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YUK-SHEE CHAN

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ON THE POSITIVE ROLE OF FINANCIAL INTERMEDIATION IN ALLOCATION OF
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Yuk-Shee Chan*

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Institute of Business and Economic Research
University of California, Berkeley CA 94720

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I. INTRODUCTION

The analysis in this paper is motivated by recent research in financial intermediation from the perspective of imperfect information in the market (e.g., Leland and Pyle [1977] and Campbell and Kracaw [1980]). In these studies, the intermediaries serve as information production agents about the qualities of firms.

As is well known in the information literature, there may not be a market for information because of the problems of the appropriation of returns to the information producers and of the reliability of the information provided. In their insightful work using a signaling model, Leland and Pyle have rationalized that the existence of the financial intermediaries can solve the above problems in markets with imperfect information. Their observation is important as it provides a foundation for further work, such as questions of the contribution of financial intermediation to the welfare of the investors. Indeed, a survey of the literature on financial intermediation (see section IV B) reveals that much of the work has not been able to attach a positive role to the intermediaries explicitly. For instance, in Campbell and Kracaw's model, an intermediary will distinguish the good firms from the bad firms, and as a result the true values of the firms will be reflected in the market; whereas, with no information production, all firms will be priced at their average value (there is no benefit from portfolio revisions in C-K's model). The net result is a wealth distribution from the owners of the bad firms to those of the good firms, which may serve the
purposes of "equity" and increase the informational efficiency of the market. But from the viewpoint of global welfare, the aggregate wealth of the owners of the firms has not changed and in fact it has decreased net of the information costs that are borne by these owners. The reason behind this is that the qualities of firms are given exogenously and they are not affected by the information acquisition process in the market. This parallels a well-known result in economics of information that when information induces no changes in production, it may have no social value (see Hirshleifer [1971] and Hakansson-Kunkel-Ohlson [1982]). The implication is unfortunate—the information acquisition activities of financial intermediation make no positive contribution to the market. We think such implication is inaccurate, because we expect an interaction between the asset market that allocates investable resources and the market of information (see Stiglitz [1975]).

Although the process of information acquisition and its effect on the behavior of the economic agents who are in charge of production (firms, entrepreneurs, etc.) has not received much attention in the finance literature, numerous studies address the question of imperfect information and resource allocation in the context of goods and service markets (e.g., see Akerlof [1970], Salop and Stiglitz [1977], Chan and Leland [1980], and reference therein). For instance, Chan and Leland have shown that the prices and qualities that the firms would offer will depend on how many consumers are perfectly informed about the price and quality decisions of the firms—the firms are induced to offer better price/quality combinations in a more informed market. We consider a
similar model of the venture capital market to see whether some of the interesting results of the earlier analysis are robust in a financial market context and to draw out the implications of the model.

In our model, an entrepreneur (or venture capitalist) selects the quality of a project (which depends on the time and effort he spends on the project) and selects his perquisite consumption from the returns on the project. Investors do not observe the entrepreneurial actions. In equilibrium, they may have rational expectation regarding these actions. Investors can learn the information about the entrepreneurs through search, which can be done at a cost that may vary among investors. Specifically, we show that the perquisite/quality combinations that are selected by the entrepreneurs (which determine the returns to investors) will depend on the distribution of the search costs among investors. When there are some zero-cost, informed investors in the market, entrepreneurs are induced to select projects with higher returns. When there are no zero-cost investors, only the projects with the lowest acceptable returns will be undertaken. The source of inefficiency can be contrasted with the model of Jensen and Meckling [1976] in which there is (agency) cost because the managers cannot convince the investors that they will restrict their perquisite consumption. In our model, when all investors are uninformed, the entrepreneurs are induced to undertake inferior projects in equilibrium. Given that the existence of informed investors has a significant effect on the allocation of resources and on the welfare of the investors, we address the question of whether some
policies (or institutions) can effectively lead the economy to a higher welfare state (net of costs of such policies), if all investors have high search costs in the initial situation.

We consider two institutions that may evolve from the initial undesirable situation: disclosure of information by entrepreneurs, and financial intermediation. We rationalize that intermediaries, as informed agents, will play a positive role to increase the welfare of investors, by inducing the entrepreneurs to offer higher return projects. So aside from information acquisition and screening of firms, there is a real contribution from the intermediaries, which we believe is a missing aspect of the received information-based approaches to financial intermediation. We then consider the equilibrium with intermediation. In particular, we consider an equilibrium in which the market for intermediation services is competitive. The distribution of returns on projects, the fees charged by intermediaries, and the number of intermediaries are all endogenous in equilibrium. We show that a competitive intermediation equilibrium will not exist in which too many investors put funds in the intermediaries or in which no one puts funds in them. This result parallels Grossman and Stiglitz's [1980] observation that there cannot be an informationally efficient market. We also show that there may be multiple competitive intermediation equilibria, which can be ranked by the Pareto criterion, with the proportion of investment funds held by the intermediaries as the measure of rank.

In section II we outline the model with a description of the entrepreneurs, the investors and their behavioral rules. In section III
we define the conditions of equilibrium and relate the equilibrium distribution of perquisite/quality combination to the distribution of investors' search costs. Section IV presents the analysis of the two institutions mentioned: information disclosure by entrepreneurs and financial intermediation. The concept and the results of the competitive intermediation equilibrium are discussed in detail. Section V is the conclusion.

II. THE MODEL

A. Entrepreneurs

Consider a market where many entrepreneurs (venture capitalists) are trying to raise funds for the projects they select in a new industry. The entrepreneurs have access to the same technological information relevant to this industry. The "quality" of the project (defined in the next paragraph) undertaken is determined by the input of the entrepreneur (quality of management)—his effort, \( e \) (for example, in research and development). If an entrepreneur's input is \( e \), the quality of the project will be \( q(e) \), which is assumed to be increasing and concave in \( e \), that is, there is a diminishing increase in quality for additional efforts. In the following, we shall consider \( q \) as the choice variable of the entrepreneurs and \( e(q) \) will be the effort needed to develop a project of quality \( q \) (\( e(0) = 0 \)). Note that our assumptions on \( q(e) \) imply \( e(q) \) is increasing and convex in \( q \).
We consider a simple one-period model, time 0 (now) and time 1. A project of quality \( q \) will generate a random return \( \tilde{X}(q) \) per dollar of investment over the period. If the investment in the project is \( I \), then the total return is \( I \tilde{X}(q) \). We assume \( I < S \), so there is a maximum scale of investment for all projects, which is the same for all ventures. But an entrepreneur may not obtain the amount of funds he needs from the market. Given that an entrepreneur has made an effort \( e(q) \) to develop a project of quality \( q \), there are economies of scale from additional investments until the optimal scale \( S \) is attained. As we shall see later, in a market with imperfect (costly) information, such economies of scale may not be exploited. For simplicity and to highlight the problem of quality choices with imperfect information, we assume all the risks involved in this new industry are project-specific and diversifiable. Let \( \tilde{X}(q) \) be the expected value of \( \tilde{X}(q) \) and assume that \( \tilde{X} \) is increasing and concave in \( q \) \( (\tilde{X}(0) = 0) \); that is, there is diminishing return to quality. Aside from the selection of the quality of the project, the entrepreneurs also determine how much to consume as perquisites from the returns to the projects, which compensate them for the effort they put in. In this model, we assume each entrepreneur will choose to consume a fraction of the returns, \( p \), as perquisites. ¹ All entrepreneurs are assumed to be holders of a well-diversified portfolio of existing assets and they have the same preference function (with regard to project selection) depending on the effort spent in the project and the expected perquisite consumption. Let \( \tilde{C} \) be the expected perquisite consumption. The entrepreneurs' preference
is presented by the function $M(C,e)$. The entrepreneurs prefer more 
consumption and less effort, so that $M_1 > 0$ and $M_2 < 0$. If an 
entrepreneur chooses to develop a project of quality $q$ and to consume 
a fraction $p$ of returns as perquisites, and if the total investment 
from investors is $I$, his utility from such choices is 
$M(pI \bar{X}(q), e(q))$. If an entrepreneur decides not to enter the market, 
his preference is $M(0,0) = M_0$.

The entrepreneurs will issue new shares at a price of one (e.g., 
thousand dollars) per share. The maximum amount they want to raise is 
$S$ but the amount they obtain will depend on how investors invest based 
on their information (discussed in section 3 below). The funds are 
raised through sealed submissions, and submissions must be made before 
total subscriptions are known. We assume that the market is competitive 
for the entrepreneurs.

B. Investors/Consumers

There are also a large number, $N$, of investors in the market, 
who are looking for new investment opportunities. The investors may 
have different investable resources for the new project; without affect-
ing the analysis, we shall normalize their initial (investable) wealth 
to be one (e.g., thousand dollars) for all investors (by increasing or 
reducing the number of investors). The investors are also assumed to be 
holders of well-diversified portfolios of existing assets and they are 
only concerned about the expected return from the investment since all 
the risks are firm-specific. That is, their expected utility from
investment can be represented by \( EU(R) = \bar{R} \), where \( \bar{R} \) is the expected return or ending wealth from the initial investment (one thousand dollars). If an investor invests in a project of quality \( q \) and the entrepreneur's perquisite consumption (fraction) is \( p \), then \( \bar{R} = (1-p)\bar{x}(q) \). (Note that we shall refer to \( \bar{R} = (1-p)\bar{x}(q) \) as the "returns" to investors from a project with quality \( q \) and perquisite fraction \( p \), which must be distinguished from the "gross return" to a project with quality \( q \), \( \bar{x}(q) \).)

The crucial assumption in a model of imperfect information is, of course, what do investors know about the returns of the projects? Like many models with imperfect information, we assume investors (as well as entrepreneurs) have rational expectations in equilibrium. The investors do not observe the quality of the projects or the perquisite consumption of the entrepreneurs, but they know the distribution (frequency) of \( \bar{R} \) in equilibrium, even though they do not know the identity of each entrepreneur.

The investors can learn about the quality and perquisite fraction of a project (here represented as \((p,q)\) of a project) by spending some search effort. The search activity may include the collection of information about the projects such as the line of business, the quality of management, and an estimation of the entrepreneurs' perquisite consumption. The total search costs of an investor is a function of the number of projects investigated. We assume that all investors perceive a minimal (zero) effort in investigating the first project, but thereafter a utility-equivalent search cost, \( \mu \), occurs in assessing each additional
project. Investors may differ in terms of search cost; those with low search costs are more efficient in analyzing the information about projects and those with high search costs are less efficient. As shown in the next section, the equilibrium distribution of \((p, q)\) combinations will depend on the distribution of search costs of the investors in the market. A special case is when there is perfect (costless) information—\(\mu = 0\) for all investors, with no surprise in that case only the optimal \((p, q)\) combinations will be offered by entrepreneurs.

Given the expectations on \(\bar{R}\) (in equilibrium), the investors with search cost \(\mu\) have two options:

1) do not enter the venture capital market, invest the money in a risky-free asset which returns \(r\) per (thousand) dollar of investment (utility is \(r\); \(r\) is the opportunity cost of investment and it is assumed these risk-free investments are always available); or

2) enter the market and search for projects (with high expected return), expending a utility-equivalent search cost \(\mu\) each time (after the first). Since there are a large number of projects in equilibrium (assume \(N\), the number of investors, is sufficiently large relative to \(S\), the maximum scale of investment for each project), we assume that investors will not revise their expectations on \(\bar{R}\) despite their search activities.

C. Optimal Investment Policy

Denote the equilibrium distribution of perquisite/quality combinations by \(\phi(p, q)\). Let \(\bar{R}_p\) be the expected return from investing
randomly in one project. All investors will enter the market if
\( \bar{R}_\phi > r \) (we assume investors will enter the market when \( \bar{R}_\phi = r \)). An
investor's searching process is as follows: the investor first selects
one project randomly among all entrepreneurs to find out \( p \) and \( q \).
For each perquisite/quality offer an investor \( i \) encounters, he may
either accept the offer and invest in the project, or reject the offer
and investigate another project, expending a search cost \( u \). Each \((p,q)\)
combination is assumed to be a random selection from \( \phi(p,q) \). It can be
shown that the optimal search policy for an investor with positive
search cost \( u \) is to reject all \((p,q)\) offers that yield levels of utility below a single critical number and to accept any offer otherwise.\(^3\)
Formally, suppose an investor with search cost \( u > 0 \) happens to have
selected a project \((p',q')\), then he will choose to have another random
selection if and only if

\[
\bar{R}' < \varepsilon
\]  

(S1)

where \( \bar{R}' = (1-p')\bar{R}(q') \) and \( \varepsilon \) is the critical level of utility de-
termined in

\[
\int_{\bar{R} > \varepsilon} [\bar{R} - \varepsilon] d\phi(\bar{R}) = u.
\]  

(S2)

\( \phi(\bar{R}) \) is the distribution of \( \bar{R} \) induced by the distribution
\( \phi(p,q) \), with \( \bar{R}(p,q) = (1-p)\bar{R}(q) \). Let \( P_\varepsilon = P(\bar{R} > \varepsilon) \) denote the
probability of selecting a project with \( \bar{R} > \varepsilon \). Equation (S2) can be
rewritten as:
\[ \left[ \frac{1}{P_c} \int_{R > \varepsilon} \phi(R) \, d\phi(R) \right] - \varepsilon = \frac{u}{P_c}, \] \hspace{1cm} (S3)

The first term in (S3) is the expected utility from following the optimal search policy (an optimal stopping rule) to search for projects until the investor finds a project with \( \bar{R} > \varepsilon \). The left-hand side of (S3) therefore represents the expected benefit of searching optimally sequentially once the investors have selected a project that yields utility \( \varepsilon \), and the right-hand side is the expected search cost of finding a project that yields greater than or equal to \( \varepsilon \), since \( 1/P_c \) is the expected number of search before the investor will find a project with return \( \bar{R} > \varepsilon \). The optimal search policy will set \( \varepsilon \) such that the expected benefit is equal to the expected cost.

Note from (S2) that \( \varepsilon \) is determined by \( \phi(\bar{R}) \) and \( u \); and given \( \phi \), \( \varepsilon \) is a decreasing in \( u \). The higher the search cost, the lower will be the optimal stopping level of utility. When \( u \) is sufficiently large such that \( \varepsilon = \min(\bar{R} \text{ offered in equilibrium}) \), the investor will find it optimal to search only once (to invest randomly).

When \( u = 0 \), it is assumed that the investors will investigate all firms and invest in the projects that yield the highest expected return. This is rational since if the search cost is zero, it costs the investors nothing to analyze the information of all projects.

D. Some Definitions

In this section we shall define some terms useful in subsequent analysis.
Let \( p(I) \) and \( q(I) \) be the solutions to the following problem, given \( I (I < S) \):

\[
\max_{p,q} (1-p)\bar{x}(q)
\]

subject to

\[
M[pI \bar{X}(q), e(q)] > M_0. \tag{1}
\]

\( p(I) \) and \( q(I) \) are the (fraction of) perquisite consumption and quality of the project that an entrepreneur can select to maximize the utility of the investors subject to the condition that the entrepreneur is not worse off than not undertaking any project. The solutions \( p(I), q(I) \) will satisfy the first-order conditions:

\[
M_1 I \bar{X}'(q) + M_2 e'(q) = 0 \tag{2}
\]

\[
M[pI \bar{X}(q), e(q)] = M_0. \tag{3}
\]

We assume that the second-order conditions for the maximum are satisfied. Define \( V(I) \) as the maximum, given \( I \), in problem (1) and

\[
p^* = p(S) \tag{4}
\]

\[
q^* = q(S) \tag{5}
\]

\[
V^* = V(S). \tag{6}
\]
p* and q* are the perquisite consumption and quality of the project that an entrepreneur can select to maximize the utility of investors subject to the no-worse-off condition for the entrepreneur. It is obvious that in a world with perfect (costless) information and a competitive market for entrepreneurs, only (p*,q*) projects will be offered by the entrepreneurs.

III. SEARCH COSTS AND EQUILIBRIUM DISTRIBUTION OF PERQUISITE CONSUMPTION AND QUALITIES OF PROJECTS

In this section we derive the equilibrium distribution of perquisite consumption and qualities of projects when there is costly information, that is, there are positive search costs for some investors. Two cases will be considered:

1) \( \mu_1 = 0 \) for \( aN \) investors and \( \mu_2 > 0 \) for \( (1-a)N \) investors; and

2) \( \mu_2 > \mu_1 > 0 \) (the proportions of investors are the same as in (1), \( a \) and \( (1-a) \) for group 1 and group 2, respectively).

A. Definition of Equilibrium

Equilibrium will be characterized by a set of perquisite/quality combinations

\[ \{(p_1,q_1);(p_2,q_2);\ldots;(p_k,q_k)\} \]

selected by the entrepreneurs. Associated with any equilibrium will be the endogenously determined number of entrepreneurs \( (M_1,M_2,\ldots,M_k) \)
selecting these combinations, and the endogenous searching behavior of the investors.

The following conditions must be satisfied in equilibrium:

(E1) **Optimal Investment Decisions**: An investor with positive search cost \( \mu_i \) will follow the optimal search policy described in section II(C) if \( \varepsilon_i > \min_k \{ \overline{R}_k \} \), where \( \overline{R}_k = (1-p_k) \overline{X}(q_k) \) and \( \varepsilon_i \) is determined in eq. (S2) when \( \mu = \mu_i \) and \( \phi(\overline{R}) \) is the equilibrium distribution of return induced from the equilibrium distribution of perquisite/quality \( \phi(p,q) \) which is characterized completely by the sets \( \{(p_k,q_k)\} \) and \( \{M_k\}, \ k = 1,2,...,K. \) We shall call these investors "searchers." When \( \varepsilon_i = \min_k \{ \overline{R}_k \}, \) the investor \( i \) will actually invest randomly in one project. We shall call them "random investors." 

An investor with zero search costs will investigate all the projects and invest in those yielding the highest expected return.

(E2) **Utility Maximization for Entrepreneurs**: The \( (p_k,q_k) \) combination chosen by entrepreneur \( k \) yields at least as great a level of satisfaction as any other \( (p,q) \) combination he could offer, given

(a) imperfect information in the market—investors do not observe the perquisites or quality \( q \) if they do not investigate information of the project,

(b) Nash behavior vis-a-vis other entrepreneurs,

(c) Stackelberg behavior vis-a-vis investors.

Conditions (b) and (c) are rules that an entrepreneur follows while contemplating a possible change in \( (p,q) \) to increase his
utility. Condition (b) says that the entrepreneur assumes that other entrepreneurs will not change their decisions if he makes a change, which is a reasonable assumption when many entrepreneurs are in the market. Condition (c) says that the entrepreneur will take into account the reactions of the investors to his changes to see whether it is worthwhile to deviate; this relates to (a) which indicates that investors will not know about any changes of an entrepreneur unless they happen to investigate his activities. A set of perquisite/quality combinations \( \{(p_k, q_k)\} \) (and the associated set \( \{M_k\} \)) will constitute an equilibrium only if there is no economic incentive for any entrepreneur to deviate from his present perquisite/quality selections.

(E3) All Entrepreneurs Have the Same Utility \( M_0 \): This is a result of free entry to and exit from the market.

(E4) The Supply of Investment Funds to the Projects is Determined by the Law of Large Numbers: As each project investigated by the investors is a random selection from \( \phi(p,q) \), the number of investors that a project will attract is random. Since a large number of investors and entrepreneurs (projects) are in equilibrium, the law of large numbers is assumed to hold; so the realized supply of funds equals the expected supply of funds to the projects, and the projects that yield the same expected return will receive the same amount of investment funds.
B. Equilibrium When the Search Costs for Some Investors is Zero:

\[ u_2 > u_1 = 0 \]

Proposition 3.1: The unique perquisite/quality combination \((p^*, q^*)\) will be offered in equilibrium if and only if

\[ u_2 < V^* - V[(1-\alpha)\bar{S}] \]

Proof: See Appendix A.

The equilibrium \((p^*, q^*)\) is the same as in the case where there is perfect (costless) information about projects in competitive markets, which maximizes the utility of each investor. There is an externality effect of the existence of zero cost \(u_1\) investors on the welfare of the high cost \(u_2\) consumers. Since only the best combinations are offered in equilibrium, everyone can invest their funds randomly and the utility from investment is \(V^*\).

Let \( Y = V^* - V[(1-\alpha)\bar{S}] \) (as in Proposition 3.1),

\( V = \) utility from a project with the lowest expected return offered in equilibrium.

We have the following two perquisite/quality combinations equilibria:

Proposition 3.2: The perquisite/quality combinations \((p^*, q^*)\) and \((p', q')\) will be offered in equilibrium if and only if \( u_2 > Y \),
where \( p' = p(I') \), \( q' = q(I') \), and \( I' \) is the investment received by the entrepreneurs offering \((p',q')\) in equilibrium. Let \( M \) be the total number of projects, \( M^* \) be the number of \((p^*,q^*)\) projects, and \( \lambda = M^*/M \). The equilibrium distribution of \((p,q)\) combinations will satisfy the following conditions:

(i) \( S = (\alpha N/M^*) + ((1-\alpha)N/M) \); and

(ii) \( I' = (1-\alpha)N/M \) ((i) and (ii) are market-clearing conditions); and

(iii) (a) \( V* - (u_2/\lambda) = V \) when \( Y < u_2 < Z \);

or

(b) \( V* - (u_2/\lambda) < V = r \) when \( u_2 > Z \)

\( Z \) is a positive number determined in Appendix B, and
\( V = (1-p')\bar{X}(q') \).

Proof: See Appendix A.

Note that in equilibrium \{\((p^*,q^*),(p',q')\)\} the \( u_1 = 0 \) investors will investigate all projects and choose the one with \((p^*,q^*)\) combination, and they will obtain utility \( V* \).

Conditions (iii) (a) and (iii) (b) imply that there is no incremental benefit from searching for a \( V* \) project once a \( u_2 \) investor has selected a project that yields a return \( V \). \( (V* - (u_2/\lambda) \) is the expected utility of searching sequentially for a \( V* \) project.)
Therefore, the \( \mu_2 \) investors are random investors in equilibrium, and their expected utility is \( \bar{V} = \lambda V^* + (1-\lambda)V \), which depends on \( \mu_2 \) (other things equal). (See Appendix B.) When \( \mu_2 < Z, V \) is greater than \( r \), the utility from investing in the risk-free asset. But when \( \mu_2 > Z, V \) equals \( r \).

C. Equilibrium when All Investors have Positive Search Costs:

\[
\mu_2 > \mu_1 > 0
\]

**Proposition 3.3:** The perquisite/quality combinations \((p^*, q^*)\) will be offered if and only if all investors have positive search costs, where \( p^* = p(I^*) \) and \( q^* = q(I^*) \) and \( I^* \) is such that \( V(I^*) = r \).

**Proof:** See Appendix A.

Significant change occurs in equilibrium results when \( \mu_1 \) is positive. Indeed, we find a degenerate distribution of perquisite/quality combinations that yield utility \( r \), which is the lowest possible to attract the investors in the market. The intuition behind these results is not hard to see. Note that the existence of \( \mu_1 = 0 \) investors insures that \((p^*, q^*)\) combinations will be offered by entrepreneurs because the zero-cost investor will always discover the "best buy" in the market. Conversely, their existence may also preclude any
entrepreneur to switch from a \((p^*, q^*)\) combination to an inferior one because in doing so they will lose all the zero-cost investors. When \(u_1 > 0\), the \(u_1\) investors will search if and only if the expected benefit is greater than the expected cost. Suppose a proposed equilibrium \(\{(p_1, q_1), (p_2, q_2)\}\) with \([(1-p_1)x(q_1)] > [(1-p_2)x(q_2)]\) and the distribution of \((p, q)\) are such that \(u_1\) investors will search for \((p_1, q_1)\) projects in equilibrium. It is easy to check that \(\{(p_1, q_1), (p_2, q_2)\}\) cannot be an equilibrium since an entrepreneur who offers \((p_1, q_1)\) can increase his perquisite consumption by a sufficiently small percentage without losing the \(u_1\) investors who happen to investigate the project (randomly). Therefore, when \(u_1 > 0\), there does not exist any multiple perquisite/quality equilibrium where some investors are searchers and some are not. The result is a degenerate distribution described in Proposition 3.3, which is analogous to Akerlof's "lemon problem" when the cost of acquiring information about used cars is prohibitive.

D. Policy Implications

From the above analysis the equilibrium distribution of the perquisite/quality combinations depends critically on the distribution of search costs of the investors. In particular, the existence of some zero-cost investors will induce some entrepreneurs to select the optimal perquisite/quality combination that maximizes the utility of the investors. When we move from the situation where all investors have positive search costs to the one where some investors have zero search costs,
there is a better allocation of resources in terms of the entrepreneurs' efforts and the utilization of the economies of scale of investment. As a result, ignoring the different search costs in the two environments, all investors will achieve a higher (or at least for the $\mu_2$ investors the same) level of utility. When $\mu_2$ is sufficiently small, indeed, the welfare of all investors will be the same as in the case of perfect costless information. To be realistic, the different search costs cannot be ignored, and an interesting question to address is: suppose we are in the initial situation where everyone has positive search cost (and the associated low welfare state), are there any policies (or institutions) that can effectively lower the search cost of some individuals to zero, thereby leading the economy to a higher welfare state, net of the costs of any such policies? Our answer to this question is the topic of section IV.

IV. TWO POLICIES: INFORMATION DISCLOSURE BY ENTREPRENEURS AND FINANCIAL INTERMEDIATION

A. Information Disclosure by Entrepreneurs

The search costs of investors include the costs of collection of information and of subsequent analysis of the information. As suggested, investors may have different search costs because of the difference in their abilities to collect and analyze information, and also perhaps because of the difference in their opportunity costs of time and effort. The entrepreneurs can do two things: make the information easily
accessible to the investors and provide detailed information with objective analysis. These will lower the search cost of the investors substantially; whether or not for some individuals the search costs would be reduced to zero is an open question. Suppose that is the case, then the welfare net of such "subsidy to search costs" may be increased. It is obvious that these additional costs incurred by the entrepreneurs will be borne by the investors eventually in a competitive market. However, there may be welfare improvement if the cost is not too large. Examples of such policies exist: firms, especially large ones, disclose information (and detailed analysis of information) voluntarily and their books are audited by accountants with the intention of verifying the information disclosed for unbiased analysis. The costs involved in the provision of such information to the investors are enormous even at a societal level, and some firms decide not to provide information to such an extent because the costs are unaffordable. This applies, in particular, to small companies that may not want to disclose detailed information either because of its cost, or because even if the information is provided, it is difficult for small investors to verify and believe. Therefore its use is limited.

In these cases, incentive exists for some individuals to form institutions to search for good projects on behalf of their clients if the costs involved in their search activities are borne by their clients. We call these institutions "financial intermediaries," which invest for their clients based on their expertise or privilege to valuable information. These institutions are usually specialized and tend
to exist for industries and firms whose private costs of transmitting information to investors in general are prohibitive. However, it does not preclude firms from providing easy access to information to these institutions if the costs are affordable. Notice that since it is less costly for the firms to convey information to the more informed or knowledgeable investors, the existence of intermediaries may induce the small firms to disclose information even though they would not do so otherwise. In fact, since the information costs of the firms and the intermediaries will be borne by the investors, it is the optimal combination of the activities of the two that will minimize the total information cost, which is expended so as to induce better allocation of resources in a world with imperfect information.

3. A Model of Financial Intermediation

1. An Integrated Theory of Financial Intermediation

There are a few well-known explanations for the existence of financial intermediaries in the literature. They can be classified conveniently in terms of the roles of the intermediaries, their functions, or contributions as follows:

(i) reduction of transaction costs (which may include costs of collection and analysis of information), e.g., Benston [1972], Benston and Smith [1977], Flannery [1974, 1978], Gurley and Shaw [1960];

(ii) portfolio optimization, e.g., Pyle [1971, 1972];

(iii) information producer/signaling agent, e.g., Leland and Pyle
[1977], Draper and Hoag [1978], Campbell and Kracaw [1980]; and

(iv) monitoring agent, e.g., Diamond [1980].

These are all plausible explanations for intermediation. However, one may find them lacking since except for the transaction cost approach the others have not been able to attach explicitly a positive role to intermediaries. For instance, consider the intermediary as an agent which distinguishes good firms from bad firms (Campbell and Kracaw). There is a wealth distribution from the owners of the bad firms to those of the good firms after the price adjustments in the market (no benefit from portfolio revisions in C-K's model). Nonetheless, on a societal level, there is no net contribution from the production of information by the intermediaries, instead there is a weight loss (information costs). The transaction cost approach, however, also has its limitation—it neglects or does not consider explicitly the imperfect information problem which is without question one of the key reasons why intermediaries exist.

Our model provides the framework to analyze the role of intermediaries in a world of imperfect (costly) information, recognizing that intermediaries obtain information at less cost than through a client's individual search. Further, our model attaches to that role a positive contribution to the economy: intermediaries increase the welfare of the investor by inducing a Pareto-preferred allocation of resources rather than the allocation when no information acquisition exists. The model integrates the transaction cost approach and the information-based
approach to financial intermediation, and emphasizes the positive contribution of intermediaries. This indeed points out the well-known fact in the economics of information: information may have no social value without changes in production decisions (see Hirshleifer [1971] and Hakansson-Kunkel-Ohlson [1982]. It is then no surprise that in the models (e.g., Campbell-Kracaw) in which the qualities of firms or production decisions are exogenously given, the intermediaries as information producers cannot play a positive role.

2. Characterization of Intermediation Equilibrium

Suppose our scenario is a world where all investors (total number of \( N \)) have positive search costs. We shall first consider the simple case when all investors have the same search cost \( \mu \) (similar to Grossman and Stiglitz's model [1980] in which all investors have the same information cost). In section IV (d) we shall consider the case with differential search costs \( \mu \) and \( \delta \) (\( \delta > \mu \)).

From the analysis in section III, if there is no information acquisition, there will be a degenerate perquisite/quality distribution \( \{(p'',q'')\} \) in equilibrium yielding the lowest acceptable utility \( r \) to all investors. Now suppose some individuals in the economy (maybe some of the investors) decide to form financial intermediaries that search for good projects and invest for the clients for a fee \( d \) per individual (to be deducted from the return on investment) to cover the search costs and possibly to make a profit. Assume

(i) the average search cost (evaluated at time 1, the same time when the fees are received, for simplicity) of a project for
each intermediary is a U-shaped function in the number of projects (n) investigated, as AC(n) in figure 1—the marginal cost first decreases and subsequently increases when more projects are investigated;  

(ii) the intermediaries divide up the total number of projects randomly among themselves for investigation, that is, every intermediary has an equal share of the "good" and "bad" projects and there is no duplication of effort on the same project. (This is assumed for simplicity only. The essential point is that all projects will be investigated (randomly) by intermediaries; to the extent there is duplication of effort, the opportunity to invest in the project (if desired) is shared equally among the intermediaries involved. The cost of duplication of effort will, of course, be reflected in the average search cost function);  

(iii) the intermediaries also get an equal share of the investors who place their funds in intermediaries;  

(iv) the intermediaries are reliable in the sense that they will pick the "best" investments for their clients (which is what their clients expect);  

(v) the market for intermediaries is competitive.

Note that given assumption (iv) the investors who desire to invest in intermediaries will go to the one charging the lowest fee, thus in equilibrium there will be a uniform fee among all intermediaries.
FIGURE 1

AVERAGE SEARCH COST OF INTERMEDIARIES

\[ AC(n) \]

\[ 0 \quad n^* \]

\[ C^* \]
We characterize a **competitive intermediation equilibrium** by the following:

(i) the perquisite/quality combinations and the number of each of them selected by the entrepreneurs $(\psi(p,q))$;

(ii) the proportion of investors $(\gamma)$ who put their funds in the intermediaries, which represents the fraction of "pseudo" zero cost investors (whose costs are paid by their clients) in equilibrium (which induces $\psi(p,q)$);

(iii) the fee charged by the intermediaries $(d)$;

(iv) the number of intermediaries $(h)$.

The important point is that the existence of intermediaries as "pseudo" zero cost investors will induce a distribution of perquisite/quality combination in equilibrium (to be characterized) by either of the following forms: (i) $((p^*,q^*))$; or (ii) $((p^*,q^*); (p',q'))$, as described in Propositions 3.1 and 3.2 (whether form (i) or (ii) will obtain depends on the parameters in the model: $U(*)$; $M(*,\cdot)$; $X(*)$; $e(*)$; $\mu(=\mu)$; $\alpha(=\gamma)$; and $N$, and on the equilibrium conditions).

The following conditions have to be satisfied in the equilibrium:

(IE1) All entrepreneurs maximize their utilities with the behavior described in (E2) (section IIIA).

(IE2) All entrepreneurs have the same utility $M_0$ (E3).

(IE3) The investors make optimal investment decisions: an investor with search cost $\mu(>0)$ will either (i) adopt the random investment strategy, or (ii) invest in the intermediaries and receive an expected return $V'' = d$. $^{10}$
Since the investors have the same search cost \( u \) and face the same opportunities, in equilibrium the distribution of perquisite/quality combination must be such that they will be indifferent as to which policy they choose, that is, 
\[ \bar{V} - d = \bar{V}, \] 
where \( \bar{V} \) as before denotes the expected utility (expected return) from the random investment policy.

(IE4) The intermediaries maximize their profits by charging the optimal service fee, given the fees charged by other intermediaries.

(IE5) The intermediaries make zero profit in the competitive market.

3. Determination of Competitive Intermediation Equilibrium when All Investors have the Same Search Cost \( u \)

Suppose \( \phi(p, q); \gamma; d; h \) is a competitive intermediation equilibrium (hereafter referred to as CI equilibrium) in which a proportion, \( \gamma \), of investors with search cost \( u \) patronize the intermediaries. The induced equilibrium distribution of perquisite/quality combinations, \( \phi(p, q) \), is determined as if there were \( \gamma N \) zero cost investors and \((1-\gamma)N\) investors with search cost \( u \) (using the notations in section III, \( u_1 = 0, u_2 = u \), and \( \alpha = \gamma \)). Let

\[ \lambda(\gamma) = \text{proportion of (p*, q*) projects in the CI equilibrium;} \]
\[ M(\gamma) = \text{total number of projects in the CI equilibrium;} \]
\[ \bar{V}(\gamma) = (1-p')X(q') = \text{expected return on inferior projects in the CI equilibrium,} \]

then

\[ \bar{V} - d = \lambda(\gamma)\bar{V}^* + (1-\lambda(\gamma))\bar{V}(\gamma) \equiv \bar{V}(\gamma) \quad (4.1) \]
Since the market is competitive, at equilibrium all intermediaries will minimize the average cost of search \((AC(n^*))\) at the output level \(n^*\), so the number of intermediaries is \(M(\gamma)/n^*\).

The total revenue to the intermediaries is \(\gamma dN\). Let \(TR(\gamma)\) be the revenue of each intermediary, then

\[
TR(\gamma) = \frac{\gamma d n^* N}{M(\gamma)}. \tag{4.2}
\]

Let \(C^* \equiv AC(n^*),\) the zero-profit condition yields:

\[
C^* = \frac{\gamma dN}{M(\gamma)}. \tag{4.3}
\]

(4.1) implies \(d\) is a function of \(\gamma\) and we can rewrite (4.3) as:

\[
F(\gamma) \equiv \frac{\gamma d(\gamma) N}{M(\gamma)} = C^*. \tag{4.4}
\]

Note that when \(\gamma = 1, \lambda(\gamma) = 1\) and this implies \(d = 0\) and \(F(1) = 0\). When \(\gamma = 0, F(0) = 0\). Since \(F(\gamma)\) is nonnegative, it must be increasing in some interval of \(\gamma \in (0,a)\), where \(a < 1\) and be decreasing in some interval of \(\gamma \in (b,1)\), where \(b > 0\). The exact shape of \(F(\gamma)\) will depend on the underlying parameters, a typical form of that is shown in figure 2.

From Proposition 3.1 we note that the unique perquisite/quality combination \((p^*, q^*)\) will be offered if and only if
FIGURE 2

DETERMINATION OF COMPETITIVE INTERMEDIATION EQUILIBRIA
\( V^* = V[(1-\alpha)S] < \mu \), for any given \( \mu \). This condition is equivalent to 
\[ \alpha < 1 - \left[ V^{-1}(V^* - \mu)/S \right] \]. We have the following proposition.

**Proposition 4.1:** There does not exist a competitive intermediation equilibrium in which \( \{1 - [V^{-1}(V^* - \mu)/S] \}^N \) or more investors choose to invest in the intermediaries or in which no investors choose to do so \( (\gamma = 0) \).

The intuition behind this is clear. When \( \{1 - [V^{-1}(V^* - \mu)/S] \}^N \) or more investors patronize the intermediaries, the perquisite/quality combinations offered are \( \{(p^*,q^*)\} \). An individual investor would be foolish to put his money in an intermediary since he could do as well by investing (randomly) himself and saving the intermediary's fee. He is willing to use the intermediary's service if no fee is charged. However, it is not economically viable for the intermediaries to provide free service since the cost of information is positive. When \( \gamma = 0 \), the perquisite/quality combinations offered are the least acceptable \( \{(p'',q'')\} \). In this case, with rational expectations, no investor is willing to use the intermediaries since he knows the intermediaries cannot possibly find some projects that yield returns higher than \( r \), even though the increase in utility is very large if they could find some \( (p^*,q^*) \) projects. The former result is analogous to Grossman and Stiglitz's observation that there cannot be an informationally efficient market—if the market is efficient, those who spend resources to obtain information will receive no compensation. The analyses in both models
reflect a fundamental conflict between the induced efficiency of the market (from information acquisition) and the incentives to acquire information. The difference is: in Grossman and Stiglitz's model, information acquisition by some individuals will make the price system more informative (informational efficiency), but in our model the search activities of the intermediaries will induce a higher degree of allocational efficiency.

Proposition 4.2: Let $F^* \equiv \max_\gamma F(\gamma)$, then

(a) when $C^*/N < F^*$, there exists a competitive intermediation equilibrium (may not be unique);
(b) when $C^*/N > F^*$, there does not exist a competitive intermediation equilibrium.

Significantly, when the total willingness to pay (for the intermediaries' service) is sufficient to cover the costs incurred by the intermediaries, a CI equilibrium will exist; otherwise, an equilibrium will not exist because it is not economically viable for the intermediaries. But the interesting observation is that there may be multiple equilibria if they exist and one would wonder whether we can rank them in the order of the investors' welfare? The answer is yes for our model.

Proposition 4.3: The larger the proportion of investment funds held by the intermediaries ($\gamma$) in a competitive intermediation
equilibrium, the higher is the level of utility attained by all
investors—equivalently, an equilibrium with a high $\gamma$ will
dominate that with a small one using the Pareto criterion.

Proof: In Appendix B.

Considering the intermediaries as informed investors in the mar-
ket, the proposition indicates that the more informed is the market, the
better off are the investors, which makes sense because a more informed
population will effectively induce a better allocation of resources by
the entrepreneurs. The economic reasons behind this are twofold: when
more "investors" are informed, (i) the total number of projects in the
market is smaller because of the exploitation of the economies of scale
in investment therefore requiring fewer intermediaries and smaller ex-
penditure on information costs; (ii) the average return of the projects
is higher because there is a higher proportion of $V^*$ projects and a
higher return on the other projects offered in equilibrium. As a re-
sult, the welfare of the investors increases. The increase in the in-
vestors' welfare will be more substantial (in a high-$\gamma$ equilibrium) if
the cost of collecting and analyzing information, $C^*$, is smaller. This
refers back to our discussion about an optimal balance of the activities
related to information provision and acquisition of the entrepreneurs
and of the intermediaries so as to minimize $C^*$. The economic principle
is simple: if one party can provide the same information at a lower
cost than the other, in a competitive market, we will expect the
formation of the institutions in the market to divide the activities in an optimal fashion. If the market for information is not competitive, some form of regulation, like information disclosure rules, may be necessary.

4. Competitive Intermediation Equilibria when There are Differential Search Costs

In this section, we consider a more general environment where there are differential search costs among investors. Suppose there are two types of investors, \( \beta N \) of them have a search cost \( \mu > 0 \) (as in previous sections) and \( (1-\beta)N \) of them have a search cost \( \delta > \mu \).\(^{11}\) Note that the first scenario we considered is a special case of this one with \( \beta = 1 \).

From Proposition 4.1, a CI equilibrium exists only if some but not too many investors patronize the intermediaries. When there are two types of investors, a CI equilibrium exists in which, given the equilibrium distribution of perquisite/quality combinations of projects and the fees charged by the intermediaries, either

(i) the investors with low search cost are indifferent between patronizing the intermediaries or investing randomly (which implies that all investors with high search costs will find it optimal to put their funds in intermediaries); or

(ii) the high search-cost investors are indifferent between patronizing the intermediaries or investing randomly (which implies that the low search-cost investors will find it optimal to search for the high-return projects on their own effort).
In case (i) all of the high search-cost (δ) investors and an (equilibrium) proportion of low search-cost (μ) investors will put their funds in the intermediaries and the other low search-cost investors will invest randomly in equilibrium. In case (ii) only an (equilibrium) proportion of high search-cost investors patronize the intermediaries, the other high search-cost investors will invest randomly, while the low search-cost investors search sequentially for V* projects at their own cost.

It can be shown that one or both types of CI equilibrium exist for any given set of parameters μ, δ, and β. The equilibrium in each category ((i) or (ii)), as before, may not be unique. But from Proposition 4.3 we note that for the equilibria in the same category, the one with the highest institutional holdings dominates the others in a Pareto sense.

Since our purpose is to illustrate that CI equilibria exist when there are differential search costs among investors, we shall limit our discussion to the equilibria in case (i).\(^{12}\)

Let \( w = \) proportion of low search-cost investors who patronize the intermediaries in equilibrium;

\[ \gamma = \text{percentage of institutional holdings}, \]

then for equilibria of type (i),

\[ \gamma = w\delta + (1-\delta) \quad (4.5) \]

or

\[ w = \frac{\gamma-(1-\delta)}{\delta}. \quad (4.6) \]
So \( w \) can be determined when \( \gamma \) is known. But the value of \( \gamma \) can be determined as in section IV(c) using (4.1), (4.2), and (4.3). The only condition that has to be satisfied is \( w > 0 \), which implies \( \gamma > 1 - \beta \) (i.e., \( \beta > 1 - \gamma \)), for any equilibrium value \( \gamma \) determined in section IV(c). This is illustrated in figure 3. In equilibria of type (i), since all the high search-cost investors will patronize the intermediaries, only the solid-lined portion of the curve \( F(\gamma) \) (domain: \( \gamma > 1-\beta \)) is applicable. For a given value of \( \beta = \beta^* \), the CI equilibrium with a low institutional holding of \( \gamma_1 \) does not exist, but the one with a high institutional holding does! This result is quite paradoxical: since the high search-cost investors are apt to patronize the intermediaries, a large number of these investors will increase the institutional holdings in a CI equilibrium, which induce Pareto-preferred allocations.

V. CONCLUSION

We have shown that in a market with imperfect information (where all investors have positive information costs), the entrepreneurs are induced to offer projects with lowest acceptable returns (undesirable allocation of resources). This is the case because in the market with no informed investors, the entrepreneurs will find it in their interests to offer some projects with lower qualities, which leads to the degeneration of the qualities of the projects undertaken. In a market containing some informed investors, the allocation will be Pareto-preferred,
FIGURE 3

DETERMINATION OF COMPETITIVE INTERMEDIATION EQUILIBRIA
WHEN THERE ARE DIFFERENTIAL SEARCH COSTS
but limited by the proportion of informed investors and also the magnitude of information costs of the other investors. It is shown that in equilibrium the higher the search costs of the uninformed investors, the lower will be the average returns of the projects offered by entrepreneurs.

Beginning with an initial situation with imperfect information, we consider two policies that may lead the investors to a higher welfare state: disclosure of information and financial intermediation. We focus our analysis on the existence of competitive intermediation equilibria and the ranking of such equilibria with the Pareto criterion. It is shown that (i) there cannot be a competitive intermediation equilibrium with very high institutional holdings, and (ii) in other cases, multiple competitive intermediation equilibria may exist, and the one with the highest institutional holdings will dominate the others in a Pareto sense.
ENDNOTES

1. The perquisites in general can be a function of \( \tilde{x}(q) \). But assuming other forms of perquisite functions will not affect our subsequent analysis.

2. The assumption that the first search cost is zero is not essential. Suppose we assume search costs are the same for the first investigation as well as the subsequent ones, all the results in the paper except Proposition 3.3 remain intact. In that case, all investors have positive search costs, and the entrepreneurs have the incentive to extract all surplus, and we show that only projects with the least acceptable return \( r \) (\( r \) is the opportunity cost of investment, the risk-free rate in the model) will be offered if equilibrium exists. However, if the search cost is positive, the investors will not enter the market of venture capital; they would rather put their funds in the risk-free asset. Therefore, equilibrium does not exist when all investors have positive search costs (including the first investigation). Note that the welfare analysis of financial intermediation is the same regardless of the assumptions, since the utility of investors is \( r \) in both cases.


4. The results are robust to the assumption of the number of classes of investors.

5. Note that in equilibrium there must be some random investors who are willing (and who find it optimal) to invest in the projects with the
lowest return. Otherwise, the entrepreneurs of these projects will not collect any investment funds and be worse off than not undertaking the project, thus violating the conditions for equilibrium.

6We assume that if an entrepreneur offers a new perquisite/quality combination, the investors who happen to investigate his activities will decide whether or not to search again using the same prior \( \phi(\bar{R}) \).

7For simplicity, the service fee \( d \) is assumed to be evaluated at time 1, the same time when the total return of investment \( \tilde{X}(q) \) is realized. If not, adjustments for time value of money are necessary.

8Note that the average cost function reflects the economies of scale on transaction costs as suggested in Benston [1972] and Benston and Smith [1977]. If the average cost is strictly decreasing for all \( n \), then the intermediary market may end up with monopoly.

9The model will be much complicated with the moral-hazard problem. One way to get around this problem, as suggested by Leland and Pyle, is that the managers of the intermediaries will hold the shares of their intermediaries to signal reliability, which is the case in our model if the investors form the intermediaries.

10From the discussion following Proposition 3.2, when there are only two classes of investors \( (\mu_1 = 0, \mu_2 > 0) \), in equilibrium the high search-cost investors will not search for \( V^* \) projects at their own cost/effort.

11The analysis can be easily extended to the case when there are many types of investors with differential search costs.
12 For a type (ii) CI equilibrium, essentially three classes of investors exist in the market: the intermediaries who are pseudo zero-cost investors, the low search-cost ($\mu$) investors, and the high search-cost ($\delta$) investors who invest randomly. It can be shown that there will either be an equilibrium with two perquisite/quality combinations or an equilibrium with three perquisite/quality combinations. To economize space, we shall not discuss this case in this paper.
APPENDIX A

Proofs of Propositions

A. We shall first establish that the perquisite/quality combinations selected by the entrepreneurs receiving an investment of $I'$ at equilibrium must be $p' = p(I')$ and $q' = q(I')$, where the functions $p(I)$ and $q(I)$ are the solutions to the maximization problem (1).

The proof is as follows: we know that in equilibrium all entrepreneurs have the same utility $M_0$, and they will either receive the investment funds (say $I'$) from (i) the uninformed investors only; or (ii) from some informed (zero cost) investors and some uninformed investors (who invests randomly in one project). Suppose (i) is the case, and suppose that there are entrepreneurs who offer $(p,q) \neq (p',q')$ to the uninformed investors only, an entrepreneur offering $(p,q)$ will find it in his interest to switch to some perquisite/quality combination(s) $(\bar{p}, \bar{q})$ such that $(1-\bar{p})\bar{X}(\bar{q}) > (1-p)\bar{X}(q)$, so that he would not lose any uninformed investors (who are assumed to be satisfied with $(p,q)$ in equilibrium), and simultaneously increase his welfare from $M_0$ to $M[I'\bar{p} \bar{X}(\bar{q}), e(\bar{q})]$. Thus $(p,q)$ cannot be offered in equilibrium.

Now suppose (ii) is the case, and suppose there are entrepreneurs who offer $(p,q) \neq (p',q')$ to the informed and some (lucky) uninformed investors and receive total investments of $I'$, it is obvious that an
entrepreneur will find it in his interest to offer some perquisite/quality combination(s) \((p^* + \varepsilon, q^*)\), which will attract all the informed investors he needs (\(S\) is the maximum) to increase his welfare from \(M_0\) to \(M[S(p^* + \varepsilon)X(q^*), e(q^*)]\). Thus the proof is complete.

B. **Proof of Proposition 3.1:** (i) When \(\mu_1 = 0\), from (A) the entrepreneurs offering projects to the zero-cost investors must offer \((p^*, q^*)\) in equilibrium. When \(\mu_2 < V^* - V[(1-\alpha)S]\), it is shown in Appendix B that \(((p^*, q^*))\) is the only equilibrium that exists.

(ii) Suppose all entrepreneurs offer \((p^*, q^*)\), we shall verify that it is indeed an equilibrium. \(((p^*, q^*))\) will be an equilibrium if there is no incentive for any entrepreneur to deviate from \((p^*, q^*)\). The entrepreneurs know that if they deviate from \((p^*, q^*)\), they will lose all the zero-cost investors and the remaining number of \(\mu_2\) investors (if they desire not to search for other projects) is \((1-\alpha)S\). Therefore, when \(\mu_2 < V^* - V[(1-\alpha)S]\), no entrepreneurs will deviate because they cannot keep the \(\mu_2\) investors without lowering their utility below \(M_0\).

C. **Proof of Proposition 3.2:** (i) When \(\mu_1 = 0\) and \(\mu_2 > 0\), from (A) if equilibrium exists, it must be of the form \(((p^*, q^*), (p', q'))\). Appendix B shows that the equilibrium as characterized by conditions (i), (ii), and (iii) in Proposition 3.2 for different values of \(\mu_2\) is unique when certain conditions are satisfied.
(ii) It is easy to verify that \(((p^*, q^*), (p', q'))\) is indeed an equilibrium. \(((p^*, q^*), (p', q'))\) will be an equilibrium if there is no incentive for any entrepreneurs to deviate from his prevailing perquisite/quality combination. First note that the \((p^*, q^*)\) entrepreneurs have no incentive to deviate, because if they do they will lose all the zero-cost investors and the remaining number of \(u_2\) investors (if they can keep them after the deviation) is \(I'\). But they have to offer at least \(V(I') = (1-p')X(q')\) to keep the \(u_2\) investors, which will not make them better off than the current situation. Also, the \((p', q')\) entrepreneurs have no incentive to change their perquisite/quality combinations. They will be better off by doing so if they can still keep the \(u_2\) investors and at the same time increase their utility, which is impossible from condition (iii)(a) and (iii)(b): if they increase \(V\), they can keep the \(u_2\) investors, but at a loss; if they decrease \(V\), they will lose all the \(u_2\) investors.

D. **Proof of Proposition 3.3:** (i) The necessity is established in the text.

(ii) We can verify that \((p'', q'')\) is indeed an equilibrium. This can be established by noting that no entrepreneurs have any incentive to deviate from \((p'', q'')\): if they increase \(V\), they will be worse off; if they decrease \(V\), all the investors who happen to investigate his project will exit from the market.
E. Proofs of Propositions 4.1 and 4.2: In text.

F. Proof of Proposition 4.3: In Appendix B.
APPENDIX B

Derivation of Equilibria and Comparative Statics Analysis

Our purpose in this appendix is to derive the equilibrium distribution of perquisite/quality combinations selected by the entrepreneurs when there are some investors with zero search cost (aN of them) and some with positive cost \( (1-\alpha)N \) of them), and then relate the equilibrium to the exogenous parameters.

When there are some zero-cost investors (who are informed), the equilibrium that will obtain is either (i) \( (p^*, q^*) \) or (ii) \( (p^*, q^*), (p', q') \), as described in Propositions 3.2 and 3.3. We have shown in the proofs that the necessary condition for the \( (p^*, q^*) \) equilibrium is \( \mu < V^* - V[(1-\alpha)S] \). Now we shall consider equilibrium (ii), and show that the distribution of \( (p^*, q^*), (p', q') \) depends on \( \mu \), and as \( \mu \) converges to \( V^* - V[(1-\alpha)S] \), equilibrium (ii) will converge to equilibrium (i), therefore the sufficiency of \( \mu < V^* - V[(1-\alpha)S] \) is established.

The following conditions must be satisfied in equilibrium (ii):

\[
S = \frac{aN}{M^*} + \frac{(1-\alpha)N}{M} \tag{C1}
\]

\[
I' = \frac{(1-\alpha)N}{M} \tag{C2}
\]

\[
\lambda[V^* - V(I')] = \mu. \tag{C3}
\]
Substituting (C2) in (C1) and using \( \lambda = M^*/M \), we have,

\[
I' = \frac{S}{(\frac{\alpha}{1-\alpha}) \frac{1}{\lambda} + 1}.
\]  

(C4)

Differentiating (C4) with respect to \( \lambda \),

\[
\frac{dI'}{d\lambda} = S(\frac{\alpha}{1-\alpha}) \frac{I'^2}{\lambda^2} > 0
\]  

(C5)

and,

\[
\frac{d^2I'}{d\lambda^2} = \frac{2I'^2S}{\lambda^4} (\frac{\alpha}{1-\alpha}) [SI'(\frac{\alpha}{1-\alpha}) - \lambda].
\]  

(C6)

So \( \frac{d^2I'}{d\lambda^2} > 0 \) if and only if \( \alpha > [(SI'/(\lambda) + 1)]^{-1} \).

\( V \) is a function of \( I' \), and we write

\[
V(\lambda) = V\left[\frac{S}{(\frac{\alpha}{1-\alpha}) \frac{1}{\lambda} + 1}\right].
\]  

(C7)

Since \( V \) is increasing in \( I' \), and \( I' \) is increasing in \( \lambda \), \( V(\lambda) \) is increasing in \( \lambda \), that is, \( V'(\lambda) > 0 \). The second derivative of \( V(\lambda) \), however, will depend on the shapes of \( V(I') \) and \( I'(\lambda) \). We shall assume that \( V(\lambda) \) is convex in \( \lambda \).

Define \( \lambda^* \) such that \( V(\lambda^*) = r \).

Equation (C3) also relates \( V \) to \( \lambda \). Let \( Q(\lambda,V,\mu) = \lambda[\lambda^*-V] - \mu \). Then (C3) can be rewritten as

\[
Q(\lambda,V,\mu) = 0.
\]  

(C8)
The derivatives of the curve : $Q = 0$ are:

\[
\frac{dV}{d\lambda} \bigg|_{Q=0} = \frac{V* - V}{\lambda} > 0 \tag{C9}
\]

and

\[
\frac{d^2V}{d\lambda^2} \bigg|_{Q=0} = -\frac{2(V* - V)}{\lambda^2} < 0. \tag{C10}
\]

Combining (C7) and (C8), we can obtain the equilibrium values of $\lambda$ and $V$, and $I'$ can be determined from the inverse of $V$. Using (C1) and (C2) the equilibrium values of $M^*$ and $M$ can be calculated; and finally from $I'$ we can obtain $p' = p(I')$ and $q' = q(I')$.

Equations (C7) and (C8) are depicted as curve (1) and curve (2) in figure 4, respectively. Note that for both curves, the largest value of $\lambda$ is 1. For curve (1), when $\lambda = \lambda^*$, $V(\lambda) = r$, and for curve (2), the smallest value of $\lambda$ it can take upon is $\lambda = \mu/(V^*-r)$ since in equilibrium $V > r$.

Note that when $V(\lambda)$ is convex in $\lambda$, the equilibrium is unique given $\mu$, $\alpha$ and other exogenous parameters.

**Comparative Statics Analysis with Respect to $\mu$ (Figure 4)**

(1) **Effect of lower $\mu$**: Notice that when $\mu$ becomes smaller, the equilibrium values of $\lambda$ and $V$ are larger. For example, when $\mu^0 + \mu^1$, the equilibrium shifts from $E_0$ to $E_1$. Therefore, when
FIGURE 4

DERIVATION OF COMPETITIVE INTERMEDIATION EQUILIBRIA

(Note: \( x \equiv \frac{v^* - V[(1-\alpha)S]}{(V^*-r)} \); \( y \equiv \frac{\mu^1}{(V^*-r)} \);
\( z \equiv \frac{\mu^0}{(V^*-r)} \).)
the search costs of the uninformed investors becomes smaller, there is a higher proportion of $V^*$ firms, and also the (net) return offered by the inferior firms, $V$, is larger. As a result, the uninformed investors will be better off than when $u$ is large.

(2) **Characterization of equilibria by $u$:** From figure 4 when $u$ becomes smaller and smaller and attains the value $V^* = V[(1-\alpha)S]$, curves (1) and (2) intersect at $\lambda = 1$ and $V = V[(1-\alpha)S]$. $u < V^* - V[(1-\alpha)S]$, as we have proved, is the necessary condition for the $\{(p^*,q^*)\}$ equilibrium. Now we have shown that it is also sufficient since when $u < V^* - V[(1-\alpha)S]$, no other equilibria can exist.

Also, when $u$ becomes larger and larger and attains the value $Z$, where $Z = \lambda^*(V^*-r)$, the equilibrium is at $\lambda = \lambda^*$ and $V = r$. The lowest value for $V$ in equilibrium is $r$, so when $u > Z$, we still have the equilibrium $(\lambda^*,r)$, but the equality condition (iii)(a) in Proposition (3.2) will be substituted by the inequality (iii)(b).

From the above analysis, the classification of equilibrium when there are some investors with zero cost ($\alpha N$ of them) and some with positive cost $u$ ($(1-\alpha)N$ of them), is as follows:

(i) when $u < V^* - V[(1-\alpha)S]$, $(p^*,q^*)$ only will be offered in equilibrium $(\lambda = 1, V = V^*)$;

(ii) when $V^* - V[(1-\alpha)S] < u < Z$, $(p^*,q^*)$ and $(p',q')$ will be offered in equilibrium $(\lambda^* < \lambda < 1, V > r)$;
(iii) when \( u > Z \), \((p^*, q^*)\) and \((p', q')\) will be offered \((\lambda = \lambda^*, V = r)\).

When all investors have positive search costs, the equilibrium is necessarily \{(p'', q'')\} \((\lambda = 0\) and \(V = r)\).

**Comparative Statics with Respect to \(\alpha\)**

The effect of an increase in \(\alpha\) is illustrated in figure 5: curve (1) shifts to the right \((\alpha_1 > \alpha_0)\), the equilibrium values of \(\lambda\) and \(V\) will increase accordingly.

**Proof of Proposition 4.3:** Let \(G(\lambda, V) = \lambda V^* + (1-\lambda)V\). \(G\) measures the utility of all investors at the equilibrium \((\lambda, V)\). Notice that when \(\alpha_1 > \alpha_0\), \(G_1 > G_0\). In a CI equilibrium, \(\alpha = \gamma\), so that a high-\(\gamma\) equilibrium will dominate a low-\(\gamma\) equilibrium using the Pareto criterion. Therefore the proof is complete.
FIGURE 5

COMPARATIVE STATICS ANALYSIS OF COMPETITIVE INTERMEDIATION EQUILIBRIA

\[ G = G_1 \]

\[ G = G_0 \]

\[ (2) \]

\[ (1) \]

\[ \alpha_0 \]

\[ \lambda_0 \]

\[ \lambda_1 \]

\[ v_0 \]

\[ v_1 \]

\[ 0 \]
REFERENCES


