TO PAY OR NOT TO PAY DIVIDENDS

by

Nils H. Hakansson*
University of California, Berkeley

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I. INTRODUCTION

Our understanding of why corporations pay dividends is currently unsatisfactory. On the one hand, received theory tells us that dividends are irrelevant (in the sense that any two arbitrarily chosen dividend policies have equivalent consequences), both in the absence of taxes (Miller and Modigliani [33]) and in their presence (Miller and Scholes [34]). On the other hand, dividends continue to flood the empirical world with cash as regularly and as consistently as the sun scorches the desert and one is hard put to characterize this pattern (currently at an annual rate of about $63 billion\(^1\)) as being founded an irrelevance. Not surprisingly, the attendant anomaly has led some, notably Black [6], to suggest that we really don't know why companies pay dividends. Something is clearly amiss.

To briefly summarize the preceding theory, Miller and Modigliani demonstrated in their classic paper [33] that dividend policy is indeed irrelevant in a world of perfect markets and in the presence of a tax differential of zero between dividends and capital gains. Among other things, this theory effectively refutes the so-called "bird in hand" fallacy.\(^2\) On the other hand, in a world in which dividends are taxed more heavily than capital gains, many authors have noted that taxable investors often prefer zero dividends to positive dividends (e.g., Brennan [11], Litzenberger and Ramaswamy [28, 29]), before-tax income being equal (although such analyses have tended to ignore how the government spends its tax take). However, Miller and Scholes [34] have documented how current tax laws in the U.S. tend to neutralize the tax consequences of differential taxation of dividend and capital gains. Specifically, they showed that most investors, regardless of their attitude
toward risk, can, by a judicious blend of dividend-paying securities, tax-sheltered investments and margin purchases, hold level (or even decrease) their tax liabilities from what they would be if all income were taxed at capital gain rates (which in their case are assumed to be zero). The key to this result is the ability of any given investor to attain the same portfolio payoff pattern under a multitude of dividend policies even when dividends are taxed to the individual and capital gains are not. However, no formal attention was given by Miller and Scholes to what would be needed to translate these results to a general equilibrium setting, nor to the various costs associated with positive dividend payments.

The dividend irrelevance notion has not gone unchallenged, of course, and a number of models in which dividends are somehow relevant have also been proposed. Ross [40], Heinkel [23], and Bhattacharya [4, 5] have developed models in which the dividend payout serves as a signal of how well the firm is doing. That is, the dividend decision is seen as a means through which management can provide information to investors about perceived changes in production opportunities, etc. This approach has intuitive appeal since it assumes that signaling is costly and that firms maximize value so that the correct incentives must be present for signaling to occur. But these models also have important structural shortcomings. First, the portfolio aspect of the investor's problem has been assumed away. Second, these models leave no room for a formal treatment of information receipt as an updating of probability beliefs. Finally, investor welfare is reflected indirectly, and imperfectly, through value maximization by firms instead of measured directly via expected utility.

A second type of alternative model is due to Kalay [24], who sees dividends as an instrument in the conflict of interest between stockholders
and bondholders. In particular, bond covenants prescribe (upper) constraints on dividend payments which are found to be tighter the greater the leverage and the company's growth potential. Valid as these constraints may be in determining corporate dividend policy, they do not explain why companies pay positive cash dividends in the first place, however, which is the central question at hand.

Finally, Feldstein and Green [16] consider a model in which risk-averse investors prefer to have a portion of the firm's profits paid out as a dividend (and subjected to ordinary taxes) over having all of the profits invested by the firm and subject only to capital gains treatment. This result, however, appears to be driven by the fact that only investors, and not firms, can make riskless (real) investments; as soon as risk-free investment is made available to the firms as well, the reason for paying dividends disappears.

The present paper will look to the information content of dividends as a substantive (although not necessarily complete) explanation for the prevalence and persistence of positive dividend policies in market economies. The notion that dividends may constitute a source of information is, of course, not new (see e.g., Miller and Modigliani [33], Black [6], Stern [41]). The basic idea is that the raising and lowering of dividends communicates information over and beyond what is provided by (mandated and nonmandated historical cost-based) earnings reports, forecasts, and other announcements. By in effect merging extant dividend theory with the theory of public information, the present paper generates several noteworthy consequences. In particular, it extends substantially the current paradigm, bringing within its domain cases in which dividend payments are beneficial to investor welfare, cases in which they are harmful, as well as new cases in which dividend policy is irrelevant. As a result, current theory becomes
in effect a special case of dividend irrelevance, which itself is a special case of the aforementioned richer set of possibilities.

The conditions separating the three cases are somewhat intertwined and not readily communicated in nontechnical language. However, the thrust of the results may be stated as follows: Whether informative or not, dividends serve no useful role when investors are substantially homogeneous, have additive utility, and markets are complete; when associated with positive costs, dividends are under these circumstances deleterious to efficiency. On the other hand, dividends are capable of improving welfare (efficiency) when they are informative provided investors have heterogeneous beliefs, utility is non-additive, or markets are incomplete, even in the presence of deadweight costs. In this context, the power of informative dividends to serve as a substitute for financial markets is especially significant; dividend announcements may under certain circumstances bring an incomplete market to or even beyond the level of efficiency that would be attained if the market were complete.

Since the model in this paper differs significantly from existing dividend models, it may be useful to summarize its key elements. First, the model does not rest on narrow assumptions: investors are risk-averse with arbitrary preferences and allocate their resources between current consumption and a portfolio of securities in a competitive, two-period setting. Second, information is treated formally in that the receipt of information signals, such as dividend announcements, causes probability beliefs to be updated via Bayes' rule. Third, in contrast to earlier studies, the analysis is one of general equilibrium rather than partial equilibrium. The last step has several important implications: for one thing, it now becomes necessary to be concerned about aggregate taxes. On top of this, the Miller and Scholes [34] notion of tax neutrality is no longer adequate but must be supplemented by
"neutralization" of endowments. Fourth, we shall take account of the dead-weight costs that arise with the payment of dividends, in particular higher flotation costs and more costly tax collection procedures.

A fifth notable property of the model is that while the number of firms in the economy may be large, it is not very large (in contrast to the number of investors). Thus, an informative dividend announcement by any given firm will cause non-negligible price changes of all securities, not only when the information in question is viewed as having industry-wide or economy-wide implications but also when it is thought to be firm-specific. The competitive assumption is thus not invoked at the firm level.

Finally, welfare is measured throughout by the ex ante expected utility that investors attain from any given dividend policy, where a policy is defined by the set of dividend levels that a given firm (or set of firms) may choose from in its announcements. This avoids reliance on firm values as a welfare indicator with its attendant problems in less than complete markets, for example. These problems are particularly relevant in the present paper, in which informative dividends are found to pay an important role in just such a context. At the technical level, the analysis extends the results in Hakansson, Kunkel, and Ohlson [22] by incorporating personal taxes; Proposition V, however, is a new result.

The paper proceeds as follows. Section II examines briefly the ability of dividend announcement to communicate incremental information and the power of share purchases to substitute for dividend payments. The basic model and its equilibrium properties are specified in Section III while Section IV examines the equilibria associated with different dividend signals. Section V develops the concept of tax neutrality in the current framework. The extended dividend irrelevance propositions are given in Section VI. Section
VII shows when a zero dividend policy is efficient while Section VIII gives conditions for the converse. The crucial tradeoff between the information contained in dividend announcements and the deadweight costs associated with positive dividends are examined in Section IX. Section X provides an example in which these tradeoffs are present and Section XI summarizes the paper.

II. ON THE INCREMENTAL INFORMATIVENESS OF DIVIDEND ANNOUNCEMENTS, AND DIVIDENDS VS. SHARE REPURCHASES

Before proceeding to the model itself, there are two more issues that need to be addressed: whether dividend announcements do convey information not communicated by other means, and whether cash dividends should be replaced by a program of share repurchases. Since space does not permit even a cursory treatment of either question, the reader is referred elsewhere for details (Hakansson [21]).

As to the first question, evidence suggesting that dividends do provide significant information to the market is given in Lintner [27], Brittain [12], Fama and Babiak [14], Fama, Fisher, Jensen, and Roll [15], Pettit [38], Laub [26], and Charest [13], while others attribute no significant information content to dividends (Watts [42], Gonedes [17]). In the former case, the information affects apparently persist after controlling for reported earnings changes (Pettit [38], Aharoni and Swary [1], Asquith and Mullins [2]) and earnings forecasts (Pemman [37]). On balance, the available evidence thus suggests fairly strongly that actual dividend announcements (changes) do provide unique information to investors in the contemporary market environment. In any case, it is evident that dividend announcements provide an opportunity for management to convey information to investors, if only because such announcements, uniquely among news releases pertaining to corporate performance, must be backed by cold cash.
As to the second item, it has been argued that the payment of cash dividends should be scrapped in favor of a program of share repurchases by the firm since repurchases presumably subject investors to capital gains treatment (with most of the gain in fact deferred) while cash dividends constitute ordinary income for tax purposes. Other reasons can be advanced as well. The real obstacle to substituting a systematic share repurchase program for cash dividends, however, lies with the tax code. A sequence of quarterly share repurchases emulating the payout pattern of cash dividends would surely be construed to be a "dividend" for tax purposes and subjected to ordinary income taxes. This would no doubt be the case even if share repurchases were made only once a year—in which case the information conveyed presumably would be less than through a quarterly cash dividend plan. In sum, then, there are good reasons for believing that cash dividends do convey information not available through other means to investors and (primarily) institutional reasons for accepting the view that (quarterly) share repurchases do not pose a realistic alternative to cash dividends in the present environment.

III. THE BASIC MODEL

Like its many predecessor studies, this paper posits a pure exchange economy with a single commodity (cash). For simplicity, the focal point will be a two-period time frame incorporating the usual assumptions. That is, at the end of period 1 the economy will be in some state \(s\), where \(s = 1, \ldots, n\). There are \(I\) consumer-investors indexed by \(i\), whose probability beliefs over the states are given by the vectors \(\pi_i = (\pi_{i1}, \ldots, \pi_{in})\), where \(\pi_{is} > 0\) for all \(i\) and \(s\) and there is agreement on the subset of states for which \(\pi_{is} > 0\). Preferences are represented by the functions \(U_{is} (c_{i1}, \omega_{is})\), where \(c_{i1}\) is the
consumption level in period 1 and $w_{is}$ is the consumption level in period 2 if the economy is in state $s$ at the beginning of that period. $U_{is}$ is assumed to be monotone increasing in each argument and strictly concave, i.e., consumer-investors prefer more to less and are risk-averse. At the beginning of period 1 (time 0), consumer-investors allocate their resources among current consumption and a portfolio chosen from $J$ securities indexed by $j$. Security $j$ pays $a_{js}^d > 0$ per share at the end of period 1 (time 1), of which $d$, in the case of common shares, is in the form of a dividend; for other types of securities, superscript $d$ will be omitted. Payoffs on common shares paying zero dividends will be denoted $a_{js}^0$. In contrast to payments, declarations of dividends are assumed to take place at the beginning of period 1 (time 0). Once declared, dividends are assumed to be paid with certainty; coupon payments on interest-bearing securities are also viewed as risk-free although the underlying securities themselves may of course be risky. The total number of outstanding shares of security $j$ held by investors (other than passive financial intermediaries) is $Z_j$. Let $z_{ij}$ denote the number of shares of security $j$ purchased by consumer-investor $i$ at time 0; his portfolio $z_i = (z_{i1}, \ldots, z_{iJ})$ then yields the payoff

$$w_{is} = \sum_j z_{ij} a_{js},$$

available for consumption period 2, if state $s$ occurs at the end of period 1. Investor endowments are denoted $(\underline{c}_i, \bar{z}_i)$ and markets, as is usual, are assumed to be competitive and perfect. If the rank of matrix $A = [a_{js}]$ is full (equals $n$), the financial market will be called complete; if not, it will be called incomplete. Aggregate wealth in state $s$ is given by

$$\bar{W}_s = \sum_j Z_j a_{js}.$$
For simplicity, and without loss of generality in terms of the primary objectives of the paper, capital gains are assumed to be taxed at a flat rate \( t_1 < t_2 \), where \( t_1 > 0 \) and \( t_2 < 1 \) is the flat rate of tax that applies to dividends and interest. The tax law is assumed to be administered symmetrically in the sense that interest and dividend payments on short positions, as well as capital losses, give rise to tax refunds at the respective rates. These assumptions imply that the after-tax payoff can be uniquely and independently determined for each security. Consequently, the numbers \( a_{js} \) will generally be assumed to represent the various security payoffs after taxes as well as after net flotation costs or on a net cash flow basis. For comparability, we also require that the firm's net assets, disregarding flotation costs, be the same at the end of period 1 independently of the dividend policy. If we think of a given firm \( j \) as raising \( x_j \) of new equity capital at the end of period 1 in the absence of a dividend, then it will need to raise, having paid a dividend of \( d_j \), \( x_j + d_j Z_j \) on a gross basis. The incremental deadweight flotation cost associated with \( d_j Z_j \) is then most easily thought of as a second subtraction from \( a_{js}^d \), over and beyond the tax effect, in calculating the net per share payoffs \( a_{js}^d \). Thus we obtain

\[
(1a) \quad W_s^d = W_s^o, \quad \text{all } s
\]

in the absence of deadweight costs and

\[
(1b) \quad W_s^d < W_s^o, \quad \text{all } s
\]

in the presence of deadweight costs from either differential taxes or incremental flotation costs or both. The aggregate deadweight costs associated with dividend payment \( d \) will be denoted by \( C = (C_1, \ldots, C_n) \), where
Thus, the presence of deadweight costs implies a reduction in the aggregate resources \( W \) available for second-period consumption.

The set of feasible second-period consumption allocations among investors in market \( A \) is given by

\[
F(A) = \{w | w_i \geq 0, w_i = z_i A, \sum_i z_i j \leq Z_j \}.
\]

Our concern will be with the relationship between \( F(A^d) \) and \( F(A^o) \), where the declared dividend \( d \) applies to some particular firm (or collection of firms).

When

\[
(2) \quad F(A^d) = F(A^o),
\]

which can only occur in the absence of deadweight costs, dividend payment \( d \) will be said to be allocationally neutral (since every feasible payoff with dividends is also feasible without dividends). When deadweight costs are present and

\[
(3) \quad F_C(A^d) \subseteq F(A^o),
\]

we shall say that the deadweight costs \( C \) associated with dividend payment \( d \) are allocationally limiting. That is, when (3) holds, certain allocations that were feasible in the absence of deadweight costs have been rendered infeasible through the appearance of deadweight costs without having added any offsetting possibilities.

Under our assumptions, each consumer-investor \( i \) maximizes

\[
(4) \quad \sum_{s, i} u_{is}(c_i, \sum_j z_{ij} a_{js})
\]

with respect to the decision vector \((c_i, z_i)\), subject to his budget constraint.
\[ c_i p_0 + \sum_j z_{ij} p_j = \bar{c}_i p_0 + \sum_j \bar{z}_{ij} p_j, \]

as a price-taker, where \( p_0 \) is the price of a unit of period 1 consumption and \( p_j \) is the price (at \( t = 0 \)) of security \( j \). Assuming interior solutions (with respect to the implicit non-negativity constraint on consumption \((c_i, w_i) \geq 0\)), the equilibrium conditions may be written

\[ \sum_s \pi_is \frac{\partial U_is(c_i, w_is)}{\partial c_i} = \lambda_i, \quad \text{all } i \]

\[ \sum_s \pi_is \frac{\partial U_is(c_i, w_is)a_{ij}}{\partial w_is} = \lambda_j p_j, \quad \text{all } i, j \]

\[ c_i + z_{ij} p = \bar{c}_i + \bar{z}_{ij} p, \quad \text{all } i \]

\[ \sum_i c_i = \sum_i \bar{c}_i, \sum_i z_{ij} = Z_j, \quad \text{all } j, \]

where the \( \lambda_i \) are the Lagrange multipliers, (8) represents the market clearing equations, and \( p_0 = 1 \) has been chosen as numeraire.

We note that an allocation \((c^*, z^*)\) which constitutes a solution to system (5)-(8) (along with a price vector \(p\) and a vector \(\lambda\)) is allocationally efficient with respect to market structure \(A\); no other allocation \((c, z)\) obtainable within market structure \(A\) can make some consumer-investors better off without making others worse off. However, there may exist trades outside the market (e.g., by the invention of new securities) which yield allocations that Pareto-dominate the market allocation \((c^*, z^*)\). When this is not the case, i.e., when \((c^*, z^*)\) is Pareto-efficient with respect to all conceivable allocations inside and outside the existing market, \((c^*, z^*)\) will be said to be fully allocationally efficient.
To be more precise, let

\[ p_{is} \equiv \frac{\pi_{is} \lambda_i}{\lambda_i} \frac{3U_{is}(c_i, w_{is})}{3w_{is}}. \]

By reference to (6), it can be seen that \( p_{is} \) denotes investor \( i \)'s shadow price of wealth in state \( s \); roughly, it represents the marginal value of receiving an additional unit of wealth in state \( s \).

It is well known that (3)-(8) plus

\[ p_{is} = p_{ls} \quad \text{all } i \geq 2, \text{ all } s \]

is a necessary and sufficient condition for the market allocation \((c^*, z^*)\) to be fully allocationally efficient. This follows because (9) insures that the marginal rates of substitution of wealth (consumption) between any two states are the same for all investors \( i \). Condition (9) plays an important role in what follows, a role which has previously not been well appreciated. A sufficient (but not necessary) condition for (9) to obtain is that the security market is complete, i.e. that the rank of \( A \) is \( n \).

IV. EQUILIBRIA WITH DIVIDENDS

As noted, it is widely recognized that dividend declarations may convey information to investors concerning future payoff patterns. In the present setting, this process may be pictured as follows.

Before trading at time 0 investors may obtain information bearing on the state that will occur at time 1 via a dividend declaration \( d \) from a set of possible signals \( D = \{d_1, \ldots, d_m\} \). We shall refer to \( D \) and its associated probabilities as a dividend policy; it may be thought of as encompassing a single firm alone or a collection of firms.
Each possible dividend signal $d$ causes consumer-investor $i$ to update his prior beliefs $\pi_i^0$ to the posterior beliefs $\pi_i^d$ via Bayes' rule. That is, if $p_i(d|s)$ denotes investor $i$'s perceived probability that dividend $d$ will be declared if $s$ is about to occur, Bayes' Theorem gives

$$\pi_i^d = \frac{p_i(d|s)\pi_i^0}{\sum_s p_i(d|s)\pi_i^0} = \frac{p_i(d|s)}{p_i(d)};$$

when $\pi_i^d \neq \pi_i^0$ for some $d$, dividend policy $D$ is said to be informative for investor $i$.

The $n \times m$ matrix of conditional probability numbers $p_i(d|s)$ for investor $i$ will be called his information structure. Whenever

$$(10a) \quad p_i(d|s) = p_i(d|s) \text{ for all } i \geq 2, d, \text{ and } s,$$

the information structures will be said to be homogeneous; if (10a) does not hold, they will be called nonhomogeneous. Finally, when there exist numbers $k_i(d)$ such that

$$(10b) \quad p_i(d|s) = k_i(d)p_i(d|s) \text{ for all } i \geq 2, \text{ all } s \text{ and } d,$$

the information structures will be said to be essentially homogeneous. This is because, as a practical matter, (10b) is only slightly more general than (10a).

Denote the maximum of (4) in the no information case, i.e., the consumer-investor's equilibrium expected utility when (all) investors make decisions on the basis of their prior beliefs $\pi_i^0$, by $V_i^0$, and let $(c_i^0, z_i^0)$ be the resulting equilibrium allocation to consumer-investor $i$. When dividend policy $D$ is in use, there will be a different equilibrium for each signal $d$. Let $V_i^d$ and $(c_i^d, z_i^d)$ represent the consumer-investor $i$'s expected utility and final
allocation when equilibrium is based on revised beliefs \( \pi_i^d \) (signal \( d \)). The probability of receiving signal \( d \) is \( p_i(d) = \sum_s p_i(d|s)\pi_i^s \); thus the relevant expected utility of consumer-investor \( i \) in the dividend case, which we label \( V_i(D) \), is

\[
V_i(D) = \sum_d p_i(d) v_i^d = \sum_d \pi_i^d p_i(d|s) u_i(c_i^d, z_i^d, \pi_i^d).
\]

Following standard practice, dividends will consequently be said to have social value or yield a Pareto-improvement if

\[
V_i(D) > V_i^O \quad \text{all } i
\]

(11)

\[
V_i(D) > V_i^O \quad \text{some } i
\]

but not otherwise.

A set of equilibrium allocations \((c^d, z^d)\), one for each signal \( d \), will be said to be informationally efficient with respect to market \( A \) and dividend policy \( D \) if there are no other allocations \((c^1, z^1), \ldots, (c^m, z^m)\) based on \( A \) and \( D \) which can make some consumer-investors better off without making others worse off. A necessary and sufficient condition (in addition to (5)-(8)) for informational efficiency to be satisfied is that the endowments \((c, z)\) are such that

\[
\frac{p_i(d) \lambda_i^d}{p_i(d) \lambda_1^d} = \frac{p_1(d) \lambda_1^d}{p_1(d_1) \lambda_1^d}, \quad \text{all } i, d,
\]

(12)

i.e., that the marginal rates of substitutions of endowments, for any two signals \( d \), are the same for all investors.

It should be noted that informational efficiency, as opposed to allocational efficiency, cannot be guaranteed. Endowments satisfying (12) may
simply fail to exist. This would typically be the case, for example, if the number of signals \( m \) exceeds the number of securities \( J \) since (12), in essence, gives rise to \( m \) equations in \( J \) unknowns.\(^{11}\)

Finally, full informational efficiency with respect to \( D \) will be said to occur if (12) holds in conjunction with

\[
(13) \quad p^d_{is} = p^d_{ls}, \quad \text{all } i \geq 2, s, \text{ and } d.
\]

This is because (13) insures full allocational efficiency for each dividend signal \( d \).

In what follows, it will sometimes be assumed that endowments represent efficient allocations without dividends, which implies

\[
(14) \quad (c^0_i, \bar{w}_i) = (\bar{c}_i, \bar{w}_i), \quad \text{all } i.
\]

V. TAX NEUTRALITY

Miller and Scholes \cite{34} observed that differential tax rates on dividends and interest on the one hand and capital gains on the other lead to the same result as if all income were taxed at (zero) capital gains rates under conditions with considerable similarity to U.S. tax laws. The ability to offset interest payments on margin purchases against dividends and the availability of tax-exempt or tax-deferred vehicles constitute the cornerstones of this phenomenon. In the context of the current paper, the preceding equivalence will be depicted as follows. Let the economy with differential taxes contain a (passive, non-taxable) financial intermediary (the MS Life Assurance Society, say)\(^{12}\) which is permitted to own, prior to trading, all risk-free bonds in the economy plus a sufficient quantity of such bonds (held short by the various consumer-investors) for investors to achieve a perfect offset between the total interest
payment on these additional bonds and the totality of taxable dividends plus the interest receipts on all risky debt securities. Against its long position in coupon bonds, the intermediary is short an equal number of discount bonds, on which holding gains are taxable to investors at rate \( t_1 \). Assuming at the same time that each investor is permitted to exchange ordinary (risk-free) bonds for discount bonds, on a one for one basis, to the point where his income taxable at \( t_2 \) based on his endowment is zero, it is demonstrated in the Appendix that each consumer-investor faced with differential taxes will in fact, when trading opens, choose the same portfolio of risky assets in equilibrium as if all income had been taxed at (the lower) rate \( t_1 \); in addition each investor will go short in the risk-free bonds sufficiently to exactly offset all income taxed at rate \( t_2 \), placing the remainder of his funds in the discount securities. This causes his after-tax equilibrium payoff pattern to be identical to the one he would have achieved with all income taxed at the lower rate \( t_1 \). Note also that total taxes paid would be the same in the two economies and that (2), which is valid in the absence of deadweight costs and in the presence of risk-free declared dividends and coupon payments, has been assumed to hold.

The intermediary and the accompanying endowment exchange we have posited gives rise to a situation which we shall refer to as tax neutrality between the single tax rate and the dual tax rate economies. It is the two-period general equilibrium analogue, for the case in which capital gains are taxed at positive rates, to the Miller and Scholes [34] tax-neutrality dividend scenario.

Whether the equilibrium prices in the two economies differ or not depends on how investors view tax payments. If taxes are looked on as separate and distinct outflows, which we refer to as the separable tax scenario, dividends
and coupons would have had no effect on security prices. On the other hand, if investors price securities on the basis of after-tax cash flows, as we assume in most of the paper (the net cash flow scenario), equilibrium prices are affected as follows. Let \( P_J \) be the price of, and \( r \) the equilibrium return on, the risk-free bonds in the single tax-rate economy. Then \( P_J \) will also be the equilibrium price of the pure discount bonds in the dual tax-rate economy. All other equilibrium prices \( P''_j \) in the dual-rate economy will be less than the corresponding single tax-rate equilibrium prices \( P'_j \) by the present value (at rate \( r \)) of two items: the incremental tax that applies to the dividend or interest coupon plus the capital gains tax on \( P'_j - P''_j \). Thus, the equilibrium price of common stock paying \( Sd \) per share, for example, will be less by

\[
(t_2 - t_1)\frac{d}{(1 + r - t_1)}
\]

and that of a bond with coupon \( i \) by \( (t_2 - t_1)i/(1 + r - t_1) \) (see the Appendix). Consequently, the before-tax return on a dividend-paying stock is an increasing function of the dividend under tax neutrality in the net cash flow scenario. The net result is that the influence of dividends on before-tax returns to investors depends critically on how investors view taxes even under tax neutrality.\(^\text{17}\)

Note that the holdings of the tax-neutralizing intermediary depend directly on \( d \). To perform its function it must therefore constantly adapt its position. While this may seem a bit farfetched in practical terms, the exactly neutralizing intermediary does provide us with a useful point of departure.

VI. DIVIDEND IRRELEVANCE UNDER UNCERTAINTY

It is natural to begin by asking under what conditions dividends are irrelevant. Adopting the assumptions of tax neutrality or an absence of
taxes coupled with zero deadweight costs from Section III, we first consider the case in which dividend declarations convey no information beyond what is already known; that is no new information. Since this implies that \( \pi^d_{is} = \pi^o_{is} \) for all \( i, s, \) and \( d \), we trivially obtain

**Proposition 1.** Dividend policy is a matter of irrelevance \( (V_i^D(D) = V_i^O, \text{ all } i) \) in a world of no taxes or tax neutrality for all signals \( d \) whenever dividend declarations convey no information.

The above can be thought of as a restatement of the irrelevance propositions of Miller and Modigliani [33] and Miller and Scholes [34] although the key words "no information" have generally not been emphasized in these studies. Recall also that tax neutrality only holds in the absence of deadweight costs and in conjunction with (2) and tax-neutral endowments.

Let us now turn to the case in which dividend declarations do convey information to investors while retaining the other assumptions of Proposition 1. As it happens, dividends will now be irrelevant only if each possible declaration causes no trading, that is, only if the change in beliefs is perfectly offset by the resulting changes in equilibrium prices in such a way that no trading becomes necessary. In the taxless case, no less than five conditions must simultaneously be present for this to hold true under arbitrary preferences and non-null information structures (see Hakansson, Kunkel, and Ohlson [22, Lemma 2]); it is readily shown that the same conditions extend to the tax neutrality case as well. Consequently, endowments must be efficient before the declaration, that is (14) must hold.\(^{18}\) Second, (10b) must be valid, i.e. the relevant information structures must be essentially homogeneous. Third, the market must exhibit full allocational efficiency in the absence of the declaration, that is, we must have
(15) \[ p_{is}^0 = p_{ls}^0, \quad \text{all } i \geq 2, \text{ all } s. \]

Fourth, signal beliefs, or equivalently prior beliefs given (10b), must be essentially homogeneous, that is, we require

(16b) \[ p_i(d) = k_i(d)p_1(d), \quad \text{all } i \geq 2, \text{ all } d, \]

where \( k_i(d) \) is the same as in (10b). Finally, utility must be time-additive, which is equivalent to saying that we must be able to write

(17) \[ U_{is}(c_i, w_{is}) = f_i(c_i) + g_{is}(w_{is}), \quad \text{all } s \text{ and } i. \]

Formally, the preceding may be stated as

**Proposition II.** Dividend policy is a matter of irrelevance \( (V_i(D) = V_i^0, \text{ all } i) \) in a world of no taxes or tax neutrality when dividend declarations convey information if and (for arbitrary preferences) only if all of the conditions (14), (10b), (15), (16b), and (17) hold, that is, if and only if

(a) the endowments represent an efficient allocation in the absence of the dividend declaration.

(b) the information structures are essentially homogeneous,

(c) the financial market is fully allocationally efficient in the absence of dividends

(d) the signal beliefs are essentially homogeneous, and

(e) the utility functions are time-additive.

While the last four of the above assumptions are somewhat strong, they have been commonly employed in financial modeling. Thus, dividend policy is indeed irrelevant in a number of popular models, both with and without taxes, in which no deadweight costs are present. Proposition II may therefore be viewed as a significant extension of the extant theory of dividend irrelevance.
VII. WHEN TO SCRAP DIVIDENDS

In the previous section, the irrelevance of dividends depended crucially on the absence of deadweight costs, both in connection with the tax structure and with the incremental sale of new shares in the presence of dividend payments. Such a benign view of these two phenomena may, of course, not be justified. We shall therefore examine the implications of positive deadweight costs on dividends.

To begin the analysis, let $F^*(A^d)$ denote the set of efficient second-period allocations $w$ in the absence of deadweight costs in the presence of dividend payment $d$. It then follows from Lemma 3 in Hakansson, Kunkel, and Ohlson [22] that the efficient allocations $F^*(A^d)$ are such that we obtain either

\[(18) \quad V_i(D) = V_i^0 \quad \text{all } i \]

or

\[(19) \quad V_i(D) < V_i^0 \quad \text{some } i \]

whenever each of the four conditions (10a), (15),

\[(16a) \quad p_i(d) = p_i(d) \quad \text{all } i \geq 2, \text{ all } d, \]

and (17) hold. That is, an informative dividend policy has no social value, independently of (14), if both the information structures and the signal beliefs are homogeneous, the financial market is fully allocationally efficient, and utility is time-additive.

Let us now introduce deadweight costs, making no other changes. This causes (1b) and possibly (3) to become valid. Let $F^*_C(A^d)$ denote the set of efficient second-period allocations in the presence of dividend $d$ and its accompanying deadweight costs $C$. Whenever (3) holds, every efficient second-period
allocation \( w^* \in F^*(A^d) \) now becomes infeasible, i.e. \( w^* \not\in F^*_C(A^d) \). This implies immediately that at least some investors must be worse off with dividends than they are without dividends, i.e., (19) or

\[
(20) \quad V_i(D) < V_i^0, \quad \text{all } i
\]

must hold whenever dividends are either 1) not informative or 2) informative in the presence of conditions (10a), (15), (16a) and (17). Formally this may be stated as

**Proposition III.** Suppose that dividends induce positive deadweight costs (via differential taxation rules and/or additional flotation costs) and that these deadweight costs are allocationally limiting, i.e., (3) is satisfied for all signals \( d \). The declaration and payment of dividends will then cause at least some investors (and possibly all) to be worse off with dividends than without dividends whenever either

- (a) the dividend declaration conveys no information, or
- (b) the dividend declaration conveys information but (10a), (15), (16a), and (17) hold, that is,
  - (1) the information structures are homogeneous
  - (2) the financial market is fully allocationally efficient in the absence of dividends
  - (3) the signal beliefs are homogeneous, and
  - (4) the utility functions are time-additive.

The preceding shows that informative dividends have nothing to contribute to the "level of efficiency" achieved by an economy with homogeneous beliefs and information structures, full allocational efficiency, and additive preferences; in fact, in such an economy the presence of deadweight costs reduces
the level of efficiency by moving it to a lower Pareto surface than that which applies in the zero dividend case.  

VIII. WHEN TO PAY DIVIDENDS

Just as there are clear-cut cases when dividends reduce efficiency, there are also clear-cut cases when positive dividends guarantee a Pareto-improvement, i.e., (11) occurs. Three conditions are pivotal for this to be true; the absence of deadweight costs, the provision of non-null information by the dividend declaration, and endowments that are efficient without the information. If in addition at least one of conditions (10b), (15), (16b), or (17) does not hold, trading must occur when the dividend signal is released and that trading can only lead to a Pareto-improvement in the taxless case (Hakansson, Kunkel, and Ohlson [22, Theorem 1]). The extension to the tax-neutrality case is again straightforward. Thus we obtain

Proposition IV. Suppose that there are no taxes or that tax neutrality prevails (so that there are no deadweight costs), that a given dividend policy conveys information, and that endowments are efficient in the absence of dividend declarations, i.e., (14) holds. The declaration and payment of dividends then yields a Pareto-improvement (except for singular preference structures) provided that at least one of conditions (10b), (15), (16b), or (17) is violated, that is, it is sufficient for (11) to hold that either

(a) information structures are not essentially homogeneous

or that

(b) the financial market is not fully allocationally efficient in the absence of dividends

or that

(c) signal beliefs are not essentially homogeneous
or that
d) the utility functions are non-additive.

Since none of conditions (a) through (d) in the above Proposition is in any sense farfetched, the case for the payment of informative dividends appears quite strong—provided there are no deadweight costs to worry about.

One question that suggests itself is whether a policy of informative dividends can bring a market that is less than fully allocationally efficient in the absence of dividends (say an incomplete market) to the equivalent of full allocational efficiency or better. The answer is in the affirmative because dividends may be able to bring an incomplete market to full informational efficiency, which implies a Pareto surface at least as high as that corresponding to full allocational efficiency without dividends in the tax neutral case.20 By reference to Section IV, two conditions are required. First, (12) must hold, i.e., the marginal rates of substitution of endowments, for any two signals d, must be the same for all investors. Second, the market must be conditionally complete, that is, it must be complete given any dividend declaration d, which implies (13). In other words, when all the states which under the revised beliefs have zero probability have been eliminated to form matrix $A^d$, conditional completeness is satisfied if the rank of $A^d = S(d)$, where $S(d)$ is the number of states with positive probability given signal d. Formally, the preceding may be stated as

**Proposition V.** Suppose that there are no taxes or that tax neutrality prevails (so that there are no deadweight costs), that a given dividend policy D conveys information, that endowments are such that

\[
\frac{p_i(d)\lambda_1^d}{d_1} \frac{\lambda_1^d}{p_i(d_1)\lambda_1^d} = \frac{\lambda_1^d}{p_i(d_1)\lambda_1^d}, \quad \text{all } i, d,
\]
and that markets are conditionally complete for each dividend signal d. The declaration and payment of dividends based on D then gives rise to full informational efficiency.

Note again that (12) is "likely" to hold if the number of dividend signals m is less than the number of securities J but not otherwise (since (12) essentially gives rise to m linear equation in J unknowns). Thus, if there are 1,000 securities and 10,000 states, we clearly have less than a complete market in the absence of information. But with 25 distinct dividend signals, for example, one can readily visualize the attainment of full allocational efficiency for each signal even in the presence of arbitrary preferences and beliefs as long as \( S(d) \leq 1000 \) for all d. Dividend signals, in other words, may serve as a direct substitute for markets. Note also that (12), if feasible, would automatically be attained if (pre-signal) trading were permitted.

IX. THE CRITICAL TRADEOFF

In view of the analysis of the two previous sections, it is apparent that the social value of an informative dividend policy is subject to two opposing forces: the conditions which promote social value in the absence of deadweight costs, and the deadweight costs themselves, which act in the opposite direction. The net result, therefore, depends on the particular tradeoffs that are attained in any particular circumstance.

Since the positive forces do not always overpower the negative ones, it will be helpful to identify the conditions which act in the positive direction. It was shown in Håkansson, Kunkel, and Ohlson [22, Theorem 2] that, in the absence of deadweight costs, it is necessary (but not sufficient) that either (10a), (15), (16a), or (17) be violated in order for a Pareto-improvement to
occur. It follows from Propositions III and IV that these conditions remain necessary when deadweight costs are introduced as long as condition (3) applies. This gives our final result.

**Proposition VI.** Suppose that dividends induce positive deadweight costs (via differential taxation rules and/or additional flotation costs) and that these deadweight costs are allocationally limiting, i.e. (3) is satisfied for all signals d. A necessary (but not sufficient) condition for dividends to yield a Pareto-improvement is that the dividend declaration conveys information and that either (10a), (15), (16a), or (17) is violated, that is, it is necessary that either

(a) information structures are not homogeneous

or that

(b) the financial market is not fully allocationally efficient in the absence of dividends

or that

(c) signal beliefs are not homogeneous

or that

(d) the utility functions are non-additive.

Again, it is noteworthy that each of conditions (a)-(d) is highly plausible so that the potential for informative dividend policies to lead to Pareto-improvements must be viewed as substantial. Since "more complete" markets also improve welfare under certain conditions (Hakansson [20]), we confirm once again, with reference to condition (b), that informative dividend policies can serve as a powerful substitute for a richer financial market.
X. AN EXAMPLE WITH DEADWEIGHT COSTS

The case in which an informative dividend policy yields a Pareto-improvement even in the presence of a deadweight loss will now be illustrated via an example with respect to condition (b) of Proposition VI (a violation of (15)). Tax rates are assumed to be as follows: $t_1 = 0$, $t_2 = .25$.

Suppose that, for simplicity, there are two consumer-investors, two securities (a stock and a risk-free bond), and three states. In the absence of an informative dividend policy, the beliefs of investor 1 are given by $\pi^0_1 = (.4, .2, .4)$ and those of investor 2 by $\pi^0_2 = (.3, .4, .3)$. Consider the (informative) dividend policy which calls for paying a dividend $d$ of either $\$4$ or $\$8$ with the following conditional probabilities:

\[
\begin{array}{c|cc}
   & 4  & 8  \\
\hline
   s & 4  & 8  \\
1 & 1  & 0  \\
2 & .6 & .4 \\
3 & .2 & .8 \\
\end{array}
\]

Bayes' Theorem now implies that the declaration of a $\$4$ dividend causes the two consumer-investors to revise their beliefs to $\pi^4_1 = (2/3, .2, 2/15)$, and $\pi^4_2 = (.5, .4, .1)$, respectively; similarly, the declaration of an $\$8$ dividend causes their beliefs to be revised to $\pi^8_1 = (0, .2, 8)$ and $\pi^8_2 = (0, .4, .6)$, respectively. In addition, the prior beliefs of both investors cause both of them to anticipate the $\$4$ dividend with probability $.6$ and the $\$8$ dividend with probability $.4$.

Suppose also that preferences are additive and state-independent, with second-period utilities $u_{12}$ and marginal utilities $u'_{12}$ for selected consumption levels given by:
In the absence of dividends the payoffs of the two securities are assumed to be the following:

\[
A^0
\]

\[
j \backslash s \quad 1 \quad 2 \quad 3 \quad Z
\]

\[
\begin{array}{cccc}
\text{Bond} & 100 & 100 & 100 & 100 \\
\text{Stock} & 52 & 102 & 202 & 100 \\
W & 15,200 & 20,200 & 30,200 & \\
\end{array}
\]

With a $4 dividend the marginal tax of $1 (0.25) is assumed to be a net deadweight loss on the stock, giving rise to the following payoff matrix:

\[
A^4
\]

\[
j \backslash s \quad 1 \quad 2 \quad 3 \quad Z
\]

\[
\begin{array}{cccc}
\text{Bond} & 100 & 100 & 100 & 100 \\
\text{Stock} & 51 & 101 & 201 & 100 \\
W & 15,100 & 20,100 & 30,100 & \\
\end{array}
\]
Similarly, with an $8 dividend, the net deadweight loss on the stock is assumed to be $2, giving rise to the following payoffs: 21

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<thead>
<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>Z</th>
</tr>
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<tbody>
<tr>
<td>Bond</td>
<td>100</td>
<td>100</td>
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<tr>
<td>Stock</td>
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<tr>
<td>W</td>
<td>15,000</td>
<td>20,000</td>
<td>30,000</td>
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It is readily verified that condition (3) is satisfied for both dividend signals.

If endowments are \( \overline{z}_1 = (50, 50) \) and \( \overline{z}_2 = (50, 50) \), there would be no trading in \( A^0 \) since the endowment allocation is in fact an equilibrium supported by the price vector \( P^0 = (90, 91.76) \). This yields the second-period consumption allocations \( w_1^0 = w_2^0 = (7,600, 10,100, 15,100) \); the resulting expected utilities are \( V_1^0 = k_1 + 3,810 \) and \( V_2^0 = k_2 + 3,420.5 \), respectively (where \( k_1 \) represents the utility of first-period consumption).

If a dividend of $4 is declared, individual 1 will alter his endowed portfolio \( \overline{z}_1 = (50, 50) \) by trading to \( \overline{z}_1^4 = (60, 36.66) \), which in turn yields consumption levels \( w_1^4 = (7,870, 9,703, 13,369) \) in the three states. Analogously, individual 2 will switch his portfolio from \( \overline{z}_2 = (50, 50) \) to \( \overline{z}_2^4 = (40, 63.34) \), which gives \( w_2^4 = (7,230, 10,397, 16,731) \). First-period consumption levels remain unchanged. The equilibrium prices are $95 for the bond and $71.20 for the stock. Expected utilities are \( V_1^4 = k_1 + 2,108 \) and \( V_2^4 = k_2 + 2,139 \), respectively.

If on the other hand an $8 dividend is declared, individual 1 will switch his portfolio from \( \overline{z}_1 = (50, 50) \) to \( \overline{z}_1^8 = (-30, 100) \), which turn yields consumption levels $7,000 and $17,000 for states 2 and 3,
respectively (state 1 cannot happen in this case). Analogously, individual 2 will alter his endowed portfolio from (50, 50) to (130, 0), which provides a certain second-period consumption level of $13,000. Again, first-period consumption levels remain unchanged. The equilibrium price vector \( P^8 \) equals (80, 128) and expected utilities are \( V^8_1 = k_1 + 6,628.8 \) and \( V^8_2 = k_2 + 5,620 \), respectively.

Weighting the two dividend-induced utilities with their respective probabilities, we obtain \( V_1(D) = k_1 + 3,916.32 > V^0_1 = k_1 + 3,810 \) and \( V_2(D) = k_2 + 3,531.4 > V^0_2 = k_2 + 3,420.5 \). This demonstrates, for the case at hand, the superiority of an informative dividend policy over a no-dividend policy even though dividends result in deadweight costs. The reason, of course, is simply that such a policy makes possible a better allocation of securities among investors.

XI. SUMMARY AND CONCLUDING REMARKS

The well-known dividend-irrelevance propositions for the taxless case (Miller and Modigliani [33]) and the tax-neutrality case (Miller and Scholes [34]) have been recast in a general equilibrium framework, extended, and embedded in a larger paradigm in which the potential of dividends to provide information was explicitly recognized. This enlarged framework led to a rich set of propositions.

The traditional case of dividend irrelevance occurs when dividend declarations fail to neither provide new information to the market participants nor to generate deadweight costs (Proposition I). The extended environment in which dividend policy is irrelevant occurs when deadweight costs are again absent but dividend declarations do provide information to investors provided
it also happens that the resulting revision in beliefs is precisely offset by security price changes in such a way as to make trading unnecessary (Proposition II). The latter only occurs if no less than five conditions are satisfied: endowments are efficient without the information, the information structures are essentially homogeneous, the financial market (without dividends) is fully allocationally efficient, the signal beliefs are essentially homogeneous and the utility functions are additive. If (allocationally limiting) deadweight costs are present when information structures and signal beliefs are homogeneous, the market achieves full allocational efficiency, and preferences are time-additive, then the payment of non-zero dividends will cause at least some, and possibly all, investors to be worse off than they would be with zero dividends, regardless of endowments (Proposition III).

Conversely, when dividend declarations convey new information to the market participants, a policy of sufficiently positive and variable dividends will often Pareto-dominate a zero dividend policy. Such is the case when endowments are efficient in the absence of dividends and one (or more) of the following holds: information structures are not essentially homogeneous, the financial market (in the absence of information) is not fully allocationally efficient, signal beliefs are not essentially homogeneous, or utilities are non-additive. Given the above, a policy of informative dividends always Pareto-dominates a no dividend policy when deadweight costs are zero (Proposition IV) and sometimes when such costs are positive as well (Proposition VI and the example); full informational efficiency may be attained even in incomplete markets (Proposition V). In view of the plausibility of each of the preceding four conditions, one should perhaps not be surprised at the prevalence and persistence of positive dividends in market economies.
In sum, we have demonstrated that dividends, whether informative or not, serve no useful role when investors have homogeneous beliefs and time-additive utility and markets exhibit full allocational efficiency—when associated with positive costs, dividends are under these circumstances deleterious to efficiency. On the other hand, dividends are capable of improving welfare (efficiency) when they are informative provided investors have heterogeneous beliefs, utility is not additive, or markets are incomplete, even in the presence of deadweight costs. In this context, the power of informative dividends to serve as a substitute for additional financial markets is particularly notable. Moreover, this demonstration was cast in a setting that does not compromise the portfolio problem, that treats information formally, that is sensitive to possible deadweight costs, that employs a general equilibrium perspective, and that measures investor welfare in expected utility.

The limitations of the framework used should also be noted. First, the analysis was based on a two-period model; it is always possible that a multi-period framework would capture additional elements of the dividend decision. Second, the concept of (strict) tax neutrality, which played a crucial role in Propositions I, II, IV, and V and is useful in terms of the insights gained, is somewhat artificial.

Third, the paper considered only two alternatives, zero dividends and an informative policy of positive dividends. An ordering of all conceivable dividend policies would of course be desirable. Limited
results in this area are actually readily obtained. Dividends policies which can be ranked by the informativeness criterion (Blackwell [8]), a criterion which produces a partial ordering, can be similarly ordered in terms of the level of economic efficiency achieved (but generally not in terms of welfare) in those cases in which deadweight costs are absent (see Ohlson [36]). Beyond this, however, comparative statements appear somewhat elusive.

Fourth, the question of whether to pay or not to pay dividends has been addressed from the perspective of the welfare implications to the owners (the consumer-investors). While this is a natural first step, the problem of what kinds of incentive schemes would be required to induce managers to carry out the wishes of the owners is equally important and remains to be addressed.

Finally, it should be reiterated that the demand for cash dividends, insofar as it is not fully accounted for by the after-tax return itself, stems in the present paper solely from the information conveyed by the applicable dividend declaration. Thus, the present paper offers no explanation for the observed preference for cash dividends over stock dividends, other things, in particular the information conveyed by the dividend declaration, being equal, that was documented by Long [31]. An explanation for this preference must await another effort.
Appendix

To distinguish the two economies, various quantities in the single tax-rate economy will be "primed" and the corresponding quantities in the dual tax-rate economy "double-primed". Thus, equilibrium prices for example will be denoted $P'_j$ and $P''_j$, respectively. Letting $f_j = d_j / d_j$, where $d_j$ is the interest rate on the risk-free bond ($J = 1$), endowment neutrality implies that

$$
\widetilde{z}''_i = \widetilde{z}'_i \quad \text{all } j < J,
$$

$$
z''_i = \sum_{j < J} f_j \widetilde{z}''_{ij} \quad , \quad z''_j = \sum_{j < J} f_j z''_j
$$

$$
\widetilde{z}_{iJ}'' = \widetilde{z}'_{iJ} - \widetilde{z}''_{iJ} \quad , \quad Z''_j = Z'_j - Z''_j
$$

where $B = J + 1$ is the pure discount bond, which in turn implies

$$
(c''_i, w''_i) = (\widetilde{c}'_i, \widetilde{w}'_i) \quad , \quad \text{all } i .
$$

In the separable tax scenario, offsetting the excess ordinary tax $k_j$ attributable to security $j$ against an allocated interest deduction benefit $b_j$, we obtain

$$
as''_j = a'_j - k_j + b_j \quad \text{all } j < J
$$

where

$$
k_j = (t_2 - t_1) d_j
$$

$$
b_j = f_j (t_2 - t_1) d_j .
$$

Let $(c', z', P')$ be a solution to system (5)-(8) in the single tax-rate economy. Then it is readily verified, by reference to (5)-(8), that the dual tax-rate economy has an equivalent solution $(c'', z'', P'')$ such that
\[
\begin{align*}
&b_j = f_j(t_2 - t_1)d_j \quad \text{all } j < J \\
&a''_j = a'_j, \quad \text{all } j < J, \quad a''_B = a'_B = a'_j.
\end{align*}
\]

(21) \[
z_{ij}' = a_{ij}', \quad \text{all } j < J, \quad \text{all } i
\]

(22) \[
z_{ij}'' = \sum_{j < J} f_j z_{ij}', z_{iB}' = z_{iJ}' - z_{ij}'', \quad \text{all } i
\]

\[
\text{\[} P_j'' = P_j', \quad \text{all } j < J, \quad P_J'' = P_J'' = P_J'.
\]

i.e.

(23) \[
(c'', w'', P'') = (c', w', P')
\]

In the net cash flow scenario, we have

\[
a''_j = a'_j - \frac{(t_2 - t_1)d_j - t_1(P_j' - P_j')}{l + r - t_1} \quad \text{all } j < J,
\]

\[
a''_j = a'_j - \frac{(t_2 - t_1)d_j - t_1(P_j' - P_j')}{l + r - t_1}, \quad a''_B = a'_B = a'_j.
\]

Allocation (21)-(23) is again a solution to the system (5)-(8) but the dual tax-rate equilibrium prices are now given by

\[
P_j'' = P_j' - \frac{(t_2 - t_1)d_j}{l + r - t_1} \quad \text{all } j < J,
\]

\[
P_J'' = P_J' - \frac{(t_2 - t_1)d_J}{l + r - t_1}, \quad P_B'' = P_J'.
\]
Footnotes

1. See Malabre [32].

2. The notion expressed by this fallacy is that a dollar in hand is worth more than a dollar retained by the firm. However, despite the difficulties of defending this argument, a number of models have recently been proposed to the effect that the market value of a dollar of (after-tax) retained earnings is less than one (King [25], Auerbach [3], Bradford [10]).

3. See also Gordon and Malkiel [19].

4. This is usually the case if the shareholder's percentage ownership after the share repurchase has been completed is more than 80% of what it was before the redemption.

5. That is, consumer-investors perceive prices as beyond their influence, there are no transaction costs, securities are perfectly divisible, and the proceeds from short sales can be invested.

6. Under current U.S. tax laws, investment interest is deductible to the extent of investment income + $10,000 and capital losses are deductible up to a maximum of $3,000 per year; any remainder can, in both cases, be offset against future years' income.

7. See the end of Section V for an alternative approach.

8. A sufficient condition for (2) and (3) to be valid, given a risk-free portfolio, is for each security's payoff in $A^d$ to differ from the corresponding payoff in $A^0$ by at most a linear transformation.
9. For other sufficient conditions, see e.g. Hakansson [20].

10. For example, the two information structures

\[
\begin{array}{ccc|ccc}
  d_1 & d_2 & d_3 & d_1 & d_2 & d_3 \\
  s_1 & .4 & .4 & .2 & .20 & .70 & .10 \\
  s_2 & .3 & .4 & .3 & .15 & .70 & .15 \\
  s_3 & .2 & .4 & .4 & .10 & .70 & .20 \\
\end{array}
\]

are essentially homogeneous but not homogeneous.

11. The simplest way to insure (12), assuming that is feasible, is to open the total existing market for a pre-signal round of trading in addition to the post-signal trading round already in use.

12. The savings component of ordinary life insurance contracts is an example of financial intermediation of this genre for the case when \( t_1 = 0 \). Pension funds represent another example with similar features.

13. This is equivalent to permitting investment interest deductions to equal investment income (interest plus dividends) in the aggregate.

14. While viewing the discount bonds as pure discount bonds keeps the exposition uncluttered, the above scenario also applies if these bonds have a coupon as long as the coupon is lower than on the bonds on the other side of the balance sheet; the crucial requirement is that the total (high coupon) interest deductible by investors equal aggregate dividend and (low coupon) interest income taxable at rate \( t_2 \). The discount bonds should also be thought of as "old" bonds, which avoids having the holding gains on these bonds taxed as ordinary income.
15. The change in the market structure caused by the intermediary may be depicted as follows. Suppose that without it there are three states and two securities with the following before-tax payoffs:

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( Z_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock ($4 dividend)</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Bond</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>W</td>
<td>15,000</td>
<td>20,000</td>
<td>30,000</td>
<td></td>
</tr>
</tbody>
</table>

Tax neutrality would now be achieved by an intermediary acquiring 150 of the 8% bonds and issuing 150 0% bonds in their place, giving rise to the following before-tax market structure:

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>( Z_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock ($4 dividend)</td>
<td>50</td>
<td>100</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Bond (8%)</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>-50</td>
</tr>
<tr>
<td>Bond (0%)</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>W</td>
<td>15,000</td>
<td>20,000</td>
<td>30,000</td>
<td></td>
</tr>
</tbody>
</table>

16. See footnote 22.

17. This invitation to schizophrenia seems to show up in the empirical literature as well: the influence of yield on before-tax returns were found to be significant in studies by Rosenberg and Marathe [39], Litzenberger and Ramaswamy [28, 30], and Blume [9], but not in Black and Scholes [7], Gordon and Bradford [18], and Miller and Scholes [35].

18. While the necessity of (12) was not established in Hakansson, Kunkel, and Ohlson [22], this is readily done.
19. For an analysis of the relationship between the "efficiency levels" attained and the degree of "fineness" of public information structures in the absence of deadweight costs, see Ohlson [36].

20. Only in the case of homogeneous beliefs and additive utility do the two Pareto surfaces coincide (see Proposition VI).

21. If the above scenario strikes the reader as somewhat coarse, it is not difficult to obtain the same $200 deadweight loss with a more realistic calculation. Let the dividend be $10 instead of $8 and assume a marginal deadweight underwriting cost of 10%, or $100. Assume also that the government "wastes" $100 of the increased tax take of $250 (.25 x 10 x 100): $25 on more complex forms and $75 on auditing dividend tax evaders. The remaining $150 in additional tax collections is rebated equally to bond and stock holders. This causes each bond's payoff to increase by 75 cents and the payoff on each share of common to decline by $2.75 (-1 - 2.50 + .75), giving rise to the following payoff structure without changing the end result:

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond</td>
<td>100.75</td>
<td>100.75</td>
<td>100.75</td>
<td>100</td>
</tr>
<tr>
<td>Stock</td>
<td>49.25</td>
<td>99.25</td>
<td>199.25</td>
<td>100</td>
</tr>
<tr>
<td>W</td>
<td>15,000</td>
<td>20,000</td>
<td>30,000</td>
<td></td>
</tr>
</tbody>
</table>

A corresponding scenario is readily constructed for the "low" dividend case.
22. It should be noted that to obtain equivalent consumption or payoff patterns in the two economies after trading, it is necessary for each investor's endowment to be neutralized as indicated only under the net cash flow scenario. Without this, welfare would end up being redistributed even when total taxes remain unchanged in the aggregate (see Hakansson [20, Theorem 2]). Under the separable tax scenario, any one-for-one exchange of the two bonds leaves the investor with equivalent starting points.
References


