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Optimal Risk Sharing and the Leasing of Natural Resources, with Application to Oil and Gas Leasing on the OCS

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ABSTRACT

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We develop the theory of optimal risk-sharing contracts. Our focus is on appropriate royalties and/or profit-sharing schedules for oil and gas leases on the Outer Continental Shelf (OCS).

Our results indicate that:

(a) more valuable tracts should carry higher royalty or profit-sharing rates;

(b) royalties tend to distort firms' decisions more than profit-sharing schemes;

(c) royalties or profit shares should increase as the risk aversion of the lessee becomes greater relative to the lessor;

(d) allowing firms to gather extensive private information on tracts prior to lease sales is not socially beneficial if risk-sharing payment schedules (such as royalties or profit shares) are optimal.
OPTIMAL RISK SHARING AND THE LEASING OF NATURAL RESOURCES,
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I. INTRODUCTION

Future natural resource discoveries will most likely take place on public lands. This is particularly true in the case of potential oil and gas resources. According to recent estimates by the U.S. Geological Survey, between ten and forty-nine billion barrels of oil are likely to be found on the outer continental shelf (OCS), the offshore lands lying beneath the relatively shallow waters adjacent to the coastline. At current prices, the potential value of these resources is staggering. But so, too, are the risks involved in their exploitation. Uncertain reserves, prices, and extraction costs have been further exacerbated by

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the formation of OPEC and by the environmental uncertainties of OCS development.

Rights to potential oil and gas resources are transferred from the public to private firms through a lease. The lease specifies the nature of the rights granted. In addition, it typically specifies a schedule of payments, such as royalties, that depends upon production or other aspects of the tract which are unknown at the time the lease is granted. Given the payment schedule, leases are granted to the highest bidder in a sealed-bid auction.

The specification of a payment schedule, conditional upon the value of the tract or some other variable, distinguishes a lease from the usual outright sale of (property) rights. Contracts with conditional payments are a natural response to an uncertain environment, in that they share the risk of uncertain value between the seller (the government) and the buyer (the firm). Both public and private sectors may benefit from a risk-sharing lease, when compared with the outright sale and its implied *caveat emptor*.

Federal offshore leases have specified that firms pay a royalty equivalent to one-sixth of the value of production. This royalty has remained unchanged since offshore leasing commenced, despite the significant changes in the value of tracts as reflected in winning bids. Leases of other countries specify quite different payment schedules, with profit rather than production often determining the amount to be paid. Clearly, alternative forms of conditional payments have differing
implications on risk sharing and on the exploration, development, and production decisions of firms.

It is the purpose of this paper to develop a general theory of optimal lease payments. We pay special attention to royalty and profit-share payments, since they are featured prominently in proposals for new federal leasing policies. Our conclusions have significant policy implications for the structure of optimal leasing contracts. Our analysis builds on earlier work on leasing by Leland, Norgaard, and Pearson [1974]. Elements of related theory are developed in the studies of insurance by Borch [1962] and Spence and Zeckhauser [1971], and in the work on agency by Wilson [1967], Berhold [1971], and Ross [1973, 1974].

II. RISK SHARING, PERFECT CAPITAL MARKETS, AND CONDITIONAL PAYMENTS

In a world with perfect and complete contingency claim markets, it can be argued that no further risk sharing through payments based on the state of nature are needed. All risks are shared through the existence of efficient capital markets, with firms' decisions unanimously backed by shareholders. 4

A variety of considerations conspire to make the actual environment diverge from the perfect market paradigm. Transaction costs and information asymmetries lead to incomplete contingency claim markets and to less than perfectly diversified portfolios. Bankruptcy costs induce investors and firms to view "specific risk" as an important determinant of share value. 5 Finally, managers tend to have (undiversified) human capital
whose returns heavily on the performance of their firm. Even if capital markets were efficient, separation of ownership and control may lead to excessively risk averse behavior by firms. Efforts by firms to diversify their leasing activities tend to suggest that specific risk does influence management decisions.  

Excessive risk aversion by firms incurs social costs. Investment in exploration and development will be insufficient. Production from leases (given a fixed production capacity) is likely to be too rapid, since risks resulting from future price uncertainty will be reduced. In addition to distorting exploration, development, and production decisions, risk aversion leads to overdiscounting for risk and, therefore, to lower bids on tracts. The government will get less economic rent than it would if socially efficient production and bidding decisions were made. And competition and, consequently, economic rent will be reduced if small firms are subject to capital constraints because of risk-averse lenders.  

These considerations suggest that further risk sharing would be socially desirable. We shall focus on the conditional-payment aspect of leases as a means to reduce uncertainty and to further the social gains from oil and gas leasing.

III . RISK SHARING AND LEASES: GENERAL STRUCTURE

In this section we develop a general characterization of competitive leasing environments. Let

\[ V = V(a, s) \]
represent the net present value of a good or tract, where \( a \) is a decision vector of actions undertaken by the firm (the lessee) and \( s \) is the state of nature.  

A lease grants the rights to the "gambles" \( V \), less a set of payments specified in the lease. Most generally, the lease could specify a payment schedule:

\[
P = P(a, s),
\]

which depends upon the action undertaken and/or the state of nature. 

Net returns to the lessee are:

\[
Z(a, s) = V(a, s) - P(a, s).
\]

The value of such a gamble to a risk-averse firm will depend upon expectations, degree of risk aversion, and the action, \( a \). Let \( B(a) \) represent the certainty amount the firm would be willing to pay for the gamble \( Z(a, s) \), rather than not have it. Then,

\[
(1) \quad EU[Z(a, s) - B(a)] = U(0),
\]

where \( E \) is the expectation operator with respect to the firm's subjective probability distribution over states of nature, \( U \) is the firm's utility function over returns, and \( U(0) \) is the firm's utility without the gamble. 

In a competitive environment, firms will submit bids equal to their certainty equivalent. Further, competition will lead firms to
choose the action \( a \) which maximizes \( B(a) \). Therefore, the amount bid, \( B \), will satisfy

\[
\text{(2)} \quad \max_a B \\
\text{subject to} \quad \mathbb{E}[Z(a, s) - B] = U(0).
\]

We presume that \( B \geq 0 \). If \( B < 0 \), the lease is not sold (or the lessee must be paid \(-B\) to accept the lease).

Net utility to the leasing firm will be zero when markets are competitive, although the actions of firms will depend upon \( Z(a, s) \) and therefore \( P(a, s) \) through equation (2). All gains are bid away, and the firm is neither better nor worse off as the payment schedule \( P \) changes.

The expected utility of the government, on the other hand, does vary as the payment schedule changes. We assume that the government chooses a payment schedule \( P(a, s) \) to maximize expected utility of economic rent, where rent is total payment \( P(a, s) + B \). The optimal schedule will

\[
\text{(3)} \quad \max_{E} \mathbb{E}[W(P(a, s) + B)]
\]

where \( \mathbb{E} \) is the expectation operator reflecting the government's probabilities, and \( W \) is the government's utility function over returns. The maximization is over the set of functions \( P \), which lead to nonnegative bids satisfying (2) and which are consistent with the government's ability to discern \( a \) and \( s \).
To facilitate analysis, we shall assume that firm and government have identical expectations:

\[ E_g = E. \]

The assumption seems warranted if government and firm share information (and its interpretation). Although information sharing is required for much of the exploration on the OCS, both government and industry agree that information in the private domain is generally better than information available to the government. We shall consider the case of asymmetric information later.

IV. OPTIMAL LEASING POLICIES WHEN ACTIONS ARE EXOGENOUSLY DETERMINED

We first consider the case where the action \( a \) is determined independently of the payment schedule \( P(a, s) \). For purposes of determining the optimal schedule, \( V \) and \( P \) can be viewed as a function of \( s \) alone, and the government’s problem is to choose a function \( P(s) = P(a^*, s) \) to:

Maximize \[ \int W[P(s) + B]d\mu(s), \]

subject to:

\[ \int [U[V(s) - P(s) - B]d\mu(s) - U(0)] = 0, \]

where \( \mu \) is the probability measure over the states of nature.
We first note that if, for all \( s \), \( P(s) \) is raised by an amount \( \Delta \), (2) implies that \( B \) will fall by the same amount. Therefore, without loss of generality we can let \( P(s) \) include \( B \), and state the optimization problem as:

\[
\max_{s} \int_{S} W[P(s)]d\mu(s),
\]

subject to:

\[
\int_{S} [U[V(s) - P(s)]d\mu(s) - U(0) = 0.
\]

Next, observe that the optimal payment \( P \) will vary only when \( V \) will vary. This enables us to restate further the optimization problem as:

\[
\max_{V} \int_{V} W[P(V)]f(V)dV,
\]

subject to:

\[
\int_{V} [U[V - P(V)] f(V)dV - U(0) = 0,
\]

where \( f(V) \) is the probability density function for the random value \( V \) derived from the probability measure \( \mu \) on \( S \).

Using standard variational techniques, it can be shown that the optimal payment schedule \( P(V) \) will satisfy

\[
(4) \quad W'[P(V)] - \lambda U'[V - P(V)] = 0, \text{ for all } V.
\]

The concavity of \( W \) and \( U \) guarantee sufficiency of conditions (4).
Results similar to (4) have been derived in a variety of contexts—for example, Borch [1962], Spence and Zeckhauser [1971], and Ross [1973, 1974].

V. PROPERTIES OF THE OPTIMAL SCHEDULE $P(V)$

We wish to examine the nature of $P(V)$. In particular, the derivative $dP/dV$ is of interest, since it can be thought of as a measure of risk sharing at any given $V$. Two cases of special interest are:

- **Case (a)** $\frac{dP}{dV} = 0$
- **Case (b)** $\frac{dP}{dV} = 1$.

In case (a), payments are invariant to different values of $V$: The optimal schedule consists of the amount bid $B$ and no further payments. The optimal lease in this situation is simply an outright sale, and the firm bears all risk. In case (b), the lessee pays a fixed amount, and the government bears all risk. A lease of this type could be thought of as a contract in which the firm is "hired" for a fixed fee to develop the lease.

As $dP/dV$ varies from zero to one, the government bears more and more risk. Thus, $dP/dV$ is a local measure (at $V$) of the extent of risk transfer from the firm to the government. This suggests a natural "global" measure of risk transfer:

\[
R(<P>) = \int \left[ \frac{dP(V)}{dV} f(V) dV \right].
\]
We now wish to relate local and global measures of risk transfer to properties of the utility functions of firm and government and, in particular, to the ratio of their absolute risk aversions.

**Theorem I**

*The less risk averse is the government, relative to the firm, the greater is the risk sharing of the optimal contract.*

**Proof:**

Define:

\[
\frac{\text{ARA}_G}{\text{ARA}_F} = \left[ -W''(P(V))/W'(P(V)) \right] \left[ \left[ -U''(V-P(V))/U'(V-P(V)) \right] \right],
\]

the ratio of absolute risk aversion of government to firm evaluated at the optimal schedule, for each \( V \).

Differentiating (4) with respect to \( V \), solving for \( dP/dV \) and substituting for \( \lambda \) from (4) gives:

\[
\frac{dP}{dV} = \frac{\text{ARA}_F}{[\text{ARA}_F + \text{ARA}_G]}, \text{ or } 1/[1 + \frac{\text{ARA}_G}{\text{ARA}_F}].
\]

The theorem follows immediately by noting \( dP/dV \) varies inversely with \( \frac{\text{ARA}_G}{\text{ARA}_F} \).

**Corollary**

The outright sale [case (a)] is optimal only if the firm is risk-neutral (\( \text{ARA}_F = 0 \)). The outright hire [case (b)] is optimal only if the government is risk-neutral (\( \text{ARA}_G = 0 \)).
Theorem I implies that there will always be some risk sharing
\(0 < \frac{dP}{dV} < 1\) if both government and firm exhibit some degree of risk aver-
sion. It shows that the optimal contract depends upon the characteristics of
the parties involved. Leases which are optimal for the federal govern-
ment, for example, will generally involve more risk sharing than leases
which are optimal for state governments, whose smaller and less diversi-
fied incomes are likely to make them more risk averse.\(^ {15}\)

Another question relevant to leasing policies is whether the opti-
mal payment schedule is linear, concave, or convex. Define \(RT_G\) and
\(RT_F\) as the risk tolerance (the reciprocal of absolute risk aversion) of
government and firm, respectively. Then we can show:

Theorem II

The optimal payment schedule \(P(V)\) is concave, linear, or convex
at any \(V\), according to whether the risk tolerance function of
the government is increasing in wealth at a lesser rate, at the
same rate, or at a greater rate than that of the firm. That is:

\[
\frac{d^2P(V)}{dV^2} \geq 0 \quad \text{as} \quad RT_G' \leq RT_F'.
\]

where primes indicate derivatives at \(P\) and \(V-P\) for government
and firm, respectively.

Proof:

See the appendix.
Corollary

The optimal payment schedule will be linear,

\[ P(V) = \alpha + \beta V, \quad \beta \in (0,1). \]

if and only if \( RT_G^t = RT_F^t \), or (by integrating)

\[ RT_G = RT_F + C \quad \text{for all} \quad V, \]

where \( C \) is a constant and \( RT_G \) and \( RT_F \) are evaluated at \( \alpha + \beta V \) and \( (1-\beta)V - \alpha \), respectively.\(^{16}\)

Thus, the linear risk sharing provided by fixed profit shares will be optimal (over the set of all possible payment schedules) only for a very restricted set of \( W, U \) pairs.\(^{17}\) A class of utility functions, for which (7) is satisfied, is:

\[ RT_G = a_G + bY, \]
\[ RT_F = a_F + bY, \]

for arbitrary \( a_G, a_F \): linear risk tolerance functions with equal slopes.

Theorem II also indicates that the convexity or concavity of the optimal payment schedule cannot be related directly to the risk averseness of the firm, relative to the government. If \( a_G > a_F \) in (8), for example, the optimal payment schedule is linear, even though the firm is more risk averse for every value of \( V \) than is the government.
Theorem I shows that the size of \( a_G \), relative to \( a_F \), affects the slope \( \beta \), not its rate of change.

For an important class of utility functions, however, there is a relation between risk aversion and convexity of \( P(V) \). This is the commonly used class of power functions:

\[
U(Y) = \frac{1}{\gamma} Y^\gamma \quad \gamma < 1
\]

\[
= \log Y \quad (\gamma = 0).
\]

The government will be less averse than the firm for comparable levels of wealth \( Y \) if \( \gamma_G > \gamma_F \). In this case,

\[
RT_G' = (\gamma_G - 1) > (\gamma_F - 1) = RT_F'.
\]

so, for functions of this class, a less risk-averse government implies a convex payment schedule: as returns increase, the government takes an ever-increasing percentage of \( V \).

Recent political and economic events have led to changes in the perceived value of oil and gas tracts. The increase in crude oil prices has raised the expected value of most leases. But the rise in expected value has been accompanied by an increase in uncertainty, both from fears about price instability and from the greater risks associated with drilling in more remote OCS areas. Not only do probability distributions over tract values tend to change with time, but they also vary across tracts. Some areas are judged better prospects than others.
How should the government alter its payment schedule to take account of differing expected value and risk? We can prove

**Theorem III**

Let $m(V)$ and $n(V)$ be two probability density functions over $V$, with

$$E_m[V - P_m(V)] > E_n[V - P_n(V)] = 0,$$

where $P_m(V)$ and $P_n(V)$ are the optimal payment schedules for the two distributions. I.e., the firm prefers the distribution $m$ to the distribution $n$, given $P_n$. Then:

(a) $P_m(V) > P_n(V)$ for all $V$.

If the government and firm have decreasing absolute risk aversion as wealth increases, then:

(b) $\frac{dP_m(V)}{dV} > \frac{dP_n(V)}{dV}$, for all $V$.

**Proof:**

See the appendix.

Theorem III has an extremely important consequence for leasing policies: tracts with greater values should have greater risk-sharing payment schedules, such as a higher royalty or larger profit share. Yet federal leases have retained a $16\ 2/3$ percent royalty, despite the fact that the average winning bid has risen from less than $2.5$ million to about
$10 million in the past twenty years. (And a tract off Florida received a bid of $128 million!)

Theorem III indicates that the crucial aspect of an environmental change is whether it raises the tract value, given current payments. Any change or combination of changes that raise the value should lead the government to invoke higher payment schedules. The opposite is true for changes that lower the certainty equivalent value of the tract. An important case of the latter is captured in the following:

**Corollary:**

A mean-preserving increase in the uncertainty of a tract's net value will lead to less risk-sharing by the government.

**Proof:** The certainty equivalent of a distribution that is riskier in the above sense will always be less for a risk-averse firm. From Theorem III, it follows immediately that the new optimal payment schedule will have a smaller rate of risk sharing $dP(V)/dV$ for each $V$.

The corollary is perhaps surprising, in that a riskier environment leads to less risk sharing. But an intuitive explanation can be given. A riskier distribution of $V$ and $P(V)$ will lower the expected utility of the government but not of the firm, since, through lower bids, the uncertainty equivalent after payments will remain as before. Since the government is now relatively less well off, it cannot share as much risk as before.
VI. ASYMMETRIC INFORMATION

The foregoing analysis has assumed that firms and the government share the same probabilities for the value of the tract to be leased. While a reasonable description of some leasing environments, such as timber sales, it is not an appropriate description of offshore oil and gas leases. The government, through the U.S. Geological Survey, typically performs geological analysis, and more recently has joined as a participant in magnetic and gravimetric surveys. But both industry and government personnel agree that the quantity of research, and the expertise in interpreting test results, is greater in the private sphere. Thus, it is of interest to see how information asymmetries, with the firm possessing better information than the government, will affect optimal payment schedules.

To model information, we presume that the firm receives an information "signal" that bears some relation to the underlying value of the tract. That is, it receives a signal

\[ I = I(V) \]

before the tract is leased. The government is presumed to know that the firm will receive an information signal, but it does not know what the signal is. If \( I(V) \) is a one-to-one function, the firm receives "perfect" information: The signal tells the firm the exact value of the tract. The opposite case is "no information," when \( I(V) \) is a constant function, and the signal \( I = I(V) \) does not vary with \( V \). The intermediate cases are when \( I(V) \) is a "many-one" function, which reduces the possible \( V \)
consistent with an information signal but does not identify the true value precisely.

As before, we assume that competition will lead the firm to bid until its expected utility gain is zero. But its expectations and therefore its bid will depend upon the information signal it receives.

Thus

\[
E_I U \left[ V - P(V) - B[I(V)] \right] = U(0) \quad \text{or}
\]

\[
\int_{V} U \left[ V - P(V) - B[I(V)] \right] f(V|I) dV = U(0) \quad \text{for all } I,
\]

where \( f(V|I) \) is the conditional distribution of \( V \), given \( I \), and \( E_I \) is the expectation operator, given \( f(V|I) \). Note that (9) generates a set of constraints for the government optimization problem, one for each possible value of \( I \).

The optimal payment program can now be formulated:

\[
\text{(10) Maximize } E_W \left[ P(V) + B[I(V)] \right],
\]

subject to:

\[
\text{(11) } E_I U \left[ V - P(V) - B[I(V)] \right] = U(0), \text{ for all } I.
\]

For any \( V \), necessary conditions for \( P^{**}(V) = P(V) + B[I(V)] \) to be optimal are:

\[
W' [P^{**}(V)] f(V) - \lambda \int_{V} U' [V - P^{**}(V)] f(V|I) = 0,
\]

for \( V \in V(I) \), where \( V(I) = \{V | I(V) = I\} \).
Now, observe that \( f(V|I) = f(V,I)/f(I) \), where

\[
f(I) = \int f(V,I)\,dV.
\]

Furthermore, since \( I = I(V) \),

\[
f(V,I) = f(V) \quad \text{for } V \in I(V)
\]

\[
= 0 \quad \text{otherwise}
\]

and, therefore,

\[
(13) \quad f(V|I) = f(V)/f(I), \quad V \in I(I(V));
\]

\[
= 0 \quad \text{otherwise}.
\]

Conditions (9) can now be written:

\[
(9') \quad \int_{V(I)} U[V - P^{**}(V)]f(V)\,dV = U(0) f(I), \quad \text{for all } I,
\]

and conditions (12) become:

\[
(12') \quad W'[P^{**}(V)] - \lambda^*_I U'[V - P^{**}(V)] = 0, \quad V \in I(I(V)), \quad \text{for all } I,
\]

where \( \lambda^*_I = \lambda_I / f(I) \).

We wish to compare the optimal payment schedule \( P^{**}(V) \) with \( P^*(V) \), the schedule which solves the symmetric information problem and satisfies

\[
W'[P^*(V)] - \lambda^* U'[V - P^*(V)] = 0;
\]

\[
\int_{V} U[V - P^*(V)] f(V)\,dV = U(0).
\]
From Theorem III, we know that if

\[ \int_V U[V - P^*(V)]f(V) > \int_V U[V - P^*(V)]f(V) = U(0), \]

then \( P^*(V) \), the schedule which is optimal, given \( f(V|I) \), \( \forall V \in V(I) \), has

\[ p^*(V) > p(V); \]

(15)

\[ dp^*/dV > dp(V) \quad \text{for all} \quad V \in V(I). \]

If the inequality in (14) is reversed, inequalities (15) are reversed.

The final link to the analysis is to show, for \( I = I(V) \) arising from large values of \( V \), inequality (14) is satisfied, while the opposite holds for \( I \) arising from small values of \( V \). First, note that \( V - P^*(V) \) is an increasing function of \( V \), since \( dp^*(V)/dV < 1 \). Therefore, since

\[ \int_V U[V - P^*(V)]f(V) = U(0), \]

the increasing property of \( V - P^*(V) \) implies that, for an information signal indicating relatively higher \( V \),

\[ \int_{V(I)} U[V - P^*(V)]f(V) > U(0)f(I), \]

or, from (13),

\[ \int_V U[V - P^*(V)]f(V|I) > U(0) = \int_V U[V - P^*(V)]f(V), \]
which is (14). For I indicating relatively lower V, the opposite holds. We summarize the above in

Theorem IV

Information asymmetries lead to an optimal payment schedule \( P^{**}(V) \) with:

\[
\frac{dP^{**}(V)}{dV} > \frac{dP^{*}(V)}{dV} \quad \text{for relatively large } V;
\]

\[
\frac{dP^{**}(V)}{dV} < \frac{dP^{*}(V)}{dV} \quad \text{for relatively small } V,
\]

where \( P^{*}(V) \) is the optimal schedule when there are no informational asymmetries.

Theorem IV indicates that the more pronounced are informational asymmetries, the more convex the optimal payment schedule should be. A simple example may shed light on the effect of information on the optimal schedule.

Example:

Assume \( V \) can take values 0, 10, 20, and 30, with probabilities .25 each.

The government welfare function \( W(P) = \log(100+P) \).

The firm's utility function \( U(V-P) = \log(10+V-P) \).

First, consider the case of symmetric information. Since both functions exhibit linear risk tolerance with the same slope, the optimal payment schedule will be linear. Solving the conditions \( W'(P) = \lambda U'(V-P) \) and \( EU(V-P) = U(0) \) yields an optimal payment schedule:
\[ P^*(V) = 1.2 + .920V. \]

Now assume the firm gets an information signal \( I(V) \), which informs the firm that \( V \) is either (0, 10) or (20, 30). Solving (9') and (12') for the optimal schedule(s) in this case yields:

\[ P^{**}(V) = 0.5 + .914V; \quad \text{when } V = 0, 10; \]
\[ P^{**}(V) = 1.8 + .926V; \quad \text{when } V = 20, 30. \]

Our example confirms the fact that higher values of \( V \) are associated with greater profit-sharing fractions, as well as fixed payments.

Will the government gain from firms gathering prior information on tracts to be leased? The answer is a qualified no. Expected utility of the government will be lower when it chooses the optimal schedule in the asymmetric information case. This follows quite simply from the fact there are more bidding constraints (one for each possible value of \( I \)), which the optimal payment schedule must satisfy. In the limit, the government will bear all risk, since perfect information implies the firm will bid an amount precisely equal to the net value of the tract: any effort by the government to alter total payments \( P^{**}(V) \) will be thwarted since the firm will change its bid to offset exactly the change.

Yet another argument against permitting firms to gather information is if it leads to information asymmetries between bidders. In this case, firms will not behave competitively, since the firm with superior information can exploit this in the lease auction with sealed bidding. Government revenues will not capture the full economic rent accruing to firms in this case, and information will have a private value exceeding its social value.
Our conclusion that the government will not benefit from permitting firms access to better information is subject to two caveats. First, if current risk sharing by the government is less than optimal, further risk sharing through permitting access to better information may increase the expected utility of rents to the government (and thereby move closer to Pareto optimality). The fact that royalty rates have remained unchanged despite the great increase in tract values seems (by Theorem III) to indicate that current risk sharing by the government is less than optimal. Second, if production decisions depend upon the payment schedule, the risk sharing resulting from information access may lead to less distorted decisions than those resulting from risk sharing through the payment schedule, with a consequent improvement in expected government utility. We now turn to the problem of interactions between payment schedules and firms' decisions.

VII. OPTIMAL PAYMENT SCHEDULES WITH ENDOGENOUS DECISIONS:
COMPLETE AND LIMITED OBSERVABILITY CASES

The previous section examined optimal conditional payment schedules \( P(V) \) when the firm's action \( a \) is determined exogenously. Typically this is not the situation: Firms are informed of the payment schedule, and choose their actions in light of it. For example, high royalties are known to affect oil and gas firm's exploration, installed capacity, and shutdown decisions.

Payments to the government must be based on an observable entity. If, for example, the government could observe both the firm's action and the true state of nature, it could specify a general payment structure
\[ P = P(a,s) . \]

If the government cannot observe the firm's action but can observe the state of nature, \( P = P(s) \). If it can only observe a variable which depends on both \( a \) and \( s \), say \( H(a,s) \), then any enforceable payment schedules must have \( P = P[H(a,s)] \). Payment schedules on oil and gas leases have been royalties based on revenue \( R(a,s) \), or profit shares based on \( V(a,s) \). But maximum efficient rates of lifting (MER's) give the government some potential to monitor the production action, in which case (if fines were levied for failure to produce at an efficient rate), \( P = P[H(a,s), a] \).  

Most generally, we assume that the government seeks a schedule \( P(a,s) \) to

\[
\text{Maximize } EW[P(a,s)]
\]

subject to

\[
EU[V(a,s) - P(a,s)] = U(0) ;
\]

\[
EU'(V-P)[(\partial V/\partial a - \partial P/\partial a] = 0 ,
\]

where the first constraint implies consistency with competitive bidding, and the second reflects the fact that the firm's choice of action will maximize its bid given the payment schedule \( P \). Limits on observability could be reflected in the problem by additional constraints, for example \( \partial P/\partial a = 0 \), if the government cannot observe \( a \) but can observe \( s \).
A crucial question is whether the lease payment schedule is consistent with Pareto optimal production decisions \( a \). When \( a \) was exogenously determined, maximizing government welfare with respect to \( P \) implied Pareto optimality, since the government maximized its utility in choosing the schedule. But while the government chooses \( P \) in the problem with production, the firm chooses \( a \). Pareto optimality then resolves into the question: "Does the firm choose the action that the government would most prefer, given the optimal payment schedule?" We can prove the following basic result:

**Theorem V:** The optimal payment schedule \( P^*(a,s) \) will be Pareto optimal if and only if it does not affect the firm's choice of action at the margin. That is, if and only if

\[
EU'(V-P^*) \left[ \frac{\partial P^*}{\partial a}(a^*,s) \right] = 0.
\]

**Proof:** See the Appendix.

**Corollary:** If the payment schedule generates Pareto optimal decisions \( a^* = \hat{a} \), then the optimal payment schedule \( P(\hat{a},s) = P[V(\hat{a},s)] \) will satisfy the properties developed in the previous section.

**Proof:** Pareto optimality implies the constraint (18) has associated Langrangean multiplier \( \lambda_2 = 0 \). With \( \lambda_2 = 0 \), the necessary conditions for the optimal payment schedule coincide with (4), given \( a \).

The following results follow directly from Theorem V.
Proposition I: If there are no restrictions on $P(a,s)$, $P^*(a,s)$ will always generate Pareto optimal decisions.

Proof: The ability to monitor $a$ implies that any $\hat{a}$ can be enforced by the government by making $P^*(a,s)$ sufficiently negative for $a \neq \hat{a}$. Differentiability of functions implies we can choose $\partial P^*(\hat{a},s)/\partial a = 0$ for all $s$. Thus, $EU'[V - P^*](\partial P^*/\partial a) = 0$ at $a = \hat{a}$, and $P^*(\hat{a},s)$ will have the properties discussed in Section II. A similar result is in Spence and Zeckhauser [1971].

Proposition II: If the government can monitor a function $H(a,s)$ and $a$, where $H(a,s)$ is sufficient information for $V(a,s)$, then Pareto optimality can be achieved by the optimal payment schedule $P^*[H(a,s), a]$.

Proof: Again we note that any action $\hat{a}$ can be enforced by making $P[H(a,s), a]$ sufficiently negative for $a \neq \hat{a}$. Given $a = \hat{a}$, the optimal schedule $P$ need only vary with $V(\hat{a},s)$. But $H(\hat{a},s)$ will signal a change in $V(\hat{a},s)$ by the assumption of information sufficiency.

Proposition III: If the government can observe $s$ but not $a$, Pareto optimality can be achieved by the optimal schedule $P = P^*(s)$.

Proof: $\partial P^*(s)/\partial a = 0$ for all $s$, implying the condition of Theorem V,
$EU'(V - P^*)(\partial P^*/\partial a) = 0$.

Proposition III shows a rather surprising result: Pareto optimality will be achievable if the government can observe only $s$. Even
though it cannot enforce an action $\hat{a}$, the firm will still choose $a^* = \hat{a}$ because the payment scheme does not interfere at the margin: $\frac{\partial P^*}{\partial a} = 0$.

In most oil and gas leasing environments, the government cannot observe the true state of nature or all actions of production on the lease, and the revenues $R(a,s)$ associated with production, are perhaps the easiest to monitor. Costs $C(a,s)$ are more difficult but not impossible to observe, implying payments based on profits

$$V(a,s) = R(a,s) - C(a,s)$$

are potentially implementable. We shall here assume that observability is "perfect," with no measurement error. A subsequent paper will consider effects of imperfect measurement.

In the oil and gas environment, $a$ can be associated with level of installed production capacity. For all $s$, we expect to find $\frac{\partial R(a,s)}{\partial a} \geq 0$, with the strict inequality holding for $s$ where substantial reserves are discovered. Under these circumstances, we can show

**Proposition IV:** A royalty system, with payments $P = P[R(a,s)]$, can never achieve Pareto optimality.

**Proof:** Royalty payment schedules specify $dP/dR > 0$. $U'(V-P)$ is positive for all $s$. Since $\frac{\partial R(a,s)}{\partial a} \geq 0$, and strictly positive for some $s$, $E[U'(V-P)\frac{\partial P}{\partial a}] = EU'(V-P)(\frac{\partial P}{\partial R} \cdot \frac{\partial R}{\partial a}) > 0$, which contradicts the necessary condition of Theorem V.
Proposition IV demonstrates that royalty schemes induce the firm to undertake too little production: Investment of any sort which would lead to greater production is to some degree stifled, with the effect being more pronounced as the royalty rate \( dP/dR \) increases.

Profit sharing plans, on the other hand, can be consistent with Pareto optimality although they will not always be:

**Proposition V:** If the optimal profit sharing schedule \( P = P(V) \) is linear \((P = a+bV)\), then Pareto optimality is achieved.

**Proof:** \( EU'(V-P)\frac{3P}{3a} = bEU'(V-P)\frac{\partial V}{\partial a} = 0 \), with the last equality following from conditions (18), which imply \( EV'(V-P)(\frac{\partial V}{\partial a}(1-b)) = 0 \), or \( EV'(V-P)\frac{\partial V}{\partial a} = 0 \). A similar result is derived by Ross [1973].

Profit sharing payments will not distort production decisions of firms if the Pareto optimal payment rule is linear. Section III indicated linear payment schedules are optimal if, for example, government and firm have utility functions with risk tolerance linear in \( Y \) with equal slopes. More generally, linear schedules are optimal if the risk tolerance function of the government is a vertical shift of the firm's risk tolerance function.

Pareto optimality is clearly a special case for profit sharing schedules. We may show:

**Proposition VI:** At the Pareto optimal payment schedule, the firm will choose too small a production level if that payment schedule is convex and too large a production level if it is concave.
Proof: See the Appendix.

When Pareto optimality does not hold, the government must trade off optimal risk sharing for better incentives. In the situation discussed in Proposition VI, the optimal payment schedule presumably is less convex or concave than the Pareto optimal schedule to motivate the firm to choose an action closer to the one desired by the government.

VIII. ASSESSMENT OF ROYALTY AND PROFIT-SHARING PLANS

A. Royalty Payments

The most common payment schedule in oil and gas leases is a royalty which specifies that the firm pay to the government a specified fraction of revenues (production times current price) from production on the tracts leased. The current royalty fraction is one-sixth. In light of the analysis previously developed, what can we conclude about the efficacy of this system?

First, we note that, in general, \( R(a, s) \) is not sufficient information for \( V(a, s) \): There does not exist a function \( G \) such that \( V(a, s) = G[R(a, s)] \). Thus, any system based on royalties cannot, in general, achieve full efficiency even when production decisions are exogenously determined. In particular, uncertain costs cannot be adequately "insured" under a royalty payment system: All cost uncertainty that is not directly dependent on revenue uncertainty must be borne by the firm. And cost uncertainties seem particularly relevant to the OCS environment.

Second, we can show that royalties are a strictly concave function of value under normal circumstances.
Concavity has two undesirable consequences. First, Theorem II indicated that concave schedules are appropriate only if the government's risk tolerance is growing less rapidly with wealth than the firm's. If both firm and government exhibit constant relative risk aversion, a common behavioral assumption, then the optimal schedule should be convex rather than concave.

The second objection to concavity is much more critical: As $V$ rises, concavity implies that $dP/dV$ falls. If the same royalty rate is applied to all leases, tracts with higher expected values will have lower expected risk sharing: $E[dP/dV]$ falls as $E[V]$ rises. This is exactly the opposite of what should be the case. Theorem III showed that risk sharing should increase as the value of a tract rises.

The percentage royalty specified for federal oil and gas leases has remained basically unchanged over a fifty-year period. Clearly, recent events have led to much higher bonus bids on tracts, indicating their value has increased. The optimal royalty rate should be raised, according to Theorem III. But it has not been.

The final problem with royalty payment schedules is their misincentive properties. Proposition IV indicates that royalties will always induce inefficient production decisions by firms. The "early shutdown" problem associated with royalties is but one aspect of this result. In general, investment in exploration and development will be less as well.22

The advantages of a royalty system are primarily in their administration. Production on a lease is relatively easy to monitor, as are
wellhead prices. Batteries of accountants and inspectors are not needed. But the real costs of inefficiency resulting from a royalty system may warrant examination of other schemes, even if these schemes incur heavier administrative costs.

B. Profit-Sharing Plans

A feature of many recent international oil leases is the specification of payments based on profits. In its purest form, a profit-sharing plan can be viewed as a payment schedule based on \( V \), the net present value of the lease. Less-than-pure profit sharing takes place when payments are based on accounting profits or on some other measure of profits that diverges from true economic profits.

Typically, profit-sharing clauses in leases have specified a fixed fraction of profits be paid to the government. The payment schedule is linear in this case: \( P(V) = \alpha V + \beta \), where \( \beta \) is the amount bid by the firm. A linear profit-payment scheme will be optimal only in the special conditions specified in the Corollary to Theorem II. Nonetheless, it does not have the undesirable concavity property of the royalty system. Nor will it tend to distort production decisions as much as royalties will, as can be seen from Propositions IV and V.

Even if linear profit-payment schemes are optimal for each lease, we showed that the fraction of profit paid should increase as the expected value of the tract increased. Most profit-sharing schemes do not permit different profit shares on different tracts. But in auctioning
leases, it would be possible for the government to specify a schedule showing the fraction profit share as a function of the bonus bid. For example, the profit share might increase by 5 percent for each million dollars bid. If the firm's bid was $5 million, the profit share would be 25 percent, and so forth. Such a scheme (which could be applied to royalty systems as well) would tend to limit the amounts of cash needed to bid effectively on valuable leases.  

In sum, profit-sharing payment schedules seem to have substantial advantages over royalty schedules. These advantages occur because profit-sharing payments not only share risk more effectively, but also because they tend to distort production decisions less. The sole disadvantage of a profit-sharing plan seems to be an administrative one—the profit base must be chosen to coincide with true economic profits as closely as possible. If the government-defined profit base is not an accurate measure of \( V \), misincentives are likely to result. And even if the profit base is accurate, the monitoring of firms may still incur substantial administrative expense. But, given the magnitude of the value of natural resources, such expenses might well be borne if efficiency is increased by even a small fraction.
FOOTNOTES


2 Guarantees tied to sales are another example of conditional payment schemes which share risk between buyer and seller.

3 Average lease bonus payments (winning bids) have risen from less than $2.5 million to approximately $10 million in the last twenty years: see Hughart [1975].

4 For a survey of stockholder unanimity results, see Leland [1973].

5 Since bankruptcy costs are real economic costs, reducing firm-specific risk could be socially useful. This contrasts with the analysis inherent in the capital asset pricing model (see Mossin [1973]), which ignores bankruptcy costs.

6 While empirical evidence is not inconsistent with capital market efficiency, this efficiency does not guarantee managerial efficiency. Managers whose human capital is strongly tied to their firm may not make decisions as if they had well-diversified portfolios.

7 Empirical and theoretical evidence indicate the expected maximum bid is an increasing function of the number of bidders. Exclusion of some bidders may well lead to firms capturing a fraction of economic rent.
The value of an oil and/or gas lease depends upon current and (discounted) future prices of oil and costs of development and production, as well as upon the reserves in the tract. The "state of nature" is a complete description of these exogenous aspects. The value of the lease will depend upon the state of nature and the "action" (production decisions) of the lessee.

The functional form of \( P(a, s) \) may be restricted because of observability requirements. See section VI.

We are assuming that the firm acts as if it had a utility function defined over returns to the lease. This is consistent with production theory under uncertainty if the returns are independent of other investments by the firm, or if returns to other assets \( j = 1, \ldots, J \) are of the form \( Z^j(s) = \alpha^j Z(s) + e^j(s) \), where \( e^j(s) \) is a project specific (independent) risk. This latter form may come closer to describing oil leases, where there are market risks common to all tracts' values and tract-specific risks of discovery.

Perfect competition would imply that all firms have identical information, which is not the OCS situation. Nonetheless, Wilson [1975] shows that if there are a large number of bidders with independent information, the winning bid will converge to its certainty equivalent.

In section VII, we show that marginal independence of the decision from the payment schedule (formalized in Theorem V) is all that is required for the analysis here to be extended to the case of actions
being endogenous. A number of conditional payment schemes will satisfy the marginal independence criterion.

13 This rests critically upon the exogenous (or marginal independence) assumption.

14 Note that with complete markets, this condition will be satisfied ex ante since marginal rates of substitution will be the same between states for all participants. Thus, any payment schedule which did not alter production decisions would satisfy (4).

15 Alaskan Native Corporations have potential oil and gas bearing lands. Because of their relatively small asset positions and their dependence on potential oil and gas revenues, they are likely to be relatively risk averse. Their lease therefore should specify lower royalty and/or profit sharing rates (with consequent higher lease bonuses).

16 A similar result is derived in a somewhat different manner by Ross [1974].

17 For any U, there exists a W such that \( W[\alpha + BV] = U[(1-\beta)V - \alpha] \), for all \( V \).

18 See the pioneering work by Wilson [1967] and later speculations by Hughart [1975]. If information gathering tends to reduce (prior) asymmetries, the argument would be reversed.

19 The MER is set for each tract by the U.S. Geological Survey; since firms' suggestions for the MER are usually accepted by the USGS, there remains some question as to whether the "controlling" of actions by the firm is real.
By "sufficient information," we mean that observing $H$ will precisely identify $V$. That is, for all $a$ and $s$, there exists a function $G$ such that $V(a,s) = G(H(a,s))$.

A royalty of fraction $\alpha$ determines a payment $P = \alpha R$, where $R$ is revenue. Since $V = R - C$, $P(V) = \alpha[V + C(V)]$, where $C(V)$ are costs of production given value $V$. Typically, costs increase less than proportionately to value, i.e., $C''(V) < 0$. But

$$\frac{d^2P}{dv^2} = \frac{d^2\alpha[V + C(V)]}{dv^2} = \alpha C''(V) < 0,$$

so payments under a fixed royalty are concave in $V$.

For a fuller examination of these and related questions, see Leland, Norrgaard, and Pearson [1974]. We are currently developing empirical estimates of the extent of distortions created by royalties.

Besides sharing risk more effectively, payment schedules requiring less "front-end cash" would enable smaller firms to compete more effectively. If there are capital constraints on such firms resulting from market imperfections, this would be a desirable effect.

"True economic profits" may be difficult or impossible to compute exactly for the base if costs are incurred jointly over a group of leases. If structures rather than tracts were leased, this problem would be eliminated.
APPENDIX

Proof of Theorem II

By definition:

\[ RT_G = -W'(P)/W''(P) = 1/\text{ARA}_G \]

\[ RT_F = -U'(V-P)/U''(V-P) = 1/\text{ARA}_F. \]

Therefore, (6) can be written:

\[(\text{II.1}) \quad \frac{dP(V)}{dV} = \frac{RT_G}{(RT_G + RT_F)}. \]

Differentiating, with respect to \( V \), gives:

\[(\text{II.2}) \quad \frac{d^2P}{dV^2} = \frac{(RT_F)(RT'_G)(dF)}{dV} - \frac{(RT_G)(RT'_F)(1 - \frac{dP}{dV})}{(RT_G + RT_F)^2}. \]

Substituting for \( dP/dV \) from (II.1) and simplifying, yields:

\[ \frac{d^2P}{dV^2} = \frac{(RT_F)(RT_G)(RT'_G - RT'_F)}{(RT_G + RT_F)^3}, \]

which has the sign of \([RT'_G - RT'_F]\).

Proof of Theorem III

By assumption, \( E_m[V - P_n(V)] > E_n[V - P_n(V)] = U(0) \). Since

\[ E_m[V - P_m(V)] = U(0), \]

it follows that:
(III.1) \[ E_m[U(V - P_m(V))] < E_m[U(V - P_n(V))]. \]

Now, (III.1) can hold only if, for some \( V \), \( P_m(V) > P_n(V) \). We shall show if it holds for some, it must hold for all. From (4),

(III.2) \[ W'[P_m(V)] = \lambda_m U'[V - P_m(V)] \]

(III.3) \[ W'[P_n(V)] = \lambda_n U'[V - P_n(V)], \text{ for all } V. \]

Take a \( V \) for which \( P_m > P_n \). By concavity of \( U \) and \( W \), (III.2) and (III.3) can only hold jointly with \( P_m > P_n \) if \( \lambda_m < \lambda_n \). But if \( \lambda_m < \lambda_n \), then, by concavity of \( U \) and \( W \),

\[ P_m(V) > P_n(V) \text{ for all } V, \]

which proves (a).

From (6), recall that

\[ \frac{dP}{dV} = \frac{1}{ARAG}, \]

where \( ARAG \) is evaluated at \( P(V) \), and \( ARAF \) is evaluated at \( V - P(V) \). Since \( P_m(V) > P_n(V) \) for all \( V \), the argument of \( ARAG \) will be larger at each value of \( V \) under schedule \( P_m \) than under schedule \( P_n \). The argument of \( ARAF \) will be smaller. Diminishing absolute risk aversion implies \( ARAG \) will be smaller at each \( V \), given \( P_m \) than \( P_n \), whereas \( ARAF \) will be larger. Therefore, at each \( V \), \( ARAG/ARAF \) is smaller, or

\[ \frac{dP_m(V)}{dV} > \frac{dP_n(V)}{dV}. \]
Proof of Theorem V

Pareto optimality requires that $\mathbb{E}[W(p,a,s)]$ be maximized, both w.r.t. $p$ and $a$, given $\mathbb{E}[U(V(a,s) - P(a,s))] = 0$. This yields necessary conditions:

(V.1) \[ W'(P(a,s)) - \lambda U'(V(a,s) - P(a,s)) = 0; \]

(V.2) \[ \mathbb{E}U'(V - P) \frac{\partial p}{\partial a} - \lambda U'(V - P) \left( \frac{\partial V}{\partial a} - \frac{\partial P}{\partial a} \right) = 0. \]

Substituting for $W'(P)$ from (V.1) into (V.2) gives

(V.3) \[ \mathbb{E}U'(V - P) \frac{\partial V}{\partial a} = 0. \]

The actual decision by the firm will satisfy constraint (18). If (V.3), the Pareto optimality condition, is satisfied as well, then

(V.4) \[ \mathbb{E}U'(V - P) \frac{\partial P}{\partial a} = 0. \]

Proof of Proposition VI

Let $\hat{a}$ be the optimal action at the Pareto optimal payment schedule $P[V(a,s)]$. Then:

(VI.1) \[ \left. \frac{\partial \mathbb{E}U(V - P)}{\partial a} \right|_{a=\hat{a}} = \mathbb{E}U'(V - P) \left( 1 - \frac{\partial P}{\partial V} \right) \frac{\partial V}{\partial a} \]

\[ = - \mathbb{E}U'(V - P) \frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial a}, \]

using (V.3).
We presume that \(-\frac{\partial V(a,s)}{\partial a}\) is larger for states with larger values \(V(a,s)\), since installing more capacity is more profitable, the greater the resources in place. Furthermore, it will switch from negative to positive at some \(\bar{s}\). So,

\[(VI.2)\]
\[-U'(V - P) \frac{\partial V}{\partial a} > 0, \quad V < \overline{V} = V(\hat{a}, \overline{s});\]

\[-U'(V - P) \frac{\partial V}{\partial a} < 0, \quad V > \overline{V}.\]

If \(P(V)\) is a strictly concave schedule,

\[(VI.3)\]
\[\frac{dP(V)}{dV} > \frac{dP(\overline{V})}{dV}, \quad V < \overline{V}\]

\[\frac{dP(V)}{dV} < \frac{dP(\overline{V})}{dV}, \quad V > \overline{V}.\]

Combining (VI.2) and (VI.3), we see that for all \(V\) (except \(\overline{V}\)),

\[(VI.4)\]
\[-U'(V - P) \frac{\partial V}{\partial a} \left(\frac{dP(V)}{dV} - \frac{dP(V)}{dV}\right) < 0,\]

or

\[(VI.5)\]
\[-\frac{3P(V)}{\partial V} EU'(V - P) \frac{\partial V}{\partial a} < -EU'(V - P) \frac{\partial V}{\partial a} \frac{\partial P}{\partial V}.\]

From (V.3), the L.H.S. of (VI.5) is zero. Therefore,

\[(VI.6)\]
\[-EU'(V - P) \frac{\partial V}{\partial a} \frac{\partial P}{\partial V} > 0, \quad \text{or}\]

\[\frac{\partial E(U(V - P))}{\partial a} \bigg|_{a=\hat{a}} > 0.\]
By concavity of $EU(V-P)$ in $a$, the firm will prefer a larger production capacity. If $P(V)$ were convex, the firm would prefer a smaller capacity.
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