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A NOTE ON THE VALUE OF INFORMATION
IN PERSONAL AND IMPERSONAL MARKETS

By

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Jaffe's (1974) provocative paper left unclear to me both the significance and generality of his conclusions. The note represents an attempt to clear these muddied waters. His distinction between personal and impersonal markets, heretofore a neglected issue in finance, is worthwhile pursuing; Jaffe has only scratched the surface. For example, insurance or loan markets, where the buyer has more information than the seller and the seller knows it, are two of the many real life examples of personal markets. The laws of an impersonal market, as typified by the standard perfect and competitive securities market, do not generally apply.

I. Personal Markets

Consider a two \((t=0, 1)\) date, \(E(e=1,2,\ldots,E)\) state, \(I(i=1,2,\ldots,I)\) individual pure exchange economy, where each individual \(i\), endowed with resources \(\{W_e^i\}\), selects state-contingent claims \(\{u_e^i\}\) so as to maximize the expected utility \(\sum_e p_e^1 u_e^i(W_e^i)\) of his future wealth with \(\gamma_e^i > 0\), \(\Psi_e^i > 0\) and \(\Psi_e^m < 0\). The final allocation must satisfy the closure conditions \(\sum_e p_e^1 = \sum_e \gamma_e^i W_e^i = W_e^m\) for all \(e\). Each individual is assumed to know
his own resources, tastes and beliefs. If, in addition, each individual knows (1) the resources of all other individuals, (2) the tastes of all other individuals, and (3) the "type" of information he and all others have, the securities market is said to be personal. By knowledge of the type of information, I mean all individuals agree on the ranking of the informativeness of all individuals in the economy.

Whether or not the market is personal, the set of Pareto-efficient allocations, \(^1\) assessed with respect to any commonly held beliefs, is independent of those beliefs. To see this, it is well known that for concave utility, an allocation is Pareto-efficient if and only if it maximizes a positively weighted sum of individual utilities subject to the closure conditions:

\[
\max_{\{u^i_e\}} \sum_{i} k_i \pi^i_u (w^i_e) - \sum_{e} \lambda^e \left[ \sum_{i} w^i_e - w^M_e \right]
\]

where \(k_i > 0\) for all \(i\) and \(\{\lambda^e\}\) are Lagrangian multipliers. The necessary and sufficient conditions for efficiency are then

\(k_i \pi^i_u (w^i_e) = k_i \pi^1_u (w^1_e)\) for all \(i\) and \(e\). Assessing Pareto-efficiency with respect to the beliefs of the best-informed individual (say individual \(i = 1\)), for example, amounts to setting \(\pi^i_e = \pi^1_e\) for all \(i\) and \(e\) so that \(k_i U^i (w^i_e) = k_1 U^1 (w^1_e)\) for all \(i\) and \(e\). This describes the exhaustive set of Pareto-efficient allocations (by varying \(\{k_i\}\)) and is independent of \(\{\pi^1_e\}\). As a consequence,

**Theorem** (personal markets without trading): In a personal market, if the endowed allocation is Pareto-efficient \(^2\) (with respect to
the beliefs of the best-informed individual), then no individual will trade and superior information about supply conditions will be valueless.

Proof: From the above argument, since each individual knows the resources and tastes of all individuals, even though he does not know the beliefs of the best-informed individual, he can calculate the set of Pareto-efficient allocations. Therefore, each individual knows the endowed allocation is Pareto-efficient. Since (1) no well-informed individual will offer to trade with a poorly informed individual unless the well-informed individual will benefit ex ante, (2) the poorly informed individual must lose ex ante in this exchange (since the endowed allocation is already Pareto-efficient), and (3) the poorly informed individual knows this and that he is poorly informed (since the market is personal), he will refuse to trade. Superior information about supply conditions, i.e., the probability distribution of \( \{W_e^M\} \), clearly has no value in a personal market if the endowed allocation is Pareto-efficient. Even if it is publicized, there will be no trade. Q.E.D.

However, in general while he knows the endowed allocation is on the contract curve, a poorly informed individual does not know which direction of movement along it will be to his advantage. For example, for the HARA class\(^3\) of utility functions, the final allocation is Pareto-efficient (with respect to any commonly held beliefs) if and only if there exist parameters \( \{\alpha_i\} \) and \( \{\beta_i \geq 0\} \) independent of \( e \) such that the final allocation must satisfy the set of equations\(^4\) \( W_e^f = \alpha_i + \beta_i W_e^M \) for all \( i \) and \( e \), where \( \Sigma_i \alpha_i = 0 \) and \( \Sigma_i \beta_i = 1 \). For constant absolute
risk aversion \((U_i(W^e) \sim -A_i \frac{W_i}{A_i})\) it is additionally required that 
\(\beta_i = A_i / \Sigma A_i\) for all \(i\); and for constant proportional risk aversion 
\((U_i(W^e) \sim \ln W^e_i \text{ or } W^e_i \cdot (1-B)^{-1})\) it is additionally required that 
\(\alpha_i = 0\) for all \(i\). Movements along the contract curve therefore amount 
to changes in \(\alpha_i\) and \(\beta_i\) for each individual. If both \(\alpha_i\) and \(\beta_i\) 
increase (decrease) individual \(i\) must be better (worse) off irrespective 
of his beliefs. However, if \(\alpha_i\) increases (decreases) and \(\beta_i\) decreases 
(increases), whether he is better off will depend upon the beliefs used to 
measure expected utility.\(^5\)

In only two cases, constant absolute and constant proportional 
risk aversion, will the private welfare implications of movements along 
the contract curve be unambiguous. In the former case, \(\alpha_i\) serves as a 
measure of relative welfare; and in the later case \(\beta_i\) serves as a mea-
sure of relative welfare. As a consequence,

Theorem (personal markets with trading): In a personal market in 
which all individuals have constant absolute risk aversion or all 
individuals have constant proportional risk aversion (with the 
same \(B\)), if the endowed allocation is not Pareto-efficient but 
individuals must trade to a Pareto-efficient allocation, then 
individuals will trade but superior information about supply con-
ditions will be valueless.

Proof: Clearly individuals must trade. Not only can they all calculate 
the Pareto-efficient set but they also know which trades within it make 
them better or worse off independent of the beliefs used to measure ex-
pected utility. Consequently, each will try to trade up to the Pareto-
efficient set by maximizing \(\alpha_i\) (constant absolute risk aversion) or \(\beta_i\) 
(constant proportional risk aversion). This strategy; being independent
of their information about supply conditions, causes such information to be valueless. Q.E.D.

However, except in these cases (where the endowed allocation is already Pareto-efficient or all individuals in the economy have constant absolute or proportional risk aversion$^6$), information about supply conditions will prove valuable since it will affect optimal trading strategies. This is true even though the market is personal and all individuals can calculate the Pareto-efficient set. For example, for quadratic utility $U_1(W^i_e) \sim \frac{1}{2} (A_1 - W^i_e)^2$ or generalized logarithmic utility $U_1(W^i_e) \sim \ln(A_1 + W^i_e)$ where $A_1 \neq 0$, information about supply conditions will be useful in assessing the private welfare implications of the trade-off available in the market for increasing $\alpha_i$ at the sacrifice of decreasing $\beta_i$. Knowing that others have superior information may be of little help since some trade is generally desirable to reach Pareto-efficiency.

II. Impersonal Markets

Assume that the economy is described as before except that the securities market is impersonal and competitive. In this case, each individual knows only his own resources, tastes and beliefs and the opportunities—prices—the market makes available to him. If $P_e$ denotes the present price of state-contingent claim $e$, then choices must satisfy the budget constraints $\sum_{e} P_e W^i_e = \sum_{e} P_e W^0_e = W^i_0$ for all $i$. 
Individuals with superior information about supply conditions can benefit from it either by making side bets with others via the securities market or by selling it to others in a prior market for information before the securities market convenes. Suppose the prior market for information, if it were utilized, did not use up aggregate resources. That is, for each state $e$, $W^M_e$ would remain the same before and after the sale of private information. Moreover, suppose the sale of private information were to bring about agreement in beliefs. Without a prior market for information, poorly informed individuals would make disadvantageous side bets. With a prior market for information, while no side bets would later be taken in the securities market, poorly informed individuals would instead deplete their wealth by purchasing superior information. They would then enter the securities market poorer but wiser.

Is it possible to organize a prior market for information so that its use is preferred by all individuals in the economy to the alternative of making side bets in an impersonal competitive securities market?

Theorem (impersonal markets): In an impersonal competitive market, given at least some disagreement, there always exists a way of redistributing resources through a prior market for information such that all individuals will be better off (with respect to the beliefs of the best-informed individual).

Proof: In an impersonal competitive securities market, each individual $i$ maximizes

$$\max_{\{W^1_e\}} \sum_{e} \pi^1_i(W^1_e) - \lambda^1_i [\sum_{e} P_i^1 W^1_e - \sum_{e} e^1_i W^1_e].$$

Since $U^1_i$ is concave, $U^1_i(W^1_e)/U^1_i(W^1_s) = P^1_i \pi^1_i(e)/P^1_i \pi^1_i(s)$ for any two states $e$ and $s$ and all $i$, are the necessary and sufficient conditions for
an equilibrium. Consequently, individuals have the same beliefs if and only if for any two states e and s, $U_i^e(W_i^e)/U_i^s(W_i^e)$ is the same for all i. Therefore, the equilibrium allocation under heterogeneous beliefs cannot be the same as the equilibrium allocation under homogeneous beliefs. Only the equilibrium under homogeneous beliefs can be Pareto-efficient (with respect to the beliefs of the best informed individual). If individuals were to trade without convening a prior market for information, they would not have the same beliefs and thus would not reach a Pareto-efficient allocation. Therefore, all individuals can be made better off by holding a prior market for information and then all entering the securities market with the beliefs of the best-informed individual. Q.E.D.

In short, the benefits from buying and selling information in a properly organized prior market for information exceed the benefits from side bets for all individuals in the economy. This may partially explain the empirical evidence supporting the speed with which the securities market digests new information, since disclosure for a price is more profitable than taking a speculative position and waiting for nature to reveal the true state. Taking a speculative position and then disclosing the information for a price will also not be preferred to pure disclosure for a price, since the prior speculative position, even by its slight effect on prices in a large market, will diminish the benefits to poorly informed individuals from later disclosure causing them to pay less for the information.

The theorem only asserts that it is possible to redistribute resources via a prior market for information so that all individuals are
better off. However, due to the special characteristics of information as a "commodity" such a prior market may be difficult to design. For a well-informed individual to benefit more from selling information than from speculating, its value (benefits its disclosure confers upon others) must be sufficiently appropriable. However, because of the Pareto-inefficiency of heterogeneous beliefs, this value need not be completely appropriable to its owner.
Footnotes

1. Pareto-efficiency, throughout this note, is measured \textit{ex ante} in terms of expected utility, rather than \textit{ex post} in terms of the utility of realized future wealth.

2. An economy in which all individuals have the same resources and tastes is a special case for which the endowed allocation is trivially Pareto-efficient.

3. The HARA class of utility functions is described by the solution to the differential equation \(-U'_1(W^e_t)/U''_1(W^e_t) = A_1 + B W^e_t\) where \(A_1\) and \(B\) (independent of \(i\)) are constants. The solution includes most popular utility functions including quadratic, exponential, logarithmic, and power utility. For a good general source see Mossin (1973, pp. 113-115).

4. These equations are known as \textit{optimal sharing rules}. \(\alpha\) may be interpreted as the amount received (paid) at date \(t = 1\) from risk-free lending (borrowing) at date \(t = 0\), and \(\beta\) as the proportion held of the market portfolio. For a more thorough analysis of sharing rules, see Rubinstein (1974).

5. As a numerical example, suppose \(E = 2\), \(U(W_e) = \ln(1 + W_e)\) and \(W_M^1 = 1\) and \(W_M^2 = 4\). Then expected utility

\[
\tilde{U}(\alpha, \beta; \pi_1) = \pi_1 \ln(1+\alpha+\beta) + (1-\pi_1)\ln(1+\alpha+4\beta).
\]
Therefore, \( \bar{U}(\frac{1}{2}, \frac{1}{4}; \frac{1}{3}) = 0.797 > \bar{U}(1, 0; \frac{1}{3}) = 0.693 \) and

\[ \bar{U}(\frac{1}{2}, \frac{1}{4}; \frac{2}{3}) = 0.678 < \bar{U}(1, 0; \frac{2}{3}) = 0.693 \]

so that the order of \((\alpha, \beta)\) strategies depends on \(\pi_1\).

6. In a third degenerate case of no aggregate uncertainty, i.e., \(W_e^m\) is the same for all \(e\), the Pareto-efficient allocation always requires \(\alpha_i = 0\) for all \(i\) regardless of tastes as long as all individuals are risk averse. Again \(\beta_i\) serves as an unambiguous measure of private welfare.

7. An economy in which all individuals have logarithmic utility functions and are identical except for their beliefs furnishes the simplest illustration. In this case, the competitive equilibrium is characterized by

\[ W_i = \frac{\pi_i}{\pi_e} W_e = \left( \frac{\pi_i - \pi_e}{\pi_e} \right) W_e + W_e \quad (\text{all } i \text{ and } e) \]

where \(\pi_e = \sum_i \pi_i / I\) and \(W_e = \sum_i W_i / I\). When all individuals have the same beliefs, all side bets \([\frac{\pi_i^e - \pi_e}{\pi_e}]W_e = 0\). For a more thorough analysis of side bets, see Rubinstein (1974).
REFERENCES

