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THE PARETO-OPTIMAL DESIGN OF
TERM LIFE INSURANCE CONTRACTS

BY

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I. INTRODUCTION

Numerous investigations have been directed toward aspects of rational life insurance purchases and optimal coverage levels under differing conditions.¹ These studies have taken as "given" the design of life insurance contracts and have focused on optimal consumer responses to available insurance opportunities.²

The present study represents a departure from this tradition in that contract design is explicitly considered. In this paper we show, under very general conditions, how life insurance contracts can be designed which lead to increases in the welfare of insurance consumers, companies, and salespersons (i.e., insurance agents). Unlike most studies, which indicate that less than full coverage is optimal when a positive loading factor is incorporated into insurance rates, we show that full coverage is quite plausible under a positive loading factor, provided that the load is incorporated into insurance rates according to the manner herein specified. Another new result of this paper is that insurance consumers will desire higher coverage levels, even though company profits and agent commissions are higher. Associated with this higher coverage is an equal or higher level of consumer welfare. Both of these results stem from the imposition of a policy fee whose magnitude is determined in accordance with the procedures outlined in this paper.
The organization of this paper is as follows. First, a theoretical framework is introduced which will be used in examining the behavior of the major parties involved in life insurance policies. Next, consumer behavior is studied within this framework, followed by a discussion of firm behavior. A set of Pareto-optimal points is then derived which forms the contract curve, and it is shown that life insurance contracts without a policy fee are Pareto-inferior to those featuring such a fee. Finally, the welfare of the insurance agent is considered and a sharing scheme is set forth by which all parties may derive increased welfare. The ultimate allocation of the welfare gain among all parties is, of course, subject to bargaining negotiations and market competition.

II. THE MODEL

The setting is a single-period two-state world in which there are three agents: a consumer, a firm, and an insurance salesperson. The consumer faces uncertainty in that his income varies beyond his control. This uncertainty arises from the unknown length of the consumer's life.

The two states in our world are denoted $S_1$ and $S_2$. If state $S_1$ is realized ($S = S_1$) the consumer has income $Y = W + H$. If state $S_2$ is realized ($S = S_2$) the consumer has income $Y = W$. The interpretation is clear. In state $S_1$ the consumer survives and earns income $H$ over and above his endowed wealth $W$. In state $S_2$ the consumer does not survive and has only $W$, his endowed wealth.
It will be assumed that the probabilities of occurrence of the states $S_1$ and $S_2$ are well defined and beyond the control or influence of all parties. Accordingly, the probabilities of states $S_1$ and $S_2$ are objective probabilities given from outside and well known. Let $\pi$ denote the probability of the event $S = S_2$; then $1 - \pi$ is the probability of the event $S = S_1$.

The consumer is endowed with a Von Neumann-Morgenstern utility function, $U(\cdot)$, defined on lotteries. In particular, his utility will be assumed to be state-dependent, represented by function $V$ if $S = S_1$ and by function $B$ if $S = S_2$.

Let the following insurance contract be offered by the firm:

\[
\begin{align*}
    k &= -P & \text{if } S = S_1 \\
    k &= -P + I & \text{if } S = S_2
\end{align*}
\]

(1)

The firm offers to cover the consumer at an amount $I$ if he dies. In return the consumer pays premium $P$ to the firm in both states of the world.

The income of the consumer that accepts the contract is:

\[
\begin{align*}
    Y &= W + H - P & \text{(if } S = S_1) & \text{with probability } 1 - \pi \\
    Y &= W + I - P & \text{(if } S = S_2) & \text{with probability } \pi
\end{align*}
\]

(2)

The above income prospects constitute the formal definition of a lottery, which will be denoted lottery $L$. Given the lottery $L$ the expected utility of the consumer when he has an insurance contract is:
U(L) = (1 - \pi) V(W + H - P) + \pi B(W + I - P) \tag{3}

If we define another lottery L' as the income prospects available to the consumer who does not buy the insurance contract,

\[ Y = W + H \quad (\text{if } S = S_1) \quad \text{with probability } 1 - \pi \tag{4} \]

\[ Y = W \quad (\text{if } S = S_2) \quad \text{with probability } \pi \]

then the expected utility of this consumer is:

\[ U(L') = (1 - \pi) V(W + H) + \pi B(W). \tag{5} \]

III. CONSUMER BEHAVIOR

Assume that consumers maximize their Von Neumann-Morgenstern expected utility function \( U(.) \), given the contract \( P = f(I) \) offered by the firm. The consumer chooses the optimal coverage, \( I \), that maximizes (3), given \( P = f(I) \), provided that \( U(L) > U(L') \). We can rewrite (3) in terms of the decision variables, \( I \) and \( P \), as:

\[ U(L) = U(I, P) = (1 - \pi) V(W + H - P) + \pi B(W + I - P) \tag{6} \]

Making the usual assumptions that utility increases with wealth, but at a diminishing rate, i.e., \( V' > 0, V'' < 0, B' > 0 \), and \( B'' < 0 \), we can determine the signs of the partial derivatives that follow.

\[ \frac{\partial U}{\partial I} = \pi B'(W + I - P) > 0 \]

\[ \frac{\partial U}{\partial P} = -(1 - \pi) V'(W + H - P) - \pi B'(W + I - P) < 0 \]
\[ \frac{\partial^2 U}{\partial I^2} = \pi B''(W + I - P) < 0 \]

\[ \frac{\partial^2 U}{\partial P^2} = (1 - \pi)\nu''(W + H - P) + \pi B''(W + I - P) < 0 \]

\[ \frac{\partial^2 U}{\partial I \partial P} = -\pi B''(W + I - P) > 0 \]

These partial derivatives are represented graphically below in figure 1.

CONSUMER UTILITY AS A FUNCTION OF INSURANCE COVERAGE AND PREMIUM LEVEL

The consumer's problem is to select the pair \((I, P)\) that maximizes (6). The pair \((I, P)\) that maximizes (6) subject to the insurance contract provisions must follow

\[ \frac{1}{dP/dI} = \left. \frac{dI}{dP} \right|_{\text{contract}} = \left. \frac{dI}{dP} \right|_{U} \]  

provided that the maximization problem is concave. Note that along any indifference curve \( U(I, P) = \bar{U} \) of the consumer,
\[
\left. \frac{dI}{dP} \right|_U = 1 + \frac{1-\pi}{\pi} \cdot \frac{V'(W+H-P)}{B'(W+I-P)}
\]

and similarly,

\[
\left. \frac{dP}{dI} \right|_U = \frac{\pi B'(W+I-P)}{(1-\pi)V'(W+H-P) + \pi B'(W+I-P)}
\] (8)

We now prove that the indifference curves are concave in the (I,P) plane. The proof proceeds as follows. First we prove that the indifference curves \( U(I,P) = \bar{U} \) are convex in the (P,I) plane, i.e., \( \frac{d^2 P}{dI^2} \bigg|_U > 0 \). Subsequently we prove that if \( \frac{dP}{dI} \bigg|_U > 0 \), then \( \frac{d^2 P}{dI^2} \bigg|_U > 0 \) implies that \( \frac{d^2 I}{dP^2} \bigg|_U < 0 \).

Differentiating \( dI/dP \) with respect to \( P \) we derive,

\[
\left. \frac{d^2 I}{dP^2} \right|_U = \frac{1-\pi}{\pi} \cdot \frac{1}{B'(W+I-P)} \left( -B'(\cdot)V''(\cdot) - V'(\cdot)B''(\cdot) \left( \frac{dI}{dP} - 1 \right) \right)
\]

\[
= \frac{1-\pi}{\pi} \cdot \frac{1}{B'(W+I-P)} \left( -B'V'' - V'B'' \cdot \frac{1-\pi}{\pi} \cdot \frac{V'}{B'} \right) > 0 \quad (9)
\]

which is clearly positive since all of its multiplicative terms are positive. Noting \( P = f(I) \), then \( dP/dI = f'(I) \). If \( f'(I) > 0 \) (and this is fulfilled in our case), the inverse function \( I = f^{-1}(P) \) exists and is well defined. Then \( \frac{dI}{dP} \bigg|_U = (f^{-1})'P = \frac{1}{dP/dI} \bigg|_U = \frac{1}{f'(I)} \) and

\[
\left. \frac{d^2 I}{dP^2} \right|_U = \frac{d}{dP} \left[ 1/f'(I) \right] = \left. \left( -f''(I) \cdot \frac{dI}{dP} \right) \right|_U = -f''(I) \left. \left( \frac{dI}{dP} \right) \right|_U = \left. \left( \frac{d^2 P}{dI^2} \right) \right|_U.
\]
Since \( \frac{dP}{dI} > 0 \), sign \( \frac{d^2I}{dP^2} \bigg|_U \) = - sign \( \frac{d^2P}{dI^2} \bigg|_U \). Therefore

\[
\frac{d^2I}{dP^2} \bigg|_U > 0 \text{ implies } \frac{d^3P}{dI^3} \bigg|_U < 0.
\]

Thus, the indifference curves are concave in the \((I,P)\) plane.

A pair \((I^*,P^*)\) that solves (7) will be the optimum for the consumer provided that

\[
U(I^*,P^*) > U(0,0), \quad (10)
\]

as entering into no contract remains an option of the consumer. In terms of indifference curves, relation (10) says that \((I^*,P^*)\) should lie on an indifference curve representing higher utility than \((0,0)\), the position of no coverage (and no cost) for the consumer.

Having delineated the conditions that give rise to optimal coverage, we turn our attention to how the optimal level of insurance varies with changes in human wealth, \(H\), nonhuman wealth, \(W\), and insurance rate structure. Consider the commonly offered linear contract \(P = f(I)\) where \(\frac{dP}{dI} = f'(I) = \frac{1}{m}\). Observe that for all linear contracts the maximization problem is concave. Then (7) may be rewritten as

\[
m = 1 + \frac{\frac{m-1}{m} \cdot V'((W+H-P) \cdot B'(W+I-P))}{V'((W+H-P) \cdot B'(W+I-P))}
\]

which implies
\[(m - 1) πB'(W + I - P) = (1 - π)V'(W + H - P)\]
\[(1 - π)V'(W + H - P) + (1 - m)πB'(W + I - P) = 0.\]

Observe that \( m > 1. \)

Let \( f(W,H,P,I,m) = (1 - π)V'(W + H - P) + (1 - m)πB'(W + I - P). \)

Then \( f(W,H,P*,I*,m) = 0 \) for all utility maximizing points \((I*,P*).\)

The following partial derivatives will be used:

\[\frac{∂f}{∂W} = (1-π)V''(•) + (1-m)πB''(•)\]
\[\frac{∂f}{∂H} = (1-π)V''(•) < 0\]
\[\frac{∂f}{∂I} = (1-π)V''(•)(-dP/dI*) + (1-m)πB''(•)(1-dP/dI*) > 0\]
\[\frac{∂f}{∂m} = -B'(•) < 0.\]

Now the manner in which optimal insurance coverage varies with human wealth is given by the sign of \(dI*/dH.\)

\[
\frac{dI*}{dH} = -\frac{\frac{∂f}{∂H}}{\frac{∂f}{∂I*}} = -\frac{(1-π)V''}{(1-π)V''(-\frac{1}{m}) + (1-m)πB''(1 - \frac{1}{m})} = \frac{m(1-π)V''}{(1-π)V'' + (1-m)^2πB''} > 0.\]

Thus, the optimal amount of insurance is increasing in the human wealth. Insurance coverage will increase less than the full amount of a rise in human wealth if \(dI*/dH < 1,\) which will be the case if\(^6\)

\[-\frac{V''(•)}{V'(•)} < -\frac{B''(•)}{B'(•)}.\] (11)
Therefore, the optimal coverage increases with human wealth, but its increase is smaller than the increase in human wealth if (11) is true, which is interpreted as the absolute risk aversion at the "living" wealth (after coverage) being lower than the absolute risk aversion of the "bequest" wealth (after coverage).

Optimal insurance coverage will also vary with nonhuman wealth levels in accordance with the sign of \( dI* / dW \).

\[
\frac{dI*}{dW} = \frac{\partial f}{\partial I*} = \frac{(1-\pi)V'' + (1-m)\pi B''}{(1-\pi)V'' + (1-m)\pi B''} = \frac{m}{\pi (m-1)B'' + (1-\pi)V''}.
\]

The sign of the above derivative will be negative if

\[-\frac{V''(*)}{V'(*)} < \frac{B''(*)}{B'(*)}.
\]

Therefore, the optimal coverage decreases in nonhuman wealth if (11) holds.

Finally we examine how optimal insurance varies with changes in the rate structure. From the partial derivatives given earlier, we know

\[
\frac{dI*}{dm} = -\frac{\partial f / \partial m}{\partial f / \partial I*} > 0.
\]

We use this information to determine that

\[
\frac{dI*}{d\left(\frac{1}{m}\right)} = \frac{dI*}{dm} \cdot \frac{d\left(\frac{1}{m}\right)}{dm} = \frac{dI*}{dm} \cdot \frac{1}{d\left(\frac{1}{m}\right)} = -m^2 \frac{dI*}{dm} < 0.
\]
This indicates that as the slope of the contract in the \((I,P)\) plane decreases, the optimal insurance coverage should increase, as shown below in figure 2. This finding is a direct consequence of the concavity of the indifference curves.

![Figure 2: Optimal Coverage as a Function of the Rate Structure](image)

**FIGURE 2**

**OPTIMAL COVERAGE AS A FUNCTION OF THE RATE STRUCTURE**

IV. FIRM BEHAVIOR

We assume that the firm's objective is to maximize expected profits. This is a rather strong assumption which appears to be at variance with the theory of the firm, under which the maximization of firm value is heralded to be a more suitable objective. Only under certain conditions will maximization of expected profits lead to maximum firm value. Main [1981] has shown that in the case of an insurance company (and more particularly in the case of a life insurance company) these conditions are approximately met. Total risk to the firm deriving
from underwriting operations is very low and can be reduced substantially through operation of the Law of Large Numbers (see, for example, Sharpe [1978, pp. 82-84]). What little insurance risk remains is almost certainly "unsystematic" risk which should be irrelevant to the firm's shareholders in a perfect capital market. Accordingly, maximization of expected profits converges toward the maximization of the value of the firm.

Expected profits associated with contract of coverage I and premium P are

\[ M(I, P, C) = -C - \pi I + P(1 - \lambda). \]  \hspace{1cm} (12)

The marketing agent (salesperson) is paid \( \lambda P \), \( C \) is a fixed administrative cost associated with each policy, and \( \pi I \) is the actuarial cost. The iso-profit curves are derived as follows:

Let \[ M(I, P, C) = A \]

\[ M(I', P', C') = B \] \hspace{1cm} (13)

Assume \( A = B \). Then

\[ -C' - \pi I' + P'(1 - \lambda) = -C - \pi I + P(1 - \lambda) \]

\[ \therefore \quad \pi'(I' - I) = [P'(1 - \lambda) - C'] - [P(1 - \lambda) - C]. \] \hspace{1cm} (14)

Let \[ P_N = P(1 - \lambda) - C \]

\[ P'_N = P'(1 - \lambda) - C'. \]

Here \( P_N \) represents the receipts of the firm net of marketing and administrative costs. Then (14) implies
\[ \pi(I' - I) = p_N^* - p_N \iff \frac{\Delta I}{\Delta p_N} = \frac{1}{\pi} \]

\[ \iff p_N = \alpha + \pi I \iff p(1-\lambda) - C = \alpha + \pi I \]

\[ \therefore p = (\alpha + C + \pi I)/(1-\lambda). \quad (15) \]

Therefore, the iso-profit curves of the firm in the (I,P) plane are straight lines with slope \( \pi/(1-\lambda) \) and varying intercept. Expected profits are given by \( \alpha \):

\[ M(I,P,C) = -C - \pi I + p(1-\lambda) \]

\[ = p_N - \pi I = \alpha. \quad (16) \]

V. PARETO-OPTIMAL POINTS

We begin by formally defining Pareto-optimality for two agents.\(^9\)

**Definition 1:** Let agent 1 be endowed with an objective function \( U(I,P) \) and agent 2 be endowed with an objective function \( M(I,P) \). Point \((I^*, P^*)\) is called "Pareto-optimal" if and only if for all \((I, P)\) such that \( U(I, P) > U(I^*, P^*) \) it is implied that \( M(I, P) < M(I^*, P^*) \) and for all \((I, P)\) such that \( M(I, P) > M(I^*, P^*) \) it is implied that \( U(I, P) < U(I^*, P^*) \).

**Definition 2:** Assuming the same objective functions as above, \((I^*, P^*)\) is Pareto-optimal if and only if \( U(I^*, P^*) \geq U(I, P) \) for all \((I, P)\) such that \( M(I, P) = M(I^*, P^*) \) and \( M(I^*, P^*) \geq M(I, P) \) for all \((I, P)\).
such that \( U(I,P) = U(I^*,P^*) \). The interpretation of the first definition is that a point is Pareto-optimal if and only if no agent can be made better off without making another agent worse off. The interpretation of the (equivalent) second definition is that a point is Pareto-optimal if and only if each agent maximizes utility at that point subject to the utility of all other agents being held constant. We will use the latter definition.

We will find Pareto-optimal points by letting the firm maximize its expected profits, \( \bar{M} \), subject to the condition that the consumer remains on the same indifference curve. Later, by changing parametrically the curve we will trace the entire Pareto-optimal frontier.

Given that the indifference curves for the consumer are concave in the \((I,P)\) plane and that the iso-profit curves for the firm are straight lines, the Pareto-optimal points will be specified by the tangency between an iso-profit curve and an indifference curve, as shown in figure 3 below. Such a tangency is specified by the condition:

\[
\frac{dI}{dP} \bigg|_{U(I,P)} = \frac{dI}{dP} \bigg|_{M(I,P)} = \bar{M}
\]

\[
1 + \frac{1-\tau}{\pi} \frac{V'(W + H - P)}{B'(W + I - P)} = \frac{1-\lambda}{\pi}
\]

\[
[\pi B'(W + I - P) + (1-\tau)V'(W + H - P)]/(1-\lambda) = B'(W + I - P)
\]

\[
\therefore B'(W + I - P) \cdot (1-\lambda-\pi) = V'(W + H - P) \cdot (1-\pi)
\]
where $P$ is defined by (15). Equation (17) specifies a Pareto-optimal point, $(I,P)$. Varying the amount of profit $M$, say through $\alpha$, the entire locus of Pareto-optimal points, i.e., the Pareto-optimal frontier, can be traced. We now present some examples of Pareto-optimal contracts under special cases.

![Diagram showing Pareto-optimal points](image)

**FIGURE 3**

**PARETO-OPTIMAL POINTS**

**Example 1: No Marketing Costs**

Say $\lambda = 0$, i.e., that there is no marketing cost (insurance agent fee). Then the contract is

$$P = \alpha + C + \pi I$$

(18)

Condition (17) still holds, with $P$ defined by (18). Condition (17) implies that $B'(W + I - P) = V'(W + H - P)$, that is, at the
Pareto-optimal point, insurance is purchased up to an amount that equalizes the marginal utility of wealth in states $S_1$ and $S_2$.

If $B' = V'$, i.e., if the bequest utility function $B$ is a translation of the "living" utility function $V$, then in this case $I^* = H$. That is, the consumer will buy full coverage. Note that the above results do not depend on the amount of profits that the company makes, as long as the consumer is not pushed to an indifference curve of lower utility than the indifference curve of no coverage.

**Example 2: Zero Expected Firm Profits**

Assume that the firm expects zero profits (say because of intense competition among firms). Then $M = \alpha = 0$. The contract is

$$P = \frac{C + \pi I}{1-\lambda}. \quad (19)$$

Condition (17) still holds with $P$ defined by (19).

**Example 3: No Fixed Costs**

Assume that $C = 0$, i.e., that there are no fixed costs per contract. Then the contract is

$$P = \frac{\alpha + \pi I}{1-\lambda}. \quad (20)$$

Condition (17) still holds, with $P$ defined by (20).

**VI. SUBOPTIMALITY OF CONTRACTS WITH NO POLICY FEE**

Suppose the contract

$$P = \mu I \quad (21)$$
is offered. We will show that such a contract results in a Pareto-inferior position. It is dominated by a contract of the form

\[ P = \frac{\alpha + C + \pi I}{1-\lambda} \]  

(15)

that results in a Pareto-optimal position.

Assume that the consumer and firm are at a position \((I^*, P^*)\) where (15) and (17) hold. Then by the proof of Pareto optimality it follows that (21) gives less to one of the agents. If the consumer is held at the same indifference curve \(U(I, P) = U(I^*, P^*)\), then the firm loses money by moving to the new contract, as shown in figure 4 below.
In figure 4, OE gives the actuarially fair cost of insurance \( \pi_I \), for all levels of coverage, I. AA' depicts the iso-profit line of zero expected firm profits, where \( C = (1-\lambda)OA \) and the salesperson's commission rate \( \lambda \) is implicit in the slope of the line, which is steeper than OE. The Pareto-optimal point \((I^*,P^*)\) is attained where the indifference curve is tangent to CC', which is the contract given by (15) where the expected profit \( \alpha = (1-\lambda)AC \). The alternative contract given by (21) is represented by OD. Note that the expected profit associated with this contract is given by \((1-\lambda)\) of the vertical distance between the zero iso-profit line AA' and the (parallel) iso-profit line BB' passing through the new tangency point of the indifference curve with the contract line (21). Clearly the expected profit deriving from this alternative contract, \((1-\lambda)AB\) is less than that expected from the Pareto-optimal contract, \((1-\lambda)AC\) and yet the consumer is no better off.

This is a direct result of the concavity of the indifference curve for the consumer contrasted with the linearity of the iso-profit curves of the firm. In some sense the whole society loses by moving from contract (15) to contract (21). The risk averse agent is not adequately covered, given the features of the risk neutral agent. A risk neutral agent can make profit by offering a contract that would move the consumer from position \((I,P)\) to position \((I^*,P^*)\).

Another finding of note is that in addition to the firm receiving higher profits under a Pareto-optimal contract of form (15) while holding the insurance consumer's utility level constant, the insurance
agent, whose total commission is \( \lambda P \) under the typically offered contract (of form (21)) receives a windfall gain from the Pareto-optimal contract design (of form (15)), which provides a commission \( \lambda P^* > \lambda P \). As there is no reason why the insurance agent should necessarily reap all of this welfare gain, we next examine the sharing rules among the firm and salesperson.\(^{10}\)

VII. SHARING RULES

From the total premium the firm keeps \( G(P, \lambda) \) and the agent receives the remainder, \( P - G(P, \lambda) \). Let \( M = -\pi I + G(P, \lambda) - C \) denote the profit received by the firm and \( A = -J + P - G(P, \lambda) \) denote the net commission received by the insurance agent, where \( J \) is a measure of the agent's effort expended to obtain the sale. Together these sum to \( A + M = -\pi I + P - J - C \). At a Pareto-optimal point,

\[
\frac{dA}{d\lambda} = -\frac{dM}{d\lambda}, \text{ i.e., } \frac{d(A+M)}{d\lambda} = 0. \tag{22}
\]

Now, \( \frac{d(A+M)}{d\lambda} = -\pi \cdot \frac{dI}{dP} \cdot \frac{dP}{d\lambda} + \frac{dP}{d\lambda} = \frac{dP}{d\lambda} (1 - \pi \cdot \frac{dI}{dP}) \). For this quantity to equal zero requires either that \( \frac{dP}{d\lambda} = 0 \) or that \( 1 = \pi \cdot \frac{dI}{dP} \). Totally differentiating (20) we derive:

\[
B''(1-\lambda-\pi) \left( \frac{dI}{dP} - 1 \right) \frac{dP}{d\lambda} - B' - (1-\pi)V'' \left( - \frac{dP}{d\lambda} \right) = 0. \tag{23}
\]

Solving (23) for \( B'(\cdot) \) gives
\[
B'(\cdot) = \frac{dP}{d\lambda} \left( B''(1-\lambda - \pi) \frac{dI}{dP} - 1 \right) + (1-\pi)V''.
\]

(24)

Since \( \frac{dI}{dP} > 1 \) at equilibrium, and as \( B'' < 0, V'' < 0, \) and \( B' > 0, \) we have \( \frac{dP}{d\lambda} < 0 \) if \( (1-\lambda - \pi) > 0. \) Therefore, if \( (1-\lambda - \pi) > 0, \)
\( \frac{dP}{d\lambda} \neq 0 \) and \( \frac{d(A+M)}{d\lambda} = 0 \) must imply \( 1 = \pi \cdot \frac{dI}{dP} \) which in turn
implies \( \frac{dP}{dI} = \pi. \) Thus, \( P = a' + C + \pi I \) is the form of a Pareto-
optimal contract offered by the firm.

In the arrangement where \( G(P,\lambda) = (1-\lambda)P, \) this corresponds to
\( \lambda = 0. \) It will give \( A = -J \) to the insurance agent and is therefore
unacceptable.

A Pareto-optimal arrangement will give a "lump sum" "X" to the
agent per contract: \( P = a + C + \pi I + X. \) Now \( A = -J + X. \) Clearly \( X \)
must not be smaller than \( J. \) The amount \( X \) is an item of bargaining
between the agent and the firm. The amount should be at least as large
as \( \lambda P \) for the agent to be as well off as he was with the Pareto-
inferior contract.

An important observation is that under a Pareto-optimal contract
of form \( P = a' + C + \pi I, \) where firm profitability and agent com-
misions are embedded in the \( a' \) component of the "policy fee" (of mag-
nitude \( a' + C), \) full insurance coverage will be purchased by an ex-
pected utility maximizing consumer, provided that (1) \( B' = V' \) (i.e.,
the bequest utility function \( B \) is a translation of the "living" util-
ity function \( V \)) and (2) the consumer is not pushed to an indifference
curve of lower utility than the indifference curve of zero coverage (see
example 1, Section V).
VIII. CONCLUDING REMARKS

The problem of insurance has attracted considerable attention in the economic literature over the past thirty years, and it is surprising that the Pareto-optimal design of life insurance contracts has not heretofore been addressed. Perhaps a reason for this oversight is that insurance is typically examined in state space rather than the \( (I,P) \) space employed here. The same results can be achieved in state space analysis, although they are not as readily apparent, and some of the results are more cumbersome to depict diagrammatically. Rather than replicate all of our results in state space, we will here only provide a diagrammatic overview of the problem. The initial part of the exposition follows closely that given by Klein [1975].

In figure 5 the endowed wealth position of the consumer in state space is given by point A, where \( W + H \) measures claims to consumption if state \( S_1 \) is revealed and \( W \) measures consumption claims if state \( S_2 \) is revealed. The 45° line is termed the certainty line. If a family holds claims somewhere along this line, their consumption status would be unaffected by the breadwinner's mortality status. Typically \( H > 0 \) so that the initial endowment point A lies beneath the 45° line.

Indifference curve 1 is simply the locus of wealth level combinations across states that yield the same level of expected utility as the endowment point A. Diminishing marginal utility is sufficient to insuring convexity of the indifference curves. Beginning at point A is the line segment AB, which represents the family's opportunity set,
assuming that it can purchase life insurance at a price equal to the reciprocal of the negative of the slope of AB. To be concrete, suppose that consumption claims in one state can be traded for consumption claims in the other state at a price of unity. The slope of AB is minus unity. Further assume that \( \pi = 1 - \pi = .5 \). Under these conditions,
the purchase of insurance would amount to the acceptance of an actuarially fair bet, i.e., one whose expected return is zero. It is well known that under these conditions the optimum for an individual with a unique utility function would be along the certainty line and the pictured consumer would move to point D and achieve a level of expected utility denoted by the higher indifference curve 2. He would give up \( AE = P \) units of income in state \( S_1 \) for \( ED = I - P \) units of income in state \( S_2 \).

If the insurance were actuarially unfair in the usual sense (where the slope of the contract is altered), the slope of AB would be less negative, and the optimum would no longer occur along the certainty line, but below it. Such a contract would not be Pareto-optimal, however. A Pareto-optimal contract in state space would consist of a horizontal line segment moving leftward from point A, for example AF, to account for the policy fee (which will be divided among fixed administrative costs, sales commissions, and profits) and a rising line segment such as FG parallel to the actuarially fair contract line AB. In the case of a unique utility function as shown here, full insurance would be purchased for any parallel contract passing through points lying between C and D.

If one of society's goals is to promote higher levels of private insurance coverage, while permitting consumers free choice as to the levels of coverage purchased, then the contract design proposed here may be one way of promoting these objectives, provided that people are risk averse and behave rationally (in the sense of maximizing expected
utility). The beleaguered insurance industry and struggling life insurance agents could only benefit by the proposed scheme, and consumers would not only purchase higher levels of coverage, but would likely have their welfare improved. The reader should be reminded, however, that no general equilibrium analysis has been performed here, and that these final observations are only provisional in nature.
REFERENCES


FOOTNOTES


\(^{2}\) These and other studies have generally shown that term insurance is the optimal form of life insurance. Indeed, as all available forms of life insurance are linear combinations of one-period term insurance and a savings plan of some sort, there is no loss of generality in focusing on term insurance, as will be done in this paper.

\(^{3}\) There is one (theoretical) exception: if the insurer incurs no marketing or administrative costs but only the actuarial costs of insurance, then one of the Pareto-optimal points would have a zero policy fee.

\(^{4}\) To avoid unnecessary complications that do not affect the major conclusions of this paper, the model employed in this presentation is timeless. A single-period, two-date model could be developed by redefining H and I as the present certainty-equivalent values of human wealth and the death benefit to be received at the end of the period, conditional upon obtaining the associated state of the world.

\(^{5}\) Under the earlier stated assumption that the objective probabilities of the occurrence of states \(S_1\) and \(S_2\) are given from outside
and known, well defined and beyond the influence of all parties, a linear contract is a reasonable form of a policy to offer. The questions relating to Pareto-optimality concern its slope and intercept.

The derivation is as follows:

\[
\frac{dI^*}{dH} < 1 \iff \frac{m(1-\pi)V''}{(1-\pi)V'' + (1-m)^2\pi B''} < 1 \\
\iff m(1-\pi)V'' > (1-\pi)V'' + (1-m)^2\pi B'' \\
\iff (m-1)(1-\pi)V'' > (1-m)^2\pi B'' \\
\iff (1-\pi)V'' > (m-1)\pi B'' \\
\iff (1-\pi)V'' > \frac{1-\pi}{\pi} \cdot \pi B'' \cdot \frac{V'}{B'} \\
\iff \frac{V''}{V'} > \frac{B''}{B'} \iff -\frac{V''(\ast)}{V'(\ast)} < -\frac{B''(\ast)}{B'(\ast)}.
\]

The derivation is as follows:

\[
\frac{dI^*}{dW} < 0 \iff (1-\pi)V'' - (m-1)\pi B'' > 0 \\
\iff (1-\pi)V'' > \frac{1-\pi}{\pi} \cdot \frac{V'}{B'} \cdot \pi B'' \\
\iff \frac{V''}{V'} > \frac{B''}{B'} \iff -\frac{V''(\ast)}{V'(\ast)} < -\frac{B''(\ast)}{B'(\ast)}.
\]
See, for example, Rubinstein [1973] and Haley and Schall [1979]. This is not to say that life insurance companies do not exhibit systematic risk; rather, that the systematic risk is unlikely to derive from underwriting operations per se. Indeed, company profits may be affected by general economic conditions. In particular, as wages go up consumers may demand greater coverage that could lead to higher profits. A more important source of systematic risk for life insurance companies is likely to derive from mismatching the "duration" of the firm's assets with that of its liabilities (see Grove [1974]). However, these considerations are properly separated from the underwriting problem and do not concern us here.

It should be noted that the unsystematic risk remaining may be relevant to the consumer, who might discount the promised benefit payment for its default risk. We assume here that the issued capital and retained earnings are sufficient to cover any "blips" in the claims distribution such that the promised death benefit can be regarded with certainty. Alternatively, reinsurance or a large insurance mutual fund could accomplish essentially the same result.

We ignore for the moment the third agent (salesperson) to facilitate the discussion. The third agent to the problem is discussed in Section VII.

Up to this point we have held the consumer's utility constant while increasing the expected profits of the firm (and commissions of the salesperson). Of course, there is no particular reason why the insurer (and salesperson) should capture all the gain arising from
Pareto-optimally designed life insurance contracts. Indeed, at the other extreme, sales commissions and profitability could have been held constant while the consumer reaped all of the gain deriving from the new contract design. More likely, the ultimate allocation of the gain will be determined by bargaining among the agents, with each of the three agents capturing a portion of the gain.