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Descriptive Theories of Financial Institutions Under Uncertainty

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Institute of Business and Economic Research

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DESCRIPTIVE THEORIES OF FINANCIAL INSTITUTIONS UNDER UNCERTAINTY

by

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DESCRIPTIVE THEORIES OF FINANCIAL INSTITUTIONS
UNDER UNCERTAINTY

by

David H. Pyle

This essay is a selective review of the received theory of financial institutions with some suggestions regarding future research on this topic. The major emphasis is placed on the positive economic theory of financial institutions. In the context of this essay, these are firms that supply financial securities and contracts that are held as assets by other sectors of the economy and that use the proceeds of these sales to finance the purchase of financial securities and contracts that are the liabilities of other sectors of the economy. The theory discussed here is stripped of much of the regulatory and legal framework surrounding such institutions. In addition to a desire to keep this essay manageable, the reason for so limiting the scope of this paper is a conviction that a reasonably complete model of a simple financial institution is a necessary precursor of useful models of the positive economic behavior of financial institutions in any specific legal, regulatory, and

In writing this paper, I have benefited from discussions with Richard Grinold, Jan Mossin, Sten Thore, and the members of my seminar at the Norwegian School of Economics and Business Administration. I am especially grateful to Professor Thore for having given me access to his lecture notes on the theory of commercial bank behavior, which I found very useful in preparing this paper.

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operational framework. While recognizing that no tractable model of a financial institution is likely to be so general as to avoid the problem of model specificity, which seems inherent in much of the existing literature, the view taken here is that many of the questions asked in the literature would be better answered in less specific models, i.e., in models that are capable of explaining additional important aspects of the behavior of the financial institution in question. 1

Another major strand of the literature on financial institutions discusses normative models for such firms. The example of the development of a positive economic model of investor behavior, e.g., Sharpe (1964), from a normative model of portfolio selection (Markowitz 1959) should make us aware of the potential for the literature on the normative theory of financial institutions to lead to fruitful improvements in the positive economic theory of such firms. Thus far, this potential has been largely unrealized.

The discussion that follows concentrates on the models of financial institutions that have incorporated uncertainty in the analysis.

A MODEL OF A FINANCIAL INSTITUTION

following

The model of a financial institution is used to illustrate the existing theory and provides a framework in which to discuss the ways in

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1. As an example (and only one of many) consider Shull's (1963) model of the bank as a multiple product price discriminator. One would be more satisfied with the connection between loan rates and the bank's monopoly power in different markets if the model that Shull used was capable of incorporating the risk characteristics of the various loans in the analysis.
which this theory might be extended. By an appropriate definition of the variables, this notation will serve for financial institutions that are subject to reserve requirements as well as for those that are not. For some uses of such models, e.g., questions regarding the effects of changes in reserve requirements, it would be better to formulate the model so that the role of reserve requirements is more transparent. For my purposes, abstraction from specific consideration of reserve requirements presents no problems.

A financial institution is thought of as a firm that can hold three types of assets: loans (L), securities (S), and cash (M). The liabilities of the financial institution consist of deposits (D) and net worth or capital (W0). The financial institution is usually constrained to hold nonnegative quantities of the assets and liabilities, although on occasion the problem has been formulated so that one or more of the asset quantities is unrestricted in sign. Not all the models reviewed consider the full set of assets and liabilities listed above.

When there are nonzero reserve requirements, the assets and liabilities affected are defined as "free" assets and "net" liabilities. For example, free cash is the cash (currency, central bank deposits, etc.) in excess of currently required cash reserves and net deposits (for a given class of deposits) are the total amount of deposits of that class times one minus the reserve requirement for that class. Interest rates on the affected assets and liabilities are adjusted so that they are the effective rates applicable to free assets and net liabilities. The analysis is concerned with the effective return received on loans (RL), the
effective return received on securities ($R_B$), and the effective return paid on deposits ($R_D$), where, for example, $R_D$ is one plus the effective interest rate on deposits.\footnote{Thus, with a nominal interest rate on $D$ of \(i_D\) and a reserve requirement of \(\rho\) on deposits, the effective return $R_D$ equals $\frac{i_D - \rho}{1 - \rho}$.}

When there are reserve requirements, note that reserve constraints become part of the set of nonnegativity constraints that are applicable to most models. In the remainder of this essay, unless it is necessary for clarity to do otherwise, assets and liabilities are referred to as loans, deposits, etc., with the presumption that the appropriate redefinition has taken place for financial institutions subject to reserve requirements.

In addition to interest costs and revenues, financial institutions are subject to fixed and variable operating costs. A comprehensive review of the literature on cost functions for financial institutions is contained in Benston (1972). A principal conclusion of the research discussed in Benston's paper is that financial institutions display economies of scale in operating costs. In part, it is the existence of these noninterest costs of selecting and servicing loan, security, and deposit portfolios and the economies of scale in these costs that provide the economic rationale for financial institutions (a point I will return to later). One should note, however, that in addition to the operating costs discussed by Benston and others the financial institution may be subject to costs.
associated with portfolio adjustments. I include in the specification of the model a cost function (C) that provides for both operating and adjustment costs. 3

Most of the existing models of financial institutions can be discussed in a framework in which two decisions, a portfolio decision and a liquidity decision, are made. None of the existing models is fully consistent with that framework. Whether the existing models are adequate to explain the behavior of a financial institution depends on which aspects of that behavior we choose to focus. It may be an empirical question as well. My viewpoint is that for many interesting questions about financial institutions the effects of both of these decisions, i.e., the portfolio decision and the liquidity decision, may be important and that to test this empirically, the relationships, e.g., asset demand functions, derived from the more simple models should be confronted with relationships derived from a more complete model of the decision process. In this two-decision framework, the financial institution is assumed to make portfolio choices at the present with the knowledge that before the end of its portfolio decision horizon the deposit-demand relationship may change, necessitating changes in the quantities of asset and liabilities held. In the following notation, these changes in assets and liabilities will be denoted by 1, b, m, and d.

3. Benston (1972) suggests that operating costs are determined by the numbers of loan and deposit accounts serviced by a financial institution. This implies that some care must be taken in interpreting the results obtained when, as will be the case for the models discussed in this essay, monetary quantities of loans, securities, and deposits are the arguments of the cost function.
The objective of the financial institution is taken to be maximization of the expected utility of terminal net worth \( W_1 \), where

\[
W_1 = R_L (L + l) + R_B (B + b) + (M + m) - R_D (D + d) - C, \tag{1}
\]

with \( C \), in general, a function of \( L, B, M, \) and \( D \) (due to operating costs) and of \( l, b, \) and \( m \) (due to adjustment costs).

Two alternative behavioral assumptions for financial institutions facing a random deposit demand relationship are considered below: deposit rate setting and deposit quantity setting. The first of these behavioral assumptions seems appropriate for many types of deposits in United States financial institutions. However, if the liability that is issued by the financial institution is traded under perfectly competitive conditions, quantity setting behavior is the only choice open to the institution.

As suggested below, the second of these behavioral assumptions seems appropriate for certain classes of financial institutions and certain types of liabilities for others.

**DEPOSIT RATE SETTING**

The deposit rate setting firm chooses (or, where rates are fixed, has chosen for it)\(^5\) the interest rate payable on its deposits over its

\(^4\) This definition of terminal net worth implies that interest revenues and costs are applicable to the quantities of assets and liabilities remaining after adjustments to changes in deposit demand. In a discrete model, the question of payments timing is somewhat arbitrary, and other timing assumptions have been used by various authors.

\(^5\) Even where rates are nominally fixed, as is the case for United States commercial banks, it appears that banks have some rate setting discretion in terms of relaxation of service charges and preferential loan rates for depositors.
decision horizon. In the two-decision framework being used here, such a firm chooses this deposit rate and the current portfolio of loans, securities, and cash, given the current level of deposits and net worth and a distribution for the deposit change that will occur before the end of the decision horizon. The financial institution must respond to the random deposit change by changes in other assets and liabilities, and these portfolio shifts may result in portfolio adjustment costs for the financial institution. For the United States, some types of deposits that appear to fit this behavioral mode are demand deposits (where in the United States the rate is set at zero and the decision is subject to that constraint) and time and savings deposits of various types.

The two-decision optimization problem for a financial institution that sets its deposit rate is

\[
\max_{L, b, m, R_D} \quad E \left\{ \max_{1, b, m} \left[ u(W_t) \right] \right\} 
\]

subject to

\[
L + B + M = D + W_0 
\]

\[
1 + b + m = d, 
\]

\[
-1 \leq L, 
\]

\[
-b \leq B, 
\]

\[
-m \leq M, 
\]

and nonnegativity constraints on \( L, B, M \), and \( D \).

The following two-asset version of this model is, for our purposes, substantially the same as the model presented by Orr and Mellon (1961) and
recently corrected by Cooper (1971). In this version of the model, the financial institution has a given amount of deposits for which the interest payable is fixed (zero for Orr-Mellon). The institution is able to choose the amounts of loans and cash it will hold. There is a known cumulative distribution for deposit changes, \( F(d) \). Loans are assumed to be completely illiquid so that the cash position must absorb any changes in deposits that occur. The cash position can be negative but, if it is, the institution is subject to adjustment (penalty) costs.

With a linear utility function, the optimization problem for the financial institution with two assets to choose is

\[
\max_{L, M} E[R_L L + M + m - R_D (D+d) - C], \quad (3a)
\]

subject to

\[
L + M = D, \quad (3b)
\]

\[
m = d, \quad (3c)
\]

where

\[
C = \begin{cases} 
0 & \text{for } M + m > 0 \\
A - a(M+m) & \text{for } M + m \leq 0 
\end{cases}
\]

Thus, the financial institution is faced with a fixed penalty cost if the random deposit change results in a negative cash position (borrowing) and a penalty cost proportional to the cash deficiency. After

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6. In the Orr-Mellon model, deposits were subject to reserve requirements. As indicated earlier, this can be handled by an appropriate redefinition of the variables. Poole (1968) presented a model that is similar to this two-asset model except that the liquidity adjustment is made in an interest-bearing asset (federal funds) and \( A = 0 \).
substitutions, this problem can be expressed as an unconstrained maximization problem in one decision variable:

$$
\max_L \left[ E(R_L) - 1 \right] L - \int_{-\infty}^{L-D} \left[ A - a(D-L+d) \right] dF(d) - (R_D - 1) [D + E(d)].
$$

This form of the objective function makes the nature of the economic trade-off implied by this model of the financial institution quite clear. An increase in loans relative to cash implies an increase in marginal revenue, but, since it influences the potential size and the likelihood of a cash deficiency, it also implies an increase in the marginal conditional expectation of penalty costs associated with a deficiency.

What are the other implications of this two-asset model regarding the behavior of financial institutions? The financial institution would engage in a simple kind of diversification in that it would hold both loans and cash at the start of the period. But this will not explain diversification among loans if they are considered to be nonhomogeneous. There is a liquidity adjustment response, but one can hardly call it optimal since it is imposed by the assumption of two assets, one of which is illiquid. Furthermore, as we will see later, this liquidity response is not optimal even for a two-asset model if the financial institution is risk-averse and if the return on cash (free reserves) and/or the cost of borrowing is random. The loan-demand function that can be derived from this optimization problem [given tractable but perhaps unrealistic assumptions regarding $F(d)$] implies that loan demand for the financial institution is independent of the deposit rate. Relaxation of the implicit assumption
that deposit markets are perfectly competitive would alter this result.
Similarly, the addition of an imperfectly elastic loan supply function
would be consistent with the view of loan market competition held by
many writers, e.g., Chandler (1938), Alhadeff (1954), and Shull (1963).

Without imperfect market assumptions, loan demand will be depend-
ent on the distribution of deposit drains. By assuming a zero-one normal
distribution for \( d \), Orr and Mellon obtain results in which the depend-
ence on the mean and variance of \( d \) is not readily apparent. However,
one could question their distribution assumption. For example, as Cooper
noted, it is not clear that the variability of the deposit drain should
be assumed to be independent of the quantity of deposits.

The most restrictive assumptions of this model of a deposit rate-
setting financial intermediary are (1) the restriction of liquidity ad-
justment to one asset and (2) the linear utility function.

Taking the model given in Equations (2a) through (2d), what is the
effect of adding a third asset? Compare the liquidity constraint given
by (2c) with that given by (3c). Even if we continue to view loans as
being completely illiquid, the objective function will not, in general,
reduce to a single-decision optimization problem such as that given by
the objective in (4).

A model by Porter (1961) does reduce to this simple form. Porter
begins by assuming that cash balances are a fixed proportion \((k)\) of
deposits so that a fixed fraction of the deposit drain will be accounted
for by a change in cash balances.\(^7\) The remaining fraction of the deposit

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7. Porter only considers deposit drains because his model is based on the
response to the "deposit-low" during the decision horizon. In the model
drain will be met first by selling securities and, if this exhausts the amount of securities available, by using loans as collateral for borrowing (negative cash position). In the context of the model I have been using (with \( k = 0 \) for simplicity, a linear utility function and non-stochastic security returns), this liquidity-response scheme leads to the following objective function

\[
\max_{L, R_0} \left\{ E(R_L) L + R_B (D + W_0 - L) - R_B \left[ \int_{0}^{L-D-W_0} dF(d) - \int_{L-D-W_0}^{-\infty} [A - a(d)] dF(d) \right] - R_B (D + d) \right\}.
\]

As long as the foregone return on securities is less than or equal to the variable cost of borrowing, the liquidity response imposed in this version of Porter's model is unobjectionable. However, if we allow for the possibility that \( R_B \) is less than \( a \), then with \( A > 0 \), there is a deposit drain large enough that the optimal response at the time of the drain would be to borrow. In the United States, evidence on the relationship between security yields and central bank borrowing rates is such that one would not like to preclude this possibility a priori.\(^8\)

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used here, this is equivalent to imposing a zero upper limit on the distribution of deposit changes.

8. Porter incorporates the supposed reluctance to borrow in the variable cost of borrowing rather than introducing it as a fixed cost. In this case, the empirical evidence on the relationship between the nominal bond return and borrowing cost does not preclude having the borrowing cost consistently greater than bond return when costs due to reluctance to borrow are included. It is not clear, however, that a bank would be twice as reluctant to borrow, say, $2 million as it would be to borrow $1 million.
The assumption that desired cash balances (or free reserves) are a fixed proportion of the amount of deposits can also be questioned. With a zero return on cash there is no reason for cash to be held. If we introduce fixed adjustment costs for changes in the securities portfolio, there will be an economic justification for holding cash balances, and the amount of cash held will depend on security yields and the adjustment costs for securities.

The fixed liquidity-response scheme becomes harder to justify if we consider security returns to be stochastic. The following simple example will illustrate this point. Suppose that \( R_L > E(R_B) \), that the distribution of deposit drains is binary with outcomes \( d_1 \) and \( d_2 \) having probabilities \( p_1 \) and \( 1 - p_1 \), respectively, that the return on securities is a random variable with outcomes \( R_{B,1} \) and \( R_{B,2} \) having probabilities \( p_2 \) and \( 1 - p_2 \), respectively, and that the two random variables are independent of each other. If the liquidity response is fixed (as in Porter's model), the optimum loan position depends on the model parameters as follows:

\[
\begin{align*}
\text{If } E(R_B) \geq a, \quad &\text{then } L = D, \\
\text{If } E(R_B) < a, \quad &\text{then } L = D + d_2.
\end{align*}
\]

However, if we allow for an optimal liquidity decision, it can be shown that the optimal loan position depends on the parameters of the model in the following, more complex way:

9. Actually, as Porter's model suggests, changes in security prices make the problem more complex than this simple example. However, it will serve to illustrate the point to be made here.
If \((a-R_{B,2})(1-p_2) + E(R_B) < R_L\), then \(L = D\).

If \((a-R_{B,2})(1-p_2) + E(R_B) > R_L\) \{ 
and \((a-R_{B,2})(1-p_2)(1-p_1) + E(R_B) < R_L\) \}
, then \(L = D + d_1\).

If \((a-R_{B,2})(1-p_2)(1-p_1) + E(R_B) > R_L\), then \(L = D + d_2\).

This, of course, is simply another illustration of the suboptimality of policies that do not use feedback in a dynamic context.

A study by Aigner and Bryan (1971) deals specifically with the liquidity adjustment problem while disregarding the question of the optimal initial portfolio. They assume a net change in a set of exogenous assets and liabilities, e.g., demand deposits, savings deposits, and nonbank loans, and maximize the expected value of the bank's profits over the remaining assets and liabilities. The returns on the various assets and liabilities that are the decision variables for this financial institution are expressed as the sum of a nominal return plus a "subjective" return. These subjective returns are related to "risk over the adjustment portfolio, disfavor of the Federal Reserve, liquidity concerns based on uncertainty regarding future deposit movements and loan demands. . . ." With the further assumption that the behavior of the subjective return component leads to decreasing marginal return for increases in assets and increasing marginal cost for increases in liabilities, Aigner and Bryan obtain demand functions from the liability adjustment process that, in general, imply adjustment of all the assets and liabilities amenable to short-run change.
In addition to the question of the adequacy of a model that deals only with the liquidity question, the Aigner-Bryan model has dealt in an ad hoc way with the two other problems mentioned earlier. Risk aversion has been introduced through the additive (and presumably non measurable) subjective element in returns rather than by introducing specific assumptions regarding the attitude of the financial institution toward risk. Furthermore, they have displayed the need for analysis in a multiperiod, dynamic context by their assumption that the subjective return element includes liquidity premia associated with future changes in the exogenous assets and liabilities.

A number of authors, e.g., Kane and Malkiel (1965), Pierce (1967), and Porter (1961), have considered models of the price-setting financial institution where the firm is presumed to optimize a mean-variance preference function. If one disregards the liquidity question, a simple, Markowitz (1959) portfolio model results. As is well known, such models can explain diversification among assets for reasons other than liquidity needs. If one incorporates assumptions that allow for imperfect competition in loan markets, this mean-variance model will combine monopoly power and risk differences in the explanation of equilibrium loan rates.

A richer analysis can be obtained by introducing stochastic deposits into the mean-variance model. Consider a two-asset model (loans and securities) similar to the Kane-Malkiel model. With illiquid loans, a shift in deposit demand must be accommodated by a change in security holdings. Assuming that both loan and security returns are random variables and that a
mean-variance objective function is appropriate, the decision problem for
the financial institution is:

$$\max_{L, B} U[E(W_1), V(W_1)],$$

Subject to $L + B = D + W_0,$

$$b = d,$$

where

$$E(W_1) = E(R_L)L + E(R_B)(B+b) - R_D[D + E(d)] - C,$$

$$V(W_1) = V(R_L)L^2 + V(R_B)(B+b)^2 + R_D^2 V(d)$$

$$+ 2CV(R_L, R_B)L(B+b) - 2CV(R_L, d)R_DL$$

$$- 2CV(R_B, d)R_D(B+b).$$

For this model, in addition to the asset diversification implied by
the variance and covariance of asset yields and the impact of deposit vari-
ability, the behavior of financial institutions is also influenced by the
covariability of the deposit drain and asset returns. If we are willing
to assume that the random element in loan yields is due only to default
and that deposit drains are determined by changes in the return-risk rela-
tionships among deposits and other assets held by depositors, it would be

10. In the following analysis, this notation applies: $E(X), V(X),$ and
$CV(X,Y)$ will represent the expected value of $X,$ the variance of $X,$ and
the covariance of $X$ and $Y,$ respectively.
reasonable to assume that the loan rate and the deposit drain are independent. It is more difficult to dismiss the idea of dependence between the return on the marketable securities held by the financial institution and the deposit drains that are a function of market returns.

As we have seen in the case of a linear preference function, the addition of a second method of liquidity adjustment makes the liquidity decision nontrivial, and the feedback from the optimal liquidity decision makes the portfolio decision more complex. A nonlinear objective function adds a dimension to this problem, because the optimal liquidity response depends on higher moments of the joint distribution of asset and liability returns as well as on the expected value of these returns. A full analysis of the optimal portfolio decision, given the feedback from the liquidity decision and assuming that the preference function is nonlinear, is not attempted here. However, to illustrate the point, suppose that the financial intermediary can meet its liquidity needs either by selling securities or by borrowing at a known rate, $a$. With $E(R_B) > a$, the opportunity loss in terms of expected return is greater if the security portfolio is reduced but, presuming a positive correlation between asset yields, the marginal effect on portfolio risk is greater if the financial institution resorts to borrowing.

To conclude the discussion of deposit rate setting institutions, I shall mention a related pair of papers by Thore (1968) and Cohen and Thore (1970). They present a normative model that takes both the portfolio decision and the liquidity decision into account. The model they employ is a two-stage linear program based on the Dantzig-Madansky (1961) technique.
It is not clear at this point how useful this linear programming model would be in answering the kind of questions that are analyzed in the positive economic models described thus far. At the least, sensitivity analysis of this model would be useful.

**DEPOSIT QUANTITY SETTING**

The quantity setting financial institution is assumed to choose the quantity of deposits it will hold, accepting in the process the return it must pay on those deposits. The most obvious case for which this behavior is applicable occurs when the deposits are in fact money market securities traded on a perfectly competitive market. In the United States, the liabilities of the financial institutions that approximate this specification include CDs, federal funds, Eurodollar Loans, finance paper, and broker call loans. However, the market for the liability in question need not be perfect for the quantity setting behavior to be appropriate; the important characteristic is that a given supply of the deposit is immediately taken by those who demand such securities. The quantity setting financial institution is presumed to have control over the timing of the net deposit accumulation or withdrawal.

Studies that view financial institutions as quantity setting firms include those by Hyman (1969), Parkin (1970), and Pyle (1968, 1971). The optimization problem for such institutions is assumed to involve only one decision, which takes place at the start of the firm's portfolio horizon. The general form of the model may be written as:
\[
\begin{align*}
\text{Max} & \quad E[U(W_t)], \\
L, E, M, D & \\
\text{Subject to} & \quad L + B + M = D + W_0,
\end{align*}
\]

where

\[
W_1 \equiv R_L L + R_B B + M - R_D D - C,
\]

and \(L, E, M,\) and \(D\) are nonnegative.

Hyman considers the case in which the return on deposits is not stochastic. An assumption, which is consistent with no uncertainty regarding deposit rates, is that the maturity of deposits matches the financial institution's portfolio horizon. With the return on loans and securities assumed to be stochastic, Hyman's model is an extension of a simple version of Markowitz' portfolio model in which the quantity of funds available for investment is a decision variable. As such, it provides the usual mean-variance justification for asset diversification. Furthermore, as Hyman shows, asset demand is a function of the deposit return, as well as of the parameters of the joint distribution of asset returns. As noted earlier, this is only true for a rate setting financial institution when there is imperfect competition in deposit markets. Perhaps the most interesting result for the quantity setting behavioral model is that, given appropriate relationships among the parameters of the decision problem, a risk-averse financial institution will have a determinant size, even with economies of scale in operating costs and perfect competition in asset and deposit markets.

To illustrate these points, consider the more general case proposed by Parkin (1970) and Pyle (1971) in which the deposit rate is also a random
variable. To keep things simple, consider a two-asset model and suppose that the loan return is stochastic but that the security return is certain (the maturity of the securities matches the decision horizon). After eliminating $B$ from the model by substituting in the balance sheet constraint, maximization of a mean-variance preference function for this case gives the following solution for the optimal quantity of loans ($L^*$):

$$L^* = \frac{E(R_L) - \frac{3C}{3L} - R_L}{\theta \{V(R_L)V(R_D) - [CV(R_L, R_D)]^2\}} \left( - \frac{E(R_D) + \frac{3C}{3D} - R_D}{CV(R_L, R_D)} \right)$$

where $\theta$ is a function of the ratio of the marginal disutility of variance to the marginal utility of expected return for the given financial institution. The dependence of loan demand by the financial institution on the deposit rate and, for the more general model, on the variance and covariance of loan and deposit rates is apparent from this result. Furthermore, the sufficiency conditions for this to be a finite, global optimum hold if the diseconomies of scale associated with increasing risk exposure at some point offset the economies of scale that Benston and others attribute to cost functions for financial institutions. Note that for any given set of assets and liabilities, if all the financial institutions trading in those assets and liabilities have the same parameter estimates, they will all hold the same relative portfolio, e.g., the percentage of assets held as loans will be the same for all institutions, while the overall size of each institution will depend on the risk aversion displayed by its managers, i.e., on $\theta$. As both Parkin and Pyle have shown, this analysis can easily be
extended to any number of assets and liabilities with qualitatively similar results.

Are there any financial institutions that behave as if they are quantity setting firms? Parkin suggests that in the United Kingdom, the discount houses display the type of behavior that is here characterized as quantity setting behavior. The deposit sources for discount houses are call loans and discounts and advances, the latter being "last-resort loans" by the Bank of England. Both of these liabilities have short maturities and together they constitute more than 90 percent of the liabilities of discount houses. Unless the decision horizon is taken to be very short indeed, discount houses are faced with a call loan rate and a bank rate that are unknown at the beginning of their decision horizon. In fact, Parkin tests a model in which both assets and these two liabilities are presumed to have uncertain rates against a model in which there is no borrowing uncertainty, i.e., a model consistent with Hyman's specification, and finds that the former model performs better. In the United States, one can suggest finance houses with finance paper as a major liability and nonbank bond dealers using call loans as a source of funds as examples of financial institutions that might be expected to display quantity setting behavior.

In recent years, United States commercial banks have been able to engage in quantity setting with respect to some of their fund sources, in particular Certificates of Deposit, Eurodollars, and, where they have been used as a more or less continuous source of funds, federal funds. This suggests that it might be useful to consider commercial banks as being mixed rate and quantity setting firms.
The single-decision model of a quantity-setting financial institution implies that there is no liquidity problem in such firms. The presumption is that a shift in the deposit demand function of the sort I considered in the case of rate setting financial institutions induces a change in the deposit rate and that the anticipation of such shifts at the beginning of the decision period is the source of uncertainty about the deposit rate over the period. But as Cootner (1969) notes in his discussion of liquidity in savings and loan associations, this need not be the optimal response. The quantity setting firm may prefer to reduce the quantity of assets held if the expected risk-return relationship for the assets at that time is not commensurate with the new risk-return relationship for deposits. For the case in which the deposits involved have a perfectly elastic demand, the choice is either to raise the rate or liquidate the assets. However, if the financial institution faces a less than perfectly elastic demand for its deposits, an intermediate solution is possible. The financial institution could choose a new quantity of deposits to hold, e.g., by not raising its deposit rate to the level necessary to retain all of the existing deposits, and could choose to liquidate some of its assets. For a quantity setting financial institution facing the latter type of deposit demand, the criticisms of single-decision models expressed in the previous section apply.

SOME SUGGESTIONS REGARDING RESEARCH

Given the state of the art in the theory of financial institutions under uncertainty as described above, what sort of research on this topic
appears promising? Within the framework of existing models, an analysis that considers both the portfolio and the liquidity problem and in which feedback from an optimal liquidity decision is allowed to influence a portfolio decision that is prior in time would be a step forward. As suggested earlier, the two-stage programming approach adopted by Thore (1968) is a move in this direction. 11

The treatment of loan decisions in the existing models of financial institutions under uncertainty is not satisfactory. Loan quantities are usually treated as being subject only to a nonnegativity constraint at the start of the portfolio decision horizon but as being completely illiquid at the time of the liquidity decision. Hester and Pierce (1968) suggest that the cost of liquidating assets is a function of the time available to arrange the disposal of the assets. This explicit treatment of liquidity as a time-dependent process is not compatible with the essentially static models considered here, but in a more dynamic treatment their approach could prove fruitful. Similarly, the assumption that the financial institution is free to choose its loan portfolio at the time of the portfolio decision is hard to accept. For example, in a deterministic, multiperiod framework, Goldfeld and Jaffee (1970) suggest that the stickiness of deposit rates at savings and loan associations is a result of the inherited mortgage portfolio with which these institutions must cope in such a

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11. The problem of modeling both liquidity and portfolio behavior of economic actors is not unique to the case of financial institutions. An analogous open problem exists for an individual with a given current wealth where the funds needed for current consumption (or the net transaction flows) and portfolio investment outcomes are both random variables.
framework. In general, where the financial institution has made loans that mature after the time at which a decision is made regarding the current portfolio of assets, the decisions are constrained by the nature of the existing portfolio and the costs of liquidating it. This, too, suggests the need for a more dynamic view of the behavior of financial institutions.

At the present time, a more dynamic approach presents serious analytical problems. For example, Fevang (1972) recently presented a dynamic programming analysis of a bank with a portfolio of loans, some of which mature in each of \( n - 1 \) future periods. He found that optimal policies for \( n > 3 \) are very complex and not well suited for the kind of comparative analysis to which we would like to subject a positive model of a bank. To obtain simple policies, e.g., a rule for choosing the incremental qualities of each of the \( n - 1 \) loan maturities to hold in the loan portfolio, he had to make very restrictive assumptions about future interest rates. The recent work on multiperiod models of portfolio selection, e.g., Fama (1970) and Hakansson (1971), provides a basis for

12. The Goldfeld-Jaffee paper clarifies some issues raised in the earlier exchange between Weber (1966, 1967) and Meyer (1967), and it illustrates the importance of inherited loans in the analysis of the behavior of financial institutions. However, as Hendershott (1971) shows, the Goldfeld-Jaffee conclusion regarding the stickiness of deposit rates is weakened considerably when one considers the macroeconomic framework as well as the microeconomic decision made by the financial institution, e.g., when one considers deposit demand in a macroeconomic framework as well as deposit supply in a microeconomic framework.

13. My understanding and interpretation of Fevang's results are based on a discussion with him. He is not responsible for any misinterpretation of those results that may have occurred. I wish to thank him for his help.
a more dynamic analysis of financial institutions in much the same way that the static portfolio models have been used as a basis for static models of financial institutions. As in the case of the static models, this approach is best suited to those cases in which the assets are marketable.

Another aspect of the behavior of financial institutions, with respect to illiquid assets, such as loans, that may deserve additional consideration, is the influence of expectations regarding future loan opportunities on liquidity decisions. As Cootner (1969) and others point out, financial institutions that anticipate random fluctuations in the loan demand function may be as concerned about this source of uncertainty as they are about random fluctuations in deposit demand and may hold liquid assets in anticipation of future loan demand needs. In the case of single-decision models of financial institutions, it is possible to view the stochastic returns on loans as a proxy for the foregone opportunity to lend at more favorable rates in the future as well as being due to default risk. This is not a very satisfactory treatment of the problem, and loan demand induced liquidity requirements remain an open area for research.

Among the models of financial institutions under uncertainty that have been discussed here, the single example of an empirical test carried out in the context of the suggested choice model under uncertainty is in the paper by Parkin (1970). In part, this uniqueness reflects the character of the assets and liabilities held by discount houses, since they are money market securities and have relatively accessible and objective data series. Parkin did not exploit the full stochastic structure of his problem, however,
due to an inability to proxy the parameters of the covariance matrix of asset and liability returns. Two lines of research are suggested by this. First, the empirical validity of the quantity setting behavioral model for United States institutions such as finance houses and nonbank bond dealers could be pursued along the lines of Parkin's work. Second, additional work on the problem of explicitly considering the covariance matrix in such an empirical test could be taken up.

In the models that we have considered here, the behavior of financial institutions is presumed to reflect expected utility maximization on the part of the managers of the institutions. It has been argued (Michaelson and Goshay (1967)) that a utility-preference approach is necessary to explain the behavior of financial institutions because of the indeterminancy of share maximization when individuals can engage in "homemade diversification." In Pyle (1968), this proposition is demonstrated for a world in which the assets and liabilities of financial institutions have risk-return relationships that are consistent with the Sharpe (1964) capital asset pricing model. However, the assumptions of that model preclude the need for financial institutions. On the other hand, if a financial institution can buy and sell some securities (loans and deposits) that, for that institution, have risk-return relationships different from those prevailing on marketable securities, and if it maximizes the value of shares traded on a mean-variance efficient capital market, the resulting loan-demand and deposit-supply functions are similar to those obtained for the single-decision, quantity setting model described earlier.  

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14. This is an example of the point made by Leland (1972): "the notion that investors (who are risk averse) control the firm . . . would seem to
A quite different view of the appropriate objective function to describe the behavior of financial institutions emerges from the recent work by Pye (1971) and Bawa (1972) on minimax policies for selling assets. Pye (1972) has used a regret function in the analysis of bank money management in the federal funds market. In an empirical section of this work, he suggests that the data on federal funds trading "appear to provide evidence for rejecting the hypothesis of risk-averse, expected utility maximization as the mode of behavior in bank money management." The investigation of regret minimization alternatives to expected utility maximization as the basis for describing other decisions by financial institutions merits consideration.  

This review concentrated on the positive economics of financial institutions under uncertainty with the implication that the users of the models described are interested in the asset demand functions and the liability supply functions of such firms. However, many of the questions that economists have about financial institutions also require specification of the behavior of those who supply assets to financial institutions and those who have a demand for their liabilities. Although there is an extensive literature on the role of financial institutions in helping to satisfy the

suggest a risk averse utility function for the firm."

15. For example, one might note that the need for liquidity to meet unexpected loan demand could be thought of as resulting from regret minimization.

16. As noted earlier, Hendershott (1971) has made the need for consideration of external market-determined phenomena clear in his treatment of the determination of savings and loan deposit rates. Also, see Emery (1971) for a consideration of banking in the framework of a capital asset market.
intertemporal desires of primary lenders and primary borrowers, there is little in the way of analytical treatment of this process. This is particularly notable in the case of the mean-variance model of individual behavior and the capital asset equilibrium derived from it. As suggested earlier, the simplifying assumptions of that description of investor equilibrium leave no room for financial institutions. With the introduction of transaction costs and information costs, a role for institutions that have economies of scale with respect to such costs could emerge. Analyses in which transactions by individuals in certain classes of securities/(in particular, restrictions on short positions in such securities)\textsuperscript{17} are steps toward capital market models that could include financial institutions. The mean-variance capital asset pricing model has served as a strong tool for integrating the theory of investor behavior and the theory of corporation finance. The incorporation of financial institutions into the equilibrium market for assets would constitute another major step in the integration of the finance literature.

\textsuperscript{17} See Black (1972), for example.
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