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Corporate Decision Making in Incomplete Markets

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I. Introduction

Stockholder unanimity is crucial for the existence of a theory of production under uncertainty. Without unanimity, firms' decisions will depend upon the weighting of diverse stockholder and managerial preferences, with the possible nonexistence of a preference ordering over actions by the firm. ¹

Several recent papers have studied the relationship between financial markets and production under uncertainty. Implicitly or explicitly, stockholder unanimity has been a property of these models. Two sets of conditions have led to unanimity. The first set, studied extensively by Rubinstein [1974a,b,c], requires identical investor expectations and tastes belonging to the linear risk tolerance (HARA) class of utility functions. The second set does not restrict individuals' characteristics, but requires the space of returns generated by changes in firms' decisions to be contained in the space spanned by marketed securities. Ekern and Wilson [1974] showed spanning was a sufficient condition for stockholder unanimity. Leland [1973] showed it was necessary as well, if investors have arbitrary tastes, and showed that the "complete markets" models of Arrow [1953] and Debreu [1959], as well as the "incomplete markets" models of Diamond [1967]

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Leland [1974], and Ekern and Wilson [1974], satisfied the spanning property. Leland [1973] further indicated that an economy characterized by competitive price uncertainty with as many securities as goods (not states) also satisfied the spanning property.

In Section II of this paper we examine implications of the competitive price environment for production decisions under uncertainty. Surprisingly, we can show the existence of a set of certainty equivalent prices. Firms, acting as if these prices were certain and maximizing profit, would undertake production plans which stockholders would unanimously approve in the uncertain environment.

While providing a rich class of environments consistent with stockholder unanimity, the identical expectations conditions and the spanning conditions seem overly restrictive. In Section III we show that there are weaker conditions implying stockholder unanimity, when it is recognized that managers may have better information about project-specific risks. This analysis restores an active role to the manager (information acquisition and decision making based on this information), which is notably lacking when identical expectations or spanning conditions are invoked.

Section IV continues the analysis of information asymmetries between manager and shareholder. If risks are no longer project-specific, will managers make decisions that are optimal for shareholders? And should managers be allowed to use "inside" information to select their own portfolios, as well as to make decisions for the firm? We show that stockholders will approve of managerial decisions in general only
II. Production Theory with Competitive Price Uncertainty

Production theory under certainty indicates that competitive, profit maximizing firms choose inputs and outputs such that the value of every marginal product is equal to the price of the input, across all firms. Comparative static and welfare analysis builds from these equilibrium conditions.

When prices are random and financial market incomplete, the certainty equilibrium conditions are vacuous. Production theory must be reconstructed in such environments, and profit maximization must be supplanted by a criterion which is meaningful under uncertainty. A natural approach is to look at stockholders' interests (since they justified profit maximization under certainty). In general, stockholders will not unanimously support any policy, and a theory of the firm does not readily suggest itself. But Ekeland and Wilson [1974] have shown that unanimity will follow given a technical "spanning" property. Leland [1973] later showed this was a necessary as well as sufficient condition for \textit{ex post} stockholder unanimity, and showed that the "complete market" models of Arrow [1953] and Debreu and the "multiplicative random technology" model of Diamond [1967] satisfied the spanning condition. Leland also suggested that another environment satisfied the spanning property: the case where there are as many firms as goods with independent prices, and where uncertainty is generated uniquely by these random competitive prices. In what follows, we develop a theory of production in such an environment. A rather startling conclusion emerges: there exist certainty equivalent prices which guide production in precisely the same manner as in the traditional theory.
Let \( r = 1, \ldots, R \) index goods
\( j = 1, \ldots, J \) index firms
\( s = 1, \ldots, S \) index states of nature
\( q^j = (q^j_1, \ldots, q^j_R) \) the production plan of firm \( j \), satisfying
a production function constraint
\( f^j(q^j) = 0 \).

\( p_{rs} = \text{price of good } r \text{ in state of nature } s \).

\( v^j = \text{stock market value of firm } j \).

By assumption, the number of firms \( J \) must be at least as great
as the number of goods (with independent prices) \( R \). For simplicity,
let \( J = R \), and let each good be produced by only one firm.

The \( S \times J \) matrix of firms' profits, \( \Pi \), will satisfy

\[ \Pi = PQ \, , \]

where \( P \) is an \( S \times R \) matrix of prices of each good across states,
and \( Q \) is an \( R \times J \) matrix whose columns are the production plans of
each firm.

Now consider \( \frac{\partial \pi^j}{\partial q^j_r} \), \( r \neq j \), the \( s \)-dimensional vector representing
the change across states of nature of firm \( j \)'s profit, when
input \( r (r \neq j) \) is changed, but other inputs remain constant. Denoting
the marginal product of \( r \) in producing output \( j \) as \( f^j_r \) (derived
from the production function \( f^j(q^j) = 0 \)), we have

\[ \frac{\partial \pi^j}{\partial q^j_r} = p_{js} f^j_r = p_{rs} \, , \quad r \neq j \, . \]
or in vector terms,

\[
\frac{\partial \pi^j}{\partial q^j_r} = P[\hat{\mathbf{f}}^j_r - \hat{r}],
\]

where as before \( P \) is the \( S \times J \) matrix of prices across states, \( \hat{\mathbf{f}}^j_r \) is a column vector of zeros with \( f^j_r \) in the \( j \)th row, and \( \hat{r} \) is a column vector of zeros with 1 in the \( r \)th row.

We can now prove Leland's Proposition IV [1973]:

**Proposition:** For all \( j \) and \( r \), \( \frac{\partial \pi^j}{\partial q^j_r} \) is spanned by \( \Pi \).

**Proof:** We must show there exists a vector \( \mathbf{c}^j_r \), for all \( j, r (j \neq r) \), such that

\[
\frac{\partial \pi^j}{\partial q^j_r} = \Pi \mathbf{c}^j_r.
\]

We propose

\[
\mathbf{c}^j_r = Q^{-1}[\hat{\mathbf{f}}^j_r - \hat{r}].
\]

\( Q^{-1} \) exists from our assumption that \( J = R \). From (1) and (4),

\[
\Pi \mathbf{c}^j_r = P Q [Q^{-1}[\hat{\mathbf{f}}^j_r - \hat{r}]] = P [\hat{\mathbf{f}}^j_r - \hat{r}] = \frac{\partial \pi^j}{\partial q^j_r},
\]

using (2). Therefore, the proposed \( \mathbf{c}^j_r \) satisfies the spanning condition (3).
Properties of Equilibrium

Stockholder unanimity with respect to small changes in firms' decisions is guaranteed by the previous proposition. Equilibrium will require that \( \frac{\partial U_i}{\partial q^j_r} = 0 \) for all investors \( i \) and decisions \( q^j_r \). From Ekern/Wilson [1974] or Leland [1973, Theorem I], it can be shown that for all stockholders \( i \),

\[
\text{sign} \left( \frac{\partial U_i}{\partial q^j_r} \right) = \text{sign} \sum_{k=1}^{J} v^k C^j_k^r ,
\]

where \( C^j_k^r \) is the \( k \)th element of the \( J \times 1 \) vector \( C^j_r \). Thus, production equilibrium requires

\[
V'C^j_r = 0 , \quad \text{or}
\]

(6) \( V' Q^{-1} [\hat{f}^j_r - \hat{r}] = 0 , \quad \text{for all } j \text{ and } r , \)

with \( V' \) the \( 1 \times J \) row vector with elements \( V^j \).

Now define

(7) \( p'^* = V' Q^{-1} \)

a \( J \)- (and therefore \( R \))- dimensional row vector. Equilibrium conditions (6) imply

(8) \( p'^*[\hat{f}^j_r - \hat{r}] = 0 , \quad \text{or} \)

\[
p'^*_f^j_r - p'^*_r = 0 , \quad \text{for all } j \text{ and } r .
\]
But conditions (8) are precisely those which would be satisfied if prices \( p_r^* = p_r^* \), \( r = 1, \ldots, R \), with certainty! Equilibrium implies firms act as if they maximized profits using \( p^* \) as prices. The prices \( p^* \), given (7), therefore may properly be called certainty equivalent prices. Furthermore, certainty equivalent profits are

\[
\pi^* = p^*Q = V'Q^{-1}Q = V',
\]

implying that decisions act to maximize market value given prices \( p^* \).

Somewhat ironically, the theory of the firm has come full circle. Under uncertainty, we rejected profit maximization and the criteria that the value of the marginal product equals the price of the input in equilibrium. But we ultimately replace it with a theory that exhibits precisely the same characteristics, with certainty equivalent prices \( p^* \) replacing the certainty prices of the traditional theory. (Of course, the certainty theory is a special case of the more general theory, as can be seen by noting that \( V' = \Pi = p'Q \) under certainty, and therefore \( p^* = V'Q^{-1} = p'QQ^{-1} = p' \).) But the more general theory does permit examination of the effects of changed expectations and/or attitudes toward risk, a set of questions beyond the purview of traditional production theory.

In principle, one could compute \( V'Q^{-1} \) to estimate the certainty equivalent prices \( p^* \). With information on production functions (marginal products), the effects of changes in stock market values \( V \) on production decisions \( Q \) could then be estimated.\(^3\)
III. Informational Asymmetry and Stockholder Unanimity Without Spanning

Recent studies have relied on two alternative sets of conditions to guarantee stockholder unanimity under uncertainty:

1) identical expectations; utility functions belonging to the linear risk tolerance class (with equal slopes); and

2) spanning of marginal return vectors by the set of traded securities.

Both conditions seem restrictive. Individuals possess different information and therefore different expectations. And, in many environments, marginal projects will have specific risks which are not spanned by currently available securities. A natural question arises: Is there some subset of conditions (1) and (2) that is not as unrealistic as either set of conditions alone, but which generates stockholder unanimity? We shall argue that there is, when information asymmetries between manager and investors are introduced.

First, we note that conditions (1) and (2) give little scope for the manager to acquire special information. Clearly, managers cannot have "better information" than investors, given the identical expectations assumption of (1). And while spanning permits different shareholders (including the manager) to have different expectations, the optimal action will be the same for any set of expectations. Why should a manager gather information if (as the model predicts) better decisions will not result? In short, current approaches to stockholder unanimity overlook
the fact that managers frequently have better (not simply different) information than investors—particularly with respect to risks that are specific to projects undertaken by their firms. In what follows, we make some initial attempts to capture the effects of asymmetric information on the theory of production and investment choice.

We consider a small investment (or marginal change in profit) whose return can be viewed as the sum of two components. The first component depends upon states $s$, which affect currently available security returns. We assume this first component is spanned by the current securities. The second component is a project-specific (independent) risk, whose value depends upon a project-specific state $e$. Letting $I(s,e)$ denote the returns to the investment, we have

$$I(s,e) = \sum_{s} w^s C^j + h_e,$$

where $C^j$ are the spanning weights.  

Thus, the investment or marginal change in profit is not spanned by the set of securities, but the unspanned residual depends only upon project-specific states whose probabilities are presumed independent of $s$. That is, for any investor $i$,

$$P_i(s,e) = P_i s P_i e.$$

The expected utility to a stockholder $i$, resulting from a firm undertaking such a small investment, is (see Leland [1973], p. 15),
(11) \[ dE_{i_1} = X_{s,e} \sum_{s,e} P_i(s,e)U'_i[R_{is}]I(s,e), \]

where \( X_{i_1} \) is the fraction of the firm undertaking the investment owned by \( i, \) and \( R_{is} \) is investor \( i \)'s return in state \( s, \) given his optimal portfolio, which will satisfy the necessary first-order conditions

(12) \[ \sum_{s \in i_1} P_{R_{is}} \pi'_j = \lambda_i \pi_i^j, \quad j = 0, \ldots, J. \]

Assuming \( j = 0 \) is a riskless asset returning \((1+r)\) in every state \( s \) (with value normalized to \( V^0 = 1 \)), then from (12),

(13) \[ \sum_{s \in i_1} P_{R_{is}}(1+r) = \lambda_i. \]

From (9) and (11),

(14) \[ dE_{i_1} = X_{s,e} \sum_{s,e} P_i(s,e)U'_i[R_{is}](\sum_j \pi'_j C_j^i + h_e) \]

\[ = X_{s,e} \sum_{j} P_i U'_i(R_{is}) \pi'_j(\sum_j \pi'_j C_j^i) + X_{s,e} U'_i(R_{is}) \sum_j P_i h_e \]

\[ = X_{i_1} \lambda_i (\sum_j \pi'_j C_j^i) + (\sum_j P_i h_e)/(1+r), \]

using (12) and (13). Since \( X_{i_1} > 0 \) for all shareholders, and since \( \sum_j C_j^i \pi'_j \) is independent of \( i, \) \( dE_{i_1} \) will have the same sign for all \( i \) if the probabilities of \( e \) are the same for all \( i. \)

But why might we expect identical probabilities here, any more than we might expect them for states \( s? \) The answer lies in informational asymmetries over \( e. \) These, after all, are project-specific risks
and, therefore, risks on which the manager is likely to have better information than investors. Formally, we can say that the manager observes a signal \( z \in Z \), which, if observed by investors, would induce them to share the manager's probabilities: for all \( e \),

\[
P_{ie|z} = P_{me|z} \quad \text{for all } i \text{ and for all } z \in Z,
\]

where \( P_{me|z} \) are the probabilities held by the manager if he receives information signal \( z \). Of course, investors do not in general observe \( z \), and their \textit{ex ante} probabilities \( P_{ie} = \sum_z P_{ie|z} P_{iz} \) will in general differ. Nonetheless, \textit{there will be stockholder unanimity for the manager's choice}. Investors would like to observe the information variable \( z \in Z \) and base their decision upon it. Their agent, the manager, can observe \( z \). Given (14) and (15), all stockholders would agree with the manager's decision to accept or reject the investment (or to increase or decrease a decision variable), given \( z \). More formally, the observation by stockholders that the manager made a specific choice would itself constitute information about \( z \), and this information would cause them to revise their probabilities \( P_{ie} \) to approve the manager's choice, even if \textit{ex ante} they themselves would have made the opposite choice without knowledge of \( z \).

We conclude that the introduction of information asymmetries with respect to specific risks broadens the class of environments for which stockholder unanimity will be present. Further, it restores the role of the manager as a producer of information with respect to specific project risks: stockholders will be willing to hire managers to make decisions on the basis of information they acquire.
IV. Inside Information and Managerial Decision Making: General Analysis

The previous section indicated that managerial decision making would be in the stockholders' interests if:

1) information pertained to specific risks only;

2) information resulted in \textit{ex post} probabilities coinciding.\(^6\)

We now wish to examine the more general situation in which better (or "inside") information may be obtained (by managers) on general as well as specific risks.\(^7\) Will stockholders still unanimously approve of managerial decisions in such an environment? And is it in stockholders' interests to prohibit managers from using this inside information for their personal portfolio adjustment, as contrasted with using it for decision making within the firm?

As before, we assume the manager receives a signal \(z \in Z\), which implies an event (a subset of \(S\)) has occurred.\(^8\) We consider two situations.

**Situation A:** The manager adjusts his portfolio in accordance with his inside information. Optimality conditions (12) become:

\[
\sum_s P_{ms} U' (R^A_{ms}) \pi^j_s = \lambda^A_m V^j_m, \quad j = 0, \ldots, J;
\]

(16)

where \(R^A_{ms}\) are returns given his portfolio which satisfies (16), given the actions chosen by firms. Assuming he manages firm \(m\), he will choose a production decision \(q^A_m(z)\) such that (from Leland [1973], p. 15)
\[(17) \quad \sum_{ms} \left. z \right|_m (R^A_{ms}) (dU^m_s / dq_m) = 0. \]

**Situation B:** The manager cannot use his information \( z \) to make portfolio choices. Rather, he uses his ex ante probabilities to select a portfolio, implying

\[(18) \quad \sum_{ms} \left. U^j \right|_m (R^B_{ms}) = \lambda^B_{jm}. \]

\( R^B_{ms} \) are returns given the portfolio which satisfies (18). \((V^j_m) \) may or may not be the same as before. The manager can and will use his information to choose an action \( q^B_m(z) \), which then will satisfy

\[(19) \quad \sum_{ms} \left. U^j \right|_m (R^B_{ms}) (dU^m_s / dq_m) = 0. \]

Since \( R^A_{ms} \) and \( R^B_{ms} \) will in general differ, so will \( q^A_m(z) \) and \( q^B_m(z) \). We wish to examine whether either of these decision functions will coincide with stockholders' interests. More specifically, if stockholders had received a signal \( z \), would they agree with the managers' decisions? Given portfolio decisions (12), would

\[(20) \quad \sum_{is} \left. U^j \right|_i (R^j_{is}) (dU^m_s / dq_m) = 0 \]

when \( q_m = q^A_m(z) \) or when \( q_m = q^B_m(z) \)? We can show the following:

**Theorem:** If securities markets are complete \((j = S)\), managers' decisions will be unanimously approved by stockholders if managers are...
not allowed to adjust portfolios to their special information (Situation B).

**Proof:** Given complete markets \((J=S)\), \(\pi = [\pi^j_s]\) has an inverse. First-order conditions (12) imply

\[
\nu^i = \nu\pi^{-1},
\]

where \(\nu^i\) is the row vector with elements \(P_{is}\frac{U'(R_{is})}{\lambda_i}\), \(s=1,\ldots,S\), and \(\nu\) is the row vector of market values of firms \((\nu^0,\ldots,\nu^J)\). (21) implies that \(\nu^i\) is the same for all investors, i.e.,

\[
P_{is}\frac{U'(R_{is})}{\lambda_i} = P_{js}\frac{U'(R_{js})}{\lambda_j},
\]

for all \(i,j\). In Situation B,

\[
P_{is}\frac{U'(R_{is})}{\lambda_i} = P_{ms}\frac{U'(R_{ms})}{\lambda_m}^{B}.
\]

Now, assume a signal \(z\) occurs. Conditional probabilities are given by

\[
P_{is|z} = \frac{P_{is}}{P_{iz}}, \quad s \in \mathcal{E}_z
\]

\[
P_{ms|z} = \frac{P_{ms}}{P_{mz}}
\]

\[
P_{is|z} = 0, \quad s \in \mathcal{E}_z
\]
where \( \mathcal{E}_z \) is the event (subset of \( S \)) consistent with signal \( z \), and

\[
P_{iz} = \sum_{s \in \mathcal{E}_z} P_{is}; \quad P_{mz} = \sum_{s \in \mathcal{E}_z} P_{ms}.
\]

Given (23) and (24),

\[
P_{is|z = i}^U(R_{is}) = P_{ms|z = m}^U(R_{ms}) \left( \frac{\lambda_i}{\lambda_m} \right) \left( \frac{P_{mz}}{P_{iz}} \right), \quad s \in \mathcal{E}_z,
\]

and therefore

\[
\sum_{s \in \mathcal{E}_z} P_{is|z = i}^U(R_{is}) \frac{d\eta^m_z}{dq^m} = k \sum_{s \in \mathcal{E}_z} P_{ms|z = m}^U(R_{ms}) \frac{d\eta^m_z}{dq^m},
\]

where \( k = (\lambda_i/\lambda_m)(P_{mz}/P_{iz}) > 0 \). It follows immediately that (19) implies (20). (Q.E.D.)

We can further show that managerial decisions made when inside information is used for portfolio adjustment (Situation A) will not generally be in stockholders' interests. From (16) and (17) it follows that

\[
\sum_s P_{ms|z = m} A(R_{ms}) (d\eta^m_z/dq^m) = \lambda_m \sum_k \lambda^k C^k = 0,
\]

given \( q^A_m(z) \), where \( C^k = \eta^{-1}(d\eta/dq) \). Similarly, from (12) and (20),

\[
\sum_s P_{is|z = i} A(R_{is}) (d\eta^m_z/dq^m) = \lambda_i \sum_k \lambda^k C^k = 0, \text{ using (27).}
\]

For \( s \in \mathcal{E}_z \), we know from (24) that \( P_{is|z} \) will simply scale up \( P_{is} \) by
(1/\pi_{iz}). If all \pi_{is} were scaled up by this factor, we could replace \pi_{is} with \pi_{is}\mid_{z}, and stockholders would still find the RHS equality in (28) holding: managerial decisions would be unanimously approved. But for \sigma \in C_{z}, \pi_{is}\mid_{z} = 0, not \pi_{is}\mid_{z} = \pi_{is}/\pi_{iz}. We conclude that, in general,

$$\sum_{i} \pi_{is}\mid_{z} U'(\alpha_{is})(d\pi_{m}/dq_{m}) \neq 0,$$

given decisions \alpha_{m}(z). Managers should not be permitted to make portfolio decisions (as contrasted with production decisions) on the basis of inside information. Although managers would benefit from so doing, it comes at the expense of stockholders because of distorted managerial production decisions.

The intuition behind this result is reasonably straightforward. Stockholders are unable to observe \(z\), and therefore can't adjust their portfolios to the information signal. Given that they have "disequilibrium" portfolios (given \(z\)), they will wish firms to make decisions that reflect this disequilibrium. If managers, too, have disequilibrium portfolios, given \(z\), they will make the correct decisions for shareholders. If managers can achieve portfolio equilibrium using their inside information, they will make decisions that are correct for them but not in the stockholders' interests.
FOOTNOTES

1 We refer to the preference aggregation problem studied by Arrow [1963]. A coherent theory of the firm may exist if the manager can behave as a "dictator." Theories assuming that firms maximize expected utility of profit (e.g., Sandmo [1971]; Leland [1972]) can be thought of as managerial theories of the firm. But they do, of course, presume stockholders have no influence, and decisions made by dictatorial managers almost surely will fail to be Pareto optimal.

2 After the results in Section II had been derived, I became aware of an excellent paper by Forsythe [1975], which duplicates many of the results in that section. Sections III and IV, hopefully, are entirely new to the profession!

3 The interested reader may have noted that some inputs (e.g., labor) may not be traded on the market and therefore will not have an equilibrium market value $V_L$. This does not pose a problem to the theory if (1) wage rates $w$ are independent of $s$ (no wage uncertainty) and (2) there exists a riskless asset. The imputed value of labor will then satisfy $V_L = (w/r)V_R$, where $w$ and $r$ are the riskless wage rate and riskless return coupon, respectively, and $V_R$ is the value of the riskless asset.

4 The interested reader can verify that our conclusions hold when

$$ I(s,e) = \left( \sum_{j} \frac{C_j}{s} \right) d_e + h_e. $$
Following Marschak and Radner [1972], one can think of an information variable \( \tilde{z} \) as a function from \( E \), the set of project-specific states, to \( Z \), the set of possible signals. Thus, the observation of a signal \( \tilde{z} \) implies an event \( E_z \) (a subset of \( E \)) has occurred, with \( E_z = \{ e | \tilde{z}(e) = z \} \). An information structure serves to partition the set \( E \) into mutually exhaustive and exclusive subsets. "Better" information can be viewed as a finer partition of \( E \); "perfect" information is when the events are singletons, i.e., the observation of \( \tilde{z} = z \) determines a unique \( e \in E \). Two (or more) persons observing a signal \( z \) will agree to probabilities over \( E_z \) if (a) \( z \) generates a finer partition than either person has using alternative information variables (or signals), and (b) their priors over \( e \) without any information are identical. Note that condition (b) does not imply they have identical expectations before observing \( z \), since they may have different expectations if they have alternative information variables before observing \( z \).

See the previous footnote for a discussion of circumstances under which information will result in \textit{ex post} probabilities coinciding.

Since we are no longer distinguishing between general and specific risks (\( s \) and \( e \)), we shall denote by \( s \) a state that includes a description of both. That is, our new \( s = (s, e) \) in the previous section.

See Marschak and Radner [1972], chapter 2, for an elegant introductory discussion of information structures.

Note that we do not require \textit{ex post} probabilities coinciding for this proof. To avoid subtle problems with conditional probabilities,
we are presuming \( P_{is}, P_{ms} > 0 \) for all \( s \). Note that spanning (without complete markets) will not, in general, assure stockholder unanimity in Situation B, since without complete markets, \( v_i^* \) is not the same for all \( i \). But the result in the previous section indicates unanimity will prevail without complete markets (or total spanning) if information is relevant only to project- (or \( d\pi/dq_\cdot \)) specific states.

It can be seen now why the previous section's result is not contradicted: Information on \( e \) (being project-specific and not affecting current portfolio returns) would not lead to changes in the manager's or the stockholders' portfolios.
REFERENCES


