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THE EFFECTS OF PURCHASING POWER RISK
ON LIQUIDITY PREFERENCE

by

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In his celebrated "Liquidity Preference As Behavior Towards Risk," Tobin (1958) shows that the risk-aversion behavior of an investor in portfolio selection can provide a basis for liquidity preference and for an inverse relationship between the demand for cash balances and the rate of interest. The problem of portfolio demand for money was further studied by Hicks (1962) and later again by Arrow (1965). In his study, Arrow has made a significant contribution in showing that the wealth effects on risk taking and the portfolio demand for cash balances are determined by the slopes of the Pratt-Arrow measures of absolute and relative risk aversion possessed in the decision maker's utility function.¹

Many interesting results and implications regarding liquidity preference and risk taking are derived in these studies. However, the analyses in these studies are based upon the maximization of the expected utility of terminal wealth in monetary rather than real terms, and hence the purchasing power risk is completely ignored. It seems incomplete to

¹The absolute and relative risk aversions are defined as 
\[-U''(W)/U'(W)\] and \[-U''(W)W/U'(W)\] respectively, where \(U'\) and \(U''\) are respectively the first and the second derivatives of the utility function \(U(W)\) with respect to wealth, \(W\). See Pratt (1964) and Arrow (1965).


analyze liquidity preference and risk taking without incorporating this very real and important element of risk frequently encountered by an investor. The purchasing power risk does add an additional dimension to the definition of risk in portfolio selection. For instance, cash with a certain nominal rate of return of zero is a riskless asset if purchasing power risk is not considered, but it becomes a risky asset if such a risk is taken into account.

The purpose of this paper is twofold. First, it intends to investigate the effects of purchasing power risk upon liquidity preference in a static model and to see how the liquidity preference in a real-value model might be different from that of a money-value model. Second, it intends to analyze investor's liquidity preference within a framework of dynamic portfolio selection under the condition of uncertain inflation.

LIQUIDITY PREFERENCE IN THE STATIC MODEL

A Portfolio Selection Model with Purchasing Power Risk. I will incorporate the risk of purchasing power into a simple single-period portfolio selection model by assuming that the investor is to maximize the expected utility of terminal wealth in real rather than monetary terms in the allocation of his initial wealth among available assets. Following Tobin's analysis, we assume that there exist only two kinds of assets, one of which yields a certain nominal rate of return of zero and is to be called "cash," while the other yields a random nominal rate of return or interest
REFERENCES


rate for the period and is to be called "bond." We also assume that there is a stochastic rate of inflation for the period.

The following notation will be employed in our analyses:

$W_0 =$ the initial wealth.

$X =$ the amount of wealth to be invested in bond.

$R =$ the stochastic nominal rate of return or interest rate on bond. Let the expected value and the variance of the random interest rate be $E(R) = \mu_r$ and $V(R) = \sigma_r^2$, respectively.

$R_a =$ the stochastic rate of inflation. $R_a > 0$ implies a decrease in purchasing power; $R_a < 0$ implies an increase in purchasing power; and $R_a = 0$ implies no change in purchasing power. Let the expected value and the variance of $R_a$ be $E(R_a) = \mu_a$ and $V(R_a) = \sigma_a^2$, respectively.

$W_1 =$ the terminal wealth in real terms.

The investor is assumed to have a quadratic utility function of wealth in real terms of the following form:

$$ U(W_1) = W_1 - aW_1^2, \quad (1) $$

where the coefficient $a > 0$ for a risk-averse investor. The utility function in (1) is defined only for the range where $W_1 < \frac{1}{2a}$, since the marginal utility is decreasing for the range where $W_1 > \frac{1}{2a}$.

\[2\text{In a money-value model of portfolio selection, a positive expected nominal rate of interest is a necessary condition for the investment in bond. However, as we shall see later, no such condition is required for the investment in bond in a real-value model.}\]
the estimates of changes in interest rate in any attempt to alter their liquidity preference and risk taking.

Our analyses have also indicated that the Pratt-Arrow measures of risk aversion could have different implications in their applications to some economic theories if the models for analysis are formulated in the real rather than the nominal terms. Finally, the question of how the purchasing power risk might affect the now well-known capital assets pricing model in equilibrium is of great importance and certainly deserves further inquiry.
Many writers have noted that there are some undesirable properties associated with a quadratic utility function. In addition to the above restriction on its applicable range, a quadratic utility function is said to have an increasing absolute risk aversion (in the Pratt–Arrow sense) which has implausible implications on portfolio selection. For instance, Arrow (1965) has shown that the investment in bond decreases as the wealth increases if a quadratic utility function is employed in the expected utility theory of portfolio selection. However, as we shall see later in this paper, a quadratic utility function of wealth in real terms does not necessarily imply this kind of portfolio selection behavior. Furthermore, the quadratic utility function is employed in the analysis so that some of my results on liquidity preference can be directly compared with those of Tobin's.

The portfolio selection problem faced by the investor is how to allocate his initial wealth between cash and bond so that the expected utility of his real terminal wealth can be maximized. With the budget constraint which specifies that the sum of cash holding and the amount invested in bond must be equal to the initial wealth, the investor's terminal wealth in real terms is a random variable and can be expressed as

$$W_1 = W_0(1 - R_a) + XR.$$  \hspace{1cm} (2)

Therefore, the portfolio selection problem is to find the value of $X$ which maximizes the following expression.
The demand for cash balances in a dynamic model is always greater than that of a static model if an increase or no change in the purchasing power is expected, since inequality (27) always holds for \( \mu_a \leq 0 \). The demand for cash balances in a dynamic model can be less than that of a static model only if a decrease in the purchasing power is expected and the sign of inequality (27) is reversed. For any given value of \( \sigma_a^2 \), the right-hand side of inequality (27) reaches its maximum value at \( \mu_a = .5 \), and hence, this is the most likely case in which the demand for cash balances in the dynamic model will be less than that of a static model.

SOME IMPLICATIONS

Our analyses, based upon a real-value portfolio selection model, have indicated that the purchasing power risk has significant effects upon an individual's liquidity preference and risk taking in both the static and the dynamic portfolio decision models. In particular, we have derived conditions under which Tobin's interest rate effects upon liquidity preference will overstate or understate the true effects when the purchasing power risk exists in the economy. In studying and drawing conclusions from the empirical relationships among the cash balances, interest rate, wealth and income, one should not ignore the importance of the levels and the fluctuations of purchasing power for the period under study. Furthermore, our results imply that influencing investors' estimates of changes in purchasing power is as important as influencing
\[
E\left[U(W_1)\right] = E\left\{W_0(1 - R_a) + XR - a\left[W_0(1 - R_a) + XR\right]^2\right\}
\]
\[
= W_0(1 - \mu_a) + X\mu_r - aW_0^2(1 - 2\mu_a + \mu_a^2 + \sigma_a^2)
- 2aW_0X(\mu_r - \mu_a\mu_a - \rho\sigma_r\sigma_a)
- aX^2(\mu_r^2 + \sigma_r^2),
\]

where \(\rho\) is the coefficient of correlation between \(R\) and \(R_a\). There is a general agreement that nominal interest rate and the rate of inflation are positively correlated, but there is so far no consensus on whether this positive correlation is perfect.\(^3\) Hence, I simply postulate that \(\rho\) is positive in my analysis.

The condition for an interior maximum is \(\partial E[U(W_1)]/\partial X = 0\), which yields the following solution for the optimal amount of wealth to be invested in bond,

\[
X = \left[\frac{1}{(\mu_r^2 + \sigma_r^2)}\right](\mu_r H + G),
\]

where

\[
H = \left(\frac{1}{2a} - W_0\right),
\]

\[
G = W_0(\mu_r\mu_a + \rho\sigma_r\sigma_a).
\]

\(^3\)See, for example, Mundell (1963) and Phelps (1965).
function is employed in the dynamic portfolio analysis. A myopic policy is optimal in a two-period dynamic real-value model only if \( k \) in (23) is equal to one. Or equivalently, \( \left( \frac{\mu_r^2 + \sigma_r^2}{\sigma_r^2} \right) > \frac{\mu_a(1-\mu_a)}{\sigma_a^2} \) to have a myopic policy in the dynamic real-value model.

Although comparative statics of the optimal dynamic solution can be obtained using (23), they will not be presented here. Instead, I will examine how the liquidity preference might be different in dynamic and static models. To make a meaningful comparison of the differences in liquidity preference in the two models, it is necessary to have a comparable initial wealth. With an initial wealth of \( \bar{w}_{T-2} \), the optimal bond holding in the static model is

\[
\hat{x}_{T-1} = \left[ \frac{\mu_r}{2a(\mu_r^2 + \sigma_r^2)} \right] \left[ 1 - 2a \bar{w}_{T-2}(1-\mu_a) \right]. \tag{25}
\]

From (23) and (25), we have

\[
x_{T-1}/\hat{x}_{T-1} = 1 + \frac{\left( \frac{1}{k} - 1 \right)}{1 - 2a \bar{w}_{T-2}(1-\mu_a)}. \tag{26}
\]

In (26), \( k > 1 \) implies that \( x_{T-1}/\hat{x}_{T-1} < 1 \). Thus, the optimal bond holding in a dynamic model is less than that of a static model if \( k > 1 \). From the definition of \( k \) in (20), \( k \) is greater than one if and only if

\[
\frac{\mu_r^2 + \sigma_r^2}{\sigma_r^2} > \frac{\mu_a(1-\mu_a)}{\sigma_a^2}. \tag{27}
\]
It can be seen from (4) that a positive amount of bond will always be held in the optimal portfolio if there is an expectation of a decrease in purchasing power (i.e., $\mu_a > 0$). However, if an increase in purchasing power is expected (i.e., $\mu_a < 0$), a positive amount of bond will be held in the optimal portfolio only if

$$
\frac{H}{W_0} + \frac{\rho_0 r \sigma_a}{\mu_r} > |\mu_a|.
$$

(5)

The solution to the problem of portfolio selection maximizing the expected utility of terminal wealth in monetary terms can be easily derived by setting $\mu_a = 0$ and $\sigma_a^2 = 0$ in (4). The result is

$$
X^* = \left[\frac{\mu_r}{(\mu_r^2 + \sigma_r^2)}\right] \left[\frac{1}{2a} - W_0\right] = \frac{\mu_r}{(\mu_r^2 + \sigma_r^2)} \left[\frac{1}{2a} - W_0\right].
$$

(6)

The expression of (6) is essentially the same as that of (3.15) in Tobin (1958).

Note that $\left[\frac{1}{2a} - W_0\right]$ in (6) is always positive, since the utility function is defined only for the range where $\frac{1}{2a} > W_0$. Therefore, as noted by Tobin and Arrow, the necessary condition for holding bond in the optimal portfolio in the money-value model is a positive expected nominal rate of interest, that is, $\mu_r > 0$.

However, in the real-value model, $\mu_r > 0$ is neither a necessary nor a sufficient condition for holding a positive amount of bond in the optimal portfolio. For instance, with the condition that $\mu_r > 0$ and an anticipation of deflation ($\mu_a < 0$), $X$ in (4) is negative, if
Substituting (20) and (22) into (21), the first-order condition for maximization yields the optimal amount to be invested in bond in the two-period dynamic real-value model as

\[ x_{T-1} = \left[ \frac{\mu_r}{2ka(\mu_r^2 + \sigma_r^2)} \right] [1 - 2kaW_{T-2}(1 - \mu_a)] . \]  

(23)

For a two-period dynamic money-value model, the optimal amount of bond holding can be determined by setting \( \mu_a = 0 \) and \( \sigma_a^2 = 0 \), that also imply \( k = 1 \), in (23). Hence, we have

\[ x_{T-1}^* = \frac{\mu_r (1 - 2aW_{T-2})}{2a(\mu_r^2 + \sigma_r^2)} = \frac{\mu_r H'}{(\mu_r^2 + \sigma_r^2)} , \]  

(24)

where

\[ H' = \frac{1}{2a} - W_{T-2} . \]

By comparing (24) with (6), we can see that a myopic policy exists in the money-value model if a quadratic utility function is employed in the dynamic portfolio decision.\(^5\) However, in the real-value model, a myopic policy needs not exist even if a quadratic utility

\(^5\)In the money-value model, a quadratic utility function exhibits a linear risk-tolerance function, defined as reciprocal of the Pratt-Arrow absolute risk-aversion function, which implies a myopic policy in the dynamic portfolio decision. The conditions for myopic policy in the dynamic portfolio theory have been derived and discussed by Mossin (1968), Leland (1968), and Hakansson (1971).
\[ |\mu_a| > \frac{H}{w_0} + \frac{\rho_{r,a} \sigma_a}{\mu_r}. \]

Hence, \( \mu_r > 0 \) is not a sufficient condition for the investment in bond. Furthermore, even with \( \mu_r < 0 \), \( X \) in (4) can be positive, if

\[ \rho_{r,a} \sigma_a > |\mu_r| \left( \frac{H}{w_0} + \mu_a \right). \]

Hence, \( \mu_r > 0 \) is not a necessary condition for the investment in bond if there exists a purchasing power risk in the economy.

However, I will concentrate my analysis hereafter only on the case where \( \mu_r > 0 \) for the reasons that \( \mu_r < 0 \) is rare in reality and the results for the case where \( \mu_r < 0 \) are symmetric to that of \( \mu_r > 0 \).

By observing \( X \) in (4) and \( X^* \) in (6), an interesting question arises: Does a consideration of purchasing power risk in portfolio selection encourage or discourage the investment in bond? To answer this question, subtract \( X^* \) in (6) from \( X \) in (4) and yield,

\[ X - X^* = \frac{G}{(\mu_r^2 + \sigma_r^2)} = \frac{w_0 \left( \rho_{r,a} \sigma_a + \mu_r \mu_a \right)}{\left( \mu_r^2 + \sigma_r^2 \right)}. \]  

(7)

It is clear from (7) that \( (X - X^*) > 0 \), if \( \mu_a > 0 \). Thus, an expectation of a decrease in purchasing power always encourages risk taking and reduces the demand for cash balances as compared with the situation in which no such an expectation exists. However, when an expectation of an increase in purchasing power exists, risk taking is encouraged (or
$$X_T = \left[ \frac{\mu_r}{2a(\mu_r^2 + \sigma_r^2)} \right] [1 - 2a \bar{w}_{T-1} (1 - \mu_a)].$$ (19)

Once the optimal portfolio policy is derived for the last period, the induced (or indirect) objective function to be maximized at the beginning of the next-to-last period can be expressed as

$$I_{T-1}(\bar{w}_{T-1}) = \text{Max } E[U(\bar{w}_T)]$$

$$= \left[ \frac{\sigma_r^2 (1 - \mu_a)}{(\mu_r^2 + \sigma_r^2)} \right] (\bar{w}_{T-1} - k \bar{w}_{T-1}^2) + \frac{\mu_r^2}{4a(\mu_r^2 + \sigma_r^2)},$$ (20)

where

$$k = (1 - \mu_a) + \frac{(\mu_r^2 + \sigma_r^2) \sigma_a^2}{\sigma_r^2 (1 - \mu_a)}.$$  

As a result, the problem of portfolio decision with purchasing power risk at the beginning of the next-to-last period can be formulated as

Maximize $$E[I_{T-1}(\bar{w}_{T-1})],$$

subject to $$0 \leq X_{T-1} \leq \bar{w}_{T-2}.$$ (21)
discouraged), and the demand for cash balances is reduced (or increased) as \( (\rho \sigma_r / \mu_r) \) is greater (or less) than \( (|\mu_a| / \sigma_a) \).

It is interesting to note that a mere existence of purchasing power risk with no direction of changes expected (i.e., \( \mu_a = 0 \) and \( \sigma_a^2 > 0 \)) encourages risk taking and reduces the demand for cash balances. Furthermore, \( (X - X^*) > 0 \), if \( \mu_a > 0 \) and \( \sigma_a^2 = 0 \). Thus, an inflation with certainty, the same as imposing a wealth tax, encourages risk taking. The effects of purchasing power risk on liquidity preference will be analyzed in more detail later.

The Wealth Effects. Arrow has shown that the investor with a quadratic utility function decreases the investment in bond as his initial wealth increases. This can be easily seen from \( \partial X/\partial W_0 < 0 \), using (6).

However, with purchasing power risk, the wealth effect on the portfolio selection could be different from what has been prescribed by Arrow. From (4), we obtain

\[
\frac{\partial X}{\partial W_0} = \frac{\rho \sigma_r \sigma_a - \mu_a (1-\mu_a)}{(\mu_r^2 + \sigma_r^2)}.
\]

We can see from (8) that \( \partial X/\partial W_0 > 0 \), if \( (\rho \sigma_r / \mu_r) > [(1-\mu_a) \cdot \sigma_a] \). Hence, the demand for bond in the real-value model will not necessarily be decreased as the initial wealth increases, even if a quadratic utility function is employed. In other words, the risky bond is not necessarily an inferior good. Therefore, the disconcerting property of \( \partial X/\partial W_0 < 0 \),
LIQUIDITY PREFERENCE IN THE DYNAMIC MODEL

The prior contributions of Tobin (1958), Hicks (1962), and Arrow (1965) to the theory of liquidity preference are static single-period models. In the following sections, the liquidity preference will be analyzed within the framework of dynamic portfolio theory. In a dynamic portfolio theory, the maximization of the expected utility of wealth at the end of a multi-period planning horizon is the criterion for portfolio policies at the beginning of each period (the time interval needs not be identical for each period). In order to carry out my analysis with simple analytical solutions in a multi-period model, I assume that both the stochastic interest rate and the rate of inflation are intertemporally identically and independently distributed.

The following notation, in addition to those previously used, will be employed in my analysis:

\[ W_t \] = the wealth at the end of the \( t^{th} \) period; \( t = 1, 2, \ldots T \).

\[ X_t \] = the amount of wealth to be invested in bond at the beginning of the \( t^{th} \) period; \( t = 1, 2, \ldots T \).

At the beginning of the terminal period (T), the portfolio decision is to maximize \( E[U(W_T)] \), subject to the constraint that \( 0 \leq X_T \leq W_{T-1} \). This terminal-period portfolio decision problem is the same as the static portfolio selection problem described in the preceding section. Assuming that the investor has a quadratic utility function of real terminal wealth and that \( \text{cov}(R, R_a) = 0 \) for simplicity, the solution to this terminal-period problem is given by
which has been noted by many previous writers as always being associated with a quadratic utility function, might not exist if the purchasing power risk is explicitly incorporated into the model. The Pratt-Arrow measures of risk aversion could have different economic implications from what have been widely accepted in the literature.

Interest Rate Effects. The effects of interest rate on liquidity preference will be analyzed in two ways: the effects of changes in interest rate expectation; and the effects of changes in interest rate variability.

1) Effects of changes in interest rate expectation. In both real-value and money-value models, an increase in the interest rate expectation has both the substitution and income effects. However, the net effect of these two conflicting forces can be obtained by differentiating $X$ in (4) and $X^*$ in (6) with respect to $\mu_r$,

$$\frac{\partial X}{\partial \mu_r} = \left[ \frac{1}{(\mu_r^2 + \sigma_r^2)} \right] \left[ \sigma_r^2 (\mu_r^2 - \mu_r^2) (R W_0^\mu_a) - 2 W_0^\mu_r^2 \right] (9)$$

and

$$\frac{\partial X^*}{\partial \mu_r} = \left[ \frac{1}{(\mu_r^2 + \sigma_r^2)} \right] (\sigma_r^2 - \mu_r^2) H.$$ (10)

\* The Pratt-Arrow measures of risk aversion have been applied to a wide range of economic problems. For instance, they have been applied to a dynamic portfolio theory by Mossin (1968A) and Leland (1968), to a theory of the competitive firm under uncertainty by Sandmo (1971) and to the effects of taxation on risk taking by Mossin (1968) and Stiglitz (1969).
respect to a *decrease* in the purchasing power expectation is positive, if an inflation is expected; while the elasticity is negative, if a deflation is expected.

2) *Effects of changes in the variability of purchasing power.*

I will now examine the absolute and the relative effects of changes in the variability of purchasing power upon liquidity preference. Differentiating $X$ in (4) with respect to $\sigma_a$ yields,

$$\frac{\partial X}{\partial \sigma_a} = \frac{\rho \sigma_r}{\sqrt{\mu_r^2 + \sigma_r^2}}.$$  \hspace{1cm} (17)

Since $(\partial X/\partial \sigma_a)$ is always positive, the demand for bond increases as the variability of purchasing power increases. In other words, the purchasing power risk has a negative absolute effect upon liquidity preference.

The relative effects of a change in the variability of purchasing power upon the demand for bond may also be found from the following elasticity,

$$\frac{EX}{\partial \sigma_a} = \frac{\sigma_a}{X} \cdot \frac{\partial X}{\partial \sigma_a} = \frac{\rho \sigma_r \sigma_a}{\mu_r H + G}.$$  \hspace{1cm} (18)

Therefore, it can be seen from (18) that the elasticity of the demand for bond with respect to the variability of purchasing power is positive.
The effect of changes in the interest rate expectation upon the investment in bond in the money-value model is quite clear from (10). As Tobin has pointed out, the optimal amount of bond holding increases with the interest rate expectation if the coefficient of variation of interest rate is greater than unity.

However, the effect of changes in the interest rate expectation upon the investment in bond in the real-value model depends upon the parameters of the distribution of interest rate and of the rate of inflation. As can be seen from (9), when a decrease in purchasing power is expected, the amount of bond holding increases with the interest rate expectation if and only if

\[
\left( \mu_a + \frac{H}{W_0} \right) > \frac{2\rho_a \sigma_a \mu_r}{\sigma_r^2 - \mu_r^2}.
\]

On the other hand, when an increase in purchasing power is expected, the amount of bond holding increases with the interest rate expectation if and only if

\[
\frac{H}{W_0} > |\mu_a| + \frac{2\rho_a \sigma_a \mu_r}{\sigma_r^2 - \mu_r^2}.
\]

To compare how Tobin's effect of interest rate expectation upon liquidity preference might be different from that of our model, we derive from (9) and (10) the following ratio,
**Purchasing Power Effects.** The effects of uncertain purchasing power upon liquidity preference will be examined in two ways: the effects of changes in the expectation of purchasing power; and the effects of changes in the variability of purchasing power.

1) **Effects of changes in the purchasing power expectation.** Similar to the changes in the interest rate expectation, a change in the expectation of purchasing power has both substitution and income effects upon the demand for bond. The net effect of these two conflicting forces can be found by differentiating $X$ in (4) with respect to $\mu_a$,

$$\frac{\partial X}{\partial \mu_a} = \frac{W_0 \mu_r}{(\mu_r^2 + \sigma_r^2)}.$$  \hspace{1cm} (15)

The sign of $(\partial X/\partial \mu_a)$ is always positive, indicating that the demand for bond increases with a decrease in the purchasing power expectation. As we have pointed out earlier, an increase in the rate of inflation, the same as imposing a wealth tax, encourages risk taking and reduces the demand for cash balances.

The relative effects of changes in the purchasing power expectation on the demand for bond can be seen from the following elasticity,

$$\frac{EX}{E\mu_a} = \frac{\mu_a}{X} \cdot \frac{\partial X}{\partial \mu_a} = \frac{W_0 \mu_r \mu_a}{\mu_r H + G}.$$  \hspace{1cm} (16)

From (4) we know that the denominator of the RHS of equation (16) must be positive. Therefore, the elasticity of the demand for bond with
\[
\frac{\partial X}{\partial \mu_r} \left/ \frac{\partial X^*}{\partial \mu_r} \right. = 1 + \frac{W_0}{H} \left[ \mu_a - \frac{2\rho \sigma_r \sigma_a \mu_r}{(\sigma_r^2 - \mu_r^2)} \right].
\]  
(11)

Tobin's results **understate** or **overstate** the effects of changes in interest rate expectation upon liquidity preference when a purchasing power risk exists in the economy, according to whether \( \left( \frac{\partial X}{\partial \mu_r} \left/ \frac{\partial X^*}{\partial \mu_r} \right. \right) \) in (11) is greater or less than unity. Thus, if there is an expectation of an increase in purchasing power \((\mu_a < 0)\), Tobin's results always overstate the effect of interest rate expectation upon liquidity preference; while if there is an expectation of a decrease in purchasing power \((\mu_a > 0)\), Tobin's results will understate or overstate the effect of interest rate expectation upon liquidity preference depending on whether \((\mu_a/\sigma_a)\) is greater or less than \([2\rho \sigma_r \mu_r/(\sigma_r^2 - \mu_r^2)]\).

2) **Effects of changes in interest rate variability.** The absolute effects of changes in the variability of interest rate upon liquidity preference in the two models can be seen by differentiating (4) and (6) with respect to \( \sigma_r \) and obtaining the following:

\[
\frac{\partial X}{\partial \sigma_r} = \left[ \frac{-1}{(\mu_r^2 + \sigma_r^2)^2} \right] \left\{ 2\mu_r \sigma_r H + W_0 [2\mu_r \mu_a \sigma_r + \rho \sigma_a \sigma_r (\sigma_r^2 - \mu_r^2)] \right\}
\]  
(12)

and

\[
\frac{\partial X^*}{\partial \sigma_r} = \left[ \frac{-1}{(\mu_r^2 + \sigma_r^2)^2} \right] (2\mu_r \sigma_r H).
\]  
(13)
It is apparent from (13) that the absolute effect of interest rate risk upon the demand for bond is negative in the money-value model. The effects of interest rate risk upon the demand for bond in the real-value model can be examined from (12). When a decrease in purchasing power is expected, the absolute effect of interest rate risk upon the demand for bond is always negative; while, when an increase in purchasing power is expected, the effect is negative only if

\[
\frac{H}{W_0} + \frac{\rho a (\sigma_r^2 - \mu_r^2)}{2\mu_r \sigma_r} > |\mu_a|.
\]

To see the difference between Tobin's effect of interest rate variability upon liquidity preference and that of ours, we obtain from (12) and (13) the following,

\[
\frac{\partial x}{\partial \sigma_r} = 1 + \frac{W_0}{H} \left[ \mu_a + \frac{\rho a (\sigma_r^2 - \mu_r^2)}{2\mu_r \sigma_r} \right].
\] (14)

Tobin's results *understate* (or *overstate*) the effects of interest rate risk upon liquidity preference when there is a purchasing power risk in the economy, according as \(\left(\frac{\partial x}{\partial \sigma_r}/\frac{\partial x^*}{\partial \sigma_r}\right)\) in (14) is greater (or less) than unity. If there is an expectation of a decrease in purchasing power, Tobin's results always underestimate the effect of interest rate risk upon liquidity preference; while if there is an expectation of an increase in purchasing power, Tobin's results will underestimate (or overstate) the effects of interest rate risk upon liquidity preference as \(\left[\frac{\rho a (\sigma_r^2 - \mu_r^2)}{2\mu_r \sigma_r}\right]\) is greater (or less) than \(|\mu_a|/\sigma_a\).