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The Limited Information Efficiency of Market Processes

by

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OF MARKET PROCESSES

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Avraham Beja*

ABSTRACT

Information efficiency concerns the inferences that a trader can draw about the information available to other traders by observing market prices. Statements of market efficiency can be partially ordered by the degree that information is reflected in prices and by the content of this information. It is shown that if prices are determined only through traders' demands, the extent of efficiency must be very limited. Any information that would occasionally be useful in an inefficient market, even in the presence of additional data, cannot be reflected in prices to a degree that would deny its value for decision-making purposes. Markets therefore cannot be efficient in the strong or semistrong form in any operational sense.

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I. INTRODUCTION

One of the key issues in the analysis of markets and prices has for some time evolved around the "information efficiency" of market systems and the question whether, and to what extent, market prices "fully reflect" traders' information. In particular, this general problem has motivated an almost overwhelming amount of empirical work within the context of the securities' markets.\footnote{An interesting and extensive sample of most aspects of the work on securities market efficiency is given in Lorie and Brealey [12]. See also Cootner [3] for a collection of many of the earlier studies that partly motivated the more recent work.} The analysis of the inferences that can be derived from prevailing prices has been typically concerned with information on the attributes of financial assets, but the underlying issues are of course much more general.\footnote{For example, Kihlstrom and Mirman [11] recently indicated the analogy between "insiders' information" as transmitted by stockmarket prices and the role of consumption goods prices as indicators of quality. Fama [5] investigated Treasury Bills prices for possible information on anticipated inflation.} The extent that market prices reflect valuable information is important not only for agents' trading behavior, but also for their investments in information - and consequently for the overall performance of the economy. Superior
information has distributional implications, and it has been suggested (cf. Hirschleifer [9] [10]) that this may lead to a socially wasteful "over investment" in information. If, however, information is well reflected in easily observable prices, traders' behavior will resemble a market with homogenous information, where speculative trading originating from diverse beliefs can hardly take place (cf. Grossman and Stiglitz [8]). Redistributational opportunities are then severely limited, and with them much of the incentive for traders to acquire costly information. The "production efficiency" of the system is then impaired, because with little acquisition of information prices will at best provide only limited signals for efficient production decisions.

"Information efficiency" as used in the present paper refers to the information reflected in prices and the inferences that can be derived from prices. Certainly, when traders' market behavior depends on the information they have and prices depend on traders' behavior, this information will somehow be reflected in prices. To be of value, a definition of "information efficiency" must specify a more precise formulation, indicating exactly how prices reflect information. Some of the formulations encountered in the literature will be reviewed briefly below, and some further definitions will be suggested. The various formulations are partially ordered by "degree" of efficiency as well as by
"strength of form". The main result of the present study is a proof that some intuitively appealing versions of information efficiency cannot prevail when prices are derived only from genuine trading behavior. The result seems to disagree with some of the positions adopted in the securities markets literature. These assert that (1) in observed markets substantial information is reflected in prices to an operationally meaningful extent, and (2) the attainability of market efficiency with respect to some information depends crucially on the range of agents to whom it is available. This paper indicates to the contrary that (1) basically useful information cannot be operationally reflected in prices, and (2) it is not the range of its availability, but the content of an information set (relative to information not in that set) which is the crucial criterion for its usefulness to the trader even when prices reflect information. Although the analysis is strongly motivated by the work on securities markets efficiency, both the exposition and the results are presented here in terms of a general exchange economy.

Some aspects of the theory of information efficiency are discussed in the next section. The model is then presented in section III, and a notion of genuine trading processes is introduced in section IV. We investigate in section V the extent that such processes can be informationally efficient, and show that this extent must be very limited,
indeed quite trivial. Some further implications of the basic result are discussed in section VI.

II. ASPECTS OF INFORMATION EFFICIENCY.

A statement of information efficiency must specify two things: what information is (presumably) reflected in prices, and how prices reflect this information. The first aspect will be discussed first. Garman [6] has recently indicated the crucial role of particularization in quantifying any version of an efficient markets hypothesis. Even when the term "efficiency" is well defined, the sentence "the market is efficient" (or "the market reflects information") is logically incomplete. It can be made precise by an explicit reference to a particular information set. Once this information set, say A, is accurately specified by the observations it includes, the statement

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Garman points out that logic also allows two other types of quantification of the statement "the market is efficient". Both of these are trivial in the present context. The "existential" quantification means that "there exists some information set with respect to which the market is efficient". It is immediately proved true (with any reasonable definition of efficiency), because market prices certainly fully reflect the (memoryless) information set which includes only current prices. The "universal" quantification means that "the market is efficient with respect to any information set". This should clearly be false, because prices at t cannot reasonably fully reflect the set of all information which is observable to traders only at t+1. Instead of the universal quantification, it would in this case be desirable to particularize the statement by an explicit reference to "the set of all information available to traders at that point in time".
"the market is efficient with respect to A" is well quantified. It may be either true or false, depending on the market system, the set A, and the adopted definition of efficiency.

Statements of market efficiency can thus be (partially) ordered according to the information set involved in the formulation. This is commonly associated in the literature with the notion of "strength". The statement "the market is efficient with respect to A" is termed stronger than the statement "the market is efficient with respect to B" if, and only if, the information set B is contained in the set A. A desirable (but by no means automatically satisfied) property of any definition of information efficiency is that "stronger" statements be associated with stronger hypotheses. That is, if by some adopted definition of efficiency it is true that "prices reflect the information set A", then for any subset B of the set A it should be also true that "prices reflect the information set B". The "weak", "semistrong" and "strong" forms of efficiency frequently encountered in the literature refer to special cases, around which many of the studies have evolved. Under the weak form, the set A includes only the past sequence of prices; under the semistrong form it includes "all obviously publicly available information" and under the strong form it includes, in addition, also some (sometimes "all") information monopolistically available only to a few
investors.\textsuperscript{4}

We now turn to the second aspect of efficiency—specifying the exact sense in which information is reflected in prices. A few definitions have been suggested in the literature. Fama's "Review of Theory and Empirical Work" [4] of securities markets efficiency has become a classical reference on this subject. His formulation is heavily couched in terms of "mathematical expectations", but he states that this "elevates the purely mathematical concept of expected value to a status not necessarily implied by the theory" and is in fact merely an "unavoidable price one must pay" to facilitate empirical work (p.384). It would be helpful, however, to state what efficiency would mean if one did not have to pay this price. Rubinstein [16] has recently suggested a formulation that does not rely on "expected values", but is in many other respects highly similar to Fama's. The essence of their approaches can perhaps be summarized as follows:

\textbf{FR (Fama-Rubinstein) Efficiency}:

The market is FR-efficient with respect to information set A if the prices it generates are identical to those generated in an otherwise identical economy where the set A accurately

\textsuperscript{4}E.g. Fama [4]. Note also Sharpe's [17] comment "...the definition of a semistrong efficiency is open to dispute (when is information publicly available? which investors are in the public? how soon must the information be available? at what price?)."
describes the information available to each and every agent. A modified, somewhat different, version may also be associated with this basic approach. Under the modified version, the market is defined efficient with respect to A if the prices it generates are identical to those generated in an otherwise identical economy where all traders know the information in A in addition to the information they already possess – i.e. letting everyone know the information in A will not affect prices.

There is, however, a basic difficulty associated with both of these formulations. FR efficiency with respect to A does not necessarily imply FR efficiency with respect to proper subsets of A. Under the FR definition, the market may thus be efficient in the semistrong form, and yet not be efficient in the weak form, or, equivalently, prices may reflect all publicly available information but not reflect the publicly available sequence of previous prices.

One’s interest in market efficiency is of course dictated by the implications of the concept, and it would be undesirable to load the definition with aspects that do not bear any interesting implications. The most common implication of information efficiency relates to traders’ (investment) behavior, and it is generally accepted that if markets are

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5 This modification was suggested to me by William Sharpe.
efficient with respect to some information investors need not use (one is almost tempted to say "should not use") this information in the determination of their optimal market position. There is abundant evidence that the implications for investment decisions are at the heart of the efficiency concept for most writers even when it is not explicit in the definition. For example, if FR-efficiency (especially in the modified version) does not apply to a trader's information, prices will change when this information becomes widely known. He can therefore anticipate that change to advantage by taking proper action in time. Rubinstein [16] explicitly associates his definition of efficiency with investors' action: "An individual perceives his new information as fully reflected in revised security prices if, and only if, no portfolio revision is his optimal strategy" (p. 815). "Trading rule" tests of market efficiency directly reflect the association between efficiency and action. 6 It is also evident in the widely accepted conclusion that, whatever the particular formulation or the statistical test adopted, the evidence on market efficiency supports a "buy and hold" portfolio policy. 7 In some studies, the idea that investors' behavior is the essence of information efficiency

6See Fama [4] for a survey of some studies where the effectiveness of trading rules based on some information is used to test how well this information is reflected in prices.

7Cf. Black [2].
is reflected in the concept of "sufficient statistic". When prices are a sufficient statistic for some information, they do not "fully" reflect this information, only sufficiently so for decision-making purposes. The general implications of either version of FR-efficiency for traders' investment behavior, outside the special structure of Rubinstein's model [16], are quite inconclusive. Even if traders can make use of certain information to increase their profits when markets are not FR-efficient, it is not at all clear what the operational implications are if FR-efficiency does prevail. Generally, there is nothing in the FR definition to indicate that when the market is FR-efficient with respect to some information, a trader could not use this information to advantage - it could be highly relevant for his own preferences even if it would not affect prices. This lack of conclusive operational implications is another deficiency in the FR definition of information efficiency.

8Cf. Kihlstrom and Mirman [11] and Grossman [7]. This interpretation is especially evident in the Bayesian approach to sufficiency.

9A purely statistical analysis of price movements, independent of any operational implications, may be of interest to some (e.g. statisticians). Alexander [1] maintains that profits from buy and hold are irrelevant to the random walk issue, stressing that the optimality of buy and hold does not necessarily imply a random walk process. Alexander is careful, however, to suggest to the "reader who is interested only in practical results ... to turn to other sources for advice" (p 351). The crucial point is, of course, that for investors the issue is not whether buy and hold is relevant for random walk theory, but rather whether random walk evidence has implications on the advisability of buy and hold.
Investors' behavior is only a means to an end, and information efficiency has sometimes been referred to in terms of investors' returns, rather than their behavior. If markets are efficient with respect to some information, then it would presumably be impossible to make "excessive profits" from its possession. The term "fair game", extensively used in market efficiency literature for the martingale property, is also indicative of the central role of returns in the efficiency concept. The question which returns are "excessive" or "abnormal" can of course be determined only in view of traders' preferences.

We conclude this section with a presentation of three alternative definitions, representing decreasing degrees of efficiency, which attempt to capture the essence of the various approaches outlined above.

Data-Efficiency:
The market is data-efficient with respect to information set A if the data in A can be accurately inferred from prices.

Action-Efficiency:
The market is action-efficient with respect to information set A if the optimal position of any investor, given the data in any information set B, the data in A, and the current prices is equal to his optimal position given only the data in B and the price.
Return-Efficiency

The market is return-efficient with respect to information set \( A \) if the expected utility of any investor, given the data in any information set \( B \) and the data in \( A \), equals his expected utility given only the data in \( B \).

The first definition expresses the notion that prices fully reflect the information in \( A \). The second indicates that trading rules which use information in \( A \) need not be considered, and the third definition concentrates on the impossibility of excessive returns originating from \( A \). The three definitions represent decreasing degrees of efficiency because it is apparent that data-efficiency implies action-efficiency, and action-efficiency implies return efficiency.\(^{10}\) For all three definitions, efficiency with respect to \( A \) implies efficiency with respect to all subsets of \( A \).

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\(^{10}\) In Rubinstein's (16) model, FR-efficiency also implies action-efficiency ("consensus beliefs imply non-speculative beliefs").

\(^{11}\) To lend the system competitive properties, it suffices to think of \( I \) classes of traders, rather than \( I \) traders, cf. Grossman [7].

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III. THE MODEL.

Consider an economy with \( I \) traders,\(^{11}\) indexed by \( i=1, \ldots, I \), trading in \( J \) assets whose price is denoted by
p=(p₁,p²,...,p_J). Each trader determines his position x=(x₁,x²,...,x_J) according to his possibilities, his tastes, and his beliefs about the environment. The relevant aspects of environmental uncertainty are represented by the set W of states of the world w, an unknown one of which obtains. Diverse beliefs about W may induce traders with similar tastes and possibilities to make different decisions. Information and inferences that change initial beliefs are central to our analysis. Traders with diverse backgrounds may draw different inferences from the same data. Following Marschak [14] [15], this is incorporated in the model by an extended description s=(w,z) of the states of nature, where w incorporates the aspects that are relevant for (final) decisions and z the aspects that are relevant for inferences.¹² Traders have their (heterogeneous) beliefs about the set S of possible (extended descriptions s of the) states of nature, which clearly also involve their beliefs about W. Information about the environment can be described as a function on S. When a trader knows that P(s)=f, he restricts his attention to those states in S that could have generated f under the function F, and correspondingly updates his probability assessments for the relevant aspects w of the states s. Diverse beliefs about S allow traders to make diverse (probabilistic) inferences about W from the

¹² Marschak calls w the "external" or "payoff relevant" state and z the state of the "information instrument".
same information. For any set \( X \) of tentative market positions (a subset of \( \mathbb{R}^J \) and any collection \( F_1, F_2, \ldots \) of functions on \( S \), let (the \( J \)-dimensional vector) \( G_i(X|F_1(s)=f_1, F_2(s)=f_2, \ldots) \) denote the \( i \)th trader's preferred position among the alternatives in \( X \), if he knows that \( F_1(s)=f_1, F_2(s)=f_2 \) etc. The formulation explicitly separates the two considerations that are involved in the trader's choice - the range of feasible alternatives, and the information that guides his choice. Traders' information is heterogeneous, and they do not all have access to the same data. Generally, there are \( N \) types of observations, where \( y_n = y_n(s) \) means that when the state is \( s \) the data \( y_n \) is generated by the observation \( y_n \). For convenience, we adopt the following notation convention. Let \( N = \{1, 2, \ldots, N\} \) denote the set of all observations, and for any subset \( A \) of \( N \) (written \( A \subseteq N \)), let \( Y_A = \{y_n, n \in A\} \) denote the

\[13\] To see how one information function on \( S \) can mean different things to different traders, consider this simple example. Suppose (the value of) \( w \) can be either \( G \) (good) or \( B \) (bad), and suppose an economic indicator can be either \( H \) (high) or \( L \) (low). Suppose further that two traders initially consider \( G \) and \( B \) equally likely, but whereas one makes no inferences from the economic indicator the other associates \( G \) with \( H \) and \( B \) with \( L \). This can be described with a model where \( S = \{GH, GL, BH, BL\} \) and \( P(w, z) = z \) regardless of \( w \), where the first trader initially assigns probability 0.25 to all states \( s \), and the other trader assigns probability 0.5 to \( GH \) and \( BL \) and zero probability to \( GL \) and \( BH \).

\[14\] With utility maximizing and price taking traders, this is the position \( x^* \) that maximizes the conditional expected utility, given \( F_1(s)=f_1, F_2(s)=f_2 \) etc., over all \( x \) in \( X \). If \( x^* \) is not unique, \( G_i(X|\ldots) \) may be defined either as the set of all utility maximizing solutions, or just one solution selected by the \( i \)th trader through any arbitrary rule.
set of observations with indexes in $A$ and $y_A = \{y_n, n \in A\}$ the set of data generated by these observations.

Let $K(i) \leq N$ denote the $i^{th}$ trader's observation set, i.e. he has access to the data $y_{K(i)}$ generated by the observations $y_{K(i)}$. Some observations may be shared by many traders, so that the observation sets $K(i)$ need not be disjoint. As noted above, diverse initial beliefs may induce traders to assign different interpretations to the same data. Public information may be defined as the intersection of all $K(i)$.

A trader's information on the environment is not restricted to his observation set alone, because he may also infer from market prices something about data that other traders know but he does not. Such inferences are possible because realized prices depend on the state of nature, and traders are aware of this (possibly through familiarity with the history of the system's behavior). Let $P$ represent the functional relationship between prices and the environment, i.e. $p = P(s)$. Again, the inferences about $S$ that traders can draw from the price $p$ have only limited implications on their inferences about the "relevant" aspect $w$ of the state $s$. In recent studies of market information, attention has been devoted to models where all traders' inferences are identical, involving complete knowledge of the "true" relationship between all ultimately observable variables and the price. Their expectations about $w$ have thus been assumed "self fulfilling", and in some cases this has even
been considered a requisite of rational behavior — hence the terminology "fulfilled expectations equilibrium" and "rational expectations equilibrium" associated with many models of markets where prices convey information. In the present study, traders' expectations need not be "self fulfilling". Their inferences about \( W \) may be limited and heterogeneous, depending on their initial beliefs about \( S \).

Besides the special role of prices in conveying information, prices naturally also have their usual role of determining each trader's set of possible market positions (demands), by affecting his wealth and the asset combinations that can be attained with this wealth. Let \( X_i(p) \) be the \( i^{th} \) trader's set of feasible market positions when the price is \( p \). \( X_i(p) \) reflects the usual budget constraint and any further restrictions that may apply (e.g. short positions, margin requirements, institutional limitations etc.).

Let \( Q_i(p|Y_A,P) \) be the \( i^{th} \) trader's position when the price is \( p \), given that he knows the data \( Y_A \) and given his inferences from the price \( p \), i.e.

\[
Q_i(p|Y_A,P) = G_i[X_i(p)|Y_A(s)=y_A,P(s)=p].
\]

This notation shows clearly how prices affect the trader's choice both through the feasible set \( X_i(p) \) and through the inferences from \( P(s)=p \). For given \( Y_A \) and \( P \), the indicated

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position is a function of \( p \), say \( D_i(p) \), mapping \((J\text{-dimensional})\) prices into \((J\text{-dimensional})\) demands. \(^{16}\) \( D_i \) is called the demand schedule or the demand correspondence of the \( i \)th trader. This demand correspondence is determined by the data he observes and by the inferences that he draws from prices. Let this relationship be denoted by \( L_i \), i.e.

\[
D_i = L_i(y_A | p),
\]

which means that when the \( i \)th trader observes the data \( y_A \) and makes inferences from prices, his demand correspondence is \( D_i = L_i(y_A | p) \), so that when the price is \( p \) his position is \( D_i(p) \), where

\[
D_i(p) = Q_i(p | y_A, p).
\]

The relationship between the environment, demands, and prices is elaborated upon in the next section.

IV. GENUINE TRADING PROCESSES.

The analysis of markets in economic theory has traditionally concentrated on equilibrium considerations (traders' constrained optimization, market clearing conditions etc.), and studies of the relationship between information and prices are no exception. The usual approach in the analysis of fulfilled expectations equilibria is thus to postulate some function \( P \) and investigate its equilibrium properties. Kreps [13] is careful to state explicitly a basic

\(^{16}\)or perhaps into subsets of \( \mathbb{R}^J \).
requirement that is implicitly adopted in such models, i.e. that "...prices (must) contain no more information than is possessed by all (traders)" (p.2). But equilibrium considerations per se—even with an explicit indication of the underlying information—do not make any reference to the way in which available information affects the determination of prices. Indeed, the standard requirement is not inconsistent with a Walrasian auctioneer who independently observes all available data and then calls the indicated price, which is indeed verified to be equilibrium. The basic problem here is that equilibrium (and thus also fulfilled expectations equilibrium) does not explicitly indicate the relationship between the participants' trading behavior and the realized prices. To capture the essence of our usual understanding of the "market" notion, the model must make some reference to this relationship. We want prices that are generated through traders' behavior. Prices that depend on the environment in a way that cannot be associated with trading behavior may be no more than a spurious property of the model.

A concept of "genuine trading processes" is introduced in this paper to rectify this difficulty. A "trade resolving process" M is defined as a function mapping 1-tuples of

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17Formally, this is stated as a requirement that P be measurable, or equivalently that there exist some function mapping observations into the realized prices.
demand correspondences into prices, i.e. \( M(D_1, \ldots, D_J) \), written also as \( M([D_i]) \), is the (J-dimensional) price vector when the first trader's demand correspondence is \( D_1 \), the second's is \( D_2 \), etc.\(^{18}\) We say that the price system is a "genuine trading process" if there exists \( M \) such that

\[
P(s) = M([L_i(Y_{K(i)}(s)) | P])
\]

for all \( s \) in \( S \).

In words, a genuine trading process represents a market where prices are fully determined by traders' demands. Two states that give rise to identical demands by all traders must give rise to the same price.\(^{19}\) We can visualize this process as if the market were operating in the following way. After observing the data in his observation set, each trader submits detailed complex limit orders, whereby his transactions in one asset may depend also on prices of other assets. In these limit orders, the desired transactions contingent on any tentative price already reflect all the inferences that he can draw from the fact that this

\(^{18}\)To be an equilibrium, a trade resolving process must of course satisfy the requirement that, for any \( I \)-tuple of demand correspondences in its domain, if \( p_0 = M([D_i]) \) then the sum over \( i \) of all \( D_i(p_0) \) equals the aggregate supply. \( M \) is associated with "genuine" trading in that price outcomes are based on the truly preferred (budget feasible) tentative positions \( D_i(p) \), and are not manipulable by false statements of these preferences. In a competitive system, the motive for such misrepresentations does not exist, because when each trader believes that his demands will have no more than negligible effect on realized prices he concentrates on getting the most preferable outcome at any given price.

\(^{19}\)In genuine trading processes \( P \) must be \( L_1, \ldots, L_I \) measurable - clearly a stronger condition than the usually assumed \( Y_1, \ldots, Y_N \) measurability.
particular price is the equilibrium realization. All orders are entered in "the book", and on the basis of these entries a price is declared. All orders applicable to the declared price are then executed.\(^\text{20}\)

V. LIMITS OF EFFICIENCY.

To what extent can prices reflect traders' information? Various special models have been proposed, with examples of equilibrium price functions that fully transmit and aggregate the information of all traders.\(^\text{21}\) It has also been pointed out, on the other hand, that when prices transmit information equilibrium need not exist.\(^\text{22}\) The purpose of this section is to investigate the information efficiency of genuine trading processes. Assuming that equilibrium prices exist, to what degree, and in how strong a form, can they reflect information? It will be shown that prices cannot really "fully reflect" any useful information.

\(^\text{20}\) Consistency with the existence of a trade resolving process is perhaps one of the most basic prerequisites that one may wish to impose on models of competitive markets. Indeed, this would be far from sufficient for the acceptability of a model as a reasonable description of observed markets. Future efforts may well be devoted to the analysis of trade resolving processes that also satisfy some additional "desirable" properties (for example, suppose \(M(D_i)=p\) and for all \(i\) \(D_i=D^*_i\) on a neighborhood of \(p\). Under what conditions would it be reasonable to assume that \(M(\{D^*_i\})=p\) ?).


\(^\text{22}\) See Kreps [13].
At best, they can trivially "reflect" only "inferior" or "basically useless" information. Roughly, inferior information is information that can be accurately inferred from other data, and basically useless information is information that would not affect traders' decisions even if prices did not reflect any part of that information. These terms will be precisely defined presently.

The following notation will sharpen the definitions and help make the exposition precise. For functions $F_1$ and $F_2$ defined on $S$, we say that $F_1$ is determined by $F_2$, or equivalently that $F_1$ transforms $F_2$, written $F_1 \triangleright F_2$, if there exists a function $H$ such that

$$F_1(s) = H[F_2(s)] \text{ for all } s \text{ in } S.$$  

It is evident that $T$ is transitive and that $A \preceq B \preceq N$ implies $Y_A \preceq Y_B$. For sets $A$ and $B$, let $B - A$ denote the set of points in $B$ and not in $A$, and let $A^* = N - A$ be the complement of $A$ in $N$. Finally, consider $D_i = L_i[Y_k(i)(s) | P]$ as a function on $S$, i.e. the state of nature determines, through the observations of the $i$th trader, the demand correspondence that characterizes his behavior. Let $D$ denote the $I$-tuple of demand correspondences of all traders, which will be termed simply "demands". Again, the realized

\[23 \text{ } F_1 \triangleright F_2 \text{ is equivalent to } "F_1 \text{ is } F_2 \text{ measurable}" \text{ e.g. Kreps [13], or to } "F_1 \text{ is coarser than } F_2", \text{ i.e. the partition of } S \text{ to equivalence classes under } F_1 \text{ is coarser than the corresponding partition under } F_2.\]

\[24 \text{ } F_1 \triangleright F_2 \text{ and } F_2 \triangleright F_3 \text{ imply } F_1 \triangleright F_3.\]
demands depend on the state s.

Some of our definitions can be neatly restated in terms of this terminology.

(1) A genuine trading process: prices are determined by demands (PTD).

(2) Data-efficiency with respect to A: the observations in A transform prices ($Y_A TP$).

Action-efficiency can be formally represented as follows: The function $P$ is action-efficient with respect to $A \leq N$ when for any $B \leq N$, $Y_A$, $Y_B$, and $p$ which are not mutually inconsistent $^{25}$

$$Q_i(p | y_{A \cup B}, p) = Q_i(p | y_B, p) \quad \text{for all } i.$$ 

For simplicity, assume that no information available to any trader is so conclusive as to completely eliminate the possibility of any price in $P(S)$ (although with some data he may assess the probability of certain prices to be arbitrarily low). $^{26}$ The equality must then apply to all $p$, and can be written simply as:

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$^{25}$I.e. the intersection of $Y_A^{-1}(Y_A)$, $Y_B^{-1}(Y_B)$ and $P^{-1}(p)$ is non-empty.

$^{26}$The purpose of this is to preclude “artificial” signaling by informed traders through statements of “irrelevant” demands contingent on prices that they know to be impossible. The same effect could be alternatively attained by assuming that such signaling (prior to taking a position based on their superior information) is contrary to the informed traders’ best interests, or by assuming that the market system generates prices that are independent of these “irrelevant” demands and that will therefore not “transmit the signal”.

21
\[ L_i(y_{\text{AUB}}|P) = L_i(y_B|P). \]

This indicates that when prices (operationally) reflect the information set A no demand correspondence depends on the data in A, hence the following lemma. Proof of this and all subsequent results is given in the appendix.

**Lemma 1:** Action-efficiency with respect to A implies that demands are determined by observations not in A \((D_{TY_A^*})\).

Since prices are determined by demands, this leads to the following basic result on the limits of information efficiency.

**Theorem 1:** In a genuine trading process, action-efficiency with respect to A implies that prices are determined by observations not in A \((PT_{Y_A^*})\).

The theorem states that, in genuine trading processes, prices that are action-efficient with respect to A are fully determined by observations not in A, and therefore cannot really "reflect" A. To prove what can be reflected by genuine trading processes, we define the following two characterizations of observation sets.

An observation set A is termed **inferior** if \(Y_A\) transforms \(Y_{A^*}\). Data in an inferior observation set can thus be accurately inferred from data not in that set.

An observation set A is termed **basically useful** if there exist states \(s^1\) and \(s^2\) which involve different preferred
positions for some trader, and which are distinguishable only through \( Y_A \), i.e.

\[
\begin{align*}
Y_A(s^1) &= y_A(s^2) = y_A^* \\
Y_A(s^1) &= y_A^1, Y_A(s^2) = y_A^2 \\
P(s^1) &= p^1, \quad P(s^2) = p^2
\end{align*}
\]

and for some \( i \) and any function \( F(s) \) such that \( FY_A \) also

\[ G_i[X_i(p^1) | Y_A(s) = y_A^1, F(s) = f] \neq G_i[X_i(p^2) | Y_A(s) = y_A^2, F(s) = f]. \]

A basically useful observation set will thus, at least under some circumstances, affect some trader's position, regardless of the information he has on observations not in that set. A basically useless observation set is defined as a set that is not basically useful. It is evident that an inferior set cannot be basically useful, and must therefore be basically useless.

When the observations in \( A \) transform prices which are determined by observations not in \( A \), \( A \) must be a set of inferior data which merely transforms other observations, i.e.

**Theorem 2:** If a genuine trading process is data-efficient with respect to \( A \), then \( A \) is inferior.

If prices are only action efficient with respect to \( A \), \( A \) need not be inferior. But then there cannot exist two states that induce different demands and that can be distinguished

\[ ^{27}\text{Because the assumed premises cannot hold for an inferior set.} \]
only through \( Y_A \) - because by theorem 1 price is determined by \( Y_A^* \) and must be identical for both states, and different demands at this price contradict action efficiency. This is stated in

**Theorem 3**: If a genuine trading process is action-efficient with respect to \( A \), \( A \) is basically useless.

All proofs are fully presented in the appendix.

If all traders are strictly risk averse utility maximizers, and if all feasible decision sets \( X_i(p) \) are convex, then data-efficiency and return-efficiency are equivalent. Return-efficiency with respect to basically useful observations is thus also impossible.

Suppose the market is action-efficient with respect to \( A \), and some subset \( B \) of \( A^* \) transforms \( A \). Then the market must be also action-efficient with respect to \( BUA \), which must therefore also be basically useless. This suggests something about the reason why data in a basically useless set \( A \) need never be used - even in an inefficient market - if sufficient data from \( A^* \) is available. It cannot be merely because the other data somehow repeats the data in \( A \) or contains its equivalent. \( A^* \) must contain some data which is in some sense "superior" to the data in \( A \). This data, say \( A^S \), cannot be reflected in prices, because then \( AUAS \) would have to be also basically useless, and so forth.

Some frequently encountered types of data are inherently
inferior. For example, all summarized balance-sheet data is
inferior information, because it merely transforms the data
in the basic accounts from which the balance-sheets were
derived. The results of this section shed a new light on the
information efficiency of the market with respect to this
kind of data. If the data may be ignored in a trader's
decision-making process, this is not because other traders
also have access to the same data, but perhaps because there
exists some other information which is superior to this
data. This superior information is, however, never fully
reflected in market prices.

VI. IMPLICATIONS AND CONCLUSIONS.

Formulations of information efficiency have been partially
ordered in this paper in two different dimensions - degree
of efficiency and strength of form. The main result of the
previous section shows that operationally meaningful degrees
of efficiency can prevail only in a trivially weak form.
Since the set ℵ of all observations is clearly basically
useful, action-efficiency in the strongest possible form is
immediately, unquestionably, and totally excluded. This is
not surprising, because the strongest form of efficiency
has always been regarded with some suspicion as "...at
best...a benchmark against which deviations...can be

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Semistrong forms of efficiency have been taken much more seriously. It is therefore interesting to note that, if published information includes a single item which would be eventually used by traders in an inefficient market even in the presence of additional data, then action-efficiency in the semistrong form is also impossible.

The above analysis is not restricted to the securities markets. Kihlstrom and Mirman [11] have recently suggested an example where a product's price conveys information on its quality. Our results indicate that in genuine trading processes this kind of information will never be sufficient to make quality inspection by a prospective buyer a useless activity.

Grossman and Stiglitz [8] have been concerned with the implications of efficiency for the gathering of costly information. Within the framework of a special model, they show that if markets are data-efficient and information is costly "markets break down", and that zero information cost is a necessary and sufficient condition for data-efficiency. Our results indicate that in genuine trading processes even zero information cost is not sufficient for data-efficiency. Since the efficiency of these processes is necessarily limited, costly observation of basically useful data is

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28 Fama [4].

29 See [8], pages 4 and 46.
economically viable even when prices may reflect some information.

The existence of equilibria in genuine trading processes when prices convey some information is a most interesting issue. Since inferences can be drawn from prices, demands need not exhibit any of the classically assumed "well behaved" properties - e.g. they cannot be automatically assumed down sloping. Classical proofs of existence, let alone uniqueness, do not directly apply. A general study of equilibrium in trading processes is, however, clearly outside the scope of the present paper.\(^{30}\) Our concern here is with the relationship between information, demands, and prices, when they exist. In this respect, note that the results on limited efficiency apply to competitive equilibrium\(^{31}\) but are not really restricted to it. Rather than represent the genuinely preferred budget feasible tentative positions, the "demands" I-tuple \(D\) can be any equilibrium solution of an I-person game with limited information, where the trade resolving process \(M\) incorporates the restrictive operational "rules of the game". With only a trivial adjustment of the action-efficiency notion, the results could then apply to a market

\(^{30}\)Within such a study, it may be of interest to explore the implications of some further restrictions on the trade resolving process (e.g. see footnotes 26 and 26).

\(^{31}\)See footnote 11.
dominated by a small number of "giants" as well as to a largely competitive system.

This paper concentrated on the primitive conceptual aspects of efficient markets theory, with no direct reference to the overwhelming amount of empirical work on the subject. This does not deny in any way, however, the many useful insights on market behavior that such work provides. The analysis seems to indicate that the task of incorporating this empirical work within a comprehensive theory of market information is far from being completed. As to the implications for traders' behavior, our results suggest that a complete dismissal of the discretionary use of non-monopolistic information in favor of policies like buy and hold may well be premature. Transaction costs should certainly be explicitly considered in the optimal decision, and they may indeed render some portfolio adjustments inadvisable. Traders should also generally allow for the information implications of prices when they take a position. But, in principle, a trader is well advised to carefully take into account any basically useful information he may have, even if it is widely shared by other traders.
APPENDIX: FORMAL PROOFS.

Lemma 1: Action-efficiency with respect to A implies $D_{TY A^*}$.

Proof: Action-efficiency with respect to A implies action-efficiency with respect to $A' = A \cap K(i)$, because for all $B$

$$L_i (Y_B | P) = L_i (Y_{BUA} | P) = L_i (Y_{BUA \cap UA} | P) = L_i (Y_{BUA} | P).$$

Hence, with $B = K(i) - A$

$$L_i (Y_{K(i) - A} | P) = L_i (Y_{K(i)} | P)$$

so that $D_{iTY K(i) - A}$ and hence $D_{TY A^*}$. QED.

Theorem 1: In a genuine trading process, action-efficiency with respect to A implies $P_{TY A^*}$.

Proof: By transitivity, $PTD$ and $D_{TY A^*}$ imply $P_{TY A^*}$. QED.

Theorem 2: If a genuine trading process is data-efficient with respect to A, then A is inferior.

Proof: Data-efficiency implies action-efficiency, hence by theorem 1 $P_{TY A^*}$, and by transitivity this and $Y_{ATP}$ imply $Y_{ATY A^*}$. QED.

Theorem 3: If a genuine trading process is action-efficient with respect to A, A is basically useless.

Proof: Assume to the contrary that A is basically useful. Then, since by theorem 1 $P_{TY A^*}$, there exist states $s^1$ and $s^2$ such that $P(s^1) = P(s^2) = p$, $Y_{A^*}(s^1) = Y_{A^*}(s^2)$, and for some i

$$G_i [X_i (p) | Y_A(s) = y_A^1, P(s) = p] \neq G_i [X_i (p) | Y_A(s) = y_A^2, P(s) = p]$$

or, equivalently

$$Q_i (p | y_A^1, P) \neq Q_i (p | y_A^2, P)$$

which contradicts $D_{TY A^*}$ and action-efficiency. QED.