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QUALITY CHOICE AND COMPETITION

Hayne E. Leland

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Hayne E. Leland is an Acting Associate Professor, Graduate School of Business Administration, University of California, Berkeley. Research for this paper was supported in part by a grant from the Dean Witter Foundation.
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by

Hayne E. Leland*

1. Introduction

Current economic theory does not offer a comprehensive explanation of quality choices by firms. For fixed product designs or quality levels, received theory explains quantity decisions made both by perfectly and by imperfectly competitive firms. But production designs typically are not fixed. Firms must choose not only how much they produce, but also the nature of what they produce. By altering design parameters and inputs per unit output, firms can choose from a wide range of alternative "quality" levels: any parking lot provides sufficient evidence that a car is not a car is not a car. And in their effect upon consumer welfare, the decisions about what is produced are as important as the decisions about how much is produced.

Traditional theory has treated different quality levels of a good as if they were different goods. The problem with this approach is that there is no metric to determine the "closeness" of different products. Without such a measuring rod, it is difficult to use the powerful tools of continuity and of marginal analysis, which have proved so useful in developing the theory of quantity selection by firms.

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To bypass this problem, Dorfman and Steiner [1954], Rosse [1972] and others have introduced quality parameters directly into demand functions: \( D = D(p, q) \), where \( p \) is price and \( q \) is "quality." This permits marginal analysis. But because it is not derived from an underlying utility framework, the approach is not promising for the study of the welfare effects of quality choices.

In this paper, we develop a comprehensive theory of quality choice using an alternative approach to consumer theory. This approach, developed by Lancaster [1966], presumes that goods themselves do not directly provide utility, but rather provide basic "characteristics" which consumers value. Related approaches have been used by Fisher, Griliches, and Kaysen [1962] to study automobile quality change, and by Muth [1966] to examine housing demand. The "household production function" approach of Becker [1965] and others utilizes similar concepts.

The characteristics approach describes each good by an \( S \)-dimensional vector, whose elements indicate the amount of each characteristic provided per unit of that good. One advantage of this analysis is that it introduces a natural metric on the "closeness" of goods. Another advantage is that it permits a simple notion of quality change: a change in the amount of each characteristic provided per unit of the good.\(^1\) Note that "quality change" by this definition may not be unambiguously good or bad. Some characteristics may be provided in greater quantities, while others in lesser. Individual may differ in whether they regard such a change as desirable. We thus avoid some of the conceptual problems encountered by
Sweeney [1975] in defining "quality."

While the characteristics formulation is familiar to students of consumer theory, many of our results draw from analysis in quite a different field: asset market equilibrium and production under uncertainty. The similarities are based on the fact that a share of stock can be viewed as a vector of returns across states of nature (rather than across characteristics). As firms change production decisions, this pattern of returns will change (i.e., there is a change in "quality" of returns). Results on the optimality of production under uncertainty have been developed in Leland [1973]. Some of what follows involves a reinterpretation of these results in the framework of characteristics and quality choice.

In Sections 1 - 4, we develop a simple model of general equilibrium with quality choice. Section 5 generates necessary conditions for quality and quantity choices to be Pareto optimal. Sections 6 and 7 introduce two concepts which are crucial to optimality: "spanning" and "competitive implicit characteristic prices." Section 8 proves that these properties are sufficient for profit maximizing firms to make decisions which satisfy the Pareto optimality conditions developed in Section 5, and shows that quality choice has aspects of a "public good." Section 9 considers Pareto optimality when spanning is not present. Section 10 examines divergences from competitive behavior and indicates that monopolies tend to underprovide quality, given their output choice. Section 11 concludes the paper.
2. Goods and Firms

We assume that there are a finite number of characteristics \( s = 1, \ldots, S \) which generate utility for at least one consumer. A good \( j \) is described by a vector

\[
c^j = (c^j_1, \ldots, c^j_S) \quad , \quad j = 1, \ldots, J
\]

where \( c^j_s \) is the amount of characteristic \( s \) provided per unit of good \( j \). For simplicity, we assume each good can be associated with a corresponding firm \( j = 1, \ldots, J \).  

"Quality" variations can now be parameterized simply. Let \( q^j \) be a design or quality parameter which can be adjusted by the firm. Changing \( q^j \) will in general change each element of the vector \( c^j \). Thus a good with variable quality is described by

\[
c^j(q^j) = [c^j_1(q^j), \ldots, c^j_S(q^j)]
\]

It is not required that individuals react unanimously to changes in the quality variable. If an increase in \( q^j \) increases the provision of all characteristics (which are presumed to be desirable) then consumers will agree unanimously that "quality has increased." But if some characteristics are enhanced, whereas others are diminished, consumers may disagree as to whether "quality" has increased or decreased.

Firms in our model are characterized by an implicit production function

\[
f^j(q^j, y^j, x^j) = 0,
\]
which relates quality \( q^j \) and output quantity \( y^j \) with input quantities \( x^j = (x^j_1, \ldots, x^j_L) \). We shall assume that \( f^j \) exhibits the usual concavity properties with respect to \( q^j \) as well as with respect to \( y^j \) and \( x^j \). Note that, for a fixed input vector \( x^j \), (1) describes a transformation curve between maximal output \( y^j \) and quality \( q^j \) per unit output.

Firms will choose the \((q^j, y^j, x^j)\) combination satisfying (1) which maximizes profits. We presume initially that output price \( p^j \) may depend upon \( y^j \) and \( q^j \). For simplicity, we assume the vector \( r = (r_1, \ldots, r_L) \) of input prices is considered constant. Thus

\[
(2) \quad \pi^j(q^j, y^j, x^j) = p^j(q^j, y^j)y^j - r^j x^j.
\]

Maximizing (2) subject to (1) with respect to \((q^j, x^j, y^j)\) yields first order necessary conditions

\[
(3) \quad \frac{\partial p^j(q^j, y^j)}{\partial q^j} y^j + \mu^j f^j = 0;
\]

\[
(4) \quad p^j(q^j, y^j) + \frac{\partial p^j(q^j, y^j)}{\partial y^j} y^j + \mu^j f^j_y = 0;
\]

\[
(5) \quad -r + \mu^j f^j_x = 0;
\]

\[
(6) \quad f^j(q^j, y^j, x^j) = 0; \quad j = 1, \ldots, J,
\]

where \( f^j_q = \frac{\partial f^j(q^j, y^j, x^j)}{\partial q^j} \), etc. Note that if \( q^j \) is fixed, equations (4) - (6) describe the usual profit maximizing conditions.
3. Consumers and Demand

Consumers are assumed to have preferences defined over bundles of characteristics. Let

\[ R_i = (R_{i1}, \ldots, R_{iS}) \]

describe a bundle consumed by \( i \), where \( R_{is} \) is the amount of characteristic \( s \). If preference rankings over a set of bundles exhibit the normal properties, consumers will possess nonsatiated quasi-concave ordinal utility functions

\[ U_i = U_i(R_{i1}, \ldots, R_{iS}) \quad , \quad i = 1, \ldots, I. \]

This formulation is consistent both with Lancaster's approach (where \( s \) indexes characteristics) and with the "state preference" approach of Arrow-Debreu (where \( s \) indexes states of nature and \( R_i \) is a vector of returns across states).

The amount of each characteristic \( s \) consumed by \( i \) depends on the bundle of goods consumed and the quality of those goods. Following Lancaster's emphasis that characteristics are in principle physically measurable, we presume that the amount of characteristic \( s \) a bundle of goods \( y_i = (y^1_i, \ldots, y^J_i) \) provides is equal to the sum of the contributions of each unit of each good.\(^5\) Thus

\[ R_{is} = \sum_{j} c^j_s (q^j_i) y^j_i \quad , \quad s = 1, \ldots, S \]

or in matrix terms
(8') \quad R_i = C(q)y_i,

where $C(q)$ is an $S \times J$ matrix with elements $c_s^j(q^j)$ and $q$ is the vector of quality decisions $q^1, \ldots, q^J$. Consumers select the bundle of goods $y_i$ which maximizes utility (7) subject to a budget constraint:

(9) \quad \sum_p^j y_i^j = \sum_r^r x_i r + \sum_j^j \theta_i^j

or in matrix terms

(9') \quad p'y_i = r'x_i + \theta_i'

where

$p' = (p^1, \ldots, p^J)$ is the vector of prices of goods $1, \ldots, J$;

$x_i = (x_i^1, \ldots, x_i^J)$ is the vector of primary goods initially owned by consumer $i$; and

$\theta_i = (\theta_i^1, \ldots, \theta_i^J)$ is the vector of fractions of each firm owned by consumer $i$.

Transposes denote row vectors. Appending the budget constraint with Lagrangean multiplier $\lambda_i$ and finding a stationary point gives first order necessary conditions for utility maximization:

(10) \quad \sum_{s=1}^S c_s^j(q^j) + \lambda_i p^j = 0 \quad ; \quad j = 1, \ldots, J \quad , \quad i = 1, \ldots, I
where $U_{is} = \partial U_{s}/\partial R_{is}$, or in matrix terms
\[(10') \quad U_{i}^{t} C(q) + \lambda_{i} p' = 0 \quad ; \quad i = 1, \ldots, I\]
where $U_{i}^{t} = (U_{i1}, \ldots, U_{iS})$. 

It is of some interest to note the similarity between conditions (10) and the portfolio equilibrium conditions in Leland [1973]. In fact, the conditions are formally identical. It is often useful to think of a commodity bundle as a "portfolio," providing an optimal balance of "returns" across characteristics. Further axioms on choice could lead to an equivalent of the expected utility theorem, with the corpus of portfolio theory becoming directly applicable. Such is not, however, our current purpose. Our immediate goal is to characterize quality choices by firms in general equilibrium.

4. Equilibrium

Equilibrium requires that individual units have no motivation to change their decisions and that markets are cleared. In our model, consumers will be in equilibrium when conditions (9) and (10) are satisfied. Note that consumers regard quality and price vectors as parameters.

Firms will be in equilibrium given conditions (3) - (6) are satisfied. Note that the derivatives $\partial p^{j}/\partial q^{j}$ and $\partial p^{j}/\partial y^{j}$ are perceived price changes, which may or may not be those which actually would occur. Models of perfect competition, for example, assume that $\partial p^{j}/\partial y^{j} = 0$. Since quality changes are not considered, this is equivalent to assuming that firms view prices as parameters.
But surely—even if $\frac{\partial p^j}{\partial y^j} = 0$ — we cannot argue that $\frac{\partial p^j}{\partial q^j} = 0$. In fact, if there were no perceived relation between price and quality, firms would be motivated to produce only the least expensive (and perhaps lowest quality) good. So a critical question affecting equilibrium and its optimality (or lack thereof) is specifying how firms perceive $\frac{\partial p^j}{\partial q^j}$. This is explored in detail in Section 7.

The final link to close the equilibrium model is a set of market equilibrium conditions

\begin{align}
(11) \quad & \sum_{i} y_{ij} = y^j, \quad j = 1, \ldots, J ; \\
(12) \quad & \sum_{i} \tilde{x}_{il} = \tilde{x}_{l}, \quad l = 1, \ldots, L. 
\end{align}

Equilibrium, then, must satisfy equations \((3) - (6)\) and \((9) - (12)\). Since our present concern is with properties of equilibrium when it exists, we shall not address the difficult problem of proving that an equilibrium does exist. 8/

5. Pareto Optimality with Quality Choice: Necessary Conditions

To examine necessary conditions for Pareto optimality, we use the standard technique of maximizing the utility of an arbitrary "first" consumer, holding other utility levels constant. For ease of notation, we consider the two consumer case with a single input \((I = 2, L = 1)\). The diligent reader can ascertain that our results hold in the general case. It should be noted that we treat the \textit{number} of firms as an exogenous variable. 9/
Optimality requires that quality, production, and distribution decisions

\[ \text{Maximize} \quad U_1(R_{11}, \ldots, R_{1S}) \]

\text{subject to} \quad \begin{align*}
(a) \quad & U_2(R_{21}, \ldots, R_{2S}) = \bar{U}_2 \\
(b) \quad & y_1^j + y_2^j = y_1^j, \quad j = 1, \ldots, J \\
(c) \quad & \sum_j x^j = \bar{x} = \bar{x}_1 + \bar{x}_2 \\
(d) \quad & f^j(q^j, y^j, x^j) = 0, \quad j = 1, \ldots, J.
\end{align*}

The maximization is with respect to \( y_1^j, y_2^j, y_j^j, x_j^j \), and \( q^j \). Rather than append constraint (b), we substitute for \( y_2^j = y_j^j - y_1^j \) directly. The Lagrangean expression is

\[
L = U_1[\sum_j c_1^j(q^j)y_1^j, \ldots, \sum_j c_S^j(q^j)y_1^j] + \lambda*[U_2[\sum_j c_1^j(q^j)(y_j^j - y_1^j), \ldots, \sum_j c_S^j(q^j)(y_j^j - y_1^j)] - \bar{U}_2]
\]

\[ + \gamma*[\sum_j x^j - \bar{x}] + \sum_j \mu^j f^j(q^j, y^j, x^j). \]

Stationary conditions are

\[
\frac{\partial L}{\partial y_1^j} = \sum_s c_s^j(q^j) - \lambda*\sum_s c_s^j(q^j) = 0;
\]

\[
\frac{\partial L}{\partial y_j^j} = \lambda*\sum_s c_s^j(q^j) + \mu^j f^j = 0;
\]
\[ \frac{\partial L}{\partial x_j} = \gamma^* + \mu^j f^j \frac{\partial L}{\partial x} = 0; \]

\[ \frac{\partial L}{\partial q^j} = s \sum s \frac{\partial c_s^j}{\partial q^j} y_1^j + \lambda^* \sum s \frac{\partial c_s^j}{\partial q^j} (y^j - y_1^j) + \mu^j f^j = 0; \]

\[ \frac{\partial L}{\partial \mu^j} = f^j(q^j, y^j, x^j) = 0; \]

\[ y_1^j + y_2^j = y^j; \]

\[ \frac{\partial L}{\partial y^*} = \sum x^j - x^* = 0; \]

where equations (13) – (18) hold for \( j = 1, \ldots, J \). We also have \( U_2 = \bar{U}_2 \),

but since \( \bar{U}_2 \) can be set arbitrarily, we do not include it directly with

the other necessary conditions.

These conditions will be sufficient as well as necessary (assuming

an equilibrium exists) if utility functions are jointly quasi-concave in

\( y_1^j \) and \( q^j \). Unfortunately, this requirement need not always be satisfied.

Utility functions involve arguments of the form \( c_s^j(q^j)y_1^j \). Even if \( c_s^j(q^j) \)

is strictly concave, the product of the terms will not necessarily be jointly

quasi-concave in \( q^j \) and \( y_1^j \).

In general, the optimality conditions (13) – (19) will not be satis-

fied by the equilibrium developed in previous sections. This is hardly sur-

prising, since received theory indicates prices must be regarded as invariant
to output—\( \frac{\partial p^j}{\partial y^j} = 0 \)—for Pareto optimality.

Even if we restricted attention to the case where \( \frac{\partial p^j}{\partial y^j} = 0 \),

there remains the term \( \frac{\partial p^j}{\partial q^j} \)—the \( j \)-th firm's perception of how its price
responds to quality change. We are reduced to the essential question: what further restrictions on competitive behavior must be satisfied if the equilibrium is to satisfy the necessary conditions for Pareto optimality?

In Leland [1973], we examined conditions under which firms would choose Pareto optimal patterns of returns across states of nature. Two properties were shown to be crucial to the optimality of asset market equilibrium: "spanning," and "competitive implicit contingency claim" prices. Both concepts are equally important to the optimality of quality choices by firms, and are examined in the following sections.

6. Spanning

Changes in patterns of characteristics consumed can occur in two ways. First, the consumer can alter the portfolio of goods he consumes. Such a portfolio change has a well-defined cost (perhaps negative). Second, firms can alter the quality of the goods they produce, thereby changing the pattern of returns to fixed bundles of goods. The personal value of such a quality change, per unit of the good consumed, will in general differ among consumers. But if every change in pattern of returns resulting from quality change can be duplicated by a change in portfolio, there will exist a common money "value" for quality change—namely, the value of the corresponding portfolio change.

Essentially, the "spanning property" says that any small change in characteristics effected by quality change can be effected by some portfolio
change of the goods consumed. Mathematically, spanning implies the existence of vectors

$$h^j(q) = [h_1^j(q), \ldots, h_J^j(q)], \quad j = 1, \ldots, J$$

such that

$$\frac{\partial c_s^j(q^j)}{\partial q^j} = \sum_{k=1}^{J} c_s^k(q^k)h_k^j(q), \quad s = 1, \ldots, S, \quad j = 1, \ldots, J.$$ (20)

or in matrix terms,

$$c_q^j(q^j) = C(q)h^j(q),$$ (20')

where $c_q^j(q^j)$ is an $S$-dimensional vector with elements $\frac{\partial c_s^j(q^j)}{\partial q^j}$.

Consider now the change in price $\frac{\partial p_1^j}{\partial q^j}$ that consumer 1 would just be willing to pay for a small change in the quality parameter of firm 1. Clearly, $\frac{\partial p_1^j}{\partial q^j}$ will be the price change which renders $\frac{\partial u_1}{\partial q^j} = 0$. Equally clearly, $\frac{\partial p_i^j}{\partial q^j}$ will in general differ between consumers. But we show below that spanning implies that $\frac{\partial p_1^j}{\partial q^j}$ is the same for all consumers.

Appending the budget constraint (9) to (7) with the Lagrangean multiplier $\lambda_i$ satisfying (10), and differentiating the resulting expression with respect to $q^j$ gives

$$\frac{\partial u_1}{\partial q^j} = \sum_{s} \left( \frac{\partial c_s^j}{\partial q^j} y_1 + \sum_{k} c_s^k(q^k) \frac{\partial y_1^k}{\partial q^j} \right) + \lambda_i \left( p_k \frac{\partial y_1^k}{\partial q^j} + \frac{\partial p_1^j}{\partial q^j} y_1 \right) = 0,$$ (21)

or
\[ \sum_{s} (\frac{\partial c_{s}^{j}}{\partial p_{i}^{j}}) + \lambda_{i} (\frac{\partial p_{i}^{j}}{\partial q_{i}^{j}}) y_{i}^{j} = 0, \text{ using (10)}. \]

From the spanning condition (20), we may rewrite (21) as

\[ \sum_{s} \sum_{k} (\begin{array}{c} k \\ s \end{array}) c_{s}^{k}(q^{k}) h_{k}^{j}(q) + \lambda_{i} (\frac{\partial p_{i}^{j}}{\partial q_{i}^{j}}) y_{i}^{j} = 0, \]

\[ \sum_{k} \sum_{s} (\begin{array}{c} k \\ s \end{array}) c_{s}^{k}(q^{k}) h_{k}^{j}(q) + \lambda_{i} (\frac{\partial p_{i}^{j}}{\partial q_{i}^{j}}) y_{i}^{j} = 0. \]

\[ \sum_{k} c_{k}^{j}(q^{k}) h_{k}^{j}(q) + \lambda_{i} (\frac{\partial p_{i}^{j}}{\partial q_{i}^{j}}) y_{i}^{j} = 0, \]

again using (10). Our assumptions of nonsatiation and interior solutions therefore imply that

\[ \frac{\partial p_{i}^{j}}{\partial q_{i}^{j}} = -\sum_{k} c_{k}^{j}(q). \]

The remarkable aspect of (23) is that the right side is independent of \( i \). Therefore spanning ensures that the price change that each consumer would be willing to pay for a small change in quality is identical for all consumers. \(^{12}\) If spanning is not satisfied, there will exist consumers who value quality changes at different prices per unit. \(^{13}\)

One can see why spanning may be a necessary condition for Pareto optimality. If rates of substitution between quality and income (and therefore between quality and inputs \( x \)) differ, trading with markets may not lead to optimal decisions. Some consumers will want more quality, others less. Further trading between individuals, as contrasted with trading in
markets, may be required for optimality. We examine this question rigorously in Sections 8 and 9.

Is spanning likely to be satisfied? There are a number of situations which imply spanning in capital asset markets (see Leland [1973], Propositions I – IV). One of these seems particularly relevant to our present study: the case of "complete markets," when the number of goods is at least as great as the number of characteristics \( J \geq S \).

With complete markets, the spanning property will be satisfied. Since \( C(q) \) will be of rank \( S \), it will possess a right inverse—that is, there exists a matrix \( A(q) \) such that

\[
(24) \quad C(q)A(q) = I_S,
\]

where \( I_S \) is the identity matrix with rank \( S \). Therefore, the vector

\[
h^j(q) = A(q)c_q^j(q^j)
\]

exists, and since

\[
C(q)h^j(q) = C(q)A(q)c_q^j(q^j) = c_q^j(q^j),
\]

we have from (20') that spanning is satisfied. Of course, spanning may still be satisfied if there are fewer goods than characteristics—but it will always be satisfied in the opposite case.
7. Competitive Implicit Characteristic Prices

From conditions (10), we have for all consumers \( i = 1, \ldots, I \)

\[
\sum_{s} \frac{U_{i,s}}{\lambda_{i}} c_{s}^{j}(q_{s}^{j}) = p_{i}^{j}, \quad j = 1, \ldots, J;
\]

or

\[
\sum_{s} v_{i,s} c_{s}^{j}(q_{s}^{j}) = p_{i}^{j}, \quad j = 1, \ldots, J,
\]

where \( v_{i,s} \equiv \frac{U_{i,s}}{\lambda_{i}} \).

Just as \( v_{i,s} \) could be interpreted as an implicit contingency claim price in the context of capital asset market equilibrium, so also can it be interpreted in the context of equilibrium with quality choice. In the present case, \( v_{i,s} \) represents the \( i \)-th consumer's implicit price per unit of characteristic \( s \). For all consumers, the price of a good will equal the sum of its characteristics weighted by the implicit price per unit of characteristic. \(^{14/}\) This is precisely the content of equation (26).

If there are as many goods as characteristics (i.e., complete markets), then the implicit prices \( v_{i,s} \) will be the same for all consumers. This follows since in matrix form (26) can be written

\[
v_{i} C(q) = p_{i}^{'}
\]

where \( v_{i} = (v_{i1}, \ldots, v_{iS}) \).

From (24), complete markets implies the existence of an \( A(q) \) such that \( C(q)A(q) = I_{S} \). From (27) we have
\[ v_i' C(q) A(q) = p' A(q), \]

or

\[ v_i' = p' A(q). \]

The right side of (28) is independent of \( i \), implying implicit characteristic prices are the same for all \( i \). If markets are not complete, the \( v_i' \)'s will not be identical, but from (27) will lie in a subspace of dimension \( S-J \).

In general (and in common with other equilibrium systems), the equilibrium implicit characteristic prices which satisfy (26) or (27) will depend upon \( y^j \) and \( q^j \), the quantity and quality decisions of the firms. But the perception of this dependence is crucial. If markets are "completely competitive," firms do not perceive their decisions affecting implicit prices \( v_i' \). Thus perfectly competitive firms perceive \( \partial v_{is}/\partial q^j = \partial v_{is}/\partial y^j = 0 \), for all \( i, j, \) and \( s \). This in turn implies

\[ \frac{\partial p^j}{\partial q^j} = \delta_{v_{is}} \frac{\partial c^j_s(q^j)}{\partial q^j} \quad ; \quad j = 1, \ldots, J \]

\[ \frac{\partial p^j}{\partial q^j} = 0 \quad , \quad j \neq k \]

\[ \frac{\partial p^j}{\partial y^j} = \frac{\partial p^k}{\partial y^j} = 0 \quad , \quad \text{for all } j, k. \]
Will \( \partial p^j / \partial q^j \) be the same, no matter whose implicit price vector is used by the firm to compute (29)? The answer is yes; if (and only if) the spanning property is satisfied. Using (20) we have

\[
(31) \quad \sum_{s} \frac{\partial c_s^j(q^j)}{\partial q^j} = \sum_{s} \sum_{k} c_s^k(q^j) h_k^j(q) \\
= \sum_k \left( \sum_s c_s^k(q^j) \right) h_k^j(q) \\
= \sum_k \left( \sum_s \left( \frac{1}{\lambda_s^j} \right) c_s^k(q^j) \right) h_k^j(q) \\
= -\sum_k h_k^j(q) \quad \text{using (10).}
\]

The right side of (31) and hence the right side of (29) are independent of \( i \), implying that--when spanning is satisfied--the firm will be able to compute a unique \( \partial p^j / \partial q^j = -\sum_k h_k^j(q) \).

With perfectly competitive characteristic prices, (30) implies the usual competitive assumption that--for any level quality--output price will be viewed as invariant to output quantity. But the assumption of perfectly competitive characteristic prices gives us more: namely, a "competitive" response of price to a change in quality. Coupled with spanning, this response is independent of whose implicit characteristic prices are used. Therefore, the manager could in principle use his own tradeoffs between money and units of characteristics to compute \( \partial p^j / \partial q^j \).
8. Optimality of Quality Choices by Firms

We shall now show that equilibrium with quality choice will satisfy the necessary conditions (13) - (19) if (and only if, in a context made precise below) the spanning property and competitive characteristic price property are satisfied. For convenience, we re-group the equations describing equilibrium, given the spanning and competitive characteristic price assumption. We have

\[
\begin{align*}
(32) & \quad L \sum_s c^j_s(q^j) + \lambda_1 p^j = 0 \quad \text{from (10)} \\
(33) & \quad p^j(q^j, y^j) + \mu^j f^j_y = 0 \quad \text{from (4), using (30)} \\
(34) & \quad -x + \mu^j f^j_x = 0 \quad \text{from (5)} \\
(35) & \quad \sum_k h^j_k y^j_k + \mu^j f^j_q = 0 \quad \text{from (3) using (29) and (31)} \\
(36) & \quad f^j(q^j, y^j, x^j) = 0 \quad \text{from (6)} \\
(37) & \quad \sum_j x^j = \bar{x} = \sum_{i=1}^l x^i_j \quad \text{from (12)} \\
(38) & \quad \sum_i y^j_i = y^j \quad \text{from (11)}.
\end{align*}
\]

Sufficiency is shown be demonstrating that the decisions satisfying equilibrium conditions (32) - (38) also satisfy the conditions (13) - (19) necessary for Pareto optimality:

Condition (13) will be satisfied when \( \lambda^* = \frac{\lambda_1}{\lambda_2} \), using (32).
Condition (14) will be satisfied when \( \lambda^* = \frac{\lambda_1}{\lambda_2} \) and \( \mu_j^* = \mu_j^1 \lambda_1 \), using (32) and (33).

Condition (15) will be satisfied when \( \mu_j^* = \mu_j^1 \lambda_1 \) and \( \gamma^* = -\lambda_1 r \), using (34).

Condition (16) will be satisfied when \( \lambda^* = \frac{\lambda_1}{\lambda_2} \), \( \mu_j^* = \mu_j^1 \lambda_1 \), using \( U_{is} = v_{is} \lambda_1 \), (31), and (35).

Conditions (17) - (19) are implied by (36) - (38).

We conclude that the decisions satisfying the equilibrium (32) - (38) will also satisfy (13) - (19), with \( \lambda^* = \frac{\lambda_1}{\lambda_2} \), \( \mu_j^* = \mu_j^1 \lambda_1 \), and \( \gamma^* = -\lambda_1 r \).

If the firm's manager uses his own implicit characteristic prices for determining the \( \frac{\partial p_j}{\partial q_j} \), we can show that spanning is a necessary as well as sufficient condition for Pareto optimality. From (3), we may substitute for \( \mu_j^1 \) in (16), yielding the necessary condition

\[
(39) \quad v_1^c y_1^c + v_2^c (y^1 - y_1^1) - \frac{\partial p_j}{\partial q_j} y_j = 0,
\]

where \( v_1^c \) is the row vector with elements \( U_{11}/\lambda_1 \), etc.

Without loss of generality we can assume the manager of the firm is individual 2. Using (29), (39) becomes

\[
v_1^c y_1^c + v_2^c (y^1 - y_1^1) - v_2^c y_1^c = (v_1^c - v_2^c) y_1^c = 0.
\]

But this condition cannot be satisfied for arbitrary \( v_1 \) and \( v_2 \) if \( c_j^q \) is not spanned. In contrast with the situation with spanning,
$v_i^j c_i^j$ will not be the same for all individuals.

The necessity of perfectly competitive characteristic prices also can be shown: otherwise, rates of substitution between output and inputs, output and quality, or quantity and inputs will differ between firms and consumers.

9. Pareto Optimality Without Spanning

In the previous section, we showed that spanning was a necessary condition for Pareto optimal quality decisions if we associate the firm's implicit prices with an individual's implicit prices (e.g., the manager). If we drop this requirement, Pareto optimality is possible when spanning is not satisfied. From (39), Pareto optimality requires

$$\partial p^j / \partial q^j = [v_i^j (y_i^j / y^j) + v_2^j (y_2^j / y^j)] c_i^j,$$

where $y_2^j = (y^j - y_i^j)$ as before. More generally, for an n-person economy, we can extend (40) to show that Pareto optimality requires the firm to act as if it had implicit prices $v_j = (v_{j1}, \ldots, v_{jS})$ such that

$$v_j = \sum_i v_i^j O_i,$$

where $O_i = y_i^j / y^j$, the share of the i-th good consumed by i. That is, the firm must act as if it had implicit prices which were a weighted average of implicit prices $v_i$, $i = 1, \ldots, I$. The weights are simply the share of the total consumption consumed by person i. 15/

If the firm uses $v_j$, and treats these prices as invariant to its decisions $y^j$ and $q^j$, the optimality conditions in Section 5 will be
satisfied. The problem, of course, is that computation of \( v_j \) will not in general be possible, since it requires knowledge of the unobservable \( v_i \)'s.

When spanning is not present, welfare may be improved by a multiplicity of quality levels. To see this, consider the division of consumers into two groups. For \( i \in I_1 \), \( v_i^j c_i^j > \partial p_j^i / \partial q_j^i \); for \( i \in I_2 \), \( v_i^j c_i^j < \partial p_j^i / \partial q_j^i \). Individuals in group one would be willing to pay more than the firm's current perception of the tradeoff between price and quality, whereas individuals in group 2 would not be willing to pay as much. If the quality of the good could be changed, say, in mid-production without fixed costs, the welfare of all consumers might be improved. When there are "transactions costs" to producing different quality levels, we cannot say a priori whether the increase in welfare resulting from different quality levels justifies such costs. A similar problem arises in the economics of uncertainty: we observe that contingency claim markets are "incomplete," but we have yet to develop a general theory which explains precisely when markets (or in our present case, goods) will exist. Lancaster [1975] has made some initial progress on the difficult question of entry and product diversification when spanning is not present. To render the problem tractable, he has had to make some very strong assumptions on technology and tastes.

When spanning is present, there is no need for further quality diversification within the industry: consumers will unanimously agree (or disagree) with the firm's tradoff between price and quality. Of course, an important problem still remains: will firms choose the price/quality tradeoff which consumers unanimously desire? The analysis in the previous section considered this problem.
10. Monopoly and Quality Choice

We have seen that, given spanning, firms will make quality and quantity decisions consistent with Pareto optimality if characteristic prices are viewed as parameters. What if these prices are perceived to depend on decisions by a firm? Can we say anything about the quality choice of a monopolist?

We shall continue to maintain the spanning hypothesis. But characteristic prices \( v = (v_1, \ldots, v_s) \) are presumed to depend on the supplies of characteristic \( s \) provided by the firm. That is,

\[
v_s = v_s(z_s),
\]

where \( z_s = c_s(q)y \), the supply of characteristic \( s \) when the monopolist chooses quality \( q \) and output \( y \). Since by (26), \( p(q,y) = \sum_s [c_s(q)y]c_s(q) \), the profit maximizing firm will maximize

\[
\pi(q,y) = p(q,y)y - TC(q,y)
\]

\[
= \sum_s [c_s(q)y]c_s(q)y - TC(q,y).
\]

First order conditions yield

\[
\frac{\partial \pi}{\partial y} = p(q,y) + y \frac{\partial p}{\partial y} - \frac{\partial TC}{\partial y} = 0,
\]

or

\[
p(q,y) + y \left( \sum_s \frac{\partial v_s}{\partial z_s} [c_s(q)]^2 \right) - \frac{\partial TC}{\partial y} = 0;
\]
\[ \frac{\partial \pi}{\partial q} = y \frac{\partial p}{\partial q} - \frac{\partial TC}{\partial q} = 0, \quad \text{or} \]
\[ y[\sum_{s} \frac{\partial c_{s}}{\partial q} + \sum_{s} \frac{\partial v_{s}}{\partial z_{s}} \frac{\partial c_{s}}{\partial q} c_{s}(q)] - \frac{\partial TC}{\partial q} = 0. \]

Since in (21) we identified \( \sum_{s} (\partial c_{s}/\partial q) \) with the amount every consumer would just be willing to pay per unit output for an increase in quality, we see
\[ y[\sum_{s} \frac{\partial c_{s}}{\partial q}] = \text{social value of change in quality}. \]

Similarly, \( p(q,y) = \text{social value of change in quantity} \). \( \partial TC/\partial q \) and \( \partial TC/\partial y \) represent social costs of a change in quality and quantity if other markets are competitive.

From (41), we see that if \( \partial v_{s}/\partial z_{s} < 0 \), \( p(q,y) > \partial TC/\partial y \) at the profit maximizing output. This is the standard result that monopolists produce too little, given their quality level.

The sign of \( \sum_{s} y_{s}(\partial v_{s}/\partial z_{s})(\partial c_{s}/\partial q)c_{s}(q) \) in (42) can be either negative or positive. If \( \partial c_{s}/\partial q > 0 \) for all \( s \), however—implying \( q \) unambiguously increases quality—then the term will be negative, implying
\[ y[\sum_{s} \frac{\partial c_{s}}{\partial q}] > \frac{\partial TC}{\partial q}, \]

i.e., the social utility of a small increase in \( q \) exceeds its social cost. In this case, the monopolistic firm tends to underprovide quality, given its output.

Even assuming concavity of a social welfare function in \( y \) and \( q \),
some care must be used in interpreting conditions \( p > \frac{\partial TC}{\partial y}; y \Sigma (\partial c_s / \partial q) \)  
> \( \partial TC / \partial q \). It does not necessarily follow that competition will lead both to greater output and to greater quality—although we cannot exclude that possibility. What we can exclude is competition leading both to lower output and lower quality. Yet it is possible that competition could lead to higher output and lower quality, or even lower output with higher quality. The correct inference as to where the monopolist "sins" depends upon complicated elasticities of several functions. 17/

17. Conclusion

While recognizing the importance of quality choice by firms, traditional economic models have been unable to examine or to explain these decisions. By using the "characteristics" approach to consumer choice, we have developed a framework for simultaneously considering quality and quantity choices by profit maximizing firms. The characteristics approach introduces a natural metric for "distance" between goods of different qualities. Marginal analysis can then be used to examine quality choice.

Our fundamental concern was with the welfare implications of firms' quality decisions. Two properties proved essential to Pareto optimality: spanning and competitive implicit characteristics prices. The spanning property assures a single "willingness to pay" per unit consumption for a small change in quality. That is, spanning guarantees consumer unanimity with respect to the tradeoff between price change and quality change. Competitive implicit prices for characteristics, presumed invariant to firms' quality and quantity decisions, were shown to guarantee that profit-maximizing firms in equilibrium will have the same tradeoff between price and
quality change as the consumers have. Spanning and competitive implicit prices were shown to be sufficient to satisfy the necessary conditions of Pareto optimality.

Competitive characteristics prices also are necessary for the optimality of quality choices. Monopolies, for example, were shown to underprovide quality, given the level of output chosen. Spanning is a necessary condition if firms are operated by managers who use the implicit prices of some individual (e.g., himself). If we permit firms to use weighted averages of individuals' prices, Pareto optimality is possible without spanning. In this case, the "public good" nature of quality decisions becomes evident. All consumers of a good are affected by quality changes, although differently. We showed that Pareto optimality requires the firm use a tradeoff between price and quality proportional to the sum of individuals' tradeoffs, weighted by their consumption of the good. The problems associated with firms implementing such a scheme are of precisely the same nature as those encountered with deciding the value of a public good. 18/

Our analysis has treated the number of different quality levels (although not their location) as a constant. When spanning is not satisfied, welfare may be improved by creating more quality levels, just as welfare under uncertainty can be improved by a movement towards more complete markets. Of course, a greater number of markets may incur resource costs which exceed the benefits resulting from a wider selection of goods. We must await the development of a theory which endogenously explains the number of markets in existence. Such a theory will permit a final assessment of quality choices by firms in differing market environments.
Footnotes

1 The reader may question the distinction between different goods versus different quality levels of the same good. Both have the property that their vectors of characteristics provided are altered. To make the distinction useful, we say two goods are different (rather than being different quality levels of the same good) if it is impossible for a firm which is set up to produce one good to change its design or quality parameter to produce the other. The distinction is not vital, however, for the analysis which follows.

2 This assumption is not restrictive given a fixed number of firms. But it does preclude a full analysis of entry and the question of whether the market provides a sufficient diversity of quality. See Section 9 for further discussion.

3 For convenience, we shall assume $q^j \in Q^j$, where $Q^j \subseteq R^1$. More general approaches, such as $Q^j \subseteq R^N$, can easily be developed.

4 Note that externalities are ruled out by assuming only quality parameter $q^j$ affects the provision of characteristics by good $j$.

5 More generally, we could allow for possible interactions through the introduction of consumption "activities," as in Lancaster [1966]. For simplicity, we use the simpler approach embodied in (8).

6 Note that factors $x = (x_1, \ldots, x_r)$ are assumed not to affect utilities. This assumption could easily be relaxed.

7 We make the strong assumption that all choices are made in the interior of choice sets. That is, we assume $y^j_i > 0$ for all $i$ and $j$. The Appendix discusses modifications required when corner solutions exist.

8 Under assumptions of spanning and competitive characteristic prices, there is similarity between this model and that considered by Debreu [1959]. Radner [1974] proves this equivalence in the context of capital asset market equilibrium with production.

9 Thus our results shed no light on the optimality of the number of different goods (and different quality levels) provided by the system. Lancaster [1975] has made progress in analyzing this question in a simplified framework. Our focus is on whether quality decisions by firms are optimal, given the number of firms (and therefore the number of different goods) as fixed.
A similar point has been made by Dreze [1972] in the context of uncertainty.

Of course, the cost of the change in characteristics resulting from a quality change may differ from the cost of the spanning portfolio of goods. In a Pareto optimal equilibrium, it can be shown that the two costs will be equal.

See the Appendix for how this conclusion may be modified when first order conditions are satisfied by strict inequalities—i.e., corner solutions.

If consumers have arbitrary vectors \( U' = (U_1, \ldots, U_n) \). If there are restrictions on tastes, spanning will not be a necessary condition for unanimity, although it clearly will remain sufficient.

This will not necessarily hold if corner solutions exist: see the Appendix.

For simplicity, we omit superscripts \( j \), since we focus on a single monopolist. Of course, the equilibrium \( v \) depends not only on \( z' \), but on suppliers of all other characteristics as well. We shall ignore such interdependencies in our analysis.

Note if \( \delta v / \delta q = 0 \), conditions (41) and (42) reduce to those of perfectly competitive characteristic prices, with consequent Pareto optimality of output and quality decisions.

The nature of our results were anticipated in part by Chamberlin [1933]: "The conclusion seems to be warranted that just as, for a given 'product,' price is inevitably higher under monopolistic than under pure competition, so, for a given price, 'product' is inevitably somewhat inferior. After all, these two propositions are but two aspects of a single one. If a seller could, by the large scale of production which is characteristic of pure as compared with monopolistic competition, give the same 'product' for less money, he could, similarly, give a better 'product' for the same money." [p. 99, 6th edition, 1948].

See Samuelson [1962] and related literature. Dreze [1972] and Dreze and de la Valley Poussin [1962] have encountered a similar public good property of private firms' decisions in different contexts. The formal similarity between our problem and that studied by Dreze [1972] is striking.
References


Appendix

Corner Solutions

To arrive at the "unanimity condition"

\[ \frac{\partial p^j_i}{\partial q^j} = -\sum_k p^k_i n^j_k(q), \]

where the right hand side is independent of \( i \), we required that the first order conditions be satisfied with equality ("interior solutions") and that \( c^j_q \) be spanned by the set of securities (i.e., by the matrix \( C(Q) \)). If optimal consumer choice involves zero consumption of some commodities ("corner solutions"), spanning alone will not guarantee unanimity.

We can, however, readily modify our criterion for (23) to be satisfied for positive consumers of good \( j \). (Note if a consumer does not consume \( j \), small quality changes in \( j \) will not affect his utility. We can exclude him from welfare considerations resulting from quality changes in \( J \).) Let \( I^j_i \) be an index set of positive consumers of \( j \). That is, \( y^j_i > 0 \) if \( i \in I^j_i \). Then the following proposition is obvious:

Let \( K^j \) be an index set of securities which span \( c^j_q \). Then if \( y^k_i > 0 \) for all \( i \in I^j_i \) and \( k \in K^j \), the unanimity condition (23) will hold for all relevant consumers (\( i \in I^j_i \)).

Note that complete markets, which guarantee spanning, will not necessarily guarantee unanimity. Unanimity is, of course, the key aspect of Pareto optimality, and "spanning" should be replaced by "unanimity" in all optimality theorems if corner solutions are possible.