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Testing International Asset Pricing: Some Pessimistic Views

by

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TESTING INTERNATIONAL ASSET PRICING:
SOME PESSIMISTIC VIEWS

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In the late sixties modern capital market theory started to be applied to the area of international investment (Grubel, (1968), Levy and Sarnat, (1970)). The original idea was to apply the Markowitz–Sharpe risk-return analysis to expost data on stock prices and exchange rates, in order to determine the optimal internationally diversified portfolios that investors of each country should hold. It has been argued that optimal portfolios should be different for investors of different countries because of exchange risk. This argument, supported by some empirical evidence, has been used to justify capital flows between countries (see, for example, Levy and Sarnat, (1975), Lee, (1969), etc.). This paper will argue that such a conclusion is inconsistent with the theoretical framework used. Moreover, the big issue is the economic definition of exchange risk. More recently equilibrium models of the international capital market have been proposed (Solnik, (1974), Grauer, Litzenbenger and Stehle, (1976)). Their underlying definitions of exchange risk are different and lead to somewhat different pricing relations and optimal portfolio selections.

Before engaging in any tests of integrated versus segmented capital markets and international pricing, it seems to be a useful time to confront the various models, especially as far as exchange risk is concerned. Empiricists consistently resort to mean-variance analysis of expost-data to test their various models and often confuse the econmic results with some obvious technical implications of the mean-variance framework.

This paper will attempt to give a fair representation of the various international asset pricing models, stressing their real economic conclusions.
It will then claim that it is very unlikely that an empirical mean-variance analysis will ever be able to discriminate between the various views of the world.¹

I. The Domestic Equilibrium Model

It might be useful to recall the technical and economic conclusions of the mean-variance (MV) asset pricing model (CAPM) and their testable implications.²

1. Efficient Portfolios

In a MV framework one can determine the efficient frontier of all risky investment; if the investor can lend and borrow in his own currency (riskless asset), he can reach a desired level of risk by simply combining the risk-free asset with a specific risky portfolio (separation property). With assumptions about market perfection and homogeneous expectations, it can be shown that this risky portfolio is the same for everyone and therefore is the market portfolio. As Fama (1976) and Roll (1976) stressed, this is the only real economic content of the model. If no risk-free asset

¹I strongly benefited from inspiring discussions with R. Roll. Because of stringent space limitations, most of the mathematical derivations had to be left out, but can be found in various published articles. A more mathematical version is available from the author; anyhow, the purpose of this paper is not to add to the proliferation of mathematical models or regression analysis but to reflect about the possibility of testing the economic content of the existing theories.

²A technical conclusion is of no empirical interest since, by the very construction of the test, we know that it must be verified independently of what is going on in the world. This part draws heavily on Roll (1976).
exists, the economic conclusion is that the market portfolio is efficient (Black, 1974).

All these models are based on expectations not realizations. An ex post MV analysis will, of course, lead to the same efficient frontier and optimal portfolio for everyone since all investors agree on realized values (equivalent to homogeneous expectations). In other words, the separation property is a technical MV necessity for any set of ex post (or homogeneous ex ante) data. The only economic test would be whether the ex post optimal portfolio is the market portfolio. Without riskless asset, the test would be whether the market portfolio is on the ex post efficient frontier.

2. Asset Pricing

Asset pricing relations are easy mathematical derivations in the MV framework once efficient portfolios are known. A technical result is that given any efficient portfolio \( m \), the famous zero-beta asset pricing relation will always apply (Roll, 1976):

\[
\bar{r}_i = \bar{r}_z + \beta_i (\bar{r}_m - \bar{r}_z) \text{ for all } i
\]  

(1)

The only economic content is to state that the market portfolio should be one of those numerous efficient portfolios (and the tangential one if a riskless asset exists). Whatever the world and structure of return look like, relation (1) will always be exactly verified for any ex post efficient portfolio \( m \). In other words, all tests of the domestic CAPM which have focused on (1) were indirect tests (and not powerful ones) of
whether the Market index used was ex post efficient. Since the only economic implications of these models lie with the composition of the optimal portfolios, the rest of this paper will focus on it, not on pricing relations which are direct by-products.

II. Integrated vs Segmented

Let's first consider the problem of international market integration in the absence of exchange risk which will be dealt with in the next section. Several authors, including myself, have attempted to test whether asset pricing was national or international. In a perfect MV international capital market the (value weighted) world market portfolio ought to be the optimal risky portfolio. To test this theory one needs a null hypothesis and the question is what would the data look like if the markets were segmented instead of integrated. Unfortunately, there is a good chance that it will not be possible to differentiate.¹ This can be seen by assuming that asset pricing is exactly domestic and looking at the apparent results of an international MV analysis.

Let's assume no exchange risk; Σ is the covariance matrix of all stocks returns and Σₖ the covariance matrix of stocks returns of country j. E is the excess return vector (over the risk free rate equal everywhere since there is no exchange risk) and M the market portfolio vector.

¹Wallingford (1974) illustrated this point in a comment on an earlier paper of mine.
Given domestic pricing the standard CAPM results hold\(^1\) and the domestic market portfolio vector \(M_j\) should be such that

\[ M_j = \lambda E_j \Sigma^{-1} E_j \text{ for all } j \]  

(2)

A researcher who would consider all countries and compute the apparent optimal international portfolio would find\(^2\) a risky portfolio vector \(X\), such that:

\[ X = \lambda \Sigma^{-1} E \]  

(3)

It has repeatedly been observed that the covariances between the national stockmarkets are very small (and slightly positive). If they were zero, \(\Sigma\) would reduce to a block-diagonal matrix of all the \(\Sigma_j\) and the apparent optimal international portfolio would have a positive investment\(^3\) in each national market portfolio \(M_j\). Indeed, according to

\(^1\)Minimizing \(X_j' \Sigma_j X_j\) s.t. \(X_j' E_j = \mu\), and aggregating over all investors with the domestic market in equilibrium.

\(^2\)Minimizing \(X' \Sigma X\) s.t. \(X' E = \mu\).

\(^3\)\(\Sigma^{-1} E\) is equal to:

\[
\begin{bmatrix}
\Sigma^{-1} & 0 \\
0 & \Sigma^{-1}_n
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_n
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{\lambda_1} M_1 \\
\frac{1}{\lambda_n} M_n
\end{bmatrix}
\]
the relative risk aversion of investors of each country, \( \lambda_i \), this portfolio could be equally weighted in each market (with identical risk aversion \( \lambda_j \)) or even identical to the value weighted world portfolio.

It is quite intuitive to expect that, as long as (expected) excess returns are positive and covariances low, an international MV analysis will always produce optimal portfolios well diversified internationally to reduce risk;\(^1\) This will be true technically even if markets are totally and physically segmented. If it is hardly possible to differentiate ex ante, it should be obvious that it will be even more difficult ex post.

The efficient way to test for segmentation would seem to be to specify the type of imperfection which might create it and study its specific impact on portfolio optimality and asset pricing (see, for example, Black, (1975) or Bauz, (1976)). The international integrated market hypothesis can only be tested against some null segmented hypothesis (a positive \( R^2 \) is not sufficient). Unfortunately the MV framework seems very inefficient in discriminating between the two even ex ante and increased sophistication in econometric methodology can be of no help.

III. Exchange Risk

Exchange risk is often mentioned as a large impediment to foreign investment. The question is whether in the absence of trade and

\(^1\)For example, a test of relation (1) using some world index could have an \( R^2 \) square of one even though everyone ignores that there exists a stock market outside of his own country (e.g., on Mars or the Moon).
capital market imperfections, exchange risk would have any importance on market equilibrium if properly measured.

Solnik (1974) presented an equilibrium model where exchange risk stemmed from differences in consumption tastes between countries. More recently, Grauer, Litzenberger and Stehle (G.L.S., hereafter) added to the analysis by studying the influence of exchange risk coming from uncertain monetary inflation.¹ While both, differences in tastes and monetary inflation exist domestically as well as internationally, the international economist is confronted with the challenge of explaining economic phenomena such as exchange rate behavior and differences in national interest rates. Furthermore, excellent data are available to the empirical researcher to test the theories.

To contrast them, the two approaches and their economic implications will be summarized separately. For convenience, the so-called monetary exchange risk will be reviewed first, knowing that total exchange risk will probably come from both monetary and real sources.

1. Efficient Portfolios and Monetary Exchange Risk

In a state preference mode of international markets, G.L.S. have studied exchange risk stemming from pure monetary uncertainty. In the G.L.S. world all individuals have identical homothetic tastes. In other words, they all consume the same good or basket independently of wealth and only differ

¹They also considered some special forms of trade imperfections.
in their propensity to lend or borrow. International trade is free and costless, therefore the ratio of all goods prices expressed in two currencies will always equal the exchange rate between the two currencies. Fluctuations in exchange rates can only be caused by pure money inflation and real goods prices will be equal everywhere. In their model the fact that investors reside in any country or use different currencies is irrelevant. Since they have the same consumption tastes, individuals face exactly the same investment and consumption opportunity set and earn the same real return on each asset. Everyone can buy his goods at the same (real) price and earn exactly the same real return on each asset.

If there exist a real risk-free asset, it is shown that money inflation (i.e. exchange risk) has no influence on equilibrium, consumption, and investment decisions. The world market portfolio will be optimal and the traditional CAPM results will obtain where returns are measured in real terms. In this model, nominal riskless bills pricing seems to be influenced by exchange risk and a theoretical price for the forward exchange is indicated, but such contracts will never be written by anyone, they are purely virtuals.¹ In this world no one will ever use nominal riskless borrowing or lending nor write a forward exchange contract.

The empirical test would be to check that the ex post optimal portfolio is the world market portfolio (with the resulting pricing formulae).

¹In real terms everyone will hold the same risky portfolio made up of stocks and nominal bills. But the net supply of nominal bills is zero so no one will hold them. Intuitively these are risky for both contractors and no one is willing to pay a premium for unnecessary risk.
The only difference with the previous case is that "some" inflation index is subtracted from nominal returns to measure covariances. However, by assumption, nominal riskless bills will be introduced as possible investments; therefore, the optimal ex post portfolio might be made up of stocks with weights which might be different from market values and nominal bills (which should not appear according to the pure monetary theory).

The obvious use of nominal riskless assets and forward contract might be explained by the nonexistence of a real risk-free asset. In this case, monetary exchange risk (or inflation risk) will affect equilibrium. In this world without real riskless assets, portfolios held by individuals will vary according to risk aversion and nominal riskless bills will generally be held. Now nominal exchange risk does affect asset choices and the portfolio composition and equilibrium asset prices will be different whether uncertain exchange rates exist or not.

It can be seen, however, that the world stock market portfolio should still be efficient. The proof could be stated as follows. Everyone faces the same ex ante opportunity set and efficient frontier (homogeneous tastes and expectations); everyone's optimal portfolio will be efficient and therefore could be described as a linear combination of two arbitrary efficient portfolios. The stock market portfolio is, in the aggregate, the sum of all individuals portfolios, given a zero net supply of riskless bills.

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1 Contrary to G.L.S. claim (section VIII "Exchange Risk and Money Illusion").

2 This is true for any portfolio on the efficient frontier. See Roll (1976).
It implies that the market portfolio is a linear combination of two efficient portfolios, and therefore is also efficient. The only economic conclusion with empirical implication is that the world market portfolio is efficient among all the stocks and exchange risk assets (nominal riskless bills).

To summarize, given homogenous tastes and expectations the separation property (same optimal risky portfolio for everyone) will always apply ex ante and therefore ex post. The test of the theory would be whether the world market portfolio is tangent or efficient. Note, however, than an appropriate inflation index has to be subtracted from nominal returns in the empirical work. If inflation is not correlated with stock market returns, it will not affect the composition of the efficient frontier. Otherwise, the optimal stock portfolio might be quite sensitive to the inflation index used in the test.

2. Efficient Portfolios and Real Exchange Risk

The previous section considered the monetary aspect of the international dimension while Solnik (1974) and (1976) focused on real differences between countries. Marked differences in tastes are assumed within a free and perfect international capital market.

To study only the real impact of consumption differences, pure (neutral) money inflation is ruled out (Solnik, (1974), p. 505, footnote 9).

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1 Indeed for any set of nominal stock return data generated by a market model, it is possible to construct one "inflation" index which will make the world market portfolio be exactly ex post efficient; its correlation with the "true" or any reasonable inflation index might be quite large.
In other words, the two definitions of exchange risk are fundamentally different since this second one considers real exchange risk stemming from differences in tastes, not pure monetary risk. Hopefully, the results of the two models are, therefore, complementary.

My original assumption about strict consumption tastes separation is not necessary if one assumes that investors have homothetic multi-consumption tastes (a standard assumption in utility theory). Every country then consumes the same commodities but in different proportions (different baskets). Let's consider a country to be defined by its tastes and, therefore, its consumption good or basket.\(^1\) As before, perfect trade markets are assumed with identical real prices for each commodity across the world. To isolate the real aspects it is also assumed that the monetary authority of each country worries about its own citizens and inflation rate, attempting (and succeeding) to keep a constant domestic inflation rate; in other words, the purchasing power of its currency in terms of the domestic consumption good is certain. Given relative price variations, exchange rates will fluctuate randomly with the relative value of national consumption baskets as long as they are different.

This is now a case where individuals have different measures of real returns according to their tastes and they face MV investment

\(^1\)For a more extensive discussion, see Solnik (1976). In fact, this model could also accommodate the case where all investors have identical tastes but face different commodity prices according to their country of residence because of uncertain transportation costs, taxes, etc. . . . . The same formulation would be used.
opportunity sets with different characteristics. It introduces some kind of heterogeneity in expectations and the separation property (ex ante or ex post) is not anymore a trivial outcome of the MV analysis. However, assuming the existence of real risk-free assets (i.e., a nominal risk-free bill in each country) and independence between exchange and stock market risk, it has been shown that a single risky portfolio will be optimal for everyone; it will be made up of the world market portfolio plus a mix of national bills which are pure exchange risk assets.

It should be stressed that the stock market portfolio is not globally optimal or efficient; it is only part of the optimal portfolio. It represents the best stock portfolio that could be chosen to be included in the total portfolio (stocks and bills). However, the market portfolio might very well be inefficient if the subset of stocks was considered alone, excluding the other (exchange risk) assets. One very unpleasant feature of the model as far as its test is concerned, is that the weights of the various national bills in the optimal portfolio will depend on net foreign investment positions and relative risk aversions of citizens of the various countries; in fact negative weights are quite possible. In other words, the composition of the optimal theoretical portfolio is hard to figure out in practice; this problem prevents any serious test of the theory.¹ Again

¹Ex post there will always be one MV optimal portfolio, or an infinite number of efficient portfolios. For these portfolios the risk pricing formulae will be exactly verified. If the theoretical composition of these portfolios is not known, no economic test can be performed.
no refinement in the statistical methodology will help solve this problem.

In this no-inflation-world, exchange risk is equal to exchange rate uncertainty. If pure monetary erosion were considered in the analysis the two models should be combined. The exchange risk which should be taken into account in the empirical tests would have to be only the exchange rate variation which is not explained by the inflation differential, where inflation is measured on each domestic consumption basket.

3. **Exchange Risk and Capital Flows**

As mentioned in the introduction, it has been argued that optimal portfolios should be different for investors from different countries because of exchange risk. Without engaging in any theoretical discussion, exchange risk is simply associated to the uncertainty about the foreign exchange rate (Levy and Sarnat, (1975); Lee, (1969)), therefore it assumes that the investor uses his own currency as the numéraire. Conclusions are simply drawn from an ex post MV analysis of stock returns and exchange rate data. The lack of economic theoretical analysis and the validity of such an ex post Markowitz analysis might be questioned. More importantly, a properly specified MV will always lead to an identical stock portfolio for everyone. In other words, the separation property is still valid technically. This can be shown by noticing that this analysis and definition of exchange risk is the ex post analog of the ex ante model described in Section III-2 (Solnik, (1974)). The mathematical separation property demonstrated in Solnik is, therefore, also valid on ex post data.
Some readers might be worried that these results depend highly on the use of continuous compounding. To remove any doubt, empirical results will now be presented using noncompounded rates of return. Clearly, the shorter the time interval used the closer the approximation to continuous compounding. While daily returns were available on tape, monthly returns were used to magnify the potential difference. Investors may invest in the domestic and foreign stocks and risk-free assets assuming perfect markets. Table 1 reports the composition of the optimal stock portfolio for investors of each country. A row indicates the nationality of the investor and a column the nationality of the asset (stock or foreign bill). The first row gives the optimal portfolio if continuous compounding is used, and the next seven if discrete monthly returns are used. Any investor will combine his own risk-free asset with this risky portfolio.

It seems clear that the ex post optimal stock portfolio is almost identical for everyone.

To summarize, as soon as the MV framework is used on ex post data, the separation property will hold internationally even if all the data come from tables of random numbers and no one holds foreign stocks. Exchange risk, real or monetary, cannot justify capital flows (in these computer runs). If some specific market imperfections are explicitly incorporated in the MV analysis the results might be different. But these differences would be caused by these imperfections, not by exchange risk.

\footnote{The weights of all stock investments sum to one.}
4. **Conclusion**

The doubts which have been expressed about our ability to test for domestic vs international pricing apply to tests of the influence of exchange risk. Three simple models have been presented here; no exchange risk (II), monetary exchange risk (III-1), and real exchange risk (III-2). With simplifying assumptions such as the existence of real risk-free assets the conclusions are simple enough to be tested. More complex models of the international world are interesting for the theoretician but lose empirical tractability. The economic conclusions of the simple alternative models might be summarized as:

1. The stock market portfolio is efficient among all stocks;
2. The stock market portfolio is efficient among all stocks and nominal bills;
3. The stock market portfolio is the stock component of the efficient portfolio (stocks and bills).

A test to discriminate between these three alternatives will not be easy to construct. For each model, returns are defined and measured differently so that it will not be possible to oppose the hypotheses in the same test. Furthermore, exchange rate and inflation uncertainty is very small compared to (and have little correlation with) stock market risk; it could be expected that, given any definition of exchange risk, it will hardly affect (ex ante or ex post) the composition of the efficient portfolios.

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1 Each imperfection in the trade or capital market might add another source of exchange risk.
This can be illustrated on the data used above. The results reported previously could be interpreted as a test of the real exchange risk model (model III-2) assuming no inflation. Similarly, Belgium was taken as the reference country and the model II was tested using nominal and real returns. Finally, the efficient set was also computed, making the assumptions of the pure monetary model (II-1), measuring real returns and introducing pure nominal bills and a real risk-free asset (zero real return). The figures reported in table 1 show how difficult it would be to differentiate between models (i.e., reject one while accepting another).

The best way to test for the effect of such a small parameter would be to try to isolate it\(^1\) and focus directly on forward exchange rates and interest rates. There, two serious problems arise. First, an exact measure of real returns is needed which requires the computation of a theoretical inflation index for each country. Second, the theoretical optimal exchange risk assets portfolio depends on variables such as net foreign investment position, relative risk aversion which are hardly measurable. If it is not possible to specify the composition of the theoretical portfolio\(^2\) or index, no formal economic tests can be performed and only qualitative although interesting results could be given.

To summarize, international asset pricing seems to be a very fruitful area for theoretical research, not empirical.

\(^1\)Fama made a similar point for testing the effect of inflation risk on asset pricing.

\(^2\)Except in the pure monetary theory which claims it should have zero weights.
### Table 1

**OPTIMAL PORTFOLIO COMPOSITION**

<table>
<thead>
<tr>
<th></th>
<th>Stock Markets</th>
<th>Foreign Bills</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>UK</td>
</tr>
<tr>
<td><strong>Continuous</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>compounding</td>
<td>.09</td>
<td>.08</td>
</tr>
<tr>
<td><strong>Model III-2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>.11</td>
<td>.08</td>
</tr>
<tr>
<td>UK</td>
<td>.09</td>
<td>.08</td>
</tr>
<tr>
<td>F</td>
<td>.09</td>
<td>.08</td>
</tr>
<tr>
<td>D</td>
<td>.11</td>
<td>.08</td>
</tr>
<tr>
<td>N</td>
<td>.08</td>
<td>.08</td>
</tr>
<tr>
<td>CH</td>
<td>.08</td>
<td>.08</td>
</tr>
<tr>
<td>USA</td>
<td>.10</td>
<td>.07</td>
</tr>
<tr>
<td><strong>Model II</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nominal returns</td>
<td>.17</td>
<td>.40</td>
</tr>
<tr>
<td><strong>Model II real</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>returns</td>
<td>.12</td>
<td>.42</td>
</tr>
<tr>
<td><strong>Model III-1 real</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>returns</td>
<td>-.07</td>
<td>.14</td>
</tr>
</tbody>
</table>

**Note:** End-of-month stock prices, exchange rates and inflation rates for seven countries were used. The period covered is March 1966 - April 1974 and the countries are: Belgium, France, Germany, Netherlands, Switzerland, United Kingdom and the United States of America.

Stock prices come from Eurofinance; exchange rates, short term money rate and inflation indices from the International Monetary Fund. For each country the efficient set was computed independently.

I am grateful to R. Roll for providing a computer program which performs an analytical derivation of the efficient frontier (see Roll, (1976)).


