SUFFICIENT AND NECESSARY CONDITIONS FOR INFORMATION TO HAVE SOCIAL VALUE IN PURE EXCHANGE*

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In 1971, Hirshleifer, in a seminal paper, elegantly contrasted the value of an information system at the private level (i.e. when access to the information is limited to a few) with the value of that same system at the social level, i.e. when all market participants have access to the information. His main conclusion was that, under pure exchange, information does not have social value, that is, either everyone is not better off with information or if someone should happen to gain in expected utility terms it could only be at the expense of a fellow. This conclusion has subsequently been "confirmed" by a number of other authors [e.g. Fama and Laffer (1971), Marshall (1974), Ng (1975), and Wilson (1975)]. Apparently, only three examples have been produced in which public information does have social value: two for the case when investors have heterogeneous prior beliefs and homogeneous posterior beliefs [Marshall (1974, p. 387), Ng (1977)] and one in which the market is either incomplete or has time-dependent preferences and homogeneous beliefs [Jaffee (1975)]; all but one of these examples also assume that the endowments represent equilibrium holdings if no information is released.

Despite being developed under very special assumptions (identical preferences of the type that imply portfolio separation, identical prior probability beliefs, identical information structures, identical endowments, and a complete market), Hirshleifer's original conclusion has taken on the prominence of a general theorem. This is remarkable only because the particular assumptions which are responsible for his result to our knowledge have never been identified, the cited examples notwithstanding. In other words, the apparent "contradiction" between the Hirshleifer result and the Marshall-Jaffee-Ng papers has never been satisfactorily resolved.
The main purpose of this communication is to set forth a simple set of sufficient conditions for public information to have value as well as necessary conditions for this to be the case. These conditions provide a sharp distinction between the single-period and the two-period cases. The sufficiency conditions derive their significance from the fact that they depend, in the single-period setting, on only three things: whether the market is fully efficient allocationally, whether information structures are essentially homogeneous, and on certain aspects of investor endowments. Thus, these conditions are independent of prior (and posterior) beliefs, of preferences, and of most aspects of investor endowments, i.e. of the major heterogeneities that may exist among investors (except to the extent that the latter impinge on the allocational efficiency on the market--see Section I). In the two-period case, however, it is also pivotal whether prior beliefs are essentially homogeneous and whether preferences are time-additive. Furthermore, it will be shown that the above conditions are very closely related to conditions necessary for information to have social value. Our results therefore represent substantial generalizations and extensions of extant papers, all of which can be seen to represent rather special cases--this applies both to the papers which have shown that information may have social value as well as to those which have shown that it does not. Our main inference is that the cases in which information has social value should not be viewed as being in any sense implausible. Thus, previous studies, which on the whole have thrown their weight behind the conclusion that public information is socially worthless in pure exchange, may be thought of as having given rise to a "biased" perspective. This is primarily due to the explicit or implicit use of a strong set of assumptions that, despite appearances to the contrary, are also closely related. Of course, the preceding does not alter the fact that there are critical differences between the private and social value of information and from this perspective it is difficult to overstate the significance of the original papers in this area.
I PRELIMINARIES

We consider a pure exchange economy with a single commodity which lasts for one or two periods under the standard assumptions. That is, at the end of period 1 the economy will be in some state s, where s = 1, ..., n. There are I consumer-investors indexed by i, whose probability beliefs over the states are given by the vectors \( \pi_i = (\pi_{1i}, \ldots, \pi_{ni}) \), where \( \pi_{is} > 0 \) for all i and s and there is agreement on the subset of states for which \( \pi_{is} > 0 \). Preferences are represented by the functions \( U_{is}(c_{is}, w_{is}) \), where \( c_i \) is the consumption level in period 1 and \( w_{is} \) is the consumption level in period 2 if the economy is in state s at the beginning of that period. \( U_{is} \) is assumed to be monotone increasing in each argument and strictly concave, i.e. consumer-investors prefer more to less and are risk-averse. At the beginning of period 1 (time 0), consumer-investors may allocate their resources among current assumption and a portfolio chosen from J securities indexed by j. Security j pays \( a_{js} > 0 \) per share at the end of period 1, where \( a_{js} > 0 \) for some s for each j, and the total number of outstanding shares is \( Z_j \). Let \( z_{ij} \) denote the number of shares of security j purchased by investor i at time 0; his portfolio \( z_i = (z_{i1}, \ldots, z_{ij}) \) then yields the payoff

\[
W_{is} = \sum_j z_{ij} a_{js},
\]

available for consumption period 2, if state s occurs at the end of period 1. Aggregate wealth in state s is given by

\[
W_s = \sum_j Z_j a_{js} > 0, \quad \forall s.
\]

Investor endowments are denoted \((c_i, z_i)\) and markets, as is usual, are assumed to be competitive and perfect. The rank of matrix \( A = [a_{js}] \) is never less than 2; if it is full (equals n), the financial market will be called complete; if not, it will be called incomplete.
Prior to trading, investors may obtain information bearing on the state that will occur at time \( t \) via a signal \( y \) from an information system \( Y = (y_1, \ldots, y_m) \). This causes consumer-investor \( i \) to update his prior beliefs \( \pi_i^0 \) to the posterior beliefs \( \pi_i^Y \) via Bayes' rule. That is, if \( p_i(y|s) \) denotes investor \( i \)'s perceived probability that signal \( y \) will be emitted if \( s \) is about to occur, Bayes' Theorem gives

\[
\pi_i^Y = \frac{p_i(y|s)\pi_i^0}{\sum_s p_i(y|s)\pi_i^0} = \frac{p_i(y|s)\pi_i^0}{p_i(y)}
\]

When \( \pi_i^Y \neq \pi_i^0 \), the information system \( Y \) is said to be non-null for investor \( i \).

The \( n \times m \) matrix of conditional probability numbers \( p_i(y|s) \) for investor \( i \) will be called his information structure. Whenever

\[(1a) \quad p_i(y|s) = p_1(y|s) \text{ for all } i \geq 2, y, \text{ and } s,
\]

the information structures will be said to be homogeneous; if \( (1a) \) does not hold, they will be called nonhomogeneous. Finally, when there exist numbers \( k_i(y) \) such that

\[(1b) \quad p_i(y|s) = k_i(y)p_1(y|s) \text{ for all } i \geq 2, \text{ all } s \text{ and } y,
\]

the information structures will be said to be essentially homogeneous. This is because, as a practical matter, \( (1b) \) is only slightly more general than \( (1a) \).

Under our assumptions, each consumer-investor \( i \) maximizes

\[
\sum_{s} \pi_i^U(s, c_i, \sum_j a_{ij})
\]
with respect to the decision vector \((c_i, z_i)\), subject to his budget constraint

\[
c_i^0 + \sum_j z_{ij}^0 = c_i^0 + \sum_j z_{ij}^0,
\]
as a price-taker, where \(P_0\) is the price of a unit of period 1 consumption and \(P_j\) is the price (at \(t = 0\)) of security \(j\). Assuming interior solutions (with respect to the non-negativity constraint on consumption, \((c_i, w_i) > 0\)), the equilibrium conditions may be written

\[
(3) \quad \sum_s \pi_{is} \frac{\partial U_i(c_i, w_{is})}{\partial c_i} = \lambda_i \quad \text{all } i
\]

\[
(4) \quad \sum_s \pi_{is} \frac{\partial U_i(c_i, w_{is})a_{js}}{\partial w_{is}} = \lambda_i P_j \quad \text{all } i, j
\]

\[
(5) \quad c_i + z_i P = c_i + z_i P \quad \text{all } i
\]

\[
(6) \quad \sum_i c_i = \sum_i \bar{c}_i, \sum_i z_{ij} = z_j, \quad \text{all } j
\]

where the \(\lambda_i\) are the Lagrange multipliers, (6) represents the market clearing equations, and \(P_0 \equiv 1\) has been chosen as numeraire. In the one period case, (3) drops out, \(P_1 \equiv 1\), and \(c_i = \bar{c}_i \equiv 0\) in (5) and (6).

We note that an allocation \((c^*, z^*)\) which constitutes a solution so system (3)-(6) (along with a price vector \(P\) and a vector \(\lambda\)) is Pareto-efficient with respect to market structure \(A\); no other allocation \((c, z)\) obtainable within market structure \(A\) can make some consumer-
investors better off without making others worse off. However, there may exist allocations outside the market (e.g., accomplished by the invention of new securities) which yield allocations that Pareto-dominate the market allocation \((c^*, z^*)\). When this is not the case, i.e., when \((c^*, z^*)\) is Pareto-efficient with respect to all conceivable allocations inside and outside the existing market, \((c^*, z^*)\) will be said to be fully Pareto-efficient or fully allocationally efficient.

To be more precise, let

\[ p_{is} = \frac{\pi_{is} \frac{\partial u_{is}(c_i, w_{is})}{\partial w_{is}}}{\lambda_i} = \pi_{is} u_{is}/\lambda_i \]

By reference to (4), it can be seen that \(p_{is}\) denotes investor \(i\)'s shadow price of wealth in state \(s\); roughly, it represents the marginal value of receiving an additional unit of wealth in state \(s\).

It is well known that (3)-(6) plus

\[(7) \quad p_{is} = p_{ls} \quad \text{all } i > 2, \text{ all } s\]

is a necessary and sufficient condition for the market allocation \((c^*, z^*)\) to be fully Pareto-efficient. This follows because (7) insures that the marginal rates of substitution of wealth (consumption) between any two states are the same for all investors \(i\). Condition (7) plays a particularly important role in what follows, a role which has previously not been well appreciated.

It is sufficient (but not necessary) for (7) to obtain that the security market be complete, i.e., that the rank of \(A\) be \(n\). Clearly,
(7) also occurs when all consumer-investors are completely identical. A third set of well-known sufficient conditions which imply (7) is the following: at least two securities in the market, one of which is risk-free; homogeneous beliefs; additive utility, with the second period's utility function being a member of the HARA-class, the exponent being the same across all consumer-investors (except in the negative exponential case).\textsuperscript{5} It is noteworthy that most studies dealing with the absence of social value of information have employed at least one of the above sufficient conditions, thus implicitly obtaining (7) [e.g. Hirshleifer (1971), Ng (1975, 1977), and Wilson (1975)].

The following follows directly from Bayes' Theorem.

**Lemma 1.** Suppose $\pi_{i1}^{y} > 0$ all $y$ for at least one $i$ ($i = 1$). Then

$$(\pi_{is}^{y}/\pi_{il}^{y})/(\pi_{is}^{0}/\pi_{il}^{0}) = (\pi_{is}^{y}/\pi_{il}^{y})/(\pi_{is}^{0}/\pi_{il}^{0}), \quad \text{all } i \geq 2, \ s \geq 2, \ \text{and } y$$

if and only if the information structures are essentially homogeneous, i.e. (1b) holds.

Paraphrased, the lemma states that whenever investors have essentially homogeneous information structures, and only then, does the revised probability ratio between any two states change by the same proportion for all investors no matter how heterogeneous their priors may be.

With this background, we are ready to take up the sufficiency question.

II SUFFICIENCY

Denote the maximum of (2) in the no information case, i.e. the consumer-investor's equilibrium expected utility when (all) investors make decisions on the basis of their prior beliefs $\pi_{i}^{0}$, by $V_{i}^{0}$, and let $(c_{i}^{0}, z_{i}^{0})$ be the resulting equilibrium allocation to consumer-investor
i. When the information system $Y$ is in use, there will be a different equilibrium for each signal $y$. Let $v_i^y$ and $(c_i^y, z_i^y)$ represent, respectively, consumer-investor $i$'s conditional expected utility and final allocation when equilibrium is based on revised beliefs $\pi_i^y$ (signal $y$). The probability of receiving signal $y$ is $p_i(y) = \sum_s p_i(y|s)\pi_i^o$; thus the relevant expected utility of consumer-investor $i$ in the information case, which we label $v_i(Y)$, is

$$v_i(Y) = \mathbb{E}_{p_i(y) v_i^y} = \sum_{i , j} p_i(y | s) U_i (c_i, z_i | y).$$

Following standard practice, information will consequently be said to have social value if

$$v_i(Y) \geq v_i^o \quad \text{all } i$$

(8)  
$$v_i(Y) > v_i^o \quad \text{some } i$$

but not otherwise.

Consider now the case in which

(9)  
$$(c_i^o, z_i^o) = (\bar{c}_i, \bar{z}_i), \quad \text{all } i,$$

i.e. in which the endowments turn out to be equilibrium allocations without information. This immediately gives

(10)  
$$v_i(Y) > v_i^o, \quad \text{all } i,$$

since rational consumer-investors will not choose an allocation worse than their endowment. Thus, whether (10) becomes (8) in this case depends solely on whether (non-trivial) trading takes place in the information case.
The conditions which rule out trading are given by:

Lemma 2. If (9) holds, it is sufficient for the receipt of information to result in no trading, i.e. to yield

$$(11) \quad (c_i^y, z_i^y) = (c_i, z_i) \quad \text{all } i \text{ and } y,$$

that the following conditions hold:

$$(1b) \quad p_i(y|s) = k_i(y)p_i(y|s) \quad \text{all } i \geq 2, \text{ all } s \text{ and } y \text{ (essentially homogeneous information structures)},$$

and

$$(12) \quad p^{0}_{is} = p^{0}_{is} \quad \text{all } i \geq 2, \text{ all } s \text{ (full allocational efficiency)}$$

and

$$(13b) \quad p_i(y) = k_i(y)p_i(y) \quad \text{all } i \geq 2, \text{ all } y \text{ (essentially homogeneous prior beliefs)},$$

and that we may write

$$(14) \quad U_is(c_i, \omega_is) = f_i(c_i) + g_is(\omega_is) \quad \text{all } s \text{ and } i \text{ (time-additive utilities)},$$

where (13b) and (14) do not apply in the single-period case. For arbitrary preferences and non-null information structures, conditions (1b) and (12) are also necessary for (11) to obtain in the single-period case while (1b), (12), (13b) and (14) are necessary in the two-period case.

Proof: Consider sufficiency and the two-period case first; it will be shown that, given (1b), (12), (13b) and (14), (11) satisfies equilibrium conditions (3) and (4) in an economy with information system $Y$. [Of course, (5) and (6) are always valid if (11) satisfies (3) and (4) and (9) holds.]
Eliminating the prices in the equilibrium conditions implied by (4) one obtains

\[ \sum_{s} (u_{is}^{y} / \lambda_{i}^{y}) a_{js} = \sum_{s} (u_{is}^{y} / \lambda_{i}^{y}) a_{js}, \quad \text{all } y, j, i \geq 2, \]

where \( \lambda_{i}^{y} \) is defined by (3). Further the preceding expression is equivalent to

\[ \sum_{s} \left[ \left( \frac{u_{is}^{y}}{\pi_{is}^{y}} \right) \left( \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} \right) - \left( \frac{u_{is}^{y}}{\pi_{is}^{y}} \right) \left( \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} \right) \right] a_{js} = 0, \quad \text{all } y, j, i \geq 2. \]

Now, if (11) is used as a trial solution for any signal \( y \), and (12) and (14) hold, then (15) will be satisfied if

\[ \left( \frac{\pi_{is}^{y}}{\pi_{is}^{y}} \right) = \left( \frac{\pi_{is}^{y}}{\pi_{is}^{y}} \right) \quad \text{all } y, s, i \geq 2. \]

This follows because (11) implies that \( u_{is}^{y} = u_{is}^{y} \) and (14) and (11) imply that \( \lambda_{i}^{y} = \lambda_{i}^{y} \) so that the trial solution of no trading is an equilibrium solution under no information:

\[ \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} = \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} = \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} = \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} \quad \text{all } y, s, i \geq 2. \]

But given (1b) and (13b) it is readily verified that (16) is satisfied. Hence, the trial solution of no trading is an equilibrium for each message \( y \) since (15) is indeed satisfied.

The sufficiency conditions for the one-period case are considered next. In this case the proof is most straightforward if one puts \( A \) equal to the identity-matrix; as is well known this entails no loss of generality in view of assumption (12). Equations (3) and (4) now reduce to

\[ \sum_{s} \left( \frac{u_{is}^{y}}{\pi_{is}^{y}} \right) \left( \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} \right) a_{js} = \sum_{s} \left( \frac{u_{is}^{y}}{\pi_{is}^{y}} \right) \left( \frac{\lambda_{is}^{y}}{\lambda_{i}^{y}} \right) a_{js}, \quad \text{all } y, j, i \geq 2. \]
Given assumption (1b) it follows from Lemma 1 that one must have

\[
\frac{(\pi_i^{y}/\pi_i^{o})}{(\pi_i^{y}/\pi_i^{o})} = \frac{(\pi_i^{o}/\pi_i^{o})}{(\pi_i^{o}/\pi_i^{o})}
\]

Hence, (11) is indeed a solution since it implies that \(u_i^{y} = u_i^{o}\) and since (12) implies that \(p_i^{o} = p_i^{o}\) for all \(i \geq 2\) and all \(s\).

Finally, necessity will be considered. Before going into the details we note that this result should be viewed as rather straightforward because (11) must hold for all utilities within restrictions (12) and (14) or (12), as the case may be. (In other words it is not ruled out that (11) may just happen to hold for certain singular preference structures.) It is therefore clear that both (16) and (17) must be satisfied if (15) is to be satisfied across different preferences. An immediate consequence is that markets must attain full allocational efficiency (12). Likewise, if (14) is not satisfied then \(\lambda_i^{y}\) can be made to depend on \(y\) in an arbitrary fashion so (14) is indeed necessary. Regarding (1b) and (13b), these conditions are necessary because (16) is equivalent to (1b) and (13b). (And, as previously noted, (16) is indeed necessary if (15) is to hold for all utilities.) This completes the necessity part of the two-period case. Finally, to complete the proof for the one-period case, one only needs to note that (1b) is necessary because of (18) and Lemma 1.

Observe that whenever (9) holds, (non-trivial) trading implies not only (10) but

\[V_i(Y) > v_i^o\]
for all who trade. Consequently, Lemma 2 implies that we have obtained a particularly simple set of sufficient conditions for public information to have social value:

**Theorem 1.** For (non-null) public information to have social value under pure exchange, it is sufficient (for all but singular preference structures) that

(i) endowments constitute equilibrium allocations without information and that **either**

(iiia) the market not be fully allocationally efficient

or that

(iiib) information structures not be essentially homogeneous

or that

(iiia) prior beliefs not be essentially homogeneous

or that

(iiib) the (two-period) utility function be non-additive

where (iiia) and (iiib) do not apply in the single-period case. That is, for (8) to be valid, it is sufficient that (9) holds and that one of conditions (12), (1b), (13b) or (14) be violated.

In the single-period case it is noteworthy that the above conditions are (essentially) independent of preferences and prior (as well as posterior) beliefs and of investor homogeneity or heterogeneity in general. What counts is the allocational efficiency of the financial market and the way the **information itself** is perceived by investors. However, when we move to two periods, certain properties of investor preferences and beliefs, in particular (13b) and (14), also enter the picture in explicit fashion.
Suppose now that the information system $Y$ is costly (in terms of real resources). Let $V_{ic}(Y)$ be consumer investor $i$'s expected utility in equilibrium with costly information (where $c$ is paid prior to the receipt of the message $y$, and thus before trading, say). Then for sufficiently small $c$, if we generalize into a setting in which all investors trade, (10) becomes

$$V_{ic}(Y) > \nu_i^o, \quad \text{all } i,$$

so that even costly information may have social value under pure exchange. That is, it may pay to exchange real resources for public information even in pure exchange.

III NECESSITY

We shall now show that either nonhomogeneous information structures or nonhomogeneous signal beliefs or (iia) or (iiib) is also necessary for (public) information to have social value under pure exchange; (9), on the other hand, does not have to hold. We begin with the following negative result.

**Lemma 3.** In a single-period setting public information cannot have social value under pure exchange when the financial market achieves full allocational efficiency and information structures are homogeneous. In the two-period case there will be no social value if, in addition, signal beliefs are homogeneous and utilities are time-additive. That is, (8) is impossible if (12) and (1a) hold in the single-period case, and (12), (1a), (14) and

$$(13a) \quad p_i(y) = p_1(y) \quad \text{all } i \geq 2, \text{ all } y$$

hold in the two-period case.
To prove the above in the two-period case, define, using (1a), the "average" allocations \((\hat{c}_{is}, \hat{w}_{is})\) as follows:

\[
(\hat{c}_{is}, \hat{w}_{is}) = \sum_y c_i^y w_{is}^y p_1(y|s), \quad \text{all } i, s.
\]

These allocations are clearly feasible since

\[
\sum_i \sum_y c_i^y w_{is}^y p_1(y|s) = \sum_i c_i^y w_{is}^y p_1(y|s) = \sum_y \sum_i c_i^y w_{is}^y p_1(y|s) = c_i^y w_{is}^y, \quad \text{all } s.
\]

Denoting the expected utility of these "average" allocations by \(\hat{V}_i\), Jensen's inequality gives

\[
(20) \quad \hat{V}_i = \sum_s \pi_i^s U(\hat{c}_{is}, \hat{w}_{is}) > \sum_s \pi_i^s \sum_y p_1(y|s) U(\sum_y c_i^y w_{is}^y) = V_i(y).
\]

Define the additional "average" first period allocations, using (13a), as follows:

\[
\hat{c}_i = \sum_s \sum_y c_i^y w_{is}^y p_1(y|s) = \sum_y \sum_s c_i^y w_{is}^y p_1(y|s) = \sum_y \sum_i c_i^y w_{is}^y p_1(y|s) = c_i^y w_{is}^y, \quad \text{all } i.
\]

These allocations are also feasible since

\[
\sum_i \sum_y \hat{c}_i^y w_{is}^y p_1(y|s) = \sum_y \sum_i c_i^y w_{is}^y p_1(y|s) = \sum_y \sum_i c_i^y w_{is}^y p_1(y|s) = c_i^y w_{is}^y.
\]

Applying (14) and Jensen's inequality one more time gives

\[
(21) \quad \hat{V}_i = f_i(\hat{c}_i) + \sum_s \pi_i^s \sum_w g_i(\hat{w}_{is}) > \sum_s \pi_i^s f_i(\sum_i c_i^y w_{is}^y) + \sum_s \pi_i^s g_i(\sum_i w_{is}^y) = \hat{V}_i.
\]
Since the no information allocations \((c_i^0, \omega_i^0)\) are fully Pareto-efficient with respect to the prior beliefs \(\pi_i^0\) when (12) holds, we must have either
\[
\hat{V}_i = V_i^0 \text{ for all } i \text{ or } \hat{V}_i < V_i^0 \text{ for some } i.
\]
Thus, by (20) and (21),
\[
(22) \quad V_i(y) = V_i^0 \quad \text{ all } i
\]
or
\[
V_i(y) < V_i^0 \quad \text{some } i,
\]
which concludes the proof for the two-period case.

The proof for the single-period case is now immediate: set \(f_i \equiv 0\) and since \(c_i^0 = c_i^Y = \hat{c}_{is} = c_i^y = 0\) it is readily seen that \(p_i(y)\) is unconstrained and that \(f_i\) is irrelevant.

From Lemma 2, we recall that (9) and somewhat weakened forms of (1a) and (13a) are sufficient conditions for (22) to obtain.

Note also that previous results concerning the lack of social value of information under pure exchange in previous papers [e.g. Hirshleifer (1971), Marshall (1974), Ng (1975), and Wilson (1975)] are in fact the special cases covered by Lemma 3.

When is everyone worse off with information? A partial answer is given in the following

**Corollary.** A sufficient condition for
\[
(23) \quad V_i(y) < V_i^0 \quad \text{all } i
\]
to hold is that (for all \(s\))
\[
(24) \quad (\hat{c}_{is}, \hat{\omega}_i) = (c_i^0, \omega_i^0) \neq (c_i^y, \omega_i^y) \quad \text{all } i, \text{ some } y.
\]
The first part of (24) implies that $\hat{v}_1 = \hat{v}_0 = \hat{v}_{0}^i$ for all $i$ and the second part insures that (20) holds with strong inequality, which in turn yields (23).

Theorem 1 and Lemma 3 now provide our second main result.

**Theorem 2.** For (non-null) public information to have social value under pure exchange, it is necessary (but not sufficient) that either

(iia) the market not be fully allocationally efficient
or that

(iiib) information structures be non-homogeneous
or that

(iiiia) signal beliefs be non-homogeneous
or that

(iiiib) the (two-period) utility function be non-additive.

where (iiiia) and (iiib) do not apply in the single-period case. That is, for (8) to be valid it is necessary that one of conditions (12), (1a), (13a), or (14) be violated.

It should be noted that the weaker conditions (1b) and (13b) cannot replace (1a) and (13a) in Lemma 3 and in Theorem 2. That is, if (1b) and (12) hold but (1a) does not (in the one-period case, say), it is possible to construct examples in which information has social value in the absence of (9). 12

IV DISCUSSION AND SUMMARY

We have demonstrated that the oft-made assertion that information does not have social value under pure exchange is incorrect unless

(iia') the financial market achieves full allocational efficiency,
and

(iiib') information structures are essentially homogeneous.
When either of these conditions is violated in the single-period case, public information may yield Pareto-superior allocations; Pareto-superiority is (except for singular preference structures) guaranteed (Theorem 1) whenever in addition

(i) endowments represent optimal allocations without information.

It is noteworthy that none of the preceding depends directly on preferences (e.g. whether they are state-dependent or not) or on the homogeneity or non-homogeneity of prior or posterior beliefs. Only the allocational efficiency of the financial market, the information itself, and initial allocations matter.

In the two-period case, (iia') and (iib') must be supplemented with the following relatively strong requirements:

(iiia') prior beliefs are essentially homogeneous and

(iiib') the (two-period) utility functions are time-additive.

Since it is rather implausible that all of conditions (iia') through (iiib') will be satisfied at the same time, it does not appear to be unrealistic to view public information as potentially valuable in a pure exchange environment. In different language, it is clear that previous authors reaching the traditional negative conclusion [Hirshleifer (1971), Marshall (1974), Ng (1975), Wilson (1975)] have in fact employed a strong and closely related set of assumptions. In addition, the significant differences between the single-period and the two-period cases have not previously been brought to light.

When all of the four conditions (iia')-(iiib') hold, trading is never optimal whenever (i) is valid (Lemma 2). Thus, these five
conditions represent something of a watershed in that information becomes a matter of indifference when they are all true. When one or more of (iia') through (iiib') is (are) violated, everyone is better off with information than without as long as (i) holds. Clearly, we can expect the same to be true for some endowments which also violate (i) but involve only "limited" trading or strongly state-dependent preferences.

When (i) alone is violated (but (1a), (12), (13a), and (14) hold), trading will occur and at least some individuals will be worse off with information than without. Some may also be better off except when everyone's average allocations of securities with information is in some sense close to the allocation that is achieved without information assuming that the first-period allocation is relatively unaffected; when this happens, the Corollary suggests that everyone will be worse off in the information case.

When none of the five conditions (i)-(iiib') hold, it follows from the preceding that the outcome is unconstrained. The availability of public information may result in Pareto-superior allocations or it may lead to Pareto-inferior allocations—as well as various intermediate cases. This is because preferences and beliefs now enter the picture even more explicitly than via (iia') and (iiib'). General statements must rely on the results of this paper or how "close" conditions (i)-(iiib') are to being satisfied.

Finally, a brief note concerning the production case may be in order. When production decisions are introduced, public information may yield allocations which range from Pareto-inferior to Pareto-superior. However, we would also expect the set of conditions which yield Pareto-superior allocation to "expand" and the conditions which imply Pareto-inferior allocations to "shrink".
Footnotes

1. Weather forecasts, crop forecasts, macro-economic forecasts, and corporate financial reports are examples of signals generated by information systems accessible to the public.

2. The single period model obtains whenever the variables $c_1$ (and quantities $c_{-1}$) in the sequel are suppressed.

3. That is, consumer-investors perceive prices as beyond their influence, there are no transaction costs or taxes, securities are perfectly divisible, and the proceeds from short sales can be invested.

4. For example, the two information structures

\[
\begin{array}{cccccc}
Y_1 & Y_2 & Y_3 & Y_1 & Y_2 & Y_3 \\
S_1 & .4 & .4 & .2 & .20 & .70 & .10 \\
S_2 & .3 & .4 & .3 & .15 & .70 & .15 \\
S_3 & .2 & .4 & .4 & .10 & .70 & .20 \\
\end{array}
\]

are essentially homogeneous but not homogeneous.

5. The HARA-class (hyperbolic absolute risk aversion) consists of the following utility functions of wealth with the properties $u'(w) > 0$, $u''(w) < 0$ (the first over at least a finite range of positive wealth):

\[
u(w) = \begin{cases} 
\frac{1}{\gamma} (w+a)^\gamma & \gamma < 1 \text{ (decreasing absolute risk aversion)} \\
-\exp(\gamma w) & \gamma < 0 \text{ (constant absolute risk aversion)} \\
-(a-w)^\gamma & \gamma > 1, \text{ a large (increasing absolute risk aversion)}
\end{cases}
\]

6. Non-trivial trading is trading that not only changes the security allocation $(c, z)$ but also the final consumption allocation $(c, w)$; trivial security trading leaves $(c, w)$ unaltered and may occur if the rank of $A$ is less than $J$. 

7. Starr (1973) has (implicitly) shown that, given (12) and (14), (completely) homogeneous beliefs are sufficient to imply that there will be no trading. However, as will be shown, these conditions can be relaxed. Varrecchia (1979) has also independently established sufficiency for special cases of (1b) and (12) in the single-period case.

8. Given (1b), (13b) is always satisfied when it is also assumed that prior beliefs are homogeneous but also in certain other cases. For example, (13b) holds when \( \pi_1 = (3/7, 1/7, 3/7) \) and \( \pi_2 = (1/7, 5/7, 1/7) \) in the presence of the (homogeneous) information structure

\[
\begin{array}{ccc}
 & y_1 & y_2 \\
 s_1 & 5/6 & 1/6 \\
 s_2 & 1/2 & 1/2 \\
 s_3 & 1/6 & 5/6 \\
\end{array}
\]

since \( p_i(y_1) = p_i(y_2) = .5 \), all \( i \). Note that what happens in the example is that condition (1a) (i.e. homogeneous information structures) combined with homogeneous signal beliefs

(13a) \[ p_i(y) = p_1(y) \quad \text{all } i \geq 2 \text{ and } y \]

have the same effect on relative prior-posterior beliefs as do conditions (1b) and (13b). More generally, (1b) and (13b) imply that

\[
\frac{p_i(y|s)}{p_i(y)} = \frac{p_1(y|s)}{p_1(y)} \quad \text{all } i \geq 2 \text{ and } s, y
\]
and these relations hold if and only if

\[
\frac{\pi_{i,s}}{\pi_{0,s}} = \frac{\pi_{i,s}}{\pi_{0,s}} \quad \text{all } i \geq 2 \text{ and } s, y.
\]

It might be added that the above analysis also indicates why we have chosen to label (13b) essentially homogeneous prior beliefs rather than essentially homogeneous signal beliefs.

9. In other words, in equilibrium there is for each signal equality across investors of marginal rates of substitution between securities and current consumption.

10. This corrects a previous statement of Fama and Laffer (1971, p. 292), who appear to have assumed homogeneous information structures but did not assume a complete market (or full allocational efficiency).

11. The Lemma can be viewed as an extension of Marshall's results. Basically, there are three generalizations in this paper: (i) the distinction between one and two periods is covered, (ii) (completely) homogeneous beliefs are not necessary, (iii) only full allocational efficiency, as opposed to complete markets, is necessary.

12. Consider the following single-period example with two states and two Arrow-Debreu securities (which implies (12)) and two investors with the following information structures (which imply (1b)):

| .6 | .3 | .1 | 12/22 | 9/22 | 1/22 |
| .4 | .4 | .2 | 8/22  | 12/22| 2/22 |
If the prior beliefs are \( \pi_1^0 = \pi_2^0 = (.5,.5) \), the implied signal beliefs are .5, .35, and .15 for investor #1 and 20/44, 21/44, and 3/44 for investor #2. The posterior beliefs are then (.6,.4), (3/7,4/7), and (1/3,2/3) for the three signals (for both investors). Suppose also that endowments are (200,0) and (0,400), respectively, and that the utility functions satisfy the following conditions:

For Investor #1:

<table>
<thead>
<tr>
<th>Wealth</th>
<th>75</th>
<th>600/7</th>
<th>100</th>
<th>120</th>
<th>150</th>
<th>1200/7</th>
<th>200</th>
<th>230</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>0</td>
<td>43.7</td>
<td>101</td>
<td>180.5</td>
<td>274</td>
<td>317.7</td>
<td>375</td>
<td>434.5</td>
</tr>
<tr>
<td>Marginal Utility</td>
<td>5.04</td>
<td>4.02</td>
<td>4</td>
<td>3.642</td>
<td>2.1</td>
<td>2.01</td>
<td>2</td>
<td>1.9</td>
</tr>
</tbody>
</table>

For Investor #2:

<table>
<thead>
<tr>
<th>Wealth</th>
<th>80</th>
<th>100</th>
<th>800/7</th>
<th>125</th>
<th>170</th>
<th>200</th>
<th>1600/7</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>0</td>
<td>81</td>
<td>138</td>
<td>180.5</td>
<td>344.5</td>
<td>405</td>
<td>462</td>
<td>504</td>
</tr>
<tr>
<td>Marginal Utility</td>
<td>4.552</td>
<td>4</td>
<td>3.98</td>
<td>3.96</td>
<td>2.375</td>
<td>2</td>
<td>1.99</td>
<td>1.67</td>
</tr>
</tbody>
</table>

The equilibrium allocations and prices are then as follows (note that (9) is violated):

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>No information</td>
<td>(100,200)</td>
<td>(100,200)</td>
<td>(2,1)</td>
</tr>
<tr>
<td>Signal 1</td>
<td>(120,230)</td>
<td>(80,170)</td>
<td>(2.4,.8)</td>
</tr>
<tr>
<td>Signal 2</td>
<td>(600/7,1200/7)</td>
<td>(800/7,1600/7)</td>
<td>(12/7,8/7)</td>
</tr>
<tr>
<td>Signal 3</td>
<td>(75,150)</td>
<td>(125,250)</td>
<td>(1.5,1.25)</td>
</tr>
</tbody>
</table>

The corresponding expected utilities are then \( v_1^0 = 238, v_1^1 = 238.21, v_2^2 = 200.27, v_3^3 = 182.67, v_2^2 = 243, v_1^1 = 137.8, v_2^1 = 323.14 \) and \( v_2^3 = 396.17 \). Consequently, \( v_1(Y) = 238.55 > v_1^0 \) and \( v_2(Y) = 243.87 > v_2^0 \).
References


