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By

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OPTIMAL FORECLOSURE POLICIES

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I. Introduction and Summary

Frequently, loans are collateralized. This gives the lender the option of foreclosing and obtaining possession of the property used as collateral if the borrower does not meet his obligations. However, even if the borrower misses a payment, the lender may not want to exercise his foreclosure option. The reason is that he may still feel that there is a good chance that the borrower will make the remaining payments. The expected present value of these payments to the lender, though now less than before, may still exceed the value of the collateral. This is especially true after taking into account foreclosure and selling costs.

The purpose of this paper is to study the conditions under which foreclosure is optimal. The associated problem of valuing a collateralized loan subject to default will also be considered. The mode of analysis is discrete, dynamic programming. The state variable on which the foreclosure decision is conditioned is taken to be the
number of periods that a payment is late. Thus, the probability of receiving the remaining payments is taken to decrease in a prescribed manner as the length of time of delinquency increases. The judgment of an experienced collection agent as to this probability might depend on other factors as well. However, it seems that this simple objective evidence would frequently be overriding. This is especially true in the case of small loans where the cost of extensive investigation is prohibitive.

In Section II the problem is specified and the general recursion relation for the dynamic programming problem is formulated. In Section III the special case is studied in which foreclosure always takes place as soon as a payment is one period late. This policy is shown to be optimal as long as the probability of receiving the late payment is less than a certain value. A method is presented for calculating this upper bound in terms of the cost of capital, the length of contract, the future anticipated decrease in the value of the collateral, and the probability of receiving future payments when payment is on time. The effect on this upper bound of changes in these parameters is investigated. In Section IV the results of Section III are applied to a case given by Robichak and Coleman [1]. This case describes a company which finances insurance premiums. If payments from the insured are late, the finance company can, in effect, foreclose by cancelling the policy to receive a refund on the unearned premium from the insurance carrier. The finance company follows the policy of foreclosing whenever a payment is one period late. Based
on data given in the case, an upper bound is calculated on the probability of receiving the late payment such that this given policy is optimal. Optimal policies are also calculated for larger values of this probability.

II. Model Specification

Let a series of equal payments be scheduled in each of the next $T$ periods. Without loss of generality, the size of each payment will be set equal to one and the value of the collateral measured in terms of this payment. If a payment has not been received by the end of the period in which it is scheduled, it is considered to be late one period. A foreclosure decision is then made at the end of the period subject to the condition that a payment is late one period. The periods are formed about the actual due dates to allow time for the customary reminders by mail or phone before the end of the period. If the payment has still not been received by the end of the following period, it is deemed to be late two periods and another foreclosure decision takes place. This process continues until either foreclosure takes place or the payment is received. The number of periods that the payment is late will be denoted by $n$.

Let $V_t(n)$ denote the maximum expected present value of the loan payments subsequent to $t$, given that an optimal foreclosure policy is used and that payment at the end of $t$ is $n$ periods late. If a payment which is $n$ periods late is received in $t$, it is assumed that a new agreement is negotiated which has an expected
present value equal to \( n + 1 + V_t(0) \). Typically, only one late payment will be received. The borrower, however, will indicate a desire to negotiate a stretched out schedule for the remaining payments. It is assumed that the lender is willing to accommodate him at no further loss in expected present value.

If foreclosure takes place at the end of \( t \), the borrower receives collateral with an expected present value to him of \( b_t \). In addition, he may receive a deficiency judgment on the back payments with an expected present value of \( \xi_n \). Sometimes by foreclosing the borrower gives up any right to the back payments. This is true, for instance, under California law when the collateral is household effects. In such a case, \( \xi_n \) would be equal to zero. By law the borrower is also usually precluded from receiving more than the back payments plus the unpaid balance of the loan from sale of the collateral. Any overage would have to be repaid to the borrower. Frequently, if the value of the collateral is in excess of the balance owed, the borrower will sell the collateral himself and make provision for the buyer to assume the unpaid balance. If such legal constraints or options apply, \( b_t \) will satisfy the constraint, \( b_t \leq 2 + V_t(0) \), which is simply a special case of the analysis to follow.

The model will be formulated so that it applies either to conventional borrowing or to leasing, where the lender has title to the collateral upon the completion of the contract. This is done by letting \((1-\beta)b_{t+1}\) represent the lender's equity in the collateral in the period following completion of the contract. For a pure leasing
contract $\beta = 0$ and for a pure loan contract $\beta = 1$. Intermediate values of $\beta$ represent cases where the leasee has an option to buy the property at a discount upon completion of the contract.

The present value this period of a dollar received next period will be denoted by $\alpha$. The probability of receiving payment next period when payment is currently $n$ periods late will be denoted by $\gamma_n$. The product $\alpha \gamma_n$ which occurs frequently in the analysis will be denoted $\gamma_n$. It will be assumed that the expected value of the receipts from late payments given by $\gamma_n$ declines with $n$. In particular, this implies that $\gamma_n \leq \gamma_1$ and $C_n \leq C_1$ for all $n$.

Given $n$ and that foreclosure does occur at the end of $t$, there are two possible chance events in the next period. Either payment is received from the borrower or it is not. In the former case, the lender receives a payoff with an expected present value of $n + 1 + V_{t+1}(0)$. In the latter case, he receives his option between foreclosing and receiving a payoff of $b_{t+1} + \xi_{n+1}(n+1)$ or not foreclosing and receiving a payoff of $V_{t+1}(n+1)$. Clearly, if he is behaving optimally, he will pick the larger of these. $V_t(n)$ must therefore satisfy the recursion relation given in (1):

(1) \[ V_t(n) = C_n[n+1 + V_{t+1}(0)] + (\alpha - C_n)\text{Max}\{b_{t+1} + \xi_{n+1}(n+1), V_{t+1}(n+1)\} \]

\[ V_t(n) = \alpha(1-\beta) b_{t+1} + C_n \]
where:

\[ n = \text{number of periods payment is late.} \]

\[ \alpha = \text{present value this period of } \$1 \text{ next period} \]
\[ (0 < \alpha \leq 1) \, . \]

\[ C_n = \alpha \gamma_n \text{ where } \gamma_n \text{ is the probability of receiving} \]
\[ \text{payment next period given } n \, . \]

\[ b_t = \text{expected present value of the collateral to the} \]
\[ \text{lender at } t \, . \]

\[ \xi_n = \text{discount factor for } n \text{ back payments upon} \]
\[ \text{foreclosure.} \]

\[ V_t(n) = \text{maximum expected present value of payments} \]
\[ \text{subsequent to } t \text{ given } n \, . \]

\[ T = \text{last period contractual payments are due.} \]

\[ \beta = \text{proportion of equity in the collateral held by} \]
\[ \text{the borrower upon completion of the contract.} \]

Given values of the parameters, the optimal, sequential foreclosure
policy is easily determined using (2), by working backward from the
horizon in the usual fashion of dynamic programming. The value of a
collateralized loan subject to default arises from this solution as
\[ V_0(0) \, . \]
III. Optimal Zero Delay Foreclosure Policies

In this section the special case will be studied in which foreclosure is always optimal whenever \( n = 1 \). In particular, a technique is derived for determining the upper bound on \( \gamma_1 \) such that this zero delay foreclosure policy is optimal. Another result of the analysis is to provide an analytical expression for the value of a collateralized loan subject to default under these conditions.

To simplify the development, it will be assumed that the value of the collateral decreases each period by a constant amount so that \( \Delta b_t = -\lambda \). Furthermore, it will be taken that \( \lambda > \gamma_1 \). For convenience, it will also be assumed that \( b_{T+1} = \xi_n = 0 \). The general plan of attack is to obtain a closed form solution for \( V_t(0) \) under the condition that foreclosure is always optimal when \( n = 1 \). Formally, this requires \( b_t \geq V_t(1) \) for \( t = 1,2,\ldots,T \). Under this same condition, an expression is then obtained for \( V_t(1) \) in terms of \( V_t(0) \). This expression for \( V_t(1) \) is substituted into \( b_t \geq V_t(1) \) to obtain for each \( t \) the maximum value of \( \gamma_1 \) which will satisfy this inequality. The form of these upper bounds is shown to be such that their minimum value is easily calculated. This least upper bound is the maximum value of \( \gamma_1 \) for a zero delay foreclosure policy to be optimal in every period.

If \( b_t \geq V_t(1) \) for \( t = 1,2,\ldots,T \) the solution to (1) for \( V_t(0) \) is given in (2).

\[
V_t(0) = \frac{C_0(1-C_0^{T-t})}{1-C_0} \left( 1 - \frac{(\alpha-C_0^0)\lambda}{1-C_0} \right) + \frac{(\alpha-C_0^0)}{1-C_0} b_{t+1}
\]
To verify that (2) is the required solution, set \( n = \xi_n = b_{T+1} = 0 \) in (1) and then substitute (2) on the right hand side of (1) to give the following:

\[
V_t(0) = C_0(1 + V_{t+1}(0)) + (\alpha - C_0) \max\{b_{t+1}, V_{t+1}(1)\}
\]

\[
= C_0\left[1 + \frac{C_0(1-C_0)^{T-t-1}}{1-C_0}\right]\left\{1 - \frac{(\alpha-C_0)\lambda}{1-C_0} + \frac{(\alpha-C_0)\lambda}{1-C_0} b_{t+2}\right\} + (\alpha-C_0)b_{t+1}
\]

\[
= C_0\left[1 + \frac{C_0(1-C_0)^{T-t-1}}{1-C_0}\right]\left\{1 - \frac{(\alpha-C_0)\lambda}{1-C_0} + \frac{C_0}{1-C_0} + 1\right\}(\alpha-C_0)b_{t+1}
\]

\[
= C_0(1-C_0)^{T-t}\left\{1 - \frac{(\alpha-C_0)\lambda}{1-C_0}\right\} + \frac{(\alpha-C_0)}{1-C_0} b_{t+1}
\]

\[
= V_t(0)
\]

It is to be noted that (2) also satisfies the initial condition

\[
V_T(0) = 0
\]

The next step in the analysis is to calculate \( V_t(1) \) in terms of \( V_t(0) \) under the condition that \( b_t \geq V_t(1) \) for \( t = 1, 2, \ldots, T \).

To do this it is first necessary to prove that \( b_t \geq V_t(1) \) implies \( b_t \geq V_t(n) \) for \( n \geq 1 \). An inductive argument will be used. Assume that \( b_{t+1} \geq V_{t+1}(n) \) for \( n \geq 1 \) which with (1) and the conditions on the parameters gives:

\[
b_t \geq V_t(1) = C_1(2 + V_{t+1}(0)) + (\alpha-C_1)b_{t+1}
\]

\[
= \alpha b_{t+1} + C_1 + C_1(1 + V_{t+1}(0) - b_{t+1})
\]

\[
\geq \alpha b_{t+1} + C_n + C_n(1 + V_{t+1}(0) - b_{t+1})
\]

\[
= V_t(n)
\]
Establishing the inequality in the next to last line depends on whether \( b_{t+1} \) is greater or less than \( 1 + V_{t+1}(0) \). If
\[ 1 + V_{t+1}(0) - b_{t+1} \geq 0, \]
it is sufficient to recall that \( C_n n \leq C_1 \).
If \( 1 + V_{t+1}(0) - b_{t+1} < 0 \), transpose \( \alpha b_{t+1} \) to the left hand side and note that \( C_n n \leq C_1 = \alpha y_1 < \alpha \lambda < b_t - \alpha b_{t+1} \). The inductive argument is completed by noting that \( b_T = \lambda > C_1 \geq C_n n = V_T(n) \) for \( n \geq 1 \).

As a special case of the result established in the last paragraph, it follows that \( b_t \geq V_t(2) \) for \( t = 1, 2, \ldots, T \) when \( b_t \geq V_t(1) \) for \( t = 1, 2, \ldots, T \). Therefore, under this condition \( V_t(1) \) may be written as given in (3):

\[
V_t(1) = C_1 (2 + V_{t+1}(0)) + (\alpha - C_1) b_{t+1} \quad t = 1, 2, \ldots, T-1
\]

The next step is to use the relation obtained in (3) to derive an upper bound on \( y_1 \) or \( C_1 = \alpha y_1 \), if the condition \( b_t \geq V_t(1) \) is to be satisfied for \( t = 1, 2, \ldots, T \). First note that \( b_t \geq V_t(1) \) for any \( C_1 (0 \leq C_1 \leq 1) \), if \( 2 + V_{t+1}(0) - b_{t+1} \leq 0 \). This is true because \( b_t - \alpha b_{t+1} \leq C_1 (2 + V_{t+1}(0) - b_{t+1}) \) must hold as the left hand side is positive and the right hand side nonpositive and using (3) this inequality may be written as \( b_t \geq V_t(1) \). In this case foreclosure must always be optimal whenever it is possible because the value of the collateral exceeds the expected present value of the loan even if payment is received. The interesting case is where \( 2 + V_{t+1}(0) - b_{t+1} > 0 \). In this case using (3), \( b_t \geq V_t(1) \) can be transformed into the following inequality where the right hand side is independent of \( y_1 \):
\[ C_1 \leq \frac{b_t - \alpha b_{t+1}}{2 + V_{t+1}(0) - b_{t+1}} \quad t = 1, 2, \ldots T-1. \]

Let \( a_t \) and \( a^* \) be defined as follows:

\[
a_t = \begin{cases} 
\frac{b_t - \alpha b_{t+1}}{2 + V_{t+1}(0) - b_{t+1}} & 2 + V_{t+1}(0) - b_{t+1} > 0 \\
1 & 2 + V_{t+1}(0) - b_{t+1} \leq 0
\end{cases}
\]

\[
a^* = \min_{1 \leq t \leq T-1} \left\{ a_t \right\}.
\]

It follows that foreclosure will be optimal for \( n = 1 \) at \( t = 1, 2, \ldots T-1 \), if \( C_1 \leq a^* \). It also follows that having \( C_1 \leq a^* \) is a necessary condition for foreclosure to be optimal for \( n = 1 \) at all \( t \). Suppose foreclosure were optimal for \( n = 1 \) at all \( t \) and \( C_1 \) were greater than \( a_t \) for one or more \( t \). Let the largest such \( t \) be \( t' \). It follows that \( V_{t'}(1) \) is given by (3) and therefore that \( b_t < V_{t'}(1) \) which is a contradiction. It is assumed throughout that \( a^* \leq \alpha \lambda \).

Otherwise, \( \alpha \lambda \) must be the upper bound on \( C_1 \) to fulfill the constraint, \( \gamma_1 < \lambda \), imposed before.

It will next be shown that the calculation of \( a^* \) is simplified by the special form of the \( a_t \). In particular, \( a^* \) will be equal to the smallest \( t \) \((1 \leq t \leq T-2)\) for which \( \Delta a_{t+1} \geq 0 \). Frequently, this will be \( t = 1 \) so that \( a^* = a_1 \). The sign of \( \Delta a_t \) may be determined directly or by use of the relation given below in (6). If \( \Delta a_{t+1} < 0 \) for all \( t \) on the range \((1 \leq t \leq T-2)\), \( a^* = a_{T-1} \).

To prove that \( a_t \) is of this form, let \( z_t = N_t/D_t \) where

\[
N_t = b_t - \alpha b_{t+1} \quad \text{and} \quad D_t = 2 + V_{t+1}(0) - b_{t+1} \quad \text{if} \quad 2 + V_{t+1}(0) - b_{t+1} > 0,
\]
\[ N_t = D_t = 1 \text{ if } 2 + V_{t+1}(0) - b_{t+1} \leq 0. \] It follows that
\[ \Delta a_t = (N_t/D_t) - (N_{t-1}/D_{t-1}) = (D_{t-1}\Delta N_t - N_{t-1}\Delta D_t)/D_{t-1}. \] Since \( D_{t-1} > 0 \), the sign of \( \Delta a_t \) will be the same as that of \( D_{t-1}\Delta N_t - N_{t-1}\Delta D_t \).

Writing this latter expression and simplifying gives:

\[
(6) \quad D_{t-1}\Delta N_t - N_{t-1}\Delta D_t = (2+V_t(0)-b_t)(1-\alpha)\Delta b_{t+1} - (\lambda+(1-\alpha)b_t)(\Delta V_{t+1}(0) - \Delta b_{t+1})
= -\lambda(\lambda+2(1-\alpha)) - \lambda(1-\alpha)V_t(0) - (\lambda+(1-\alpha)b_t)\Delta V_{t+1}(0).
\]

If \( \Delta(D_{t-1}\Delta N_t - N_{t-1}\Delta D_t) \geq 0 \), it follows that \( \Delta a_{t+1} \geq 0 \), if \( \Delta a_t \geq 0 \); and that \( \Delta a_{t-1} < 0 \), if \( \Delta a_t < 0 \). Therefore, to prove that \( a_t \) has the required property, it suffices to show that \( \Delta(D_{t-1}\Delta N_t - N_{t-1}\Delta D_t) \geq 0 \).

This may be done as follows:

\[
\Delta(D_{t-1}\Delta N_t - N_{t-1}\Delta D_t) = -\lambda(1-\alpha)\Delta V_t(0) - (1-\alpha)\Delta b_{t+1}\Delta V_t(0) - (\lambda+(1-\alpha)b_t)(2V_{t+1}(0))
= -\lambda+(1-\alpha)b_t \Delta^2 V_{t+1}(0)
= (\lambda+(1-\alpha)b_t)(1-C_0+\lambda(\alpha-C_0))C_0^{T-t}
\geq 0.
\]

In the third line (2) is used to obtain \( \Delta^2 V_{t+1}(0) \).

The next step in the analysis is to determine the effect of changes in the parameters on \( a^* \). In doing this, it will be assumed throughout that \( 1 + V_t(0) > b_t \) for all \( t \). In other words, the value of the collateral is taken always to be less than the value of the bond when payments are on time. This guarantees that default is a bad and not a good.
Differentiating $a_t$ with respect to $\gamma_0$ shows that the sign of $\partial a_t/\partial \gamma_0$ is the same as that of $-\partial V_{t+1}(0)/\partial \gamma_0$. The sign of $\partial V_{t+1}(0)/\partial \gamma_0$ is seen to be positive by a simple inductive argument. Writing $V_t(0)$ in recursive form and differentiating gives:

$$\frac{\partial V_t(0)}{\partial \gamma_0} = a \gamma_0 \frac{\partial V_{t+1}(0)}{\partial \gamma_0} + \alpha(1 + V_{t+1}(0) - b_{t+1}).$$

Therefore, if $\partial V_{t+1}(0)/\partial \gamma_0 \geq 0$, it follows that $\partial V_t(0)/\partial \gamma_0 > 0$. Since $\partial V_t(0)/\partial \gamma_0 \geq 0$, it follows by induction that $\partial V_t(0)/\partial \gamma_0 > 0$ for all $t$. Therefore, the sign of $\partial a_t/\partial \gamma_0$ must be negative for all $t$. As $\partial a_t/\partial \gamma_0 < 0$ for all $t$ it must also be true that $\partial a_t/\partial \gamma_0 < 0$ which was to be determined. By a similar argument, it can be shown that $\partial a_t/\partial \alpha < 0$. These results are as expected. An increase in $\alpha$, or an increase in $\gamma_0$ (when $b_t < 1 + V_t(0)$) raises the expected present value of the payments on the bond. As the future payments on the bond become more valuable, foreclosure becomes less desirable. Therefore, the maximum value of $C_1$ under which it is always optimal to foreclose as soon as possible must fall.

To determine the effect of a change in $\lambda$ calculate $\partial a_t/\partial \lambda$ obtaining:

$$\frac{\partial a_t}{\partial \lambda} = [D_t(l+(1-\alpha)(T-t))] - N_t \frac{\partial V_{t+1}(0) - b_{t+1}}{\partial \lambda} D_t^2.$$

Using (2) to calculate $\partial (V_{t+1}(0) - b_{t+1})/\partial \lambda$ shows it to be negative so that $\partial a_t/\partial \lambda > 0$ for all $t$. Therefore, $\alpha^*$ must increase with an increase in $\lambda$. This should also be expected. An increase in the
loss in value of the collateral over time makes foreclosures at any
given time more desirable.

IV. An Example

In this section the results derived in Section III will be
applied to a case presented by Robichek and Coleman [1, p.536-40] in
their casebook on financial institutions. This case concerns an
industrial loan company incorporated in the State of California
called Premium Finance. The business of Premium consists of loaning
money to the purchasers of automobile, liability, and fire insurance
policies for payment of the insurance premiums. Robichek and Coleman
report that agents frequently prefer financing through a third party
such as Premium as it alleviates the agents' fear of losing control
of their accounts, if their customers were to begin dealing directly
with the insurance carrier. Under the plan with Premium, the insured
buys a policy from an agent in the normal manner. Premium then pays
the insurance premium to the insurance carrier and notifies the
carrier of Premium's right to cancel the policy and to receive the
return portion of the unearned premium in the event of default by
the insured. In most instances, the loan is repayable to Premium
in ten equal monthly installments with a down payment of 25 percent.
An add-on financing charge of 5 to 15 percent is used to arrive at
the value of the contract.

The following example is provided in the case for a typical
contract. Assume a premium of $100. The down payment would then be $25.
This would make the value of the loan $75, and with a $10 finance charge the total value of the contract would be $85. The insured would be required to pay $8.50 in each of the next 10 months. If the first payment were not received at the end of the first month, the insured would be sent a warning. If the payment still has not been collected in the following two weeks, a cancellation notice would be sent to both the insured and the insurance carrier. The carrier would then refund to the finance company the unearned premium, which at that point would be $77. The same procedure would be followed in subsequent months if the payment were not received on time. However, the refund on the unearned premium would decline by approximately equal amounts each month.

The model developed earlier is easily fit to this example. The periods may be set equal to one month intervals starting one-half month after the contract is initiated. Thus, if the first payment is not received, the first foreclosure decision takes place at the end of the first period which is one and one-half months after the contract was initiated. The value of the collateral to Premium at any time is the value of the unearned premium they can obtain as a refund measured in terms of the monthly payment. The value of \( b_1 \) therefore is equal to 77/8.5 or approximately $9. Since \( b_1 \) decreases by equal amounts each period, the value of \( \lambda \) must be \( b_1/10 \) or $.9.

It is stated in the case that Premium obtains over 80 percent of their capital from short-term bank loans at 6 percent. It therefore seems reasonable to assume that Premium's pretax cost of capital
is around 8 percent. This translates into a monthly discount factor for \( \alpha \) of approximately .9935. Using this \( \alpha \), values of \( a^* \) were calculated for three different values of \( Y_0 \): .999, .99, and .95. For each of these values the sign of \( \Delta a_2 \) as calculated by (6) was positive. Therefore, in all three cases \( a^* = a_1 \). Using (4), these values of \( a^* \) were calculated to be .57, .59, .62 for \( Y_0 = .999, .99, \) and .95, respectively. As expected from the results at the end of Section III, \( a^* \) and \( Y_0 \) are inversely related. What is of interest here is that \( a^* \) is not very sensitive to a reasonable choice of \( Y_0 \). This is also true for \( \alpha \). This is significant since no information on the value of \( Y_0 \) is provided in the case. Thus, whether or not Premium's zero delay foreclosure policy is optimal has been reduced to an assessment of the probability, \( Y_1 \). If the probability of receiving a payment which is one period late is less than about .59, their existing policy is optimal.

It is of interest to also calculate the optimal policies for this example when \( Y_0 = .99 \) and \( Y_1 > .59 \). For instance, taking \( Y_1 = .70 \) it turns out using (1) that foreclosure is optimal when \( n = 2 \) for \( t = 2, 3, 4, 5, 6 \) and when \( n = 1 \) for \( t = 7, 8, 9 \) as long as \( Y_2 < .36 \). Taking \( Y_1 = .80 \) and \( Y_2 = .40 \), foreclosure is optimal when \( n = 3 \) for \( t = 3, 4, 5, 6 \), when \( n = 2 \) for \( t = 7, 8 \) and when \( n = 1 \) for \( t = 9 \) as long as \( Y_3 < .25 \). In general, in this case, the critical \( n \) at which foreclosure is optimal declines as time increases. It appears that this may not always hold especially when \( T \) is large. Conditions under which the optimal policy will necessarily have this general form is an area for further study.
Footnote

Though $a^*$ is not very sensitive to either $\gamma_0$ or $\alpha$, it is quite sensitive to $\lambda$. In particular, if $\lambda$ is reduced from .9 to .8, the value of $a^*$ falls from .59 to .32 when $\gamma_0 = .99$.

Reference