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THE DYNAMICS OF GOVERNMENT POLICY IN AN INFLATIONARY ECONOMY:
AN "INTERMEDIATE-RUN" ANALYSIS

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Recent developments in macroeconomic theory stress the intrinsic
dynamics of the system imposed by the government budget constraint. The
financing of any budget deficit or surplus must involve changes in the
net claims of the private sector against the government and these changes,
in turn, lead to changes in the other endogenous variables. Consequently,
the system can be in equilibrium only when the government's budget is
balanced and the stock of government debt ceases to change.

Building on a number of previous contributions (e.g., Ott and Ott
1965 and Christ 1968), Blinder and Solow (1973) recently presented two
models analyzing the stability and the long-run equilibrium properties of
fiscal policy under alternative assumptions about the method used by the
government to finance its deficit. The first model follows earlier
authors and makes an assumption that Blinder and Solow rightly regard as
somewhat contradictory: namely, that, although net investment is taking
place, capital stock is treated as remaining constant. This is in direct
contrast to the treatment of financial assets, the adjustment of which
is explicitly incorporated into the analysis. Indeed, changes in the
stock of financial assets form the basis for the dynamics of the system.

Blinder and Solow, therefore, postulate a second model in which
the dynamic relationship between investment and capital stock is explicitly
included. In this model, equilibrium is reached when the accumulation of
real capital (net investment), as well as the accumulation of financial

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assets, ceases, so that saving has been reduced to zero. While this analysis is clearly an important advance over previous models, it is somewhat unsatisfactory as an equilibrium for what is essentially a short-run Keynesian model. This is because, although the model fully incorporates the demand effects of investment, no supply effects are accounted for. Thus, the long-run equilibrium of zero net investment in the second model is obtained by driving down aggregate demand and income to the level where savings and hence net investment are reduced to zero.\footnote{This can be seen quite readily from the following simplified version of their IS Equation, which abstracts from wealth effects and interest payments on bonds

\[ Y = C(Y) + I(r, K) + G , \quad 1 > C' > 0, \quad I_1 < 0, \quad I_2 < 0, \]

where \( Y, r, K, \) and \( G \) are as defined in the text. Consider a monetary policy which holds \( r \) constant. Differentiating this equation with respect to \( K \) we see

\[ \frac{\partial Y}{\partial K} = \frac{I_2}{1-C'} < 0 \]

Implying that as capital stock is being accumulated, the level of investment and level of income will fall.}

While this may appear to be consistent with the equilibrium of neoclassical growth theory, in which the capital stock grows at the rate of growth of population, there is an important difference. The neoclassical equilibrium is reached via a process of \textit{capital deepening} in which the supply effects of the additional capital stock are fully taken into account.\footnote{For a long-run analysis of monetary-fiscal policies in a neoclassical model, see Foley and Sidrauskis (1971). Apart from the fact that they are concerned with the long run, the emphasis of their analysis is quite different from that presented here in that they assume full employment.} This tends to be a longer-run adjustment process and is not incorporated into Keynesian-type macromodels.
In this paper we analyze the dynamics and equilibrium properties of what one might call an "intermediate-run" model. On the one hand, we view labor as the only variable factor of production. We also recognize the demand effects of investment and the fact that investment implies a change in the capital stock that must be absorbed in savers' portfolios. On the other hand, we ignore the supply effects of the increase in the capital stock. Thus, we formulate a model in which the dynamics of capital accumulation on portfolio behavior are taken into account, in a way precisely analogous to the treatment of financial assets, without getting into the complications of adjustment on the production and output side. This is done by applying a technique previously used by Sargent (1973a) and working with appropriate quantities relative to the stock of capital. Defining units in this way, we differ from Blinder and Solow in that we do not require the accumulation of capital to cease in equilibrium. A steady-state with a positive savings rate is possible. This is obviously desirable for an essentially Keynesian analysis, which ignores the supply effects of investment.

The second modification we make is to relax the Blinder-Solow assumption of fixed prices. Instead we allow the rate of price inflation to be determined by a Phillips curve, embodying the "expectations hypothesis" of wage determination, with the expectations being endogenously throughout and pay relatively little attention to considering alternative methods of financing the government budget deficit. In many respects the model analyzed here resembles that outlined in Ando (1974). But his paper is essentially an exposition of the theoretical structure of the MPS model and he is not concerned with examining any of its analytical properties.

An equilibrium in which $K$ is growing is precisely analogous to the notion of a "quasi-equilibrium" as it has been termed by Hansen (1970) - a steady state in which prices are growing at a constant rate.
determined. We also include an expanded financial sector along the lines developed by Tobin (1969), in which bonds, money, and claims on physical capital are, in general, imperfect substitutes in portfolio allocation. Extending the model in this way enables us to integrate the Blinder and Solow analysis, which stresses the dynamics of the system, with a more adequate portfolio model, and with the Friedman (1968)—Phelps (1968) analysis of the long-run rate of inflation. One result of particular significance to emerge from such a synthesis is that while the Phillips curve may yield a natural rate of unemployment in the steady state and thus imply an indeterminate rate of inflation, a unique equilibrium rate of inflation can be established by considering the equilibrium conditions in the asset markets.

AN INTERMEDIATE-RUN MACROECONOMIC MODEL OF A CLOSED ECONOMY

The analysis presented in this paper is based on a continuous time model of a closed economy. As we have already indicated, we call this an intermediate-run model, because we deal with the demand and portfolio effects of increases in the capital stock, but not with their supply effects. Specifically, at any given time, labor is assumed to be the only variable factor of production, while physical capital is a fixed factor in production, although its quantity changes as new investment takes place.

The development of the model proceeds in several stages. First, we consider a number of relationships describing the output sector, the wage-price-employment sector and the financial sector of the model.
These together provide a set of equations describing the instantaneous equilibrium of the system in terms of a number of variables, such as the supplies of the assets and expectations, which describe the state of the system. Next, we consider the intrinsic dynamics of the system arising from the financing of the government budget deficit, the accumulation of capital, and the formation of expectations. These relationships describe the evolution of the system over time. Finally, for the reasons already given, we respesify the system in terms of quantities per unit of physical capital.

**Output Sector**

In the product market, consumption is assumed to be a function of real disposable income and real wealth. Real disposable income is defined to be the after-tax value of the sum of the real value of output and the real value of interest payments on the government debt, where for simplicity a proportional tax rate \( u \) is assumed. Following common practice, capital gains on existing assets are ignored in our definition of disposable income. As discussed more fully below, there are three financial assets in the model, so that real wealth is the sum of the real value of claims on capital, the real value of government bonds, and the real value of outside money. Investment is specified as a function of the real return on claims on capital, real income, the quantity of physical capital. This investment function can be justified in terms of the theory of investment formulated by Foley and Sidrauski (1971). The basic idea involved is that increases in the capital stock are determined by the existing stock of physical capital and by the price of claims on existing capital relative
to the price of new capital. As we indicate later, this relative price is itself a function of real income, the real returns on capital claims, and the quantity of physical capital. Therefore, we write the investment function directly as a general function of these three factors. Our formulation can still be justified when the assumptions of the Foley and Sidrauski theory break down; see Ando (1974). For example, it can be justified under the assumption of all equity financing by firms or alternatively if the conditions of the Modigliani and Miller theorem on capital structure with no corporate taxes hold. The final component of aggregate demand is real governmental expenditures, which we take to be exogenously given.

With these definitions and assumptions, product market equilibrium requires the following equation to hold at all times. Partial derivatives are denoted by numerical subscripts; time derivatives are signified by dots.

\[ Y = C[(Y + B/P)(1 - u), A] + I[r_k, Y, K] + G \]  

\[ 0 < C_1 < 1, \quad C_2 > 0, \quad I_1 < 0, \quad I_2 > 0, \quad I_3 < 0 \]  

where

\[ Y \quad \text{Real output.} \]

\[ B \quad \text{Nominal value of interest payments on government debt} \]

\[ \text{(assumed to be perpetual bonds paying$1 per bond).} \]

\[ u \quad \text{Proportional tax rate.} \]

\[ A \quad \text{Real net private wealth.} \]

\[ r_k \quad \text{Real rate of return on claims to physical capital.} \]
$K$ = Quantity of physical capital.

$G$ = Real government expenditure.

$C$ = Real consumption.

$I$ = Real investment.

$P$ = Price level.

The restrictions on the functions $C(.)$ and $I(.)$ given in (1) are self-explanatory. The other requirement we introduce is that $C$ is homogeneous of degree one in disposable income and wealth, and that $I$ has a similar property with respect to $Y$ and $K$. This is necessary to enable us to work in units per capital.

**The Wage-Price-Employment Sector**

The rate of money wage inflation is determined by an extended Phillips curve of the form:\(^4\)

$$\frac{\dot{W}}{W} = \alpha_0 + \alpha_1 \left( \frac{E - \bar{E}}{E} \right) + \beta \pi \quad \alpha_1 \geq 0 \quad 0 \leq \beta \leq 1$$

(2)

where

$W$ = Money wage rate.

$E$ = Actual demand for labor.

$\bar{E}$ = Full employment supply of labor.

$\pi$ = Expected rate of price inflation.

Thus, the coefficient $\alpha_1$ defines the short-run tradeoff between the

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\(^4\)See, e.g., Smith (1970) and Turnovsky (1972) for further discussion and empirical estimation of this version of the "expectations hypothesis" of wage determination.
unemployment rate \( (E - \bar{E})/\bar{E} \) and the rate of wage inflation for a
given rate of price inflation, while \( \beta \) measures the extent to which
inflationary expectations are reflected in current wage changes.

Next we assume that current prices are based on a constant
markup on unit labor costs, so that

\[
p = \frac{\text{GNP}}{Y}, \quad \theta \geq 1
\]  

(3)

where \( \theta = \) mark-up factor.

Differentiating (3) with respect to \( t \), we obtain the rate of
price inflation as

\[
p = \frac{\dot{p}}{p} = \frac{\dot{W}}{W} + \frac{\dot{E}}{E} - \frac{\dot{Y}}{Y}
\]  

(4)

Furthermore, we assume that average labor productivity grows at the
constant rate \( \rho \), so that

\[
\frac{\dot{Y}}{Y} - \frac{\dot{E}}{E} = \rho
\]  

(5)

With capital fixed in the short run, we assume that real income and
employment are related by

\[
E = F(Y, K, \rho) \quad \text{where } F_1 > 0
\]  

(6)

This equation is an employment function showing the number of units of
labor required to meet the production of \( Y \) and forms the basis of
empirical studies of labor productivity by Wilson and Eckstein (1964),
Brechling (1965), and others. For levels of employment sufficiently
close to the full employment level \( \bar{E} \), we can obtain the following
linear approximation to (6)

\[ \frac{E - \bar{E}}{E} = \frac{F_1}{F} \left( \frac{Y - \bar{Y}}{\bar{Y}} \right) = k \left( \frac{Y - \bar{Y}}{\bar{Y}} \right) \tag{7} \]

where \( \bar{Y} \) is the full employment level of output given the current level of capital stock. Thus, in the neighborhood of full employment, the excess demand for labor (as a percentage) is proportional to the excess demand for goods, the factor of proportionality being the full employment elasticity of demand for labor, denoted by \( k \).

Combining equations (2), (4), (5), and (7), we obtain the following price adjustment equation

\[ p = \alpha_0 + \alpha_1 \left( \frac{Y - \bar{Y}}{\bar{Y}} \right) + 2\pi - \rho \tag{8} \]

where the constant of proportionality \( k \) in (7) has been absorbed in \( \alpha_1 \). This equation, which describes how current output prices respond to disequilibrium in the product market and inflationary expectations, can be viewed as a "reduced form" for the wage-price-employment sector.\(^5\)

\(^5\) An alternative specification which results in a price adjustment equation of the same form as (8) can be obtained by assuming that firms choose employment levels so as to equate the real wage rate to the marginal product of labor

\[ \frac{W}{P} = H_1 \] , where \( H_1 \) is the marginal product of labor.

This implies that the rate of price inflation is

\[ \dot{p} = \frac{P}{P} = \frac{\dot{W}}{W} - \hat{\rho} \] , where \( \hat{\rho} = \frac{H_1}{H_1} \).

With labor being the only variable short-run factor of production, the proportionality relationship (7) is still obtained. The net result is an expression for \( p \) which is the same as (8), except that the productivity growth rate is based on marginal rather than average productivity.
The Financial Sector

The financial sector is virtually the same as that discussed by Tobin (1969). Real asset demands are assumed to be functions of real income before taxes, expected real rates of return on assets, and real wealth. The notion that the level of real wealth, the own real rate and the real rates of return on other assets influence the demand for assets is straightforward. Real income is introduced as a determinant of the transactions demand for assets. Money in the model is outside money and is noninterest bearing, so that its expected rate of return is the negative of the expected rate of inflation. The supply of money in nominal terms is \( M \), so that its real value is \( M/P \). Because we abstract from a fractional reserve banking system, \( M \) refers to "high-powered money." The interest-bearing government debt is assumed to be in the form of perpetual bonds paying $1 per bond. With the number of bonds outstanding being \( B \), their real quantity and hence the real amount of total interest paid on the government debt is \( B/P \), as already noted above. Moreover, letting the nominal interest rate on bonds be \( r \), the real value of bonded debt is \( B/P \). The nominal value of a claim on one unit of physical capital is \( P_K = qP \) where \( q \) is the ratio of the price of existing claims on capital to the price of currently produced goods. Thus the real value of claims on the total stock of capital is \( qK \).

In equilibrium the nominal value of the claims on capital must equal the discounted present value of the expected nominal future earnings from owning the physical capital stock. That is, letting \( r' \)
denote the nominal rate of return on capital we must have

$$P_k(t)K(t) = \int_t^\infty [P_t^*(\tau)Y(\tau) - W_t^*(\tau)E(\tau)]e^{-r'(\tau-t)} d\tau$$  \(9\)

where \(P_t^*(\tau)\) denotes the price expectation for time \(\tau\), made at time \(t\), and \(W_t^*(\tau)\) is defined similarly. With inflationary expectations \(\pi\), and labor productivity growing at rate \(\rho\), we assume

\[
P_t^*(\tau) = P(t)e^{\pi(\tau-t)}
\]

\[
W_t^*(\tau) = W(t)e^{(\pi+\rho)(\tau-t)}.
\]

Moreover, from the productivity growth equation (5) we have

\[
E(\tau)/E(t) = Y(\tau)e^{-\rho(\tau-t)}/Y(t).
\]

Using these facts, together with the price markup assumption given in (3), equation (9) simplifies to

$$P_k(t)K(t) = \lambda P(t) \int_t^\infty Y(\tau)e^{-(r'-\pi)(\tau-t)} d\tau$$

where \(\lambda = (\theta - 1)/\theta > 0\). Finally, adding the assumption that \(Y(\tau)\) is expected to remain constant throughout the indefinite future, we obtain the following relationship between the real value of the capital stock and the real rate of return, \(r_k = (r' - \pi)\) on the claims on this capital

$$qK = \frac{\lambda V}{r_k}.$$  \(10\)

Equilibrium in the financial markets requires that the following equations hold at all times.
\[ M/P = L(Y, r^*_k, r-\pi, -\pi, A) \] (11a)

\[ B/Pr = J(Y, r^*_k, r-\pi, -\pi, A) \] (11b)

\[ qK = N(Y, r^*_k, r-\pi, -\pi, A) \] (11c)

where

- \( L \) = Demand for real balances of outside money.
- \( J \) = Demand for real outside bonds.
- \( N \) = Demand for real claims on capital.
- \( (r-\pi) \) = Real rate of return on bonds.

These demands are subject to the aggregate wealth constraint

\[ A = M/P + B/Pr + qK. \]

As pointed out by Tobin, the wealth constraint imposes the following self-explanatory conditions on the partial derivatives of the asset demand functions

\[ \begin{cases}
  L_i + J_i + N_i = 0 & i = 1, \ldots, 4 \\
  L_5 + J_5 + N_5 = 1 & \text{where } 0 < L_5 < 1, 0 < J_5 < 1, 0 < N_5 < 1.
\end{cases} \] (1)

In addition to (1), we follow Tobin and assume

(ii) The demand for each asset varies positively with its own real rate of return and nonnegatively with the rates on other assets; that is, all assets are gross substitutes.

(iii) Capital is not a transactions substitute for money, so that

\[ N_1 = 0, \text{ and } L_1 = -J_1 > 0. \]

(iv) The asset demand functions are homogeneous of degree one in income and wealth.
Equations (1), (8), (10), (11a), (11b), (22) define six independent instantaneous equilibrium conditions. Provided that the Jacobian of this system is non-zero, the set of equations can be solved for the six variables \( Y, q, r_k, r, A, \) and \( p \), in terms of \( M, B, K, \pi \), and other exogenous variables and parameters. Other variables such as the level of employment and the instantaneous rate of wage inflation, can then be determined.

The Dynamics of the System

To complete the model, we need to describe its evolution over time. Recalling the investment function, it is clear that the accumulation of physical capital is given by

\[
K = I(r_k, Y, K)
\]  

(13)

The dynamics of inflationary expectations is assumed to be generated by a simple adaptive process

\[
\pi = \gamma(p - \pi)
\]  

(14)

where \( \gamma > 0 \) is constant.

Finally, the accumulation of financial assets is described by the government budget constraint (expressed in real terms)

\[
\frac{\dot{M}}{P} + \frac{\dot{B}}{P_r} = G - uY + (1-u)\frac{B}{P}
\]  

(15)

This equation asserts that the sum of government expenditure plus interest payments on the government-bonded debt less tax receipts must be financed
either by changes in the stock of outside money, or outside bonds, or both. Thus, the system still possesses one degree of freedom and to close it, we must specify a policy describing how the government finances its deficit. Once this is done, the dynamics of the model and its ultimate equilibrium will be uniquely determined assuring there are no singularities.

Summary of the Model and its Respecification in Per Unit of Capital Form

For reasons we have already discussed (chiefly because we wish to focus on the intermediate run), it is convenient to analyze the system relative to the stock of capital. The homogeneity assumptions we have introduced for the C, I, L, J, N, functions enable us to scale the income and wealth variables in this way. Using these homogeneity assumptions and the following redefinitions of variables and functions:

\[ \frac{Y}{K} = y, \frac{\bar{y}}{K} = \bar{y}, \frac{A}{K} = a, \frac{M}{KP} = m, \frac{B}{KP} = b, \frac{G}{K} = g, \]
\[ \frac{C}{K} = c, \frac{L}{K} = \ell, \frac{J}{K} = j, \frac{N}{K} = n, \frac{I}{K} = i, \]

the system of equations describing the instantaneous equilibrium and the price adjustment mechanism may be written as

\[ y - c[(1-u)(y+b), a] - i(r_k, y) - g = 0 \] (16a)
\[ \ell(y, r_k, r-\bar{\pi}, -\bar{\pi}, a) - m = 0 \] (16b)
\[ j(y, r_k, r-\bar{\pi}, -\bar{\pi}, a) - b/r = 0 \] (16c)
\[ a = q + m + b/r \] (16d)
\[ q = \lambda y/r_k \] (16e)
\[ p = \alpha_0 + \alpha_1 \left( \frac{y-\bar{y}}{\bar{y}} \right) + \beta \bar{\pi} - \rho \] (16f)
These six equations determine \( y, r_k, r, a, q, \) and \( p \) in terms of \( m, b, g, \) and \( \pi \). With quantity variables being expressed in per unit of capital form, it is necessary to transform the dynamic equations into a comparable form, yielding:\(^6\)

\[
\dot{k} = \frac{K}{K} = l(r_k, y) \quad (17a)
\]

\[
\dot{\pi} = \gamma(p - \pi) \quad (17b)
\]

\[
\dot{m} + \frac{b}{r} = g - uy + (1-u)b - (m + b/r)(p + k) \quad (17c)
\]

together with the policy specification for \( m \) and/or \( b \). In equation (17c), the quantity \( g - uy + (1-u)b \) is the government budget deficit, including interest payments on outstanding bonds, expressed in real terms per unit of physical capital. The remaining term, \( - (m + b/r)(p + k) \) measures what one might loosely call a "real inflationary and growth tax" levied on the real stock of existing outside government debt. Thus, equation (17c) asserts that, even if the government's budget is balanced in the sense that \( G - uY + (1-u)B/P = 0 \), so that nominal debt ceases to be issued, the real value per unit of physical capital of this debt will fall as long as capital is being accumulated and/or prices are rising. However, for convenience, we refer to the entire righthand side of (17c)

\[^6\]Equation (17c) is obtained by dividing (15) by \( K \) and noting that \( \dot{m} = M/K - m(p + k) \) and \( \dot{b} = \dot{b}/K - b(p + k) \).
as the real budget deficit in capital units, since this is the amount to be financed by changes in $\text{m}$ or $\text{b/r}$.

Equations (16)-(17) provide the complete model.

**COMPARATIVE STATIC PROPERTIES OF THE INSTANTANEOUS EQUILIBRIUM**

By substituting for $a, q$ into equations (16a)-(16c), the instantaneous equilibrium of the system can be reduced to three equations, determining the three variables $y, r_k$, and $r$ in terms of the dynamically evolving variables $m, b$, and $\pi$, as well as other exogenous variables, such as $g$. Once these variables are determined, the instantaneous rate of price adjustment can then be determined recursively from (16f). As well as being of some interest in their own right, these results are necessary for the stability and steady-state equilibrium analysis undertaken later in this paper.

**Partial Derivatives with Respect to $\pi$**

The instantaneous response of the system to a change in the expected rate of price inflation is given by the following system of equations:

\[ L_y + J_y = -N_y = (1-n_5) \lambda/r_k > 0 \]
\[ L_r + J_r = -N_r = -n_2 + n_5 b/r^2 > 0 \]
\[ L_{\pi} + J_{\pi} = -N_{\pi} = -n_2 - (1-n_5) \lambda \pi/k > 0 \]
\[ L_{\pi} + J_{\pi} = -N_{\pi} = -n_3 - n_4 > 0. \]

\[ \text{For the evaluation of the various partial derivatives, it is useful to note that} \]

\[ L_y + J_y = -N_y = (1-n_5) \lambda/r_k > 0 \]
\[ L_r + J_r = -N_r = -n_2 + n_5 b/r^2 > 0 \]
\[ L_{\pi} + J_{\pi} = -N_{\pi} = -n_2 - (1-n_5) \lambda \pi/k > 0 \]
\[ L_{\pi} + J_{\pi} = -N_{\pi} = -n_3 - n_4 > 0. \]
\[
\begin{bmatrix}
    c_y & c_{r_k} & c_r \\
    l_y & l_{r_k} & l_r \\
    j_y & j_{r_k} & j_r \\
\end{bmatrix}
\begin{bmatrix}
    \partial y / \partial \pi \\
    \partial r_k / \partial \pi \\
    \partial r / \partial \pi \\
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    \ell_{\pi} \\
    j_{\pi} \\
\end{bmatrix}
\]  
(18)

which we write as

\[Dx = z\]

where

\[c_y = 1 - c_1(1-u) - c_2 \lambda / r_k - i_2 > 0.\]
\[c_{r_k} = c_2 \lambda y / r_k^2 - i_1 > 0.\]
\[c_r = c_2 b / r^2 > 0.\]
\[l_y = \ell_1 + \ell_5 \lambda / r_k > 0.\]
\[l_{r_k} = \ell_2 - \ell_5 \lambda y / r_k^2 < 0.\]
\[l_r = \ell_3 - \ell_5 b / r^2 < 0.\]
\[j_y = j_1 + j_5 \lambda / r_k \geq 0.\]
\[j_{r_k} = j_2 - j_5 \lambda y / r_k^2 < 0.\]
\[j_r = j_3 + (1-j_5)b / r^2 > 0.\]
\[\ell_{\pi} = \ell_3 + \ell_4 \geq 0.\]
\[j_{\pi} = j_3 + j_4 \geq 0.\]

Apart from the condition on \(c_y\), which ensures that the economy's IS curve has the usual downward slope, the signs on the other elements of \(D\)
follow from the restrictions given earlier. With these assumptions, the only indeterminate elements are \( j_y \) in the matrix \( D \), and \( \xi, j, \) in the vector \( z \).

Let \( |D| \) denote the determinant of the matrix \( D \). Five of the six terms in the sum that gives its value are negative. Alternative sufficient conditions to ensure that \( |D| \) is negative—and hence ensure that the instantaneous equilibrium can be solved uniquely for \( y, r, r_k \)—are

A.1a) The elasticity of the demand for money with respect to income is greater than or equal to the absolute value of the elasticity of the demand for money with respect to the rate of return on capital claims, or:

A.1b) The impact of wealth on consumption is negligible (i.e., \( c_2 = 0 \)).

Since the first of these two conditions in particular is most likely to be met, one can assume with some confidence that \( |D| < 0 \), and henceforth we assume this to be so.

Given either of the above conditions, sufficient conditions for \( \partial y/\partial w \) to be positive are

A.2 The own-rate elasticity of the demand for money is greater than or equal to the absolute value of the elasticity of the demand for money with respect to the return on bonds, and,

A.3 The absolute value of the elasticity of the demand for money with respect to the return on bonds weighted by the fraction of wealth held as capital is greater than or equal to the absolute value of the elasticity of money demand with respect
to the return on capital claims, weighted by the fraction of wealth held as bonds.

Furthermore, A.3 (which is not very intuitive) can be replaced by the above assumption A.1b. Thus, on balance one would expect $\partial y/\partial \pi$ to be positive, as indeed was almost certainly the case in the simple one period analysis of price expectations studied recently by Turnovsky (1974).

Next, consider the sign of $\partial r/\partial \pi$. One would expect this derivative to be positive, and a sufficient condition to ensure this will be so is:

$$A.4 \quad \ell_3 + \ell_4 = 0.$$  

Roughly speaking, as long as the difference between the own-rate derivative of money demand and the absolute value of the derivative of money demand with respect to the bond rate is small, $\partial r/\partial \pi$ will be positive. It is also likely that $\partial r/\partial \pi$ will be less than unity, implying that an increase in inflationary expectations will lead to a lower real return on bonds (instantaneously). Tobin (1969) with his assumption of fixed output, was able to show that this result always holds. In our case, however, where output is endogenously determined, this result requires the imposition of additional restrictions. One reasonably mild sufficient condition which will ensure $\partial r/\partial \pi < 1$ is:

$$A.5 \quad \text{The elasticity of bond demand with respect to wealth is greater than or equal to its elasticity with respect to income.}$$

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$^8$ A.2 is stronger than it need be, since $\ell_3 + \ell_4 \geq 0$ will suffice.

$^9$ A.5 is stronger than it need be since $j_1 + j_5 \lambda / r_k \geq 0$ will suffice.
On the other hand it is not possible, on the basis of simple suffi-
ciency conditions such as those we have been considering, to determine
the effect of a change in inflationary expectations on the real return to
claims on capital. The reason for this inherent indeterminacy is as fol-
lows. As we have just seen, it is likely that $\partial y/\partial \pi$ is positive. It
can be shown that the relative price of claims on capital, $q$, is also
likely to increase with anticipated inflation. Since $r_k$ depends directly
on $y$ and inversely on $q$ [see (10)], it is thus not surprising that the
sign of $\partial r_k/\partial \pi$ is indeterminate and will depend upon which of these ef-
fects dominates. This is again in contrast to the Tobin fixed output model,
in which only the $q$ effect operates, implying $\partial r_k/\partial \pi < 0$, unambiguously.

Finally, from the price adjustment equation (16a), it is clear
that given our assumptions on $\alpha$, $\beta$,

$$\text{sgn } (\partial p/\partial \pi) = \text{sgn } (\partial y/\partial \pi).$$

Partial Derivatives with Respect to $m$ and $b$

The partial derivatives with respect to changes in the stocks of
the financial assets can be obtained analogously by differentiating the
instantaneous equilibrium conditions. Since these calculations are simi-
lar to those given in (18) there is no need to report the full system.
Assuming $|D| < 0$, it can be shown, without further restrictions, that
$\partial y/\partial m$, and hence $\partial p/\partial m$ are both positive, as one would expect. Perhaps
rather surprisingly, one cannot show $\partial r/\partial m < 0$, without further assump-
tions. The simplest sufficient--but by no means necessary--condition is
A.1b, namely, that the impact of wealth on consumption is negligible.
Finally, the instantaneous effect of an increase in the money supply on the real return to capital is ambiguous, although intuitively one would expect $\frac{\partial r_k}{\partial m} < 0$. This, however, cannot be established without imposing several restrictions on the elasticities.

As one might expect, the partial derivative of the bond rate with respect to the bond-capital ratio $b$ is likely to be positive. Sufficient conditions to ensure this is so are given by assumptions A.1a and A.5 above. However, the sign of $\frac{\partial y}{\partial b}$, and hence that of $\frac{\partial p}{\partial b}$, is indeterminate. The reason for this is that on the one hand an increase in $b$ has, as we have just seen, the contractionary effect of raising $r$. On the other hand, more bonds means higher interest payments and thus higher disposable income, and this is expansionary. It also gives rise to wealth effects, the net effects of which are ambiguous and add to the overall indeterminacy. As in previous cases, the effect on $r_k$ is ambiguous, although if bonds and equity are close substitutes one would expect $\frac{\partial r_k}{\partial b} > 0$.

Summary

The results of the above comparative static analysis of the instantaneous equilibrium are summarized in the following table, in which the probable signs of the various derivatives are reported. We have also included the instantaneous derivatives with respect to government expenditure $g$, as these are required in subsequent analysis. These, however, are self-explanatory, and we do not discuss them further.
TABLE 1

SUMMARY OF PROBABLE SIGNS

<table>
<thead>
<tr>
<th>Derivative of</th>
<th>With respect to</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\pi$</td>
</tr>
<tr>
<td>$y$</td>
<td>+</td>
</tr>
<tr>
<td>$p$</td>
<td>+</td>
</tr>
<tr>
<td>$r$</td>
<td>+</td>
</tr>
<tr>
<td>$r_b = r - \pi$</td>
<td>-</td>
</tr>
<tr>
<td>$r_k$</td>
<td>?</td>
</tr>
</tbody>
</table>

THE STABILITY OF THE SYSTEM

It will be recalled from the second section that the dynamics of the system are specified by equations (17). However, since we are concerned with the system expressed per unit of physical capital, the evolution of $K$ itself does not concern us except as it affects the real budget deficit expressed in terms of units of capital. Therefore, substituting (17a) into (17c), we can analyze the dynamics in terms of equations (17b) and (17c). Provided the system is stable, capital stock, and thus real income and the real supplies of financial assets, will all be growing at the same rate in the intermediate-run equilibrium which we are considering. That rate is $i(r_k^*, y^*)$, where * denotes the equilibrium values of the endogenous variables which are arguments of $i$.

We can restrict our discussion of stability to the evolution of price expectations and the dynamics of financial asset accumulation. It will also be recalled from the second section that the dynamic adjustment
of the model is not uniquely determined until one specifies a policy
describing how the government finances its deficit. There are many ways
in which this might be done and the policies we consider below, while
quite plausible, are chosen in part for their analytical convenience.

We have expressed the dynamics of financial asset accumulation in
terms of $\dot{m}$ and $\dot{b}$, the rates of change of the real supplies of financial
assets per unit of capital, and we shall specify our policies in terms of
these real quantities as well. In practice, the monetary authorities may
tend to formulate their policies in terms of the nominal supplies of their
financial assets, namely $M$ and $B$, although $\dot{m}$ and $\dot{M}$ (and likewise
$\dot{b}$ and $\dot{B}$) are related by

$$\frac{\dot{m}}{m} = \frac{M}{M} - \frac{\dot{P}}{P} - \frac{\dot{K}}{K} = \frac{M}{M} - (p + 1) \tag{19}$$

It should be realized that while we are perfectly at liberty to specify
our policies directly in terms of $\dot{m}$ and/or $\dot{b}$, by doing so, we are im-
plcitly imposing an adjustment such as that given in (19) on the nominal
supply of the asset.

In specifying how the government finances its deficit, we shall
define the deficit to be purely bond financed if $\dot{m} = 0$, so that

$$\frac{\dot{b}}{r} = g - uy + (1-u)b - (m + b/r)(p + i).$$

Thus, pure bond financing does not mean that the nominal supply of money
is held constant; on the contrary it grows at the rate
\[ \frac{M}{M} = p + i(r_k, y). \]

Clearly in an inflationary economy in which capital is accumulating, the natural base for defining a neutral or passive monetary growth policy is one which in equilibrium would maintain portfolio balance. Given our specification of the asset demand functions, portfolio balance in equilibrium requires \( m \) to be constant, thus suggesting \( m = 0 \) as the appropriate benchmark policy. \(^\text{10}\)

At the other extreme, we shall define the deficit to be purely money financed if \( b = 0 \), so that

\[ m = g - uy + (1-u)b - (m + b/r)(p + i) \]

with identical remarks applying to this notion as just given for the pure bond case.

We shall consider the stability of the system under these two extreme assumptions. One could consider the intermediate case in which some fraction of the deficit is financed by bonds (over and above that required to maintain \( b = 0 \)), with the rest financed by money. As one would expect, this turns out to be an average of the two extremes being considered and we do not give it separate treatment.

\(^{10}\) This portfolio balance condition follows from our assumption that asset demand functions are homogeneous of degree one in wealth. Optimum portfolio rules in which asset demand is proportional to wealth are obtained when individuals are presumed to maximize expected utility with utility functions which display constant proportional risk aversion. See e.g. Hakansson (1970) and Merton (1971).
Pure Money Finance Policy

Solving the instantaneous equilibrium conditions for \( y, r_k, r \), the real government deficit per unit of capital, which we shall denote by \( d \), can be expressed in the form

\[
g - uy + (1-u)b - s(p + i) = d[m, b, \pi, g] \quad (20)
\]

where \( s \geq 0 \) denotes the real stock of government debt, per unit of capital. In the case where the government's deficit is purely money financed, the dynamics can be described by the following pair of differential equations in \( m, \pi \)

\[
m = d[m, \bar{b}, \pi, g] \quad (21a)
\]

\[
\pi = \gamma[p(m, \bar{b}, \pi, g) - \pi] \quad (21b)
\]

where \( \bar{b} \) denotes the fact that \( b \) is held constant.

Linearizing the pair of equations (21) about their equilibrium, the necessary and sufficient conditions for this system to be locally stable are

\[
\frac{\partial d}{\partial m} + \gamma \left( \frac{\partial p}{\partial \pi} - 1 \right) < 0 \quad (22a)
\]

\[
\frac{\partial d}{\partial m} \left( \frac{\partial p}{\partial \pi} - 1 \right) - \frac{\partial p}{\partial m} \frac{\partial d}{\partial \pi} > 0 \quad (22b)
\]

All the partial derivatives in (22) can, with the aid of (20), be obtained from the instantaneous comparative static analysis reported earlier. These
earlier results fall short of ensuring that these inequalities will be satisfied. Nevertheless, one would expect the system to be stable.

One set of conditions sufficient for stability, which the previous comparative statics results suggest are likely to be met, is

\[
\begin{align*}
\frac{\partial p}{\partial m} > 0, & \quad \frac{\partial p}{\partial \pi} < 1 \\
\frac{\partial d}{\partial m} < 0, & \quad \frac{\partial d}{\partial \pi} < 0
\end{align*}
\]

(23)

The positive sign of $\partial p/\partial m$ is obtained directly from table 1 and always applies. The incomplete adjustment of instantaneous price movements to changes in inflationary expectations, while tending to be supported by available empirical evidence, need not hold. In particular, given $\partial y/\partial \pi > 0$, it will cease to be true if the coefficient on price expectations in the price equation (i.e., $\beta$) approaches one. In this case, $\partial d/\partial m < 0$ becomes a necessary condition for (22a) to hold.

Differentiating $d$ with respect to $m$, $\pi$, we can express $\partial d/\partial m$, $\partial d/\partial \pi$, in terms of the instantaneous derivatives from the previous section.

\[
\begin{align*}
\frac{\partial d}{\partial m} = & - u \frac{\partial y}{\partial m} - s \left[ \frac{\partial p}{\partial m} + \frac{1}{1} \frac{\partial r}{\partial m} + \frac{1}{2} \frac{\partial y}{\partial m} \right] - (p+1) \left( 1 - \frac{b}{r} \frac{\partial r}{\partial m} \right) \\
\frac{\partial d}{\partial \pi} = & - u \frac{\partial y}{\partial \pi} - s \left[ \frac{(p+1)}{\partial \pi} \right] - (p+1) \frac{\partial s}{\partial \pi} \\
= & - u \frac{\partial y}{\partial \pi} - s \left( \frac{\partial p}{\partial \pi} + \frac{1}{1} \frac{\partial r}{\partial \pi} + \frac{1}{2} \frac{\partial y}{\partial \pi} \right) + (p+1) \frac{b}{r} \frac{\partial r}{\partial \pi}
\end{align*}
\]
On intuitive grounds one would expect an increase in the supply of money with government bonds held constant to reduce the government deficit, and (24a) indicates that this will almost certainly be so. Given the signs reported in table 1, and introducing the mild restriction \( p + i \geq 0 \), all partial derivatives in (24a) are negative, with the exception of \( \partial r_k / \partial m \), which is ambiguous. However, as was argued earlier, if claims on physical capital are a sufficiently close substitute for bonds in investors' portfolios, \( \partial r_k / \partial m \) will be negative as well, in which case \( \partial d / \partial m < 0 \) unambiguously.

The sign of \( \partial d / \partial \pi \) is somewhat less determinate. On the one hand, an increase in \( \pi \) raises income and with it tax receipts. It also raises the current rate of inflation and almost certainly the rate of investment, thereby raising the "real inflationary and growth tax" on existing debts. However, for given levels of \( m \) and \( b \), it will also almost certainly raise the nominal interest rate on bonds, thereby reducing the value of existing bonds, and hence the base on which this "inflationary and growth tax" is levied. On balance, it still seems reasonable to expect \( \partial d / \partial \pi < 0 \) as we have suggested.

By evaluating the terms involved in (23), the following alternative set of sufficient conditions for stability can be obtained

\[
\frac{\partial r_k}{\partial m} < 0, \quad \frac{\partial r_k}{\partial \pi} < 0
\]

\[
\frac{\partial p}{\partial \pi} < 1 \quad r > (\pi + i) \frac{b/r}{\delta} > 0
\]

(25)
None of the constraints on the derivatives appearing in (25) are ensured by the instantaneous comparative static results, although it does seem reasonable that they should be met. The constraint on the nominal rate of interest on bonds also seems likely to be satisfied for plausible choice of parameters. Taking the ratio of government bonds to total government debt \((b/r_a)\) to be about 0.5, and the growth rate of capital \(i\) to be about 0.03, it is clearly satisfied for realistic values of the remaining parameters \(r, \pi\).

One may conclude with some confidence that the pure money finance policy will be stable. However, the possibility of instability cannot be ruled out, especially if the coefficient of price expectations, \(\beta\), in the price equation tends to unity and expectations adjust rapidly to actual price changes \((\gamma \text{ large})\). Note however, as \(\gamma \to \infty\), so that inflation is perfectly anticipated, we do not necessarily get instability, as is the case in some (but not all) long-run models of money and growth.\(^{11}\) In this case, \(\partial d/\partial m\) would have to be negative and sufficiently large in absolute value to offset the destabilizing behavior of the wage-price sector. In either case, the stability conditions (23) highlight the role played by price movements and expectations in determining the stability of a purely money-financed government deficit.

Before leaving our discussion of the pure money finance policy, it is worth pointing out that essentially the same analysis also holds for a single open market operation (described by a substitution of bonds

\(^{11}\) See, e.g., Burmeister and Dobell (1970).
for money for a given level of the budget deficit). For example, an open
market sale of bonds implies a once-and-for-all increase in $b$; thereafter
$b$ stays constant at its new level, with all subsequent government defi-
cits or surpluses so generated being financed by changes in the money
supply. The dynamics are thus virtually identical to that of our pure
money finance policy.

**Pure Bond Finance Policy**

The other extreme, where the government deficit is financed by a
pure bond policy can be analyzed similarly. In this case the dynamic
equations are

$$
\cdot b = d(m, b, \pi, g) \tag{26a}
$$

$$
\cdot \pi = \gamma(p(m, b, \pi, g) - \pi) \tag{26b}
$$

with $m$ now referring to the fixed stock of money per unit of capital.
The necessary and sufficient conditions for local stability are

$$
\frac{\partial d}{\partial b} + \gamma \left( \frac{\partial p}{\partial \pi} - 1 \right) < 0 \tag{27a}
$$

$$
\frac{\partial d}{\partial b} \left( \frac{\partial p}{\partial \pi} - 1 \right) - \frac{\partial p}{\partial b} \frac{\partial d}{\partial \pi} > 0 \tag{27b}
$$

and again the relevant partial derivatives can be obtained from the pre-
vious section. Now, however, one can be much less certain that these con-
ditions will be met.

Analogous to (23), the following are sufficient conditions for

**stability**
\[ \frac{\partial p}{\partial b} > 0, \quad \frac{\partial p}{\partial \pi} < 1 \]

(28)

\[ \frac{\partial d}{\partial b} < 0, \quad \frac{\partial d}{\partial \pi} < 0 \]

two of which are the same as in the pure money policy, but the other two are quite indeterminate. As discussed previously, \( \partial y/\partial b \) (and \( \partial p/\partial b \)) cannot be signed on a priori grounds, as an increase in bonds has several offsetting effects on aggregate income. It will raise interest rates (contractionary), increase interest payments to the private sector and thus raise disposable income (expansionary) and increase wealth (indeterminate). Consequently, its net effect on output and inflation cannot be determined unambiguously without further knowledge of relevant parameter values.

Also, while it may seem reasonable to suppose that an increase in \( b \) with \( m \) held constant will reduce the government deficit, the effect is again more in doubt than is the corresponding effect of an increase in the supply of money. This can be readily seen from the following expression for this effect

\[ \frac{\partial d}{\partial b} = (1-u) - u \frac{\partial y}{\partial b} - s \left( \frac{\partial p}{\partial b} + i_1 \frac{\partial r_k}{\partial b} + i_2 \frac{\partial y}{\partial b} \right) - (p+1) \left( 1 - \frac{b}{r^2} \frac{\partial r}{\partial b} \right) \]

(29)

One of the key destabilizing factors is the net interest payment on the outstanding government debt. It is clear that this will be increased by
an increase in \( b \), thereby tending to raise the deficit. It is also apparent that given the likely signs for \( \partial r_k/\partial b, \partial r/\partial b \), it is almost certainly necessary for \( \partial y/\partial b > 0 \) in order to ensure \( \partial d/\partial b < 0 \).

Without going into the stability conditions (28) in any more detail, it is clear that pure bond financing is much more likely to be unstable than is a purely money financed budget deficit. If \( \partial y/\partial b > 0 \) and if the interest payments from the public to the private sector are not too large, then one may be reasonably confident that bond financing will be stable. This, however, is an open empirical question about which relatively little information is currently available.\(^{12}\)

INTERMEDIATE-RUN EQUILIBRIUM PROPERTIES

The steady-state solution to this intermediate-run model consists of the six instantaneous equilibrium conditions (16), together with the stationary solutions to the three dynamic equations (17a-c) and the policy specification. The latter relationships provide three extra equations

\[
p = \bar{p}
\]

\[
g - uy + (1 - u)b - (m + b/r)(p + i) = 0
\]

\[
m = \bar{m}, \text{ or } b = \bar{b},
\]

\(^{12}\) In this connection, we feel that Blinder and Solow may be perhaps excessively confident in concluding \( \partial y/\partial b > 0 \). They deduce this by claiming that \( \partial y/\partial b \) is analogous to an ordinary multiplier for transfer payments, which the U.S. empirical evidence suggests is between 1 and 2. But this analogy is not entirely correct. It is certainly true that one effect of an increase in \( b \) is to increase disposable income and in this respect it is similar to an increase in transfer payments. But the increase in \( b \) has wealth effects, not shared by transfer payments, and these are what create the indeterminacy.
giving a total of nine independent equations to be solved for the nine variables \( y, r_k, r, q, a, p, \pi, m, \) and \( b \). Eliminating \( p, q, a, \) the equilibrium (for which (23) and (28) are sufficient conditions for uniqueness) can be written more compactly as

\[
\begin{align*}
y = c[(1-u)(y+b), \lambda y/r_k + m + b/r] - i(r_k, y) - g &= 0 \\
\lambda[y, r_k, r-\pi, -\pi, \lambda y/r_k + m + b/r] - m &= 0 \\
j[y, r_k, r-\pi, -\pi, \lambda y/r_k + m + b/r] - b/r &= 0 \\
(1-\beta)\pi &= \alpha_0 - \rho + \alpha_1 \left( \frac{y-y}{y} \right) \\
g - uy + (1-u)b - (m + b/r)(\pi+i) &= 0
\end{align*}
\] (30a)

(30b)

(30c)

(30d)

(30e)

together with \( b = \bar{b}, \) or \( m = \bar{m}, \) depending upon whether a pure money or pure bond policy is being specified. The first three equations are just the condensed instantaneous equilibrium conditions; (30d) is the steady-state equilibrium price (and price expectations) adjustment relationship, while (30e) is the requirement that in equilibrium, the real stock of financial assets per unit of physical capital must be constant.

There are several features of this equilibrium to which attention should be drawn. First, unlike the Blinder and Solow long-run equilibrium, our intermediate-run equilibrium is one in which positive savings and investment is occurring. Second, the stationary solution is what is sometimes referred to as a "quasi-equilibrium" in that it is characterized by a constant rate of inflation, rather than a constant price level. Furthermore, because of the normalization we have adopted, all real quantities are also growing at the constant rate \( i(r_k, y). \)
Third, rewriting (30e) in the form

\[ p = \pi = \frac{g - uv + (1-u)b}{m + b/r} - i(r_k, y) = \frac{\dot{m} + \dot{B}/\dot{r}}{\dot{M} + \dot{B}/\dot{r}} - \frac{\dot{K}}{\dot{K}} \]  

(30e')

it becomes apparent that the equilibrium rate of inflation equals the difference between the rate of growth of nominal government debt and the rate of growth of real physical capital. This is a conclusion of some importance which we wish to stress. Looked at in this way, we see that the only way the government can reduce the rate of inflation over the intermediate-run is to reduce the rate of growth of its nominal debt relative to that of the real physical capital stock. Intuitively, the reason for this result is that, in steady-state equilibrium, portfolio balance can be maintained only if the supplies of all assets grow at the same real rate. From this it follows that the equilibrium rate of inflation must equal the difference in growth rates between the nominal supplies of assets—and in particular, the nominal government debt—and their common real growth rate.\(^{13}\)

---

\(^{13}\) As a quick preliminary check on the equilibrium relationship (30e), we regressed the rate of inflation in the U.S. (measured by the percentage change in the CPI) against the rate of growth of U.S. government debt. In order to make the regression conform as closely as possible to that of the theoretical closed model, all international flows of reserves and holdings of assets have been omitted. In practice of course these could be significant determinants of inflation, even for a relatively closed economy such as the U.S. Thus, the rate of growth of government debt was taken to be the ratio of its current deficit to outside debt, defined to be outside money (currency plus member bank reserves other than vault cash) and outside bonds (net held by U.S. public). Denoting this variable by \( z \), we used ordinary least squares to estimate

\[ \pi = \alpha_0 + \alpha_1 z \]

where according to (30e), \( \alpha_1 \) should be close to unity. As we are
The fourth property of some interest is the fact that even if \( \beta = 1 \)—so that (30d) implies a natural rate of unemployment and thus the absence of a steady-state tradeoff between inflation and unemployment (or excess demand)—the system still implies a determinate equilibrium rate of inflation. This is again given by (30e') with \( y \) set equal to
\[
\hat{y} = (\alpha_1 + \rho - \alpha_0)\bar{y}/\alpha_1
\]
the corresponding "natural rate of output."

**Some Comparative Static Properties**

To analyze the equilibrium comparative static behavior of the system, we can differentiate (30) with respect to the parameter of interest and solve for the resulting set of partial derivatives. This turns out to be a cumbersome exercise and for the properties with which we are most concerned, namely, the comparative inflationary and expansionary effects of the two alternative extreme means of financing increases in government expenditure, it is more convenient to work in terms of the

---

concerned with an equilibrium relationship, this regression was run using 3-year moving averages on annual data extending over the period 1956-73.

We must emphasize that this is an extremely crude exercise.

(1) Since \( z \) is an endogenous variable, which on the basis of our instantaneous comparative static results is likely to be negatively correlated with \( \pi \), the OLS estimates will tend to be downwardly biased. (2) The smoothing of the data also introduces problems of autocorrelation, which we did not attempt to correct. (3) There are the problems of specification already mentioned.

With these reservations in mind the resulting equation is
\[
\pi = 0.0148 + 0.607z \quad R = 0.48, \quad D.W. = 0.74
\]
\[ [3.21] \quad [3.81]\]
with the t-statistics being in brackets. Given the downward bias of OLS, the coefficient on \( z_t \) tends to support the equilibrium relationship (30a).
stationary solutions to the dynamic functions described in (21) and (26). These equations, which of course embody implicitly the instantaneous equilibrium conditions (30a)-(30c), yield equilibrium solutions for the pairs \((\pi, m), (\pi, b)\) respectively. The corresponding value of output can then be readily obtained from the steady-state price relationship (30d), (except of course when \(\beta = 1\), when we know it is fixed).

Looking first at the case of a purely money-financed government deficit, the equilibrium levels of \(\pi, m\), are given by the stationary solutions to (21), namely

\[
\begin{align*}
d(m, \overline{b}, \pi, g) &= 0 \\
p(m, \overline{b}, \pi, g) - \pi &= 0
\end{align*}
\]  

(31a) 

(31b)

For simplicity, let us assume that the sufficient conditions for stability (23), which we have already argued are likely to be met, are in fact satisfied, so that

\[
d_1 < 0, \quad d_3 < 0, \quad p_1 > 0, \quad (p_3 - 1) < 0
\]

(32)

We have already seen from table 1 that an increase in government expenditure will increase the instantaneous level of output and rate of inflation. Moreover, it seems reasonable to expect that for given \(m\) and \(b\), it will increase the instantaneous government deficit, although this result cannot be established unambiguously. Accepting the latter result, we have the further restrictions

\[
d_4 > 0, \quad p_4 > 0
\]

(33)
Using (32) and (33), we can readily show that with pure money financing, an increase in government expenditure will be unambiguously inflationary, the effect being

\[
\left( \frac{\partial \pi}{\partial g} \right)_m = \frac{p_4 d_4 - p_4 d_1}{d_1 (p_3 - 1) - p_1 d_3} > 0
\]  

(34)

The corresponding impact on the equilibrium supply of money is given by

\[
\frac{\partial m}{\partial g} = \frac{p_4 d_3 - d_4 (p_3 - 1)}{d_1 (p_3 - 1) - p_1 d_3} < 0
\]  

(35)

and is ambiguous. While intuitively one would expect \(\partial m/\partial g > 0\), it is interesting to note that in the "natural rate" case where \(\beta = 1\) and hence \(p_3 > 1\), (but the system is stable), (35) implies \(\partial m/\partial g < 0\). The reason for this rather counter-intuitive result is that, starting from equilibrium, an increase in \(g\) will raise the rate of inflation, both directly [see (33)] and indirectly, through its induced effects on inflationary expectations. Moreover, the latter indirect effects are now destabilizing, so that the only way for equilibrium to be restored is if \(m\) is ultimately reduced, thereby providing an offsetting deflationary and stabilizing effect. This can be seen immediately by differentiating (31b) with respect to \(g\).

With pure bond financing, the equilibrium levels of \(\pi\), \(b\) are determined by the stationary solutions to (26), namely,

\[
d(\bar{m}, b, \pi, g) = 0
\]  

(36a)

\[
p(\bar{m}, b, \pi, g) - \pi = 0
\]  

(36b)
so that

\[
\left( \frac{\partial \pi}{\partial g} \right)_b = \frac{d_4 p_2 - d_2 p_4}{d_2 (p_3 - 1) - p_2 d_3} > 0
\] (37)

where the stability condition (27b) implies \(d_2 (p_3 - 1) - p_2 d_3 > 0\). If we now add the conditions \(p_2 > 0, d_2 < 0\), which we showed in (28) form part of a set of sufficient conditions for stability, but which we nevertheless argued are somewhat tenuous, then we have unambiguously

\[
\left( \frac{\partial \pi}{\partial g} \right)_b > 0
\] (37')

The effect on the equilibrium supply of bonds is

\[
\frac{\partial b}{\partial g} = \frac{p_4 d_3 - d_4 (p_3 - 1)}{d_2 (p_3 - 1) - p_2 d_3}
\] (38)

implying \(\text{sgn} \left( \frac{\partial b}{\partial g} \right) = \text{sgn} \left( \frac{\partial m}{\partial g} \right)\) if both methods of financing are stable.

A related question of some importance is to determine the comparative inflationary effects of the two modes of deficit financing. This is obtained by subtracting (37) from (34), yielding

\[
\left( \frac{\partial \pi}{\partial g} \right)_m - \left( \frac{\partial \pi}{\partial g} \right)_b = \frac{[d_4 (p_3 - 1) - p_4 d_3][p_1 d_2 - d_1 p_2]}{[d_2 (p_3 - 1) - p_2 d_3][d_1 (p_3 - 1) - p_1 d_3]}
\]

from which it follows that

\[
\text{sgn} \left[ \left( \frac{\partial \pi}{\partial g} \right)_m - \left( \frac{\partial \pi}{\partial g} \right)_b \right] = \text{sgn} \left\{ \frac{\partial m}{\partial g} \left( \frac{\partial d}{\partial m} \frac{\partial y}{\partial g} - \frac{\partial y}{\partial m} \frac{\partial d}{\partial g} \right) \right\}
\] (39)
Let us consider the more plausible case where $\partial m/\partial g > 0$ (and equivalently $\partial b/\partial g > 0$). In this case, the comparison reduces to

$$\left( \frac{\partial \pi}{\partial g} \right)_m - \left( \frac{\partial \pi}{\partial g} \right)_b > 0 \quad \text{according as} \quad \frac{\partial d}{\partial m} \frac{\partial y}{\partial b} > \frac{\partial y}{\partial m} \frac{\partial d}{\partial b}$$

(40)

which, recalling the fact that $\partial y/\partial m > 0$ and adding the further weak restriction $\partial d/\partial m < 0$, can be written in relative form

$$\left( \frac{\partial \pi}{\partial g} \right)_m - \left( \frac{\partial \pi}{\partial g} \right)_b > 0 \quad \text{according as}$$

(41)

$$\frac{\partial d}{\partial b} \left/ \frac{\partial d}{\partial m} \right. > \frac{\partial y}{\partial b} \left/ \frac{\partial y}{\partial m} \right.$$  

In economic terms this says that if the instantaneous impact of bond financing on the deficit relative to the impact of money financing on the deficit is algebraically greater than (less than) the impact of bond financing on output relative to the output effect of money financing then money financing will be more (less) inflationary than bond financing. The intuition behind this result can be seen most clearly in the special case where output effects are equal, so that $\partial y/\partial b = \partial y/\partial m$, and where $\partial d/\partial b < 0$, as well. In this case, (41) asserts that money financing will be more inflationary than bond financing if and only if the instantaneous reduction in the budget deficit resulting from an increase in $b$ exceeds that from a marginal increase in $m$.  

In fact, if the output effect of bond financing is positive but less than the output effect of money financing, it is almost certainly the case that bond financing will be more inflationary than money financing for $3m/3g > 0$. For example, with $3y/3b = 3y/3m$, a sufficient condition to ensure this is\textsuperscript{14} 

$$\frac{3r}{3b} - \frac{3r}{3m} > \left( \frac{3s_1}{(p+1)^{b/2}} \right) \left( \frac{3r_k}{3b} - \frac{3r_k}{3m} \right)$$

Our earlier comparative statics suggest that the left side of this inequality is positive. Furthermore, the right side is likely to be negative and, even if it is positive, it is small (the term in brackets was probably less than 0.05 for the U.S. in 1973).

The various conclusions stemming from (41) would of course all need to be reversed as $\beta + 1$ and the sign of $3m/3g$ changes.

The effects of increased government expenditure on the equilibrium rate of inflation can be used to determine corresponding impacts on output. First, provided $\beta \neq 1$ so that a steady-state tradeoff exists between inflation and output, we immediately deduce

$$\text{sgn} \frac{3y}{3g} = \text{sgn} \frac{3\pi}{3g} \quad (42)$$

with the comparative effects satisfying

\textsuperscript{14}For $3y/3m > 3y/3b > 0$, we also require that the quantity $(1-u-p-i)$ is nonnegative which is surely the case for the U.S.
\[
\text{sgn}\left(\frac{\partial y}{\partial \psi}_m - \frac{\partial y}{\partial \psi}_b\right) = \text{sgn}\left(\frac{\partial \pi}{\partial \psi}_m - \frac{\partial \pi}{\partial \psi}_b\right)
\] (43)

Thus, the comments made with respect to the impact of increased government expenditure on inflation apply to output as well.\footnote{In particular (37') and (41) give answers to the Blinder-Solow question "Does fiscal policy matter?" in the context of our model. The analysis of (41) suggests that there are plausible circumstances in which bond-financed fiscal policy matters more than money-financed fiscal policy.} With \( \beta = 1 \), we know \( y = \gamma \), implying
\[
\frac{\partial y}{\partial \psi} = 0
\] (44)
irrespective of how the deficit is financed.

The "Fisher Effect"

Another comparative static question which has attracted recent attention from monetary economists is the Fisherian proposition that nominal interest rates fully incorporate inflationary expectations. This proposition has been the subject of detailed empirical investigation and has also been examined within the context of fairly simple, integrated macroeconomic models.\footnote{For empirical evidence, see, e.g., Yohe and Karnosky (1969) and Pyle (1972); theoretical analyses within the context of integrated short-run macroeconomic models are given in Sargent (1972) and Turnovsky (1974).} On the whole, these studies suggest that the nominal interest rate adjusts only partially to inflationary expectations, at least within the fairly short-run they usually consider. Thus, real rates of return tend to fall with increases in inflationary expectations,
a conclusion which tends to be confirmed by the instantaneous comparative statics we presented earlier.

However, Fisher was probably concerned with a longer-run relationship than that considered by these models and certainly longer than that given by an instantaneous model. We therefore briefly consider the relationship between inflationary expectations and interest rates within the context of the steady-state equilibrium of our model. In considering such steady-state relationships, it should be realized that, in equilibrium, both the rate of inflation (and equivalently its expectation) and the rate of interest are endogenously determined. It therefore makes no sense to calculate and interpret derivatives such as $\partial r/\partial \pi$ in the usual comparative static way. Recognizing this, authors such as Sargent (1973b) reinterpreted the Fisher proposition to assert that in equilibrium the real rate of interest is independent of the systematic part of the money supply.

To consider this hypothesis one can differentiate the equilibrium system (30) with respect to an exogenous shift in the supply of an asset, say money, and calculate $\partial r_k/\partial \phi$, $\partial r_b/\partial \phi$ (where $r_b = r - \pi$). From the general structure of this system we can see that there is no reason for $\partial r_k/\partial \phi = \partial r_b/\partial \phi = 0$ and there is little point in reporting the actual calculations. Some idea of the issues involved can be obtained by considering the example where $\beta = 1$ (so that $y = \gamma$) and where there are no wealth effects in consumption ($c_2 = 0$). These, incidentally, were two of the critical assumptions made by Sargent, who showed that the existence of a natural rate of unemployment and the validity of this version of the Fisherian proposition are intimately related.
In this case, differentiating (30a) with respect to the exogenous shift parameter $\phi$, we have

$$c_1 (1-u) \frac{\partial b}{\partial \phi} + i_1 \frac{\partial r_k}{\partial \phi} = 0.$$  

With pure money financing $\frac{\partial b}{\partial \phi} = 0$, implying $\frac{\partial r_k}{\partial \phi} = 0$, so that the real return on capital is indeed independent of systematic shifts in the supply of money. With pure bond financing, however, $\frac{\partial b}{\partial \phi}$ is presumably negative, in which case $\frac{\partial r_k}{\partial \phi} < 0$. The effect on the real rate of interest on bonds is somewhat more complex and involves differentiating the asset demand functions, together with the steady-state government budget constraint. Even with the special assumptions we are making, we deduce that in general $\frac{\partial r_b}{\partial \phi} \neq 0$; moreover the effect on the real rate of interest will vary with the mode of government finance. One case in which $\frac{\partial r_b}{\partial \phi} = 0$ is if bonds and equity are perfect substitutes and all incremental government debt is money financed.

**Some Policy Implications**

This model has some definite policy conclusions to offer for "intermediate-run" inflationary control. Recalling (30e'), the equilibrium rate of inflation equals the difference between the rate of growth of nominal government debt and the rate of growth of real physical capital. Thus, the only way to reduce the rate of inflation over the intermediate run is to reduce this difference. One way we can be fairly sure this can be achieved is to reduce the level of government expenditure and accommodate the resulting surpluses by withdrawals in the money supply. However,
as long as $\beta \neq 1$, this will have the effect of simultaneously reducing
equilibrium output and employment and this is presumably undesirable.
Moreover, from the price equation we have

$$\frac{\partial y}{\partial p} = \frac{\partial p}{\partial \omega}$$

where $\omega$ is a parameter denoting exogenous monetary policy. That is, the
relative impact of monetary and fiscal policy on output is the same as it
is on the rate of inflation, so that it is impossible to achieve independent target values for these two policy objectives by using conventional monetary and fiscal instruments.

However, our analysis does suggest a positive policy. We have
already seen that when $\beta = 1$, the equilibrium output is $y = \hat{y}$, the
"natural rate" of output. In this case a cut in government expenditure
will lower the rate of inflation without reducing the equilibrium level
of output and employment. This suggests some kind of indexation policy
which results in $\beta = 1$, so that wages and ultimately prices are fully
adjusted for expectations (or perhaps more operationally for past price changes). Such a policy, together with a contractionary fiscal policy,
should have the effect of eventually reducing the rate of inflation with-
out affecting equilibrium output or employment, although it will of course
have contractionary short-run effects.

There is one serious difficulty—even at the purely theoretical level—with such a policy. This involves the instability of the system.
In the preceding section it was shown that with $\beta = 1$, the wage-price sector becomes unstable. In this case, only if the asset adjustment
process is sufficiently stable will the overall system not diverge. Thus, an indexation policy of the type we are suggesting, which in effect sets $\beta = 1$ and thereby induces instability into the wage-price sector, is feasible only as long as the remainder of the system is sufficiently stable to be able to accommodate this destabilizing influence. Otherwise it introduces the risk of setting off an unstable hyperinflationary process. This risk may be severe especially if $\gamma$ (the coefficient of expectations adjustment) is large.

SUMMARY

In this paper we have analyzed the dynamics of an "intermediate-run" model of an inflationary economy. It is intermediate run in the sense that the model takes full account of the dynamics of capital accumulation on demand and on portfolio behavior in a way precisely analogous to the treatment of financial assets, but ignores the longer-run supply effects. This is done by normalizing appropriate quantities relative to the stock of physical capital. The most important conclusions of our analysis can be summarized as follows.

The stability of the system depends crucially upon how the government finances its deficit, as well as upon the degree to which wage and price changes incorporate inflationary expectations and the speed with which these expectations respond to past price changes. With what we call pure money financing (a policy in which the real supply of government bonds per unit of capital is held constant), one can be reasonably sure that the system will be stable provided inflationary expectations are not
fully reflected in current price changes. However, it is possible for the wage-price sector to be destabilizing, in which case the instability of the entire system cannot be dismissed. This possibility arises as the coefficient of price expectations in the price equation tends to unity and expectations adjust rapidly to actual price changes. In this case, the dynamics describing the accumulation of assets would have to be "strongly stable" in order to compensate for the destabilizing influence of the wage-price sector.

On the other hand, even with a stable wage-price sector, stability becomes less clear when the deficit is entirely bond financed. One of the destabilizing factors is the interest owing on outstanding bonds. This interest needs to be continually financed. A further destabilizing element is the fact that the impact of an increase in bonds on output is indeterminate. This is due to the fact that, while an increase in bonds will have a contractionary effect through higher interest rates, they also have an expansionary effect in the form of higher interest payments and larger disposable income. If these expansionary effects dominate, one can again be reasonably confident that the system will be stable.

Assuming that the system is stable, its steady state is one in which there is a constant rate of inflation and all real quantities grow at the same constant rate as capital. This equilibrium rate of price change is shown to equal the difference in the growth rate of nominal government debt and the rate of growth of real physical capital. Moreover, even when $\beta = 1$ so that there is no steady-state tradeoff between inflation and output, a determinate equilibrium rate of inflation is obtained; equilibrium conditions in the asset markets determine a unique point on the vertical steady-state Phillips curve.
In considering the comparative static properties of this equilibrium, we show that under plausible conditions a money-financed increase in government expenditure will be unambiguously inflationary, although the corresponding effect under bond financing is somewhat less determinate. However, with some added restrictions, notably that \( \frac{\partial y}{\partial b} > 0 \), one can be fairly confident that this latter case will be inflationary as well.

The comparative inflationary effects of these two modes of finance are also considered and assessed in terms of their relative effects on the government deficit on the one hand, and their relative output effects on the other.

We also briefly consider the equilibrium "Fisher Effects" and reach the following conclusions. In general the real rates of return depend crucially upon the mode of government finance. Where bonds and equity are imperfect substitutes, the real rate of interest on bonds will respond to shifts in the money supply. The Fisherian proposition will hold with respect to the real return on capital in the case where the incremental debt is money financed; it will not hold with bond financing. This same proposition applies to the real rate of interest on bonds, in the limiting case where bonds and physical capital are perfect substitutes in investors' portfolios.

Finally, we use the equilibrium properties of the model to draw some intermediate-run policy implications. The fact that the equilibrium rate of inflation equals the difference in the growth rate of nominal government debt and the growth rate of physical capital implies that the only way the rate of inflation can be reduced is to reduce this difference.
One way this can be done, without simultaneously reducing equilibrium output and employment, is to introduce an indexation policy which sets $\beta = 1$, while at the same time cutting government expenditure. In this case, equilibrium output becomes fixed by the natural rate of unemployment and thus is unaffected by any contractionary fiscal measures. However, one serious difficulty of such a policy must be stressed. It introduces an instability into the wage-price sector and, unless the remainder of the system is sufficiently stable, its implementation may generate an unstable inflation.
LIST OF REFERENCES


