio of bonds. Thus, large investors may not require a mean return much in excess of the default-free rate. Let the default-free rate, plus any excess which may be required, be denoted by \(r\). The mean return available on poorer quality securities must be at least equal to \(r\), if they are to provide an attractive investment.

Set \(r\) equal to the mean return in the expression given above. With some obvious algebraic simplification, the following relation is obtained:

\[
\frac{r + \alpha \lambda}{1 - \alpha} = \frac{c + P_{t+1} - P_t}{P_t}.
\]

Setting \(\rho\) equal to \((r+\alpha \lambda)/(1-\alpha)\) on the left-hand side of this
equation and solving for $P_t$ gives:

$$P_t = \frac{c + P_{t+1}}{1 + \rho} \quad \text{where} \quad \rho = \frac{r + \alpha \lambda}{1 - \alpha}.$$  

This relation must hold for any year in the future so that $P_{t+1} = (c + P_{t+2})/(1 + \rho)$, $P_{t+2} = (c + P_{t+3})/(1 + \rho)$, . . . , etc. Use these relations to substitute successively for $P_{t+1}$, $P_{t+2}$, . . . , etc. and take $P_T = 100$, corresponding to maturity. In this fashion, the following relation for $P_t$ is obtained:

$$P_t = \frac{c}{1 + \rho} + \frac{c}{(1 + \rho)^2} + \ldots + \frac{c + 100}{(1 + \rho)^T}.$$  

This is simply the familiar discounted present value expression for the price of a bond.\(^1\)

This last expression provides the main result of this paper. A bond subject to default must yield at least $\rho = (r + \alpha \lambda)/(1 - \alpha)$ for it to be an attractive investment relative to default-free securities. This required yield has been expressed in terms of the two basic parameters describing an investor's anticipations about default: the probability of

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1. The relation for $P_t$ can easily be adapted to handle values of $r$, $\alpha$, and $\lambda$ which change in different years. Let $r_1, \alpha_1, \lambda_1; r_2, \alpha_2, \lambda_2; \ldots$ represent the values for the 1st, 2nd, . . . years following $t$, and let $\rho_1, \rho_2, \ldots$ equal $(r_1 + \alpha_1 \lambda_1)/(1 - \alpha_1), (r_2 + \alpha_2 \lambda_2)/(1 - \alpha_2), \ldots$. The discount rates $1/(1 + \rho), 1/(1 + \rho)^2, \ldots$ would then be replaced by $1/(1 + \rho_1), 1/(1 + \rho_1)(1 + \rho_2), \ldots$ etc. The yield on a bond will be an average of $\rho_1, \rho_2, \ldots$ of the same form developed in the theory of the term structure of interest rates.
default, \( \alpha \), and the mean expected loss rate if default occurs, \( \lambda \).

**GAUGING PARAMETERS**

Relating the yield on a lower quality security to anticipations about its default provides considerable help in judging the adequacy of its risk premium. However, an investor is still faced with the problem of arriving at what he considers to be reasonable estimates of the default parameters. Two aids in assessing the probability of default are available. One is simply to check the probability of default that a given value of \( \alpha \) implies for longer periods of time. What at first glance seems to be a reasonable value of \( \alpha \) may turn out to imply what seems to be an unreasonably large probability of default over a ten- or twenty-year time span. The second aid is to examine actual default experience in the past for creditors with specific characteristics. As calculating the longer run default probabilities for a given \( \alpha \) is a matter of simple logic, this will be presented first. Then, the record of default experience on rated bonds will be examined.

Given the probability of default over a one-year period, \( \alpha \); the probability of default over a two-year period is easily calculated. The probability of the bond not defaulting over the first year is \( 1 - \alpha \) and this is also the probability of it not defaulting in the second year, given that it does not default in the first. The probability of not defaulting over both years is therefore the product of these two probabilities or \( (1-\alpha)^2 \). Over the two years, the bond must either default or not default, so these two probabilities must add to one. As the probability
of not defaulting is \((1-\alpha)^2\), the probability of default must be \(1 - (1-\alpha)^2\). Similarly, it follows that the probability of default over \(t\) years must be \(1 - (1-\alpha)^t\). Using this formula and various values of \(\alpha\), the probability of default for various time spans is given in Table 1. The values in the first column are the assumed values for \(\alpha\) for each row.

**TABLE 1**

**RELATION OF PROBABILITY OF DEFAULT TO TIME SPAN OF ISSUE OUTSTANDING**

<table>
<thead>
<tr>
<th>Default Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>.001</td>
</tr>
<tr>
<td>.005</td>
</tr>
<tr>
<td>.01</td>
</tr>
<tr>
<td>.02</td>
</tr>
<tr>
<td>.03</td>
</tr>
</tbody>
</table>

For small \(\alpha\), it can be seen that the default probability over \(t\) years is approximately \(\alpha t\).

Extensive data on default experience for corporate bonds in the prewar period have been provided by Hickman.\(^2\) He gives the number of defaults experienced on all large outstanding issues during the twenties and thirties, broken down by agency rating, two years prior to default.

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He also provides data from which estimates can be made of the average number of issues outstanding in each decade for each agency rating. Dividing the number of defaults in each decade by the average number of comparable issues outstanding (times 10) gives the estimates for \( \alpha \) provided in Table 2. For each rating and each decade there were approximately 200 issues outstanding. The agency ratings of I, II, III, and IV correspond to the Moody's ratings of Aaa, Aa, A, and Baa, respectively. The values reported for 1920-1939 are averages of those for 1920-1929 and 1930-1939.

**TABLE 2**

<table>
<thead>
<tr>
<th>Period</th>
<th>Agency Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>1920-1929</td>
<td>.0012</td>
</tr>
<tr>
<td>1930-1939</td>
<td>.0042</td>
</tr>
<tr>
<td>Estimated ( \alpha )</td>
<td>.003</td>
</tr>
</tbody>
</table>

Hickman has also presented some data which can be used to make inferences about the value of the mean loss rate, \( \lambda \). For the defaults on large issues, he gives the present value at time of default of the future receipts obtained, discounted at 6 percent. The present value of these receipts divided by the aggregate par value of all such defaulted issues gives an estimate for \( \lambda \) of .48. This is based on approximately 700 defaults with an aggregate par value of about $10 billion. Unfortunately,
these data are not segregated with respect to junior and senior liens. Therefore, it is not possible to obtain an estimate of this factor on \( \lambda \).

In the postwar period, default experience on bonds with the four highest ratings has been sharply different. There has been virtually no subsequent default on bonds rated Baa or better. To provide specific evidence on this assertion, all firms with issues rated Baa or better by Moody's in 1950 and maturing in more than ten years were examined.\(^3\) This included industrials, rails (excluding equipment trust certificates), and utilities. In all, there were 410 firms distributed as follows: Aaa (41), Aa (95), A (149), and Baa (125). Default did not occur in the subsequent decade on any of these issues. Some were retired at the call prices stated in their indentures. Even in these cases, it appeared that the surviving company was meeting its obligations in 1960.

This test was repeated using the firms rated Baa or better in 1960. There were 514 firms distributed as follows: Aaa (57), Aa (117), A (184), and Baa (156). None of these firms defaulted over the subsequent decade. However, there were some famous defaults, or near defaults, over the period which warrant discussion. The bonds of the New York, New Haven, and Hartford; the Boston and Maine; the Lehigh Valley; and the Penn Central all defaulted. The bonds of the first three and the New York Central were rated, at best, Ba in 1950 and subsequently. Bonds of the Pennsylvania were rated Baa through 1952 and then less. The Pennsylvania's bonds would rate as a default if a twenty-year rather than ten-year time span had been used from 1950. The bonds of Lockheed were rated Baa in 1960 but did not default due to intervention by Congress.

\(^3\) I am indebted to Ahmet Tezel for aid in gathering these data.
Based on observation for the postwar period, an estimate for $\alpha$ on Baa bonds of .001 might be quite reasonable, and even a bit conservative. This is true even if every conceivable marginal case is counted as a default, since $\alpha = .001$ corresponds to one default per hundred per decade. This contrasts sharply with the value of .023 obtained in Table 2 for the prewar period. Such a sharp discontinuity is not unreasonable, given the significant increase in knowledge since the thirties on preventing major depressions. The recent crisis period also indicated some willingness by the government to intervene directly to prevent default by large corporations.

Though it appears that the default rate has dropped sharply in the postwar period, this does not seem to have been reflected in a narrowing in the spread between the Baa and Aaa yields. During the sixties, a yield spread of 75 basis points was quite typical. Letting $r = .06$ be the Aaa rate, and taking $\lambda = .5$, shows that a 75 basis point spread requires $\alpha = .013$. This is obtained by solving the equation $.0675 = (.06 + .5\alpha)/(1-\alpha)$ for $\alpha$. Such an $\alpha$ is well in line with prewar experience on Baa bonds but not the experience in the postwar period. An $\alpha$ of .001 corresponds to a spread of only 5 basis points (i.e., $\rho = (.06 + .5(.001))/.999 = .0605$).

It might be argued that the excess spread represents additional mean return required above that available on Aaa bonds to bear the additional risk of default. This does not seem reasonable. Fraine and Mills$^4$ found little evidence in Hickman's data of a relationship between

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average realized return and bond quality. In any case, the spread observed appears too large to explain on this basis. Rather, it seems that pension fund managers and insurance companies could increase their mean return with little offsetting increase in risk by some switching into lower quality securities. The model and data presented here may provide some help for them in deciding whether or not this is true.

CONCLUSION

A simple model has been presented for judging the adequacy of the premium yield available on bonds subject to default. The required inputs are the yield available on default-free securities, the probability of default, and the mean expected loss rate if default were to occur. Data on actual default experience for rated bonds has been presented to aid in arriving at reasonable estimates of the default parameters for such securities. These data indicate a sharp drop in default experience in the postwar period. Recent yield spreads for Aaa and Baa rated bonds appear reasonable, based on default experience in the twenties and thirties. Based on more recent default experience, however, these spreads appear unreasonably high. This suggests that managers of fixed-income portfolios may want to consider some switching into lower quality bonds. The model and data presented here should provide an aid in making such a decision.
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