Research Program in Finance

Graduate School of Business Administration

Asset Substitution, Inflation, and Interest Rates

By

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Institute of Business and Economic Research

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RESEARCH PROGRAM IN FINANCE AT THE
WALTER A. HAAS SCHOOL OF BUSINESS,
UNIVERSITY OF CALIFORNIA, BERKELEY

The Research Program in Finance in the Walter A. Haas School of Business at the University of California has as its purpose the conduct and encouragement of research in finance, investments, banking, securities markets, and financial institutions. The present reprint and working paper series were established in 1971 in conjunction with a grant from the Dean Witter Foundation.

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WORKING PAPER NO. 2

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November 1971

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ABSTRACT

Using a general equilibrium model for financial assets, it is shown that, in the short run, the effect of the expected rate of inflation on the real rate of return on a bond depends on the maturity and liquidity of the bond in question and on its substitutability for capital and for money.

In his classic work on the effects of inflation on interest rates, Irving Fisher (1930, pp. 399-451) suggested that the nominal rate of interest rises (falls) by the expected rate of inflation (deflation) so that the real rate of return on capital remains constant. A number of contemporary authors (e.g. Gibson [1970], Mundell [1963], and Yohe and Karnosky [1969]) assume that "arbitrage" will ensure that the real rate of return on bonds will equal the real return on capital.
Using a general equilibrium model for assets which was introduced by Tobin (1969), I will extend some of Tobin's results to show that the effect of expected inflation on the real rate of interest on a bond will be greater the greater the substitutability of bond for money and the less the substitutability of the bond for capital.

THE MODEL

To analyze the role of asset substitution in determining the effect of expected inflation on real interest rates, I will use a four-asset model similar to Tobin's money-securities-capital model (Tobin 1969, pp. 23-26). The economy will be assumed to have one private sector and four assets: homogenous physical capital (and claims to this capital), money, and two classes of debt securities. One of the debt securities will be called "bills" and the other "bonds."\(^1\) The money, bills, and bonds are assumed to be the debt of the government. Let the expected real rates of return on capital, money, bills, and bonds be \(r_k\), \(r_m\), \(r_t\), and \(r_b\), respectively. Let \(r_m\), \(r_t\), and \(r_b\) be the nominal rates of interest on money, bills, and bonds, respectively, and let \(R\) be the marginal efficiency of capital relative to reproduction cost.\(^2\) Let the price of currently produced goods (consumer goods and capital goods) be \(P\) and the market price of existing capital goods be \(qP\).\(^3\) Let \(W\) be

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1. In the subsequent analysis, bills and bonds will be differentiated on the basis of liquidity and maturity.
2. If \(X\) is the total product of capital evaluated at commodity prices, then \(R = X/PK\).
3. The factor \(q\) is the ratio of the price of existing capital to the price of currently produced goods and can differ from one.
real wealth and \( Y \) real income (in terms of commodity prices) and \( q \), \( K \), 
\( M \), \( T \), and \( B \) the nominal or market value of capital, money, bills, and 
bonds, respectively. Let \( \pi_e \) be the expected rate of change of commod-
ity prices.

The portfolio balance equations are assumed to be homogenous in
wealth with the proportion of real wealth held in each of the four assets
being a function of the vector of real rates of return, \( \hat{r} \), and, because
of transactions demand for money, on the ratio of income to wealth, \( Y' \):

\[
\begin{align*}
    k(\hat{r}, Y')W &= qK. \\
m(\hat{r}, Y')W &= \frac{M}{p}. \\
t(\hat{r}, Y')W &= \frac{T}{p}. \\
b(\hat{r}, Y')W &= \frac{B}{p}.
\end{align*}
\]

Real wealth is defined as:

\[
W = qK + \frac{M}{p} + \frac{T}{p} + \frac{B}{p}.
\]

Given this wealth constraint, one of the portfolio balance equations is
redundant. The equations for the real rates of return are:

4. The vector \( \hat{r} \) is \((r_k, r_m, r_t, r_b)\) and \( Y' = \frac{Y}{W} \).

5. In this formulation, bonds are assumed to be short-term securities
since their value is not affected by changes in \( r_b \). This assumption
will be dropped later.
\[ r_k = \frac{R}{q}, \quad (1.6) \]
\[ r_m = r'_m - \pi_e, \quad (1.7) \]
\[ r_t = r'_t - \pi_e, \quad (1.8) \]
\[ r_b = r'_b - \pi_e. \quad (1.9) \]

For the short-run (i.e., the period during which \( q \) can differ from one) interpretation of this model, there are eight endogenous variables \((q, \bar{w}, r_k, r_m, r_t, r'_t, r_b, \text{ and } r'_b)\) to be determined by the eight independent equations. Using a three-asset version of this model and assuming (1) that the partial elasticity of the demand for money with respect to income is positive but does not exceed one, (2) that securities absorb any changes in the transactions demand for money, (3) that own-rate derivatives are positive, and (4) that cross-derivatives are nonpositive, Tobin (1969, p. 25) found that the partial derivative of the real return on capital with respect to the expected rate of change of prices and the partial derivative of the real return on securities (i.e., the single bond in the three-asset model) with respect to the expected rate of change of prices were both negative. With these assumptions, it can be shown that this decrease in real rates of return with increases in expected inflation holds for capital and both types of debt securities in the four-asset model. In the analysis which follows, we will be chiefly concerned

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6. The nominal rate of interest on money is assumed to be an exogenously determined constant.
with the relative size of these three partial derivatives (i.e., \( \frac{\partial r_k}{\partial \pi_e}, \frac{\partial r_t}{\partial \pi_e}, \frac{\partial r_b}{\partial \pi_e} \)).

INFLATION AND THE REAL RATE OF RETURN ON SECURITIES

For a comparative statics analysis of the four-asset model, let us drop equation (1.1). To differentiate between bills and bonds on the basis of liquidity, let bills absorb any changes in the transactions demand for money. With appropriate substitutions using the equations for the real rates of return and the stated assumptions, we obtain the following system of equations for the three partial derivatives in question:

\[
Dx = v
\]

(2.1)

where

\[
D = \begin{bmatrix}
m_1 + (m_3 y' - m) \frac{k}{r_k} & m_3 & m_4 \\
t_1 - (m_3 y' + t) \frac{k}{r_k} & t_3 & t_4 \\
b_1 - b_3 \frac{k}{r_k} & b_3 & b_4
\end{bmatrix}
\]

and

\[
x = \begin{bmatrix}
\frac{\partial r_k}{\partial \pi_e} \\
\frac{\partial r_t}{\partial \pi_e} \\
\frac{\partial r_b}{\partial \pi_e}
\end{bmatrix}
\]

\[
v = \begin{bmatrix}
m_2 \\
t_2 \\
b_2
\end{bmatrix}
\]

7. Let the partial derivatives of the portfolio proportions \( k, m, t, \) and \( b \) with respect to their arguments be \( k_1, m_1, t_1, \) and \( b_1 \) \((i = 1, 3)\) where, e.g., \( k_1 = \partial k/\partial r_k, \) \( k_2 = \partial k/\partial r_m, \) \( k_3 = \partial k/\partial r_t, \) \( k_4 = \partial k/\partial r_b, \) and \( k_5 = \partial k/\partial y'. \) Using this notation the assumptions stated in the text are: (1) \( m_3 y' - m \leq 0, \) (2) \( m_5 = -t_5, \) \( k_5 = b_5 = 0, \) (3) \( k_1 > 0, m_2 > 0, \) \( t_3 > 0, b_4 > 0, \) and (4) all other \( k_1, m_1, t_1, \) and \( b_1 \leq 0. \)

8. If the bonds are presumed to be perpetual securities rather than short-term securities, the final column of \( D \) is
The solution for a given partial derivative of a real rate of return with respect to the expected rate of change of prices is

$$\frac{\partial r_j}{\partial \pi_e} = \frac{|D_j|}{|D|} \quad j = k, t, b$$

(2.2)

where $D_j$ is the matrix formed by substituting the vector $v$ for the appropriate column in $D$ and $|D_j|$ is the determinant of the matrix.

To evaluate the relative magnitude of the effect of inflation on the three real rates of return $r_k$, $r_t$, and $r_b$, I will add the assumption that the wealth-compensated change in the demand for asset $i$ with respect to the real rate of return on asset $j$ is equal to the wealth compensation change in the demand for asset $j$ with respect to the real rate of return on asset $i$ (i.e. the matrix $M$ where

$$M = \begin{pmatrix}
    k_1 & m_1 & t_1 & b_1 \\
    k_2 & m_2 & t_2 & b_2 \\
    k_3 & m_3 & t_3 & b_3 \\
    k_4 & m_4 & t_4 & b_4
\end{pmatrix}$$

is symmetric). With this assumption the "adding-up" requirement may be

8. (Cont’d.)

$$\begin{pmatrix}
    m_4 + (m_5 Y^t - m) b \\
    t_4 - (m_5 Y^t + t) b \\
    b_4 + (1-b) b
\end{pmatrix}_{r_b}$$
written as

\[ \sum_{i=1}^{4} z_i = 0; \quad z = k, m, t, b. \]

To compare the effect of expected inflation on capital and bonds, we need to compare the determinants \(|D_k|\) and \(|D_b|\). By using various row and column operations and both versions of the "adding-up" requirements, the difference between the two determinants may be written as

\[
|D_b| - |D_k| = \begin{pmatrix}
(m_3 Y' - m) \frac{k}{r_k} & m_3 & m_2 \\
-(m_5 Y' + t) \frac{k}{r_k} & t_3 & t_2 \\
-b & \frac{k}{r_k} & b_3 & b_2
\end{pmatrix}
\]

(3.1)

Since the determinant on the right-hand side (r.h.s.) of equation (3.1) is positive, the effect of expected inflation on the real rate of return on short-term, illiquid bonds is greater than the effect of expected inflation on the real return on capital.

In the case of perpetual bonds, the difference between the two determinants may be written as

\[
|D_b| - |D_k| = \begin{vmatrix}
-(1-k) + \frac{b}{r_b} & b & -\frac{k_3}{k} & -\frac{k_2}{k} \\
-(m_5 Y' + t) \left(\frac{k}{r_k} + \frac{b}{r_b}\right) & t_3 & t_2 \\
-k & (1-b) & \frac{b_3}{b} & \frac{b_2}{b}
\end{vmatrix}
\]

(3.2)
Without further assumptions, the sign of the determinant on the r.h.s. of equation (3.2) is indeterminate. However, for the two determinants and therefore for $\partial r_b/\partial \pi_e$ and $\partial r_t/\partial \pi_e$ to be equal, it is sufficient to assume that the real rates of return on capital and bonds are equal ($r_k = r_b$) and that perpetual bonds and perpetual capital are equally good substitutes for both bills and money (i.e. the partial elasticity of substitution of capital for money [bills] equals the partial elasticity of substitution of bonds for money [bills]). These assumptions are consistent with Fisher's view regarding the substitutability between bonds and capital. It was on the basis of these assumptions that he concluded that the effect of expected price changes on the real rate of return on bonds was equal to their effect on the real rate of return on capital. As shown above, this will not be true in general for bonds with finite maturity dates.

To compare the effect of expected inflation on bonds and bills, we need to know the sign of the difference between $|D_t|$ and $|D_b|$. After suitable manipulation, this difference may be written as

$$|D_t| - |D_b| = \frac{k}{r_k} \left[ \begin{array}{ccc} 0 & -k_2 & k_1 \\ -m_2 & m_2 & -m_1 \\ t_2 & -t_1 & -t_1 \end{array} \right] + \left[ \begin{array}{ccc} -m_3 & 0 & 0 \\ t_2 & t_2 & t_2 \\ -b_1 & -b_1 & -b_1 \end{array} \right]$$ (3.3)

The first determinant on the r.h.s. of equation (3.3) is clearly positive. Therefore, a sufficient condition for the effect of the expected rate of
price change on the real rate of return on bills to be greater than its
effect on the real rate of return on bonds is
\[ m(t_2 b_1 - t_1 b_2) + t(b m_1 - b m_2) + b(m_2 t_1 - m_1 t_2) \geq 0. \]  
(3.4)

Let \( E_{ij} \) be the absolute value of the partial elasticity of substitu-
tion of asset \( i \) with respect to asset \( j \). By multiplying both sides
of inequality (3.4) by the positive quantity \( -k \frac{r_m}{m + b} \) (assuming
\( r_k > 0 \) and \( r_m < 0 \)), the sufficient condition may be expressed in terms
of the partial elasticities of asset substitution
\[ E_{32} E_{41} - E_{31} E_{42} + E_{42} E_{21} + E_{41} E_{22} - E_{22} E_{31} - E_{21} E_{32} \geq 0. \]  
(3.5)

Therefore, the effect of expected inflation on the real rate of return
on bills will be greater than its effect on the real rate of return on
bonds if bonds are at least as good a substitute for capital as bills
or money (i.e. \( E_{41} \geq E_{31} \) and \( E_{41} \geq E_{21} \)) and bills are at least as
good a substitute for money as bonds (i.e. \( E_{32} \geq E_{42} \)). Furthermore,
the greater the absolute value of the partial elasticity of bonds with
respect to capital relative to the absolute value of the partial elastic-
ities of bills and money with respect to capital and the greater the
absolute value of the partial elasticity of bills with respect to money
relative to the partial elasticity of bonds with respect to money, the
larger the difference between \( \frac{\partial r_b}{\partial \pi_e} \) and \( \frac{\partial r_t}{\partial \pi_e} \) will be. If the
bonds are assumed to be perpetual bonds, the only effect on the analysis
of the size of \( \frac{\partial r_b}{\partial \pi_e} \) relative to \( \frac{\partial r_t}{\partial \pi_e} \) is that \( |D_t| \) will be
smaller than was the case for short-term bonds. Thus, the results I have reported for the case of short-term bonds will apply a fortiori to the case in which bonds are long-term securities.

CONCLUSIONS

Using a set of assumptions which seem to me to be plausible, I have shown that in the short run the effect of a given expected rate of inflation on the real rate of return on a bond depends on the maturity and liquidity of the bond in question and on its substitutability for capital and money. 9

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9. See Pyle (1971) for empirical evidence which suggests that the nominal interest rate on six-month commercial paper is affected less by expected price changes than is the nominal interest rate on one-year government securities.
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