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Dynamic Market Processes and the Rewards to Up-to-Date Information

by

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UP-TO-DATE INFORMATION**

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I INTRODUCTION

The disparity between observed behavior in financial markets and the theory of adjustment processes is striking. In the standard models of "tatonnement", traders first enter into a sequence of "tentative contracts" as a prelude to "discovering" a binding one which is mutually satisfactory. Trading takes place only after a (possibly unbounded) random number of iterations, during which no new information presumably reaches the traders—who are assumed to respond to all price announcements on the basis of fixed preferences. But in real financial markets price adjustments and trades occur concurrently and with high frequency, the role of the "specialist" indicates that supply and demand do not always match, and the arrival of new information as time progresses is of the essence. One result of this discrepancy is that "descriptive" models of actual financial markets fail to capture important empirical phenomena. A second consequence is a lack of foundations for developing (normative) proposals concerning alternative schemes for the operation of financial markets.

In fairness, economists have generally not claimed that tatonnement and similar processes are models of actual market operations. Rather, they represent a convenient vehicle for examining the question of price adjustment and for gaining insights concerning existence, uniqueness, and stability of equilibria. Our orientation in this paper, however, demands that we eventually recognize that contracting, whether tentative or binding, consumes real time, and that the study or design of realistic market processes take account of the arrival of new information during "the search for equilibrium". The main purpose of this paper is to delineate some of the issues involved in moving in this direction.

Our focus will be on processes which permit "trading out of equilibrium" (see e.g. Negishi [11], Clower [3], Arrow and Hahn [1, Ch. 13], Hurwicz, Radner Reiter [7]) and involve both quantity and price adjustments.\footnote{This is in contrast to processes involving only "Keynesian" quantity adjustment or rationing (e.g. Benassy [2]), which are primarily motivated by labor markets.} Our analysis
concentrates not on the existence of equilibria, but on the path of trades generated when the market is out of equilibrium.

The basic structure presented in section II deals with a pure exchange environment in a joint market. Section III shows the allocation-efficiency of a general class of market processes. Our primary concern will be with comparisons involving communication demands (section IV), the rewards to information about the "present state of the market" (section V), distributive effects of the market process (section VI), and the impact of real time (section VII).

II DYNAMIC MARKET PROCESSES

Suppose there are $J$ assets, indexed by $j=1,\ldots,J$, traded in a given "market-place". $R^J$ denotes the $J$-dimensional Euclidean space, $0^J$ the origin, and $R^J_+\cup\{0^J\}$ the (strictly) positive subset of $R^J$. When the (relative) prices in the marketplace are given by $p\in R^J_+\cup\{0^J\}$, $H(p)=\{x\in R^J: xp=0\}$ is the set of "trades" which are consistent with that price. Let asset 1 be a numeraire (money), and let $P=\{p_1\}$ denote the set of positive prices. The set $H$ of all possible market trades is then given by $H=\{x\in R^J: x\in H(p),p\in P\}$.

Time is a central element in our analysis. All exogenous variables (e.g. agents' preferences) are assumed fixed at any point in time, but may generally vary between different time points.

A Market Process $F$ is defined as follows. Given:

1. $t\in T$ - a point in time
2. $p\in P$ - a price
3. $X=(X_i, i=1,2,\ldots,I)$ - orders

$X_i$ is a set of orders (e.g. market orders, limit orders) submitted by the $i$th trader. Orders are stated as proposed market trades, i.e. when $x\in X_i$ then $x\in H(p)$ for some $p\in P$. For notational convenience, we represent "no order" as an order for "no trade" $X=\{0^J\}$; thus $X_i$ is non-empty for all $i$.

2The effect of segmented markets will be considered in a separate paper.
The market process is a mapping from the triple of time, price, and orders, to the triple of time, price, and trades, i.e. \( F(t, p, X) = t', q, y \) where

1. \( t' \epsilon T \) - a new point in time
2. \( q \epsilon P \) - the new price
3. \( y = (y_1, y_2, \ldots, y_T), y_i \epsilon \mathbb{H} \) - trades.

The market is thus described as an iterative process evolving through time. Each iteration generally involves both price adjustments and quantity adjustments (trades).

We shall be interested in market processes \( F \) satisfying the following properties.

**P1. Time-feasibility:** \( t' \geq t \).

**P2. Trade-feasibility:** \( y_i = a_i x_i, x_i \epsilon X_i, 0 < a_i < 1 \).

**P3. Market clearing:** \( \Sigma_i y_i = 0 \).

**P4. Compatibility with the relevant price range:** for all \( i \), \( y_i \epsilon \mathbb{H}(p') \), \( p' = bp + (1-b)q, 0 < b < 1 \).

**P5. Non-anticipatory prices:** \( X_i = \{0\} \) for all \( i \) implies \( q = p \).

It may be useful to note the following points with respect to the stated properties.

In real-world markets the execution of any order would have to take a positive time, so that \( t' > t \). \( t' = t \) is an abstraction which will be helpful in making comparisons with previously studied models of markets where exogenous variables are assumed fixed throughout the market's adjustments.\(^3\)

Property P2 states that no trader may be forced to trade more than his orders specify, but he may have to be rationed: \( a_i = 1 \) is "desired", but may be infeasible. One source for the infeasibility of trades which fully satisfy all submitted orders may be the failure of some traders to submit detailed orders.

\(^3\)The word "time" (and the index \( t \)) have often been used to indicate successive iterations in processes where the environment is kept fixed, e.g. price iterations in classical tatonnement or quantity iterations in Keynesian models such as in Benassy [2] or Grandmont and Larroque [5]. Such iterations are represented in our framework by \( t' = t \), and are distinguished by an index \( n \).
contingent on all possible prices. Even with a complete specification of tentative orders, a market-clearing "equilibrium" may fail to exist except for extremely idealized economies.\(^4\) Trades with \(a_i > 1\) cannot be imposed because they may turn out to be infeasible with respect to the \(i\)th trader's endowment, since every non-trivial trade must contain some negative element ("payment"). Quantity "rationing", with \(a_i < 1\), may thus be in some cases the only alternative to no trade (which is essentially an extreme rationing of all traders at a given iteration). When the preferences of traders are monotone, the arrangement that orders which cannot be fully satisfied may be partly executed is clearly to the traders' advantage: if \(x_i\) is "desired" over \(0_i\) then \(a_i x_i\) --which is a convex combination of \(x_i\) and \(0_i\)-- is preferred to "no trade" and is thus at least "acceptable". Indeed, when \(x_i c_H(p')\) "cannot be satisfied" this suggests that, in some sense, \(p'\) is "too low" for some asset \(j\) that the \(i\)th trader wants to buy or "too high" for some asset that he wants to sell. It therefore seems all the more to his advantage to accept the trade \(y_i\) as a partial fulfillment of his order.

Property P3 does not exclude a "government-like specialist" who can maintain arbitrary long or short positions in some assets. This can be introduced into the model by the addition of "artificial" traders. In particular, note that no symmetries are presumed in the definition of \(F\), so that the specialist can be given special treatment.

Property P4 requires that all trades executed at a given time be compatible with one price between \(p\) and \(q\).\(^5\) "New-price compatibility", where \(b=0\) and \(y_i c_H(q)\), is an attractive choice in this range. An interesting alternative is "old-price compatibility", where \(b=1\) and \(y_i c_H(p)\). \(p'=bp+(1-b)q\) is the effective trading price, or in short the "trading price". When \(0<b<1\), the new price is an

\(^4\)In particular, note that when traders make inferences from prices (prices "reflect" the information available to some traders), equilibrium need not exist even in classical economies (e.g. Green [6] or Kreps [8]).

\(^5\)If the mechanics of trade involves matching of "bid" and "ask" prices and quantities (e.g. Garman [4], this excludes the simultaneous matching of two pairs with a different matching price for each pair.
extrapolation of the trend expressed by the difference between the old price and the effective trading price.

An extrapolation of the "price trend" is anticipatory in nature. Property P5 restricts the anticipatory power of the market system to inferences that can be derived from submitted orders. Anticipatory prices which contradict P5 may be effective for expediting trades in some circumstances, where economic developments indicate even before new orders are submitted that the old price will no longer be relevant. However, such anticipatory prices seem to impose prohibitively excessive demands on the capabilities of the market process, and are excluded from our analysis. In part, this is reflected in the suggested domain for F, which does not admit any variables beyond t, p, and X.

Throughout this study, the term market process refers to processes F which satisfy properties P1-P5.

III ALLOCATION EFFICIENCY

Classical economic analysis concentrates on the ability of prices to identify and sustain Pareto-efficient allocations. In this section, we show that all market processes F satisfy this important property.

A crucial concept in the classical treatment of efficiency is "competitive behavior" on the part of traders. We start with a formal definition of this concept within the context of our framework.

Classical competitive behavior. Let \( w_i \) denote the endowment of trader \( i \). Then \( w_i \in M_i \), where \( M_i \subseteq \mathbb{R}^J \) is the set of possible positions for trader \( i \) (this includes the extent of permissible short positions and any other institutional constraints which may apply). We assume that all sets \( M_i \) are bounded from below. Let \( u_i \) be an (expected) utility function representing the preferences of the \( i^{th} \) trader at a given point in time over possible positions in \( M_i \).\(^6\) For \( p \in \mathbb{P} \), define \( X_i^0(p) = \ldots \)

\(^6\)We shall have more to say later about the appropriate "choice" of a utility function, and how it is influenced by the way the market operates.
termed the **competitive behavior at** \( p \) of the \( i \)th trader—as the trade \( x \) that solves the following problem.

Maximize \( U_i(w_i + x) \)

subject to \( xp=0 \), \((w_i + x)e_{M_i}\).

(1)

Formally, the concept is well defined regardless of the environment or the market structure. By way of motivation, one usually considers traders who believe that the influence of their orders on the effective trading price will be no more than negligible. But to evaluate the economic plausibility of the classical competitive behavior, one must also consider the organization of the market. For traders to base their orders on the solution to problem (1), they must be confident that, at an effective price \( p \) where any trading takes place, their order \( X_i^0(p) \) will be fully satisfied. This is consistent with two (essentially equivalent) market processes \( F^e \) and \( F^t \). We give a partial specification of these.

**Equilibrium process \( F^e \).** Suppose \( X_i = \bigcup_p X_i^0(p) \). If \( F^e(t, p, x) = t', q, y \) then

1. \( t' = t \)
2. \( y \in X_i^0(q) \).

The trades in process \( F^e \) are sometimes described as the trades generated at the limit of a sequence of iterations in a tatonnement adjustment of prices. This is made explicit in the following.

**Tatonnement process \( F^t \).** Suppose \( X_i = X_i^0(p) \). If \( F^t(t, p, x) = t', q, y \) then

1. \( t' = t \)
2. either \( y_i = 0_i^J \) for all \( i \), or \( q = p \) and \( y \in X_i \).

With this introduction, the classical theorem on the efficiency of the competitive price system can be stated as follows.

**Classical efficiency theorem.** For a class of economies with convex preferences and opportunity sets (concave \( U_i \) and convex \( M_i \)),

1. If \( X_i^0(p) = \{0_i^J\} \) for all \( i \) at some \( p \), then \( w = w_1, w_2, \ldots, w_i \) is Pareto-efficient.
2. For every Pareto-efficient \( w = w_1, w_2, \ldots, w_i \), there is \( p \in R^J \) such that \( X_i^0(p) = \{0_i^J\} \) for all \( i \).

The role of processes \( F^t \) or \( F^e \) in the classical efficiency theorem is implicit in the central role played by the classical competitive behavior \( X_i^0 \). If the
traders cannot reasonably assume that \( y_i e X_i \), then \( X_i^0 \) cannot be attractive to the \( i \)th trader even if he believes that his orders will not influence prices.

For general market processes, we must introduce a generalized notion of competitive behavior. Basically, we define as "competitive" the optimizing behavior of traders who assume that their orders will have a negligible influence on the effective trading price, but not necessarily on their own trading at that price.

The expectations of traders with respect to possible rationing of their orders are described by probability distributions. For \( p \in P \) and \( x \in H(p) \), let \( G_i(a|x,p) \) denote the \( i \)th trader's assessment of the probability that his trade \( y_i = a_i x_i \) will satisfy \( a_i < a \) if the effective trading price is \( p \).

**Generalized competitive behavior at \( p \) of the \( i \)th trader is denoted by \( X_i^C(p) \) and defined as the order \( x \) that solves the following problem.**

\[
\begin{align*}
\text{Maximize } & E[U_1(w_1+ax)] = \int_a U_1(w_1+ax) dG_i(a|x,p) \\
\text{subject to } & xp = 0, (w_1+x) e M_i.
\end{align*}
\]

Appropriate continuity assumptions on \( G_i(a|x,p) \) as a function of \( x \), and the lower bounds on \( M_i \), will ensure that \( X_i^C \) exists. The following theorem shows that the allocation efficiency of the price mechanism is not restricted to the process \( F^e \) or its equivalents.

**Generalized efficiency theorem.** For a class of economies with convex preferences and convex opportunity sets,

1. If \( X_i^C(p) = \{0^J\} \) for all \( i \) at some \( p \), then \( w = w_1, w_2, \ldots, w_I \) is Pareto-efficient.
2. For every Pareto-efficient \( w = w_1, w_2, \ldots, w_I \), there is \( p \in P \) such that \( X_i^C(p) = \{0^J\} \) for all \( i \).

The proof follows immediately from the fact that \( X_i^C(p) = \{0^J\} \) if and only if \( X_i^O(p) = \{0^J\} \). This establishes that all market processes maintain the allocation efficiency properties of the price mechanism in a competitive environment. Comparative evaluation of alternative processes must therefore concentrate on properties other than allocation efficiency.
IV COMMUNICATIONS CHARACTERISTICS--MARKET ORDERS AND LIMIT ORDERS

One dimension for the comparison of market processes is the communication requirements of the system. The tatonnement process \( F^t \) is characterized by extremely low communication requirements. At each iteration, traders need only submit orders for possible trades at the "called" price \( p \) -- a trader who fails to submit in advance tentative "limit orders" for trades at alternative prices will never "miss" an opportunity for a desirable trade. This is of course relevant only under the abstraction of iterations in "fictitious time", with \( t' = t \). When iterations take real time, the large number of iterations which may be needed before any trading takes place will practically inhibit all trading (all the more so when the environment changes with time).

In the equilibrium process \( F^e \), on the other hand, the market is presumed to achieve an unrationed trade in one iteration (as a market-clearing new price \( q \) is presumed to exist). But this is attained at the cost of extremely high communication requirements. All traders are assumed to submit at each iteration a complete specification of tentative orders for all possible prices. In securities markets terms, this implies an infinite number of limit orders for infinitesimal quantities, conditional on infinitesimal differences in prices. The (implicit) cost of placing orders makes these requirements prohibitively expensive.

The preceding points to a clear tradeoff between the communication requirements at each iteration and the number of iterations required to achieve unrationed trading. When iterations take time, failure to trade at a given iteration usually represents a real cost to traders. "Market orders", which specify a quantity to be sold, say, regardless of the effective trading price, are an obvious demonstration of this cost. The evaluation of alternative market processes, possibly with rationing, should explicitly take into consideration this tradeoff.

The notion of "orderly markets" is closely associated with this problem. In orderly markets, as much trading as possible is presumed to take place at effective
trading prices that are as "close" as possible to the previous price. Limit
orders, which are often conditional only on prices which are not too divergent
from the prevailing price, indicate that traders are aware of the fact that the
effective trading price may differ from the initial price, but yet believe that
the market process will generally be "orderly". This sheds light on the com-
promise adopted in organized markets between communication and trading.

V THE VALUE OF INFORMATION ON POSSIBLE RATIONING

In classical analysis, prices are assumed to adjust until no rationing of
orders is necessary. The analysis of the information available to traders is
then limited to information about the environment, which influences their pref-
erences over the various portfolio positions that they can attain. The purpose
of this section is to show that when the market process allows for some rationing
of orders there is also substantial value to up-to-date information about the
state of the market. Later, we shall argue that this information, too, might
affect the preferences of sophisticated traders over the portfolios that they can
attain in one iteration of the market process. For the present, however, we
assume that these preferences, represented by the utility functions $U_i$, are fixed.\(^7\)

Consider the trader's choice of orders as represented in the optimization
problem (2). Uncertainty about the rationing coefficient $a_i$ restricts his actions,
because his orders must be fully backed by resources even if at the end they are
only partly satisfied. Suppose, for example, that he wishes to buy a quantity
$x_j > 0$ of some asset $j$, for which he is willing to pay in money. If this order is
rationed by $a_i<1$, he would like to invest the money that was spared in a certain
quantity of another asset $k$. However, he cannot afford to place orders for both
purchases, because his resources are limited. If he can get up-to-date informa-
tion on the rationing that is going to take place, he will organize his orders
accordingly, and not "waste resources" to back orders which cannot be executed.

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\(^7\)As in a hypothetical situation where only one iteration of trading is
assumed to take place.
Formally, let $L$ denote some information on the rationing that is going to take place, given the orders that were submitted to the market by other traders. By the laws of probability $E_a[U] = E_L E_a[U | L]$. The utility level attainable by a trader with no access to $L$ can therefore be written as

$$V_0 = \max_{E_L E_a[U | L]}$$

over the constraints in problem (2). On the other hand, a trader with access to the information $L$ will choose $x$ to maximize his conditional expected utility only after he observes $L$. The expected value of his attained utility (over all possible realizations of $L$) is thus given by

$$V_L = E_L \max_{E_a[U | L]}$$

where the maximization is over the same constraints. Clearly $V_L > V_0$---a well known inequality in the economics of information.\(^8\) Equality holds only in trivial problems which are of no interest here.

The above analysis indicates the advantage of "the man on the spot" when orders may be rationed. A place in the exchange may be worth the price not only because it reduces the necessary number of tentative orders, but also (and perhaps even mainly) because it permits a better organization of the orders in view of up-to-date information on the state of the market.

**VI DISTRIBUTIVE EFFECTS: WHO GETS WHAT**

We have seen that the trades that take place at any given iteration or sequence of iterations depends to a great extent on the market process in use. The trades attained via process $F^e$, or equivalently via a sequence of iterations in process $F^t$, are just special cases out of a large number of alternative trades attainable via other processes. In any given situation, the trades generated by one process will be preferred by some traders, while others will prefer the trades generated by another process. An interesting dimension in the comparison of

\(^8\) E.g. Marschak [10].
market processes therefore concerns the **distributive implications**: does one process systematically favor a certain type of traders over other types, relative to another process?

In this section, we examine this aspect by an informal comparison of the trades generated by the classical tatonnement process $F^t$ with the trades generated by a sequence of iterations of an alternative process.

Loosely speaking, the alternative process may be thought of as a tatonnement process with intermediate trading—we shall therefore call it the Simple Intermediate Trading process—or SIT for short. Thus, we visualize an auctioneer who calls prices, to which investors respond by stating their desired trades at those prices. "Maximum" trading at these prices is accomplished by the auctioneer through a rationing of orders (according to some rule based on P2). A new price is then called with the goal of correcting perceived supply and demand imbalances. The process continues until all participants no longer wish to trade at the most recently called (equilibrium) price $p^*$. For the present, we assume that the SIT process, like tatonnement, takes place in fictitious time. Formally, $b=1$ in P4 and $t'=t$ (see P1).

The reasons for selecting the SIT process for analysis at this juncture are several. First, it contains the essential elements of the standard non-equilibrium processes (see e.g. Arrow and Hahn [1971, Ch. 13]). Second, it departs only slightly from the usual tatonnement process; in particular, both processes utilize the same message space and may be characterized as making low communication demands. Third, many properties of the SIT process are readily illustrated (in the two asset case) via the familiar Edgeworth box. These aspects make the SIT process a natural first candidate for the comparisons we will make between non-equilibrium trading processes and the usual tatonnement process.

**Naive Trading Behavior**

To provide the simplest possible comparison with the $F^t$ process, we also assume in this sub-section that investors in the SIT process base their orders
on fixed preferences throughout the (non-equilibrium) trading "period". We will see in the next sub-section that this is generally not an optimal strategy; accordingly, we term it naive behavior. However, some of the results in this sub-section will be seen to hold when the "fixed preference" assumption is relaxed in the next sub-section.

Consider, for simplicity, the case of two assets and two (types of) investors with convex indifference curves such that all equilibria under tatonnement are unique. The current position is given by point $0$ in Fig. 1, which was an equilibrium point under the previous indifference curves $I^1_0$ and $I^2_1$, respectively; the related equilibrium price $p^0_0$ is represented by the slope of line $00'$. With the arrival of new information, the first investor's indifference curves shift to $I^1_1$, $I^2_2$ etc. Thus, investor 1 is an "informed" trader with some up-to-date information. $CC'$ shows the resultant "contract curve". The standard equilibrium process $F^e$ yields point $E$ as a solution, with the new equilibrium price $p^e$ represented by the slope of line $OE$. Recall that, in this process, all trading occurs at this price.

Under the SIT process, the old equilibrium price $p^0_0$ would repeatedly be called until an imbalance in offers to buy and sell occurs. This happens when the new information is received by investor 1, who in this example then wishes to sell some of his holdings in asset 2. Since investor 2 is still happy at point $0$, no trade is executed at $p^1_1=p^0_0$ but the auctioneer, seeing asset 1 in excess demand and asset 2 in excess supply at $p^1_1$, now lowers the price of asset 2 to $p^2_2$. Under the assumptions of this sub-section, investor 1 now makes an offer to move from point $0$ to point $A^1_1$, his most preferred position under the terms proposed by the auctioneer. Similarly, investor 2 would offer to trade from point $0$ to point $A^2_2$ at price $p^2_2$. Faced with these two offers, the auctioneer executes investor 2's offer in full and investor 1's in part (the

\[9\] Recall that the price of the first asset is 1.
Fig. 1

Investor 1  Asset 1

Fig. 2a  Fig. 2b  Fig. 2c
fraction \( a_{1j} = \frac{OA_1}{OA_2} \) for both assets), causing a non-equilibrium trade from point \( O \) to point \( A_2 \) at price \( p_2 \). In the process, investor 1 moves from indifference curve \( I_{1}^{1} \) to (the higher) curve \( I_{1}^{3} \) while investor 2 moves from curve \( I_{1}^{2} \) to (the higher) curve \( I_{2}^{2} \).

Seeing asset 1 in excess demand and asset 2 in excess supply even at \( p_2 \), the auctioneer decreases the price of the second asset even more before calling \( p_3 \). The investors again respond with their most preferred offers, a trade is executed, a new price is called, etc. The previous price is repeated only when excess demand and supply is zero; if no non-zero offers are then made (in round \( m+1 \), say), equilibrium has been reached at \( p^* = p_{m+1} \). Fig. 1 represents the case when \( p^* = p_3 = p_4 \), i.e. when the third price called by the auctioneer turns out to cause offers which clear the market; investors then trade from point \( A_2 \) to point \( M \) on the contract curve in the third round, where they remain.

Under the assumptions of the present sub-section, the positions of the investors will converge to a point on the contract curve under any reasonable rule for declaring price changes followed by the auctioneer (such as making price changes proportional to excess demand, as in some tatonnement processes). The (final) equilibrium point reached, and the equilibrium price vector \( p^* \), clearly depends on the price sequence \( p_1, p_2, \ldots, p_m = p_{m+1} = p^* \). Clearly, we generally have \( p^* \neq p_e \).

We shall now characterize some of the properties of the SIT process under naive investor behavior using our simple example as a reference:

N1. Only "price-lines" which intersect the interior of the region of Pareto-superior positions, and hence the interior of OBDO in Fig. 1, lead to trades.

N2. The path of investor holdings falls entirely within region OBDO since both investors never wish to move outside that region for any given price vector.

N3. After each trade, the region of positions which remain Pareto-superior is a proper subset of the previous region of such points, that is, trading
results in monotone shrinking of the region of remaining desirable trades.

N4. Suppose that the price changes from $p_0$ to $p^*$ under the SIT process are monotonic in each asset and that equilibrium under the $F^e$ process is unique (for each initial endowment). Then the SIT process leaves the "informed trader", who is a seller of the asset whose price decreases and a buyer of the asset whose price increases better off (and the other party worse off) than he would have been under the $F^e$ process (see Fig. 1 and Fig. 2a).

N5. Under "almost continuous" price changes, the SIT process would follow a path which falls inside curve OD only by some distance $\varepsilon > 0$. This implies (in Fig. 1) that investor 1 would extract essentially all surplus from trade.

N6. A necessary, but not sufficient, condition for the seller of the asset whose new equilibrium price ($p^e_{2n}$) falls short of its old equilibrium price ($p^e_{20}$) to be worse off under the SIT process than under the equilibrium process with recontracting is that $p^e_{2n} < p^e_{2e}$ for some $p_n$ under the SIT process (see Fig. 2b and 2c). That is, at some point the auctioneer in the SIT process must, at a minimum, call a price which differs from $p_0$ by more than $p^e_{2e}$ does.

N7. For each sequence of trades executed under the SIT process, there exists a corresponding sequence of $F^e$ equilibria generated by false indifference curves.

It is not difficult to show that the preceding results readily generalize to the case of $J$ assets and $I$ investors. However, in this case there are many feasible allocations from which the auctioneer must select one. Several alternatives are available: maximization of the "dollar" value of trading, maximization of the average "fraction" $a_i$, maximization of the minimum component of $a_i$ over all investors, etc. Each rule would generally lead to a different trading path and final equilibrium even though each rule, by in some sense "maximizing" trading, takes large steps in the "direction" of equilibrium for any given sequence of price calls.
Property N1 is basic and requires no comment. Properties N2 and N3 are noteworthy because they imply that all trading moves the investor to a point "between" their present position and the contract curve, i.e. without "overshooting", or along a path without "reversals", albeit at difference prices, of previous trades.

Properties N4, N5, and N6 will be discussed more fully in the next subsection since they have important implications for investor strategies, the concept of "orderly markets", and the design of trading processes in general. Finally, property N7 reminds us of the possibility for investors to choose strategies that generate trading paths via sequential $F^\epsilon$ processes that are identical to those obtained by the SIT process. This is relevant when the informed trader is "large" and can advantageously depart from competitive behavior.

**Sophisticated Trading Behavior**

The naive investors in the previous subsection made offers as if they were going to be executed in full, even though they knew that only some would be and that others would only be partially executed, and as if the current price would be the final price. In the $F^\epsilon$ process where there is but a single round of trading, and this without rationing, such an assumption is plausible, and is in fact consistent with investor rationality. But when there are several rounds of intermediate trading (with possible rationing), the concept of investor strategy comes into play. For each $p_n$ called by the auctioneer and each possible order $x_n$, the investor will assess a probability distribution over the degree of order execution, $a_{in}$, as reflected in (2). But he will also have a probability distribution over the remaining price sequence and over how long the process will continue. Taking each of these into account he will change his current behavior to maximum advantage. This means that the induced preferences he will apply at any iteration $n$, $U^*_n$, will differ from $U_i$, the basic preference function that he
would apply if he knew that the process were reaching the end-point for sure in one iteration. Furthermore, since $u^*_i \not\in$ one can conceive of circumstances when convergence to equilibrium does not take place under rational investor adaptation. Only if $u^*_i \not\in$ in some fashion as $n \to \infty$ does convergence seem assured.\(^{10}\)

Assuming convergence, trading paths such as those illustrated in Fig. 2 are clearly possible in the present case as well. Property N7 is still valid but properties N1, N2, N3, and N5 of the previous model no longer hold. That is, the trading path may move outside the region OBDO. The reason is that an investor acting in accordance with (his currently relevant) induced preferences may choose to experience a "loss" on the basis of his final preferences (in order to take advantage of expected future price patterns, say).\(^{11}\)

Property N4 now holds only under the following more stringent conditions:

**Proposition.** Suppose that the $F^r$ process has a unique equilibrium, that the SIT process reaches an equilibrium, and that the price sequence in the SIT process is monotone, where $p_n = p^*$. Then investors $i$ for whom

\[
\begin{align*}
&w_{ij,n^{-1}} > w_{ij,n^{-1}} \text{ whenever } p_{jn} > p_{ji,n^{-1}} \\
&\text{and} \\
&w_{ij,n^{-1}} < w_{ij,n^{-1}} \text{ whenever } p_{jn} < p_{ji,n^{-1}}
\end{align*}
\]

for all $j$ and $n$, with strict inequality holding for some assets $j$ and some iteration $n < m$, are better off in the SIT equilibrium than under the $F^r$ equilibrium.

Conversely, those for whom the signs are reversed are worse off in the SIT process than under tatonnement. Property N6 is similarly modified in the present setting.

The proposition makes intuitive sense in that those who buy on the way up and sell on the way down are better off with intermediate trading than without. What makes the assertion non-trivial is the fact that generally $p^* \neq p_e$.

\(^{10}\) As noted earlier, questions of convergence will not be addressed in this paper.

\(^{11}\) See also Arrow and Hahn [1, pp. 328-329].
One implication of our analysis is that those with "better" information about the sequence of future prices or about the conditional distributions of \( a_i \) tend to gain via an intermediate trading process of the SIT type compared to how they would do in a tatonnement process (recall also our discussion in Section V). But insisting on an \( F^t \) process may not be the best solution for the "less informed", as property N7 reminds us.

A central concept underlying the operation of actual organized markets is the notion of an "orderly market". The person normally charged with maintaining an "orderly market" is the specialist. This requirement is backed by numerous rules and is generally interpreted as including, among other things, the demand that the specialist "... maintain a continuous market with price continuity" (Leffler and Farwell [9, p. 212]). Commodity futures markets even impose limits beyond which prices cannot change in one day. It is interesting that this intuitively appealing institutional concept also tends to favor not only those with "better" information about the state of the market, but also those with access to "frequent" and up-to-date information about the environment, as reflected in properties N4 and N5 and in the Proposition.

**VII RESOLUTION IN REAL TIME**

To achieve some basic comparisons between tatonnement and intermediate trading processes, we have so far been forced into viewing the transformation of orders into trades as taking place in a fixed environment, along "fictitious time". When this unrealistic assumption is relaxed, the limitations of the classical approach become even more apparent.

It is clear that as preferences continually change (e.g. as a result of new information), the tatonnement process will generally not converge. Indeed, the lack of assured convergence to a Pareto-efficient allocation is common to all market processes with property P5 in a real-time framework.\(^{12}\) Even so, the SIT

\(^{12}\) Including even the \( F_e^t \) process with its exuberant demands on communication.
process appears to have some definite virtues in this context. The search of
the tatonnement process for a price which (if found) would attain Pareto-efficiency
in one swift move prohibits possible, though less ambitious, Pareto-improvements,
and in the absence of convergence no trades ever occur. In the SIT process, on
the other hand, trading generally takes place at each iteration, moving investors
closer to the current contract curve. Moreover, this movement is in some sense
"maximal". Finally, the communication demands placed on the system are no greater
than under ordinary tatonnement. On each of these counts, the SIT process also
compares favorably relative to the stochastic adjustment process, adapted to real
time, of Hurwicz, Radner, and Reiter [7].

When equilibrium is "out of reach", even the most basic criteria for the
evaluation of market system performance are unclear and need a careful examina-
tion. If coming "close" to Pareto-efficiency at "low" cost should be seen as
desirable, just as coming closest to the elusive rabbit in dog racing is a measure
of winning, then processes such as SIT deserve further investigation.

VIII CONCLUDING REMARK

This paper was motivated by the lack of correspondence between observed
market behavior and the theory of adjustment processes. It has attempted to
formulate a general model of market processes which encompasses previously
studied models as well as a rich class of alternatives. One of these, the SIT
process, was selected for comparison with ordinary tatonnement, both under
fictitious time and real time resolution. While the analysis tended to raise
more question than it answered, the study of generalized adjustment processes
shows some promise for improving the understanding, and eventually also the
operation, of organized financial markets.
References


