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THE IMPACT OF THE GOVERNMENT SECTOR ON FINANCIAL EQUILIBRIUM AND CORPORATE FINANCIAL DECISIONS
BY
SASSON BAR-YOSEF
YORAM LANDSKRONER

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THE IMPACT OF THE GOVERNMENT SECTOR ON FINANCIAL EQUILIBRIUM AND CORPORATE FINANCIAL DECISIONS

by

Sasson Bar-Yosef and Yoram Landskroner
School of Business Administration
Hebrew University of Jerusalem

and

School of Business Administration
University of California, Berkeley

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Research Program in Finance
Graduate School of Business Administration
Institute of Business and Economic Research
University of California at Berkeley
The mean-variance Capital Asset Pricing Model (CAPM), derived initially by Sharpe and Lintner, considers only the private sector of the economy and is based on a number of explicit and implicit assumptions. One of these assumptions, which is related to the fact that the government sector does not enter into the analysis, is that there are no taxes, either corporate or personal. A number of studies have relaxed this assumption and have incorporated the effects of taxation into the determination of financial equilibrium and valuation of the firm. These studies are, however, incomplete since they consider only one aspect of the economic role of the government—that of a taxing agent—while ignoring its complementary role of wealth distribution. These studies consider taxes exogenous to the system.

The fundamental valuation theorem derived by Modigliani and Miller (hereafter MM), [1958] is that given complete and perfect markets, capital structure will have no effect on the value of the firm. In a later paper, MM [1963] focus on the effects of corporate taxation and conclude that the tax system introduces a market imperfection that affects the value of the firm, which becomes an increasingly linear function of financial leverage. Two subsequent studies, Hamada [1969] and Rubinstein [1973], deal with the tax effects on the value of the firm, but, unlike MM, their analysis was carried out in the context of financial equilibrium. Hamada was the first to derive MM's propositions on capital structure and the cost of capital, with and without corporate taxes, in the mean-variance framework of the CAPM. Similarly,
Rubinstein provides proof of MM propositions for the case of risky debt; this differs from MM who assumed that firms issue only risk-free debt. The result of MM, as well as that of the subsequent studies mentioned above, is that the gain in value from leverage is given by the product of the corporate tax rate and the market value of the firm's debt. Miller [1977] modifies this result by introducing personal as well as corporate taxes into the analysis. He assumes two types of personal tax rates: the tax rate on income received from holding stocks, and the higher rate on income from bonds. The implication of considering personal taxes is that the gain from leverage is lower than in the original model.

All these studies, dealing with the effects of taxation on the value of the firm and financial equilibrium, have a major shortcoming in common: their treatment of the government economic role is incomplete. These papers consider taxes as a cost incurred by firms and individuals but fail to consider the effects of the expenditure side of the government budget. The government budget as a whole, taxes on the one side and expenditures on the other, will affect individuals' wealth directly and indirectly through its effect on the general price level.

In our model the government collects taxes from firms and individuals and distributes transfer payments to the public. We thus consider explicitly the full role of the government sector, which is treated as endogenous to the system, and determine its total effect on the equilibrium prices of financial assets. A second purpose of this
paper is to analyze the effects of the government sector on the optimal capital structure and the value of the firm.

Following the framework of Landskroner and Liviatan [1981], we derive in Section 2 financial equilibrium results for risky securities (including risky debt), as well as a nonmarketable asset which is issued by the government. In Section 3, two related macroeconomic constraints are introduced: (1) the relationship between output, prices, and money in the economy, and (2) the government budget. The effects of these constraints on the pricing of financial assets are examined simultaneously and a modified CAPM is derived. Section 4 examines the effects of the financing decision on the valuation of the firm when the government is endogenous to the system. Section 5 presents some concluding remarks.

2. THE GOVERNMENT SECTOR AND FINANCIAL EQUILIBRIUM

In this paper we expand the analytical framework of financial equilibrium to incorporate the government sector as well as the private sector which consists of both firms and individuals. In the analysis, following Landskroner and Liviatan [1981], we make the usual assumptions of the original CAPM, but also include taxes, a nonmarketable asset, and price uncertainty. Within this framework we assume three types of marketable securities issued by the private sector: a real (indexed) bond linked to the general price level, a nominal bond (which is risky in real terms), and common stocks. We also assume a nonmarketable asset
issued by the government—these are the net (of personal taxes) transfer payments distributed to individuals, which have a random return unique to each individual.

The budget constraint of the \( k \)th investor may be described as:

\[
V_k = C_k + m_k + \sum_i S_{ik} + B_{fk} + B_{nk}
\]  

(1)

where \( C_k \) is the consumption in real terms of the \( k \)th investor; \( m_k \) are his nominal money balances; \( S_{ik} \) is the market value of the \( i \)th stock held by the \( k \)th investor, \( B_{fk} \) is the value of his holdings of the real bond; and \( B_{nk} \) his holdings of the nominal bond.

Equation (1) explicitly introduces money balances into the portfolio of the individual. This will serve later as part of the macroeconomic constraint. Following Fama and Farber [1979], the money balances are split into two components representing the two roles of money: one is consumption of liquidity services, \( \frac{m_k (r_n - 1)}{r_n} \) where \( r_n \) is the nominal risk-free interest rate; and the second is equivalent to an investment in a nominal bond, \( \frac{m_k}{r_n} \). Thus, equation (1) can be rewritten as follows:

\[
V_k = C_k^* + \sum_i S_{ik} + B_{fk} + B_{nk}^*
\]  

(1a)

where \( C_k^* = C_k + \frac{m_k (r_n - 1)}{r_n} \) represents the total consumption of the individual and \( B_{nk}^* = B_k + \frac{m_k}{r_n} \) represents his total investment in
"nominal bonds." Since this paper concentrates on the investigation of financial equilibrium, only the last three terms on the RHS of (1a) should be considered, assuming throughout the paper that the consumption decision $C_k^*$ has already been made. Therefore, real wealth of the $k^{th}$ individual at the end of the period can be written as:

$$W_k = \sum_{i} S_{ik} (R_i - R_f) + V_k^* R_f + B_{nk} (R_n - R_f) + p_k \Pi$$  \hfill (2)

where $V_k^* = V_k - C_k^*$ is the "investable" wealth of the individual; $R_i$ is the random return (one plus the rate of return) on the $i^{th}$ stock:

$$R_i = \frac{Z_i}{S_i}$$  \hfill (3)

with $Z_i$ representing net income to the $i^{th}$ firm stockholders,

$$(Z_i = (X_i - R_n B_i)(1-t) - tB_i), \quad X_i \text{ is its net operating income}, \quad R_n \text{ is one plus the risky real interest rate paid on the firm's nominal debt}, \quad B_i, \quad \text{and } t \text{ is the uniform corporate tax rate}. \quad \text{Note that since our analysis is of a single-period horizon, we have included the repayment of the principal of the debt in the flow. This is unlike previous studies which fail to do so (see Rubinstein [1973] for example). We also assume that the current value of the bond, } B_i, \text{ is its par value. The other variables in equation (2) are: } R_f \text{ is one plus the certain real interest rate on the indexed bond; } p_k \text{ is the random nominal net transfer payments received by the } k^{th} \text{ investor;}^2 \quad \Pi \text{ is the random index of purchasing power of money } (\Pi = I_0/I_1 \text{ where } I_0 \text{ and}$$
\( I_1 \) are the general price levels at the beginning and end of the period, respectively. As \( R_n \) is the real return on the nominal bond, with a certain nominal rate \( r_n \), \( R_n = r_n I \). This means that the firm's debt is risky in real terms, indicating that all firms have the same risk on debt which results from uncertain inflation.

Maximizing the expected utility of wealth given by equation (2), assuming a mean-variance utility function, and aggregating over investors (see the Appendix for the derivation) yields the equilibrium return relationships for the risky securities. Thus, for the \( i \)th stock we obtain:

\[
AE(R_i - R_f) = S \text{cov}(R_i, R) + B_n^* \text{cov}(R_i, R_n) + \text{cov}(R_i, p\Pi) = \text{cov}(R_i, W) \tag{4}
\]

where all variables without subscripts denote aggregate market values:

\( S = \sum \sum S_{ik} \) is the aggregate market value of all stocks; \( R \) is the real return on the stock market portfolio, \( R = [(X - BR_n)(1-t) - tB]/S \)

where \( X = \sum X_i \), \( B = \sum B_i \); similarly \( B^* = B + \frac{m}{r_n} \) where \( m = \sum m_k \); and \( p = \sum p_k \). The RHS of equation (4) can be greatly simplified by recognizing that it equals the covariance between the return on the \( i \)th stock \( R_i \) and the aggregate risky final real wealth \( W \), where \( W = SR + B_n^* R_n + p\Pi \); and \( E \) and \( \text{cov} \) denote the expectation and covariance operators respectively. The term \( A = \sum \left[ \frac{a_k}{2\delta_k} - E(W_k) \right] > 0 \) is a factor common to all stocks and is determined by an average of investors' attitudes towards risk, where \( a_k \) and \( \delta_k \) are coefficients of the individual's utility functions (see Appendix). This weighted risk aversion factor is equal to the market price of risk (MPR), which is the market
risk premium for bearing the risk of the market portfolio. Summing (4) over all stocks yields:

\[ AE(R-R_f) = \sum_n B_n^* \text{Var}(R_n) + \sum_n \text{cov}(R_n, R_n) + \text{cov}(R_n, \Pi) = \text{cov}(R, \Pi) \] (5)

The MPR denoted as \( \gamma \), where \( \gamma = A^{-1} \), is given by

\[ \gamma = E(R-R_f)/\text{cov}(R, \Pi) \] (5a)

Thus, unlike the original CAPM where the single source of risk was the market variability, two additional sources of risk are introduced: uncertainty about interest rates, and transfer payments. As will be seen in the next section, both of these risks are related to inflation uncertainty, which is determined in part by the government economic activities.

A major implication of the introduction of a nonmarketable asset, unique to each investor, which was first derived by Mayers [1972], concerns individuals' portfolio composition. When the opportunity set of the investors is expanded, all investors will no longer have the same portfolios. Thus, the separation theorem, stating that the optimal portfolio of all investors will be a combination of the riskless asset and the market portfolio, will no longer hold.

In a way similar to the above procedure we derive the equilibrium return for the nominal bond:

\[ AE(R_n - R_f) = B_n^* \text{Var}(R_n) + S \sum_n \text{cov}(R_n, R_n) + \text{cov}(R_n, \Pi) = \text{cov}(R_n, \Pi). \] (6)
As can be seen from the RHS of (6), the same types of risk which determine the return of stocks also determine the return of the nominal bonds.

3. THE MACROECONOMIC CONSTRAINT AND THE GOVERNMENT BUDGET

In the previous section we derived financial equilibrium after introducing the factors of risky debt and a nonmarketable asset with inflation and the government treated as exogenous to the economic system. Thus we had no theoretical basis to relate rates of return uncertainty (on stocks and bonds) with uncertainty about transfer payments and inflation. Also, we were unable to determine the effect of the government's economic activities, i.e., taxation and the distribution of transfer payments, on the pricing of capital assets. To analyze these effects, two related macroeconomic constraints are imposed: (1) similar to Landskroner and Liviatan [1981], we define the relation between output, prices, and money in the economy; and (2) we specify the government budget (balanced or unbalanced). These constraints make the government and inflation endogenous to the system and define the relationships between the random variables in the model.

3.1 The Macroeconomic Constraint

The macroeconomic constraint considered here, which relates output, prices, and money, is the well-known Quantity Theory of Money in its Cambridge formulation:
\[ \Pi(m+g) = KX \]  \hspace{1cm} (7)

where \( K \), a positive constant, is the famous "Cambridge" \( K \), a factor of proportionality equal to the reciprocal of the velocity of money in the Fisher formulation of the theory. The other additional variable, \( g \), is the difference between the government expenditures (the transfer payments) and receipts (corporate and personal taxes); it may be positive (implying a budget deficit), zero (balanced budget), or negative (budget surplus).\(^4\) It can be described as:

\[ g = p - T \]  \hspace{1cm} (8)

where \( p = \sum_k p_k \) is the total net (of personal taxes) transfer payments of the government and \( T = \sum_i T_i \) is the total, in nominal terms, of corporate taxes collected, where corporations are assumed to be taxed uniformly on their real income, and interest payments are tax deductible, that is: \( T_i = \frac{1}{\Pi} (X_i - R'B_i) \) where \( R'_n = R_n - 1 \).

Equation (7) implies that real national income (LHS of equation) is equal to some proportion of the real national output, \( X \). In a sense this is a conservation of value equation for the economy as a whole, thus an increase in real national income can result only from changes in the real sector; on the other hand, changes in the monetary sector, i.e., changes in government activities, by affecting \( g \), will be offset by changes in the purchasing power of money (we shall elaborate on this point more fully later in this section).
Let us now determine the effect of the constraint on the pricing of capital assets, that is, treat the government and inflation as endogenous. This is done by substituting equation (7) in the pricing equations (4) through (6), noting that \( W = (1+K)X \). This yields for the \( i \)th stock:

\[
AE(R_i - R_f) = \frac{(1+K)(1-t)}{S_i} [\text{cov}(X_i, X) - \rho_i \beta_i \text{cov}(\Pi, X)].
\] (9)

The terms in the brackets on the RHS of (9) represent the risk of the \( i \)th stock. This consists of two parts: (i) a covariance term of real output of the \( i \)th firm with that of the economy, which is similar to the result of the original CAPM, and (ii) the covariance of the purchasing power of money index with the real output in the economy, which is a measure of the degree to which real output is a hedge against inflation. The factor of proportionality in the latter is determined by the amount of debt issued by the firm. Assuming both covariance terms to be positive, an increase in the debt will reduce the risk of the stock. This result may seem to contradict previous results in the literature (see Hamada [1969]); however, one should note that a change in \( \beta_i \) will also affect \( S_i \) in equation (9) and thus the total effect of increased debt is not clear. Once we have analyzed the question of capital structure (the next section), we shall be able to determine the relationship between \( \beta_i \) and the required real return \( E(R_i) \).

In a similar way, we can derive the equilibrium expected risk premium for the stock market:
\[ AE(R - R_f) = \frac{(1+k)(1-t)}{s} \{ \text{var}(X) - r_n B \text{ cov}(\Pi, X) \}, \quad (10) \]

and for the nominal bond:

\[ AE(R_n - R_f) = (1+K)r_n \text{ cov}(\Pi, X). \quad (11) \]

Note that the risk of the nominal bond is measured by a single factor, because the only source of uncertainty for the bond is inflation uncertainty.

3.2 Balanced Government Budget

Next we want to determine explicitly the effects of the government's economic activities on financial equilibrium.

Let us first consider the case of a balanced budget, i.e., \( g = 0 \). For this case, equation (7) can be rewritten as

\[ \Pi = \frac{K}{m} X. \quad (7a) \]

Thus, inflation uncertainty is determined solely by uncertainty about real output, when \( \frac{K}{m} \) is a positive constant. Substituting (7a) in (10), we obtain the following equation for the MPR:

\[ \gamma = \frac{SE(R - R_f)}{(1+K)(1-t)(1-b)\text{var}(X)}. \quad (12) \]

That is, the measure of risk of the stock market portfolio is determined by the variance of real output. Note that \( b = r_n B \frac{K}{m} \), and \( b < 1 \) as long as the nominal national income \( \left( \frac{m}{K} \right) \) is greater than the aggregate final value of the nominal bonds \( (r_n B) \).
Now substitute (7a) and (12) in equations (9) and (11) to obtain a modified CAPM for stocks and risky bonds. For stock \( i \) we obtain:

\[
E(R_i - R_f) = \frac{S_i}{S} \lambda [\beta_i - b_i]
\]

where \( \lambda = \frac{E(R - R_f)}{(1-b)} \) is a factor common to all securities, but where

\[
\beta_i = \frac{\text{cov}(X_i, X)}{\text{var}(X)}
\]

and \( b_i = r_{n_i m} \), are factors unique to each stock and can measure its risk. \( \beta_i \), is a measure of the \( i \)th stock's systematic risk, similar to that in the original CAPM. In addition however, as was seen in equation (9), this is adjusted by a term proportional to the firm's debt, \( b_i \). Equation (13), however, differs from equation (9) in that all uncertainty is limited to one factor. Thus, although any particular investor is still facing an additional source of uncertainty—his net transfer payments—the market will not award any premium for this government-produced uncertainty. This is so because risk premium is awarded to real factors, which in this case is the only source of uncertainty in a market context. Thus the government, provided it balances its budget, has no effect on the pricing of stocks.

Similarly we may obtain the modified CAPM for risky debt:

\[
E(R_n - R_f) = \frac{S}{B} \lambda \frac{b}{1-t}
\]  

That is, the risk of the bond is proportional to the total outstanding corporate debt. As the risk increases, so does the required return of
the bond, and is unrelated to the uncertainty of the real factor. Of course, the uncertainty of the real factor will affect the required return on risky debt through its effect on the common market factor \( \lambda \), and on \( S \).

3.3 Unbalanced Government Budget

The analysis above indicates that as long as the government maintains a balanced budget it has no direct impact on risk premia of securities.\(^5\) However as a practical matter, the government may in many instances, operate an unbalanced budget, i.e., \( g > 0 \). A deficit in the budget, \( g > 0 \), would imply that the government finances its activities partly through taxation and partly by printing money. In this case, the economy has two sources of price uncertainty: a real source, associated with the variability of \( X \), and a second source, associated with the variability of the increase in the money supply, \( g \). This (uncertain) monetary factor may be expressed as

\[
\phi = \frac{1}{1+g/m}
\]  

(15)

and, thus, the equation for the purchasing power of money may now be rewritten, following equation (7) as:

\[
M = \frac{X}{m} X \phi.
\]  

(16)

In order to simplify our results and make them more explicit, we need to specify the relationship between the real factor, \( X \), and the monetary factor, \( \phi \). Let us assume first that they are independent.
This assumption might imply that the government is pursuing some economic goals such as economic growth, full employment, or income distribution, but not price stabilization. Under this assumption, following logic similar to that used previously, we may derive the risk premia for the risky securities.

First the risk premia for stock \( i \) is now given as:

\[
E(R_i - R_f) = \frac{S}{S_1} \lambda' [\beta_i - b_i E(\phi)]
\]  \hspace{1cm} (17)

where \( \lambda' = \frac{E(R - R_f)}{(1-bE(\phi))} \).

Similarly, for the nominal bond one obtains:

\[
E(R_n - R_f) = \frac{S}{B} \lambda' \frac{b}{(1-t)} E(\phi).
\]  \hspace{1cm} (18)

The risk of both types of assets, stocks as well as nominal bonds, will be affected by the expected value of the monetary factor, \( \phi \). By comparing the results for stocks in equation (13) versus the results in equation (17), one observes that an increase in the government deficit, which will increase the inflation rate (decreasing \( \phi \)), will increase the risk premia of a stock of a levered firm, provided that \( B_i/B_i > B \) (the debt per unit of risk of the stock of the firm is greater than this ratio for the market as a whole). On the other hand, a comparison of equation (18) to equation (14) indicates that a similar increase in the government budget deficit will reduce the risk premia of the nominal bond. This result merits an explanation: the value of the nominal
bonds is inversely related to the value of transfer payments. Thus, as the expected inflation increases, the value of the nominal bond that serves as a hedge against inflation will also increase and its required rate of return will decrease.

A second extreme case of government policy is that of complete price stabilizing: the government through its budget policies eliminates price uncertainty, and \( \Pi \) equals some positive constant. Thus, from equations (9) and (10), we obtain for the \( i^{th} \) stock:

\[
E(R_i - R_f) = S_i \frac{S_i}{E(R - R_f)} \frac{\text{cov}(X_i, X)}{\text{Var}(X)}
\]

This result is identical to the original CAPM, except that in our model the sources of uncertainty are specified.

Using equation (11) yields for the nominal bond:

\[
E(R_n) = R_f.
\]

That is, when \( \Pi \) is completely stabilized, the nominal bond is identical to the real bond and we have a single riskless bond as in the original CAPM. Note also that in (20) we have derived the classic Fisher equation relating the nominal interest rate and the inflation rate to the real interest rate.

4. CAPITAL STRUCTURE AND THE VALUE OF THE FIRM

One of the more important implications of the existence of a government (and its tax laws) in financial theory is its effects on the
value of a given firm under alternative financial structures. It has been shown in previous articles, e.g., MM, Hamada, and Rubinstein, that the value of the firm will increase linearly with the value of its debt. However, all previous studies on the issue of corporate cost of capital and capital structure have considered only one side of the government role—that of a taxing agent—but have ignored the transfer payments distributed to the public by the government and their effect on the general price level. Here we analyze the validity of these conclusions in light of our results obtained in the previous sections, in which the dual role of the government has been considered under a macroeconomic constraint. In our results of equations (13), (17), and (19), the equilibrium return on common stock is shown to be a function of the firm's debt, as well as that of the total debt of the market. These results, which are different from the original CAPM, call for a re-examination of the effects of the firm's capital structure on its value.

Suppose that there are two firms that are identical with respect to all their real decisions and therefore face an identical distribution of operating income. The capital structures of these two firms, however, are different. For simplicity, assume that one of them has no debt (unlevered—denoted as \( u \)), and the other one finances its assets by issuing risky debt as well as equity (levered firm—denoted \( l \)). Let us first consider the case of a balanced government budget. The required rate of return on the unlevered stock, following equation (13), should equal the expected return defined in (3):
\[
E(R_u) = R_f + \frac{S}{S_u} \lambda \beta_u = \frac{E(X_u)(1-t)}{S_u}
\]  
(21)

where as before \( \beta_u = \frac{\text{cov}(X_u, X)}{\text{Var}(X)} \).

Similarly, the required return for the levered firm is:

\[
E(R_L) = R_f + \frac{S}{S_L} \lambda [\beta_u - b_L] = \frac{E(X_u-R_B)(1-t) - tB_L}{S_L}
\]  
(22)

Where by assumption \( X_u = X_L \) and also \( \beta_u = \beta_L \).

Noting that by definition \( V_L = S_L + B_L \), where \( V_L \) is the total value of the firm, and substituting for \( E(X_u)(1-t) \) from equation (21) in equation (22) and rearranging yields:

\[
R_f V_L = R_f S_u - E(R_n)B_L(1-t) - tB_L + S\lambda b_L + R_f B_L.
\]

Substituting for \( E(R_n) \) from equation (14) and rearranging yields the relationship between the values of the levered and the unlevered firms:

\[
V_L = V_u + tB_L(1 - \frac{1}{R_f})
\]  
(23)

where \( S_u = V_u \).

Consider now the case of an unbalanced budget where the real and monetary factors are either independent or completely dependent. Following the same procedure as before, equations (17), (3), and (18) (or alternatively equations (19), (3), and (20)) yield a result identical to that we obtained in equation (23). In its general form, this result corresponds to that obtained previously for the cases of risky as well
as risk-free debt:

\[ V_L = V_u + tB_L. \]

The value of the levered firm increases linearly with the amount of its outstanding debt, and this tax subsidy is directly affected by the corporate tax rate. In our case, the gain from leverage is smaller than the previously determined \( tB_L \), by an amount equal to the present value of this tax subsidy, i.e., \( tB_L/R_p \). The reason is that in a finite horizon model like ours, which is a single-period model, one also has to consider the repayment of principal of the bond at the end of the period. This was overlooked in previous studies also carried out in a single-period framework, such as Rubinstein [1973]. In a sense, our result is similar to that of Miller [1977] who also concluded that, in the face of personal taxation, the gain from leverage will be smaller than that obtained in previous studies. His finding, however, is a result of differential the introduction of personal taxation. It is interesting that the valuation formula remains essentially unchanged, which means that the value of a levered firm will still be determined by the financing policy of the firm and the taxation policy of the government, but not by its expenditure policy (transfer payments) which do not affect the firm directly.

Of course the government budget policy as a whole may affect the values of the levered as well as the unlevered firm, but it will not affect their relative values, that is, the value of one in terms of the other.
4.1 Leverage and the Expected Return on Equity

MM [1958, 1963] in its second proposition states that the expected rate of return on the common stock of a levered firm increases in proportion to the debt-equity ratio of the firm. That is, the required return on equity will increase with financial risk. This proposition was later proved by Hamada [1969] using the CAPM framework. It is of interest to examine MM's proposition for the case of risky debt assumed in this paper as well as the case assumed by Rubinstein [1973].

Let us consider the case of a balanced budget, and for comparability with Rubinstein also assume no repayment of principal of debt, i.e., we define the rates of return as:

\[
R_k = \frac{(X - R'B)}{u} \frac{(1-t)}{S_k} \quad (3a)
\]

where \( R'_n = R - 1 \),

and

\[
R_u = \frac{X(1-t)}{u} \frac{1}{S_u} \quad (3b)
\]

The relationship between the value of the levered and the unlevered firms in this case is

\[
V_k = S_u + tB_k \quad (23a)
\]

or

\[
S_k = S_u - (1-t)B_k.
\]
The expected return for the levered firm is given by:

\[ E(R^*_L) = R^*_L + \frac{S}{S_L} \lambda (\beta_u - b_L) \]  \hspace{1cm} (13a)

From equation (13a) one may be inclined to conclude that as a firm increases its debt financing, thus increasing \( b_L \), it will reduce the required rate of return on equity, \( E(R^*_L) \). This conclusion, which seems to contradict previous results, overlooks the fact that a change in the debt will also affect the value of the firm's equity as can be seen clearly in equation (23a). We now wish to derive the conditions under which the expected rate of return on equity will increase with risky debt; stated somewhat differently, under what conditions will we have:

\[ E(R^*_L) > E(R_u) \]

which, following equation (13), can be expressed as:

\[ \frac{\beta_u - b_L}{S_L} > \frac{\beta_u}{S_u} \]

where the only difference between the levered firm \((L)\) and the unlevered \((u)\) is their capital structure. Using the definitions (3c) and (3b) we obtain:

\[ \frac{\beta_u}{S_u} > \frac{N}{1-t} \]

where \( N = \frac{r}{n} \frac{K}{m} \).

Rewriting equation (19), the expected rate of return on the nominal bond
as:

\[ E(R_n) = R_f + S\lambda \frac{N}{1-t} \]  \hspace{1cm} (14a)

and equation (13) for the unlevered firm as:

\[ E(R_u) = R_f + S\lambda \frac{\delta_u}{s_u} \]  \hspace{1cm} (13b)

yields the result:

\[ E(R_u) > E(R_n) \text{ iff } E(R_u) > E(R_n). \]  \hspace{1cm} (24)

That is, the expected rate of return on the equity of a levered firm increases with debt if and only if the expected return on the unlevered firm is greater than the expected return on the risky debt. This result is identical to the one implied by Rubinstein's model for the case that the debt of all levered firms has the same risk.

Assuming now, as we did throughout the paper, that the principal on debt is repaid, the condition in equation (24) is modified to:

\[ E(R_u) > E(R_n) \left( 1 - \frac{t}{(1-t)R_f} \right)^{-1} \]  \hspace{1cm} (24a)

That is, a discounting factor is added to the RHS of the inequality.
5. CONCLUDING REMARKS

This paper first determines the effects of the government's economic activities (expenditures in the form of transfer payments to the public and tax collection from corporations and individuals) on the pricing of capital assets. Under a conservation of value equation in the form of the quantity theory of money, two cases are considered: a balanced and an unbalanced government budget. In the first case, where all price uncertainty has a real source, the modified CAPM derived here states that the expected return on equity will be determined by a risk factor consisting of two parts, and a factor common to all securities determined by market risk aversion and total debt in the economy. The risk of stock consists of a measure of systematic risk similar to that of the original CAPM, but the sources of uncertainty are specified, and a second part that depends on the extent of the firm's debt financing. The expected return on nominal risky debt, on the other hand, is determined only by a single risk factor related to the total debt in the economy; this follows the fact that the return on bonds is unrelated to the real factor (output), unlike the return on stocks. In the second case, where the government has an unbalanced budget, price uncertainty will have two sources: monetary and real. If these two are independent, the expected value of the monetary factor will affect the rates of return on risky securities. If these factors are, however, completely dependent, then the result for common stock is identical to the original CAPM and the nominal bond is identical to the index bond since in this case both are riskless.
The second purpose of this paper was to determine the effect of the government on the optimal capital structure of the firm. The interesting finding here is that the result remains essentially unchanged in comparison to previous results reported in other papers which considered only taxation, not the full economic role of the government as this paper does. Thus, the difference between a levered firm and an identical unlevered firm will be determined linearly by the corporate tax rate and the amount of debt outstanding. Finally, we derived explicitly the relationship between the rates of return on levered and unlevered firms for the case of risky corporate debt. The result obtained here is that the expected return on equity will increase with debt, as would be expected, only if the expected return on the equity of the unlevered firm is greater than that of the risky bond.
APPENDIX

The individual's expected utility can be expressed as:

\[ F = J(C, m) + E[U(W)] \]  \hspace{1cm} (1A)

where \( J(C, m) \) and \( U(W) \) are strictly concave utility functions of consumption and wealth, respectively. Subscripts \( k \) for individuals are dropped except where required for clarity. The individual maximizes the expected utility in (1A) subject to the budget constraint (2). This constrained maximization yields the first-order conditions with respect to \( (C, m, S_i, B_n^*) \).

In order to obtain explicit equilibrium results we assume a specific form of the utility function of the individual. Since in this paper we are mainly interested in financial equilibrium, we will only specify the utility function of wealth. Assuming a quadratic utility function of wealth, which is common in a mean-variance framework like ours:

\[ U_k(W_k) = \alpha_k W_k - \delta_k W_k^2 \]  \hspace{1cm} (2A)

where, by construction \( \alpha_k > 2\delta_k W_k \) and \( \delta_k > 0 \) in order to be consistent with the assumptions of nonsatiation and risk aversion. Thus the expected utility of wealth for the \( k \)th individual is:

\[ E[U(W)] = \alpha E(W) - \delta \{ \text{var}(W) + [E(W)]^2 \} \]  \hspace{1cm} (3A)
where the expected value of wealth can be written as:

$$E(W) = \sum_{i} S_i E(R_i - R_f) + \frac{\sigma}{\sqrt{n}} E(R_n - R_f) + \nu R_f + E(p\Pi).$$  \hspace{1cm} (4A)

And the variance is:

$$\text{Var}(W) = \sum_{i} \sum_{j} S_i S_j \text{cov}(R_i, R_j) + \frac{\sigma^2}{n} \text{Var}(R_n) + \text{Var}(p\Pi)$$

$$+ 2 \frac{\sigma}{\sqrt{n}} \sum_{i} S_i \text{cov}(R_i, R_n) + 2 \sum_{i} S_i \text{cov}(R_i, p\Pi) + 2 \frac{\sigma}{\sqrt{n}} \text{cov}(R_n, p\Pi)$$ \hspace{1cm} (5A)

where $\text{cov}(*)$ denote covariance terms between real random returns. The first-order condition of the $k^{th}$ investor for the $i^{th}$ stock is given by:

$$\frac{\partial E[U(W)]}{\partial S_i} = \frac{\partial E[U(W)]}{\partial E(W)} \cdot \frac{\partial E(W)}{\partial S_i} + \frac{\partial E[U(W)]}{\partial \text{Var}(W)} \cdot \frac{\partial \text{Var}(W)}{\partial S_i} = 0.$$ \hspace{1cm} (6A)

From this condition we are able to derive the return relation for the $i^{th}$ stock as:

$$E(R_i - R_f)\left[\frac{\gamma}{2\delta} - E(w)\right] = \sum_{j} S_j (R_j - R_f) + \frac{\sigma}{\sqrt{n}} \text{cov}(R_i, R_n) + \text{cov}(R_i, p\Pi)$$

$$\hspace{1cm} \text{(for all i)}$$ \hspace{1cm} (7A)

where $\left[\frac{\gamma}{2\delta} - E(w)\right] > 0$, since it is proportional to the expected marginal utility of wealth. Summing (7A) over all investors yields an equilibrium return equation for the $i^{th}$ stock:
\[ AE(R_i - R_f) = S \text{cov}(R_i, R) + E^* \text{cov}(R_i, R_n) + \text{cov}(R_i, \pi) \quad \text{(for all i)} \quad (4) \]

where all the variables without subscripts denote market values and are defined in the text.

In a similar way we may derive the equilibrium return for the nominal bond. The first-order condition of the \(k\)th individual, for the nominal bond is:

\[ \frac{\partial E[U(W)]}{\partial B_n^*} = \frac{\partial E[U(W)]}{\partial E(W)} \cdot \frac{\partial E(W)}{\partial B_n^*} + \frac{\partial E[U(W)]}{\partial \text{Var}(W)} \cdot \frac{\partial \text{Var}(W)}{\partial B_n^*} = 0 \quad (8A) \]

which yields the return relation:

\[ E(R_n - R_f) = \frac{\alpha}{25} - E(\omega) = B^* \frac{\text{Var}(\pi)}{n} + \sum_{i} S_i \text{cov}(R_n, R_i) + \text{cov}(R_n, \pi). \quad (9A) \]

Summing this equation over individuals yields the equilibrium return relationship for the nominal bond:

\[ AE(R_n - R_f) = B_n^* \text{Var}(R_n) + S \text{cov}(R_n, R) + \text{cov}(R_n, \pi). \quad (6) \]
FOOTNOTES

1 These assumptions are: (1) Individuals (investors) are single-period expected utility of terminal wealth maximizers who make their investment decisions on the basis of mean and variance of return. (2) Individuals can borrow and lend at a risk-free rate. (3) Investors have homogenous expectations about future returns. (4) All assets are marketable (perfectly liquid). (5) There are no transaction costs and no taxes. (6) Capital markets are perfect and no bankruptcy exists.

2 The net transfer payment of the kth investor, \( p_k \), is a function of the government budget—the next section will elaborate on this relationship.

3 It should be noted that Landskroner and Liviatan [1981] focus on the effects of inflation on financial equilibrium. They treat inflation as endogenous but do not analyze the effects of the government economic policy.

4 Mayshar [1977] uses a somewhat similar framework; i.e., individuals' budget constraint (as equation (2) above) and a balanced government budget (our equation (8)). However, Mayshar addresses himself to a different problem—the subsidization of risky private projects by the government.

5 Throughout this paper we abstract from income redistribution effects.
REFERENCES


