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INFORMATION, MANAGERIAL CHOICE, AND STOCKHOLDER UNANIMITY

by

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I. Introduction

Stockholder unanimity plays a key role in the theory of production. Under certainty, profit maximization is justified as being in the interest of all stockholders. But the introduction of uncertainty renders profit maximization meaningless, since production decisions do not give rise to a unique level of profits. A fundamental question is whether stockholders with diverse expectations and attitudes towards risk will support unanimously a particular set of production decisions. Without unanimity, firms' decisions will depend upon the weighting of diverse stockholder and managerial preferences, with the possible nonexistence of a preference ordering over actions by the firm.¹ And without a theory of the firm, a substantial fraction of positive economics disappears.

Several recent papers have studied the relationship between financial markets and production under uncertainty. Implicitly or explicitly, stockholder unanimity has been a property of these models. Two sets of conditions have resulted in stockholder unanimity.

The first set, studied by Wilson [1968] and Rubinstein [1974], requires identical investor expectations and tastes belonging to the linear risk tolerance class of utility functions. The second set of conditions requires the space of returns generated by marketed securities to contain the marginal returns resulting from small changes in the firms' decisions. Ekeren and Wilson [1974] showed that spanning was a sufficient condition
for "ex post" unanimity. Leland [1973] showed it was necessary as well, and showed that, with an additional competitive price assumption, spanning assured "ex ante" stockholder unanimity. A number of economic environments are consistent with the spanning property, including the "complete markets" models of Arrow [1964] and Debreu [1959], and the incomplete markets models of Diamond [1967], Leland [1974, 1975], and Forsythe [1975].

While providing a rich class of environments consistent with stockholder unanimity, both sets of conditions seem overly restrictive. Individuals possess different information and different expectations. And investment projects or other decisions resulting in marginal changes in returns typically have project-specific risks which are not spanned by currently marketed securities.

Furthermore, the two sets of conditions consistent with unanimity give little scope to the role of the manager. Clearly managers cannot have "better information" than investors if expectations are identical. And while spanning permits different expectations, the optimal decisions by the firm will be independent of any particular set of expectations, including those of the manager. Why should a manager gather information if (as these models predict) better decisions will not result? In short, current approaches to stockholder unanimity are inconsistent with the fact that managers typically have (and are urged to obtain) better information about firm-specific risks. While informational asymmetry has been considered in models of principal-agent and hierarchical relationships (Ross [1973], Mirrlees [1976]), it has not been considered in models where there are multiple stockholders (principals) with divergent tastes and expectations.
In this paper we show that there are weaker conditions than previously considered which are consistent with stockholder unanimity, when it is recognized that managers may have better information than investors about project-specific risks. Not only does the analysis extend the conditions under which stockholder unanimity will hold, but also it is consistent with managers having an active role in acquiring and using information for decision making.

II. Securities Markets and Portfolio Choice

We consider a simple two-period model similar to Leland [1974]. In the first period, investors select portfolios of securities and firms make investment/production decisions. Returns to portfolios, which may depend upon firms' decisions, are realized and consumed in the second period.

Let \( S = (1, \ldots, S) \) denote a set of market-relevant states and let \( J = (1, \ldots, J) \) represent a set of securities.\(^3\) A security \( j \in J \) is defined by a pair \((\pi^j, V^j)\), where

\[
\pi^j : S \to R; \quad V^j \in R_+.
\]

\( \pi^j(s) \) is the (gross) return of security \( j \) in state \( s \in S \), and \( V^j \) is the market value of the security given current investment/production decisions. Define

\[
\pi(s) = [\pi^1(s), \ldots, \pi^J(s)]
\]

\[
V = [V^1, \ldots, V^J]
\]
as the \( J \)-dimensional vectors of security returns given \( \varsigma \) and of security values, respectively. We assume the returns are linearly independent.

Finally, let \( I = (1, \ldots, I) \) represent a set of investors. Each \( i \in I \) is characterized by a triple

\[
(x_i, p_i, U_i)
\]

where \( x_i \in \mathbb{R}^J_+ \) is the vector of initial security holdings; \( p_i \in \mathbb{R}^S_+ \) is the vector of subjective probabilities with components \( p_i(s), \sum_S p_i(s) = 1 \); and \( U_i: \mathbb{R} \to \mathbb{R} \) is the \( i \)-th investor's von Neumann-Morgenstern utility function defined over final period wealth.

Investors choose portfolios \( x_i \in \mathbb{R}^J \) which maximize the expected utility of end period wealth \( R_i(\varsigma) \), where

\[
R_i(\varsigma) = \pi(\varsigma)x_i.
\]

The portfolio \( x_i \) must satisfy the investor's budget constraint

\[
(x_i - x_i)\varsigma \leq 0.
\]

Assuming \( U_i \) is strictly increasing and concave, necessary and sufficient portfolio optimization conditions are

\[
(2) \quad \sum_\varsigma p_i(\varsigma)U'[R_i(\varsigma)]\pi(\varsigma) = \lambda_i \varsigma, \quad \text{for all } i \in I,
\]

where \( U'(\cdot) \) is the marginal utility of end period wealth, and \( \lambda_i > 0 \) is the Lagrangian multiplier associated with the constraint (1).
III. Investment Projects

A subset $K \subseteq J$ of securities represents shares of firms. The return vectors of these securities will depend upon the production/investment decisions of firms' managers. Consider now a managerial decision which alters the return vectors of an arbitrary firm $k$'s shares by a small amount. We shall term such a decision an investment project, although clearly this label is a proxy for any managerial decision which effects a marginal change in the firm's returns.

The investment's returns will in general vary with $s \in S$, the states relevant to the securities market as a whole. But the project's returns also may depend upon a set of project-specific states $E = \{1, \ldots, E\}$. Project-specific states have two properties:

a) Only the project's returns vary with $e \in E$. Security and portfolio returns do not depend upon project-specific states, implying

$$
\pi(s,e) = \pi(s);
$$  

$$
R_i(s,e) = R_i(s).
$$

b) Investors' subjective probabilities over the states in $E$ are independent of their probabilities over the states in $S$. That is, for all $i \in I$,

$$
p_i(s,e) = p_i(s)p_i(e)
$$

over the set of all states $\Omega \subseteq S \times E$. An example of a project-specific state might be the mechanical integrity of the investment project: while
investors may differ on their probabilities of mechanical breakdown, they all agree that these probabilities are independent of \( s \in S \) and therefore of securities' returns.

The investment project's returns can be described by a function

\[ I: \Omega \rightarrow \mathbb{R}, \]

where \( I(s,e) \) is the change in the \( k \)th firm's returns resulting from the project, given market state \( s \) and project-specific state \( e \).

We shall focus on investment returns which are separable. Returns are said to be separable if, for each \( e \in E \), there exists a vector \( C(e) \in \mathbb{R}^J \) such that

\[ I(s,e) = \pi(s)C(e) \]

for all \((s,e) \in \Omega\). This property implies that the project's returns can be separated into two independent factors: a market risk component \( \pi(s) \), and a project-specific risk component \( C(e) \). If markets are quasi-complete, with the number of securities \( J \) equal to the number of market-relevant states \( S \) (but still less than the total number of states \( SE \)), then any set of investment returns can be put in form (5). But even when markets are quasi-complete, it should be noted that securities' returns do not span the investment's returns. This is because project-specific risks are not present in currently marketed securities.

Therefore stockholder unanimity cannot generally be expected with respect to the investment decision. But in the subsequent section we shall show that stockholder unanimity will result if managers have
superior information about project-specific risks. Furthermore stockholders will urge the manager to obtain this superior information.

IV. Managerial Information and Investment Decisions

In a world with no transactions costs, shareholders could be consulted with respect to each decision a firm might undertake. The managerial role would be limited to implementation only, with no decision-making powers. Yet it is manifestly clear that managers do have considerable responsibility in making investment, production, and financing decisions.

An important reason for the active managerial role lies in the transactions costs associated with information. The manager typically knows a great deal about the firm's operations. This information would be useful to shareholders in making decisions for the firm. But it is costly to communicate. Rather than make decisions without information, or incur the costs of information communication, stockholders grant managers the authority to make decisions for them. But this independence of managerial decision making poses a fundamental question: Given their "inside" information, will managers make decisions which are approved by shareholders?

Let \( m \in I \) denote the manager of an arbitrary firm \( k \) which is considering an investment. The investment will alter the firm's returns by a small amount \( I(s,e) \). We shall consider two information environments. In the first, information is available to the manager which is relevant only to project-specific states. We prove that stockholders will unanimously support managerial investment decisions if the manager has superior information and if returns are separable. In the second environment,
information on market states is also available to the manager. We show that quasi-complete markets \( J = S \) is a sufficient condition for stockholder unanimity. We prove these assertions below.

A. Environment I: Project-Specific Information

We model this environment by assuming the existence of a set of signals \( Z \) and a managerial information process \( f \) which generates a signal \( z \in Z \) for each possible project-specific state:

\[
(6) \quad f: E \rightarrow Z.
\]

Observation of \( z \in Z \) identifies a subset of project-specific states, labeled \( E(z) \), which are consistent with the signal:

\[
(7) \quad E(z) = \{ e \in E | f(e) = z \}.
\]

Upon receipt of a signal \( z \), the manager will modify his subjective probabilities to reflect this information:

\[
(8) \quad p_m(e|z) = \frac{p_m(e)}{p_m(z)}, \quad e \in E(z)
\]

\[= 0, \quad e \notin E(z),\]

where

\[
(9) \quad p(z) = \sum_{e \in E(z)} p_m(e).
\]

Note that, given (4) and (6), \( p_i(s|z) = p_i(s) \), for all \( i \in I \) including the manager.
The managerial information process is said to provide better information than that available to investors if, given that investors could observe \( z \in Z \), they would all agree with the manager's conditional probabilities for project-specific states.\footnote{7} That is, for all \( i \in I \),

\[
p_i(e|z) = p_m(e|z).
\]

(10)

We now may prove the following:

**Proposition I:** If project returns are separable and if managers possess better information than investors, then stockholders will unanimously support managerial investment decisions.

**Proof of Proposition I:**

Given \( z \in Z \), the manager will undertake the investment project if and only if it raises his expected utility. From Leland [1974, p. 132] or Ekern-Wilson [1974, p. 175], the change in the manager's expected utility resulting from acceptance of the project when signal \( z \) is received will be

\[
d[E U_m | z] = \sum_{s,e} p_m(s)p_m(e|z)U'_m[R_m(s,e)]I(s,e)
\]

\[
= \sum_{s,e} p_m(s)p_m(e|z)U'_m[R_m(s)]\pi(s)C(e)
\]

\[
= \sum_{e} p_m(e|z)C(e) \sum_{s} p_m(s)U'_m[R_m(s)]\pi(s)
\]

\[
= \sum_{e} p_m(e|z)C(e)V,
\]

using (5), rearranging terms, and using (2). Assuming the manager owns shares in his own firm, \( \lambda < 0 \). Define
\[(12) \quad Z^+_m = \{ z \in Z \big| \sum_e p_m(e|z)C(e)V \geq 0 \} \].

\(Z^+_m\) is the acceptance-strategy set of the manager: he will undertake the investment if and only if \(z \in Z^+_m\), since these signals imply a rise in his expected utility.

Consider now an arbitrary investor \(i \in I\) who is a stockholder in firm \(k: x_i^k > 0\). He cannot observe \(z\), but the manager acts as his agent. Assume he could specify an acceptance-strategy set \(Z^+_i\) to the manager. Clearly he would choose \(Z^+_i\) such that

\[(13) \quad Z^+_i = \{ z \in Z \big| d[E_iU_i|z] \geq 0 \} \].

Following the steps in (11), we can show

\[d[E_iU_i|z] = \lambda_i x_i^k \sum_e p_i(e|z)C(e)V.\]

Since \(\lambda_i x_i^k > 0\),

\[Z^+_i = \{ z \in Z \big| \sum_e p_i(e|z)C(e)V \geq 0 \} \].

Using (10),

\[Z^+_i = \{ z \in Z \big| \sum_e p_m(e|z)C(e)V \geq 0 \} \]

\[= Z^+_m,\]

from (12). Thus the optimal investment strategy for every investor coincides with the manager's. (QED)

Clearly it is in stockholders' interests to have the manager gather and act upon superior information, since a strategy based on receiving a signal
z will have higher expected utility to all investors than a strategy which is invariant to (ignorant of) z.  

B. Environment II: Market and Project-Specific Information

We now consider the case, in addition to project-specific information (6), the manager has market-state information as well. Let Y be another set of signals related to market-states by the information process

\[ g: S \to Y. \]

As before, we define

\[ S(y) = \{ s \in S \mid g(s) = y \}, \]

and note that for all \( i \in I, \)

\[ p_i(s \mid y) = \frac{p_i(s)}{p_i(y)}, \quad s \in S(y); \]

\[ p_i(s \mid y) = 0, \quad s \notin S(y), \]

where

\[ p_i(y) = \sum_{s \in S(y)} p_i(s). \]

The manager can observe \( y \) and \( z \) before undertaking his investment decision. He will undertake the project if \( d[F_m | y, z] > 0, \) where
\begin{align}
(17) \quad d[E_{m}|y,z] &= \sum_{s,e} p_m(s|y)p_m(e|z)U_m'[R_m(s)]\pi(s)C(e) \\
&= \sum_{e} p_m(e|z)C(e) \left( \sum_{s} p_m(s|y)U_m'[R_m(s)]\pi(s) \right) \\
&= \sum_{e} p_m(e|z)C(e) \left( \frac{1}{p_m(y)} \right) \sum_{s \in S(y)} p_m(s)U_m'[R_m(s)]\pi(s),
\end{align}

where the last line uses (15). Similarly, if an arbitrary investor \( i \) could observe \( y \) and \( z \) before making his investment decision, he would find

\begin{align}
(18) \quad d[E_{i}|y,z] &= \sum_{e} p_m(e|z)C(e) \left( \frac{1}{p_i(y)} \right) \sum_{s \in S(y)} p_i(s)U_i'[R_i(s)]\pi(s).
\end{align}

The investor and manager will agree on the investment decision if and only if \( d[E_{i}|y,z] \) has the same sign as \( d[E_{m}|y,z] \) for all \((y,z) \in (Y \times Z)\). This in turn requires

\begin{align}
(19) \quad \text{sign} \sum_{s \in S(y)} p_i(s)U_i'[R_i(s)]\pi(s) &= \text{sign} \sum_{s \in S(y)} p_m(s)U_m'[R_m(s)]\pi(s)
\end{align}

for all \( y \in Y \). From the portfolio optimization conditions (2), we know

\begin{align}
(20) \quad \sum_{s \in S} p_i(s)U_i'[R_i(s)]\pi(s) &= \lambda_i V,
\end{align}

for all \( i \in I \) including \( i = m \). Although (20) implies (19) in the case \( S(y) = S \) (i.e. the case where the signal provides no information), (20) does not imply (19) when \( S(y) \) is strictly contained in \( S \). Thus, we conclude that "inside" managerial information with respect to market risks
as well as to project-specific risks will not produce stockholder unanimity in general, even when the separability condition holds. But we can prove the following

**Proposition II:** If markets are quasi-complete \((J = S)\) and managers have inside information on both project-specific and market risks, stockholders will unanimously support managers' investment policies.

**Proof of Proposition II:**

Quasi-completeness \((J = S)\) implies separability of returns \((5)\) for any investment. It also implies from \((2)\) that marginal rates of substitution between wealth in any two states is the same for all investors, or

\[
\frac{p_i(s)U_i[R_i(s)]}{\lambda_i} = \frac{p_m(s)U_m[R_m(s)]}{\lambda_m}
\]

for all \(i \in I\) and \(s \in S\). It follows immediately from \((21)\) that, since \(\lambda_i/\lambda_m > 0\), \((19)\) must hold for all \(y \in Y\) and \(z \in Z\). Condition \((19)\) was shown to imply unanimity. \((Q.E.D.)\)

V. Conclusion

Stockholder unanimity plays a key role in the theory of production under uncertainty. But previous models have required strong assumptions to assure unanimity, conditions which at the same time have minimized the role of the manager.

Both the identical expectations assumptions underlying the capital asset pricing models, and the spanning conditions invoked by Ekern and Wilson [1974] and others, seem unlikely to be satisfied. Informational differences preclude identical expectations. And investment projects or
other decisions changing returns typically contain project-specific risks which are not present in the market.

We have shown that stockholder unanimity nonetheless will result, if managers have superior information about project-specific risks and project returns are separable. Separability implies that project returns can be decomposed into two independent elements: an element which depends upon the state of the market as a whole, as reflected in securities' returns; and an element which depends upon the independent, project-specific state.

The separability of returns into market risks and independent risks bears resemblance to models considered by Sharpe [1963] and Malinvaud [1973]. These models and ours are consistent with the existence of underlying factors, of which only a subset affects market returns as a whole. In such environments, Malinvaud has shown that complete contingent claim markets are not required for efficient exchange. Our analysis has a similar implication for production theory: complete markets or spanning are not required for stockholder unanimity, and hence for a theory of production under uncertainty.

In contrast with previous models of stockholder unanimity, the environment we have examined is consistent with the manager having an active information-gathering and decision-making role. The manager's project-specific information is useful for all stockholders, in the sense that stockholders would prefer to base their decisions on information rather than to act without it. The information, however, may be costly to communicate to the stockholders. To avoid bearing these communication costs,
but to utilize the manager's superior information, stockholders grant the manager decision-making powers. In the simple environment we have considered, the manager will choose an investment strategy based on his inside information which will be optimal for all stockholders.
FOOTNOTES


1 We refer to the problem of aggregating preferences which was studied by Arrow [1963]. A coherent theory of the firm may exist if the manager is assumed to be a dictator: see Sandmo [1971] and Leland [1972b]. Also, an interesting recent paper by Hart [1977] suggests some game-theoretic notions of equilibrium which do not require stockholder unanimity.

2 "Ex post" unanimity refers to unanimity towards marginal changes when investors' portfolios are optimal given current production decisions. "Ex ante" unanimity refers to unanimity towards marginal changes when investors' portfolios are arbitrary. Leland [1973] shows that the conditions which assure "ex ante" unanimity imply that optimal decisions maximize the stock market value of the firm. Here we study ex post unanimity only.

3 Although we develop our results with a finite number of states of nature, the analysis is readily extended to an infinite number of states.

4 Under slightly stronger conditions, an optimal portfolio satisfying (2) can be shown to exist: see Leland [1972a].

5 We assume firms which are financed entirely by equity, or by equity and risk-free debt. If debt is risky, our presumption that the decision affects only the returns of shareholders is unrealistic.

6 If there exists a riskless asset, then a special case of (5) is

\[ I(s,e) = \pi(s)C + h(e), \]

where \( C \in \mathbb{R}^J \). In this case, the investment project's returns can be decomposed into the sum of a market risk and an independent project-specific risk.

7 If, as argued by Harsanyi [1967], different subjective probabilities follow solely from different information, then

\[ p_{i}(e) = p(e|I_i), \]
where $I_i$ is the information received by investor $i$. Assume that a signal $z \in Z$ is "superior," in that it is a sufficient statistic for any $I_i$. Then

$$p(e|I_i, z) = p(e|z) \quad \text{for all} \quad i \in I,$$

which implies (10).

8 It also follows that if implicit personal contingency claim prices are competitive (see Leland [1973]), the strategy $Z_i^+$ will also maximize the value of the firm's stock.

9 Our argument would not be altered if the manager received the signal $y \in Y$ before making his portfolio (as well as investment) decision. In this case, we simply substitute $p_m(s|y)$ for $p_m(s)$ in (17), (19), (20), and (21), leaving $p_i(s)$ unchanged for $i \neq m$. Assuming the manager is "small" relative to the market, the prices of securities $V$ will be in- variant to the signal $y$. 
REFERENCES


