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Ex Post Stockholder Unanimity:
A Complete and Simplified Treatment

By
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EX POST STOCKHOLDER UNANIMITY:
- A COMPLETE AND SIMPLIFIED TREATMENT

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I. INTRODUCTION

Of central concern in the neo-classical theory of the firm is the identification of conditions under which a firm's stockholders agree on the desirability of alternative production (and financing) plans. In a world of certainty and perfectly competitive markets, Fisher demonstrated that stockholders agree that firms should choose plans that maximize market values. The result has been generalized to settings of uncertainty, and related results are usually referred to as ex ante unanimity. The dual assumptions of perfectly competitive markets and "spanning" result in a simple argument about the nature of consumption opportunity sets in which value-maximization is unanimously agreed upon. "Spanning" of course means that the space of available return patterns is unaffected by firms' plans, and "perfectly competitive markets" translates into a requirement that implicit prices are, or at least are perceived to be, unaffected by changes in plans. As has been emphasized by DeAngelo [81], the idea behind Fisher's result is now straightforward, since individuals' consumption opportunity-sets are isomorphic to a firm's value. For this reason the conclusion is relatively robust, and it even permits extensions to multi-period environments (see Grossman and Stiglitz [77]).

The presumption of an invariant implicit price system is a severe restriction in a general equilibrium setting. Partly because of this, there is a literature on stockholder unanimity in markets that are not perfectly competitive. The significant contributions by Ekern and Wilson [74], Radner [74], and Leland [73] demonstrated that unanimity might be present under conditions other than perfectly competitive markets. The key result developed in the aforementioned papers can be stated briefly as follows: provided that the spanning condition is met, and that endowments in firms' shares are non-negative and utility-maximizing, stockholders will be unanimous in their
evaluation of the desirability of a change in a firm's plan in a contemplated direction. Because the endowed (pre-change) holdings are equilibrium holdings, the analysis is referred to as ex post unanimity. It should further be noted that the existing literature presents the result as a local one; i.e., it relates to a "small" change in plans. Nothing can be said generally about how two distinct plans compare with respect to expected utility across individuals, even if there is spanning and the ex post condition is met relative to one set of plans.

This paper reconsiders the ex post analysis. A complete and simplified analysis is pursued, and many unnecessary restrictions in the literature are eliminated. The added generality manifests itself in several ways.

First, restrictions on individuals' endowments are relaxed; in particular, endowments in shares need not be non-negative. The analysis therefore pertains not only to how shareholders react to a contemplated change, but also to how all other individuals in the economy evaluate the contemplated change. (In the existing literature, long and short endowments imply a disagreement). In a related vein, all firms -- not just one firm -- can contemplate a change in plans, and the total (joint) impact on changes in individuals' expected utilities is evaluated. The removal of these constraints permits a "co-ordinate-free" analysis that is not solely concerned with a change in one particular (a specific firm's) vector of payoffs in the securities-states tableau. Conclusions can be expressed in terms of individuals' endowed consumption plans and economy aggregates, rather than individuals' ownership positions in a specific firm and the impact any change in the firm's plan might have. What fundamentally counts is the following: (i) the change in aggregate outputs and the related impact on the implicit price system; (ii) the change in the space spanned by the securities-states tableau; and (iii) individuals' endowed consumption plans and their changes in values.
Second, the role of a "small" change in plans is clearly identified. Existing literature relies on the condition as one of those sufficient for unanimity. However, I explain the reason for and the extent of the condition's necessity.

Third, the spanning condition deserves a similar comment; its necessity has not been subject to systematic analysis.

Fourth, the relationship between the values of consumption plans and unanimity is considered. Unanimity about a change in plans may be desirable or undesirable; the question is how these binary outcomes relate to changes in the values of plans, especially when the changes are nonlocal.

While the analysis is directed toward a complete treatment of ex post unanimity, with particular emphasis on the identification of necessary conditions, it is performed without any mathematical or analytical complications. The analysis relies solely on elementary concepts and is in most respects more direct than the usual kinds of developments. Differential calculus, Lagrangean multipliers, first-order optimality conditions, etc., are all unnecessary to the development of key insights.

II. PRELIMINARIES

The setting considered is a one-period economy of the simplest (and standard) kind. There are \( s = 1, \ldots, S \) states, \( i = 1, \ldots, I \) individuals, and \( j = 1, \ldots, J \) firms. Individuals' preferences are defined on consumption plans \( c_i = (c_{i1}, \ldots, c_{iS}) \). If a bundle \( c_i' \) is strictly preferred to a bundle \( c_i'' \), this is denoted by \( c_i' \succ c_i'' \). In a similar fashion, weak preference is denoted by \( c_i' \succeq c_i'' \).

Further, more is presumed to be preferable to less; i.e., if \( c_i' \succeq c_i'' \) then \( c_i' \succeq c_i'' \), and \( c_i' \geq c_i'' \) implies \( c_i' \succeq c_i'' \). Each firm \( j \) is characterized by its output-vector \( y_{ij} = (y_{i1j}, \ldots, y_{iSj}) \geq 0 \). The output patterns of all firms in the
economy are represented by the \( J \times S \) matrix ("securities-states tableau")

\[
Y = [y_{js}].
\]

Markets are not assumed to be complete, and individuals trade in firms' shares. Let \( \theta_i = (\theta_{i1}, \ldots, \theta_{ij}) \) denote individual i's shareholdings, and let \( \bar{\theta}_i \) denote his endowments in shares. Without any loss of generality, total supply of shares for each firm \( j \) can be set equal to one; that is, \( \Sigma_i \theta_{ij} = 1 \) for all \( j \). Any plan \( \theta_i \) of shareholdings results in a consumption plan \( c_i \) via the relationship

\[
c_{is} = \Sigma_j \theta_{ij} y_{js}, \quad \text{for all states } s,
\]

or \( c_i = \theta_i Y \) for short.

Let \( P_j \) be the market value of firm \( j \). Let \( \nu = (\nu_1, \ldots, \nu_S) > 0 \) be any (equilibrium) implicit price system. The market values of firms can be stated in the usual fashion as:

\[
P_j = \Sigma_s v_{js} v_s \quad (\text{all } j).
\]

Individuals' budget constraints are given by

\[
\Sigma \nu \theta_{ij} \leq \Sigma \nu \theta_{ij}.
\]

Let \( C(Y) \) be the vector space generated by \( Y \):

\[
C(Y) \equiv \{ c | c = \theta Y, \text{ all } \theta \}.
\]

The budget constraints can now be stated equivalently as

\[
\Sigma_s c_{is} v_s \leq \Sigma_s \bar{c}_{is} v_s,
\]

where \( \bar{c}_{i} \equiv \bar{\theta}_i Y \) and, additionally, the spanning requirement, \( c_i \in C(Y) \), holds. Thus, while \( \nu \) is unique if and only if markets are complete, this is of no particular concern since \( c_i \nu \) (and \( Y \nu \)) are unique across all implicit price systems consistent with returns implied by \( (Y, P) \). (This is, of course, well known.)

Given any matrix of production plans \( Y' \) and preference structures,
\((\{y^i_{-i}\}, v')\) constitute an equilibrium allocation and implicit price-vector if, and only if, the following three conditions are met:

(i) \(c^i \in C(Y')\), all \(i\);

(ii) if \(c^i \in C(Y')\) and \(c^i \succeq c'^i\), then \(c^i v' > c'^i v'\);

(iii) \(\sum c^i = 1Y' = \sum \theta^i Y'\).

Condition (i) is the spanning requirement; (ii) requires that \(c^i\) be utility-maximizing relative to the implicit price system \(v'\), and (iii) states the balancing equations, i.e., total consumption equals total production. Note further that, given any equilibrium \((\{y^i_{-i}\}, v')\), there is also a characterization in terms of \((\{\theta^i_{-i}\}, \{p^i\})\). The latter quantities are simply transformations of the former via the matrix \(Y'\): \(c^i = \theta^i Y'\) and \(p' = Y'v'\).

III. CONDITIONS NECESSARY AND SUFFICIENT FOR EX POST UNANIMITY

The general problem can be summarized as follows. Given an existing tableau of (single-primed) production plans, \(Y'\), a related equilibrium is denoted by \((\{\theta^i_{-i}\}, \{p^i\})\), and \((\{c^i_{-i}\}, v')\). A production change from \(Y'\) to \(Y''\), results in a different equilibrium \((\{\theta^i_{-i}\}, \{p^i\})\), and \((\{c^i_{-i}\}, v'')\). Under what conditions will all individuals agree that the contemplated change is for the better (or worse)? The issue is addressed within an ex post framework, i.e., the condition \(\theta^i = \theta^i\) (all \(i\)) is imposed. In other words, the contemplated change in production plans is evaluated given that the financial markets are currently in equilibrium.

It will be useful to distinguish between positive and negative unanimity; their respective definitions are

positive unanimity: \(c^i \gtrless c'^i\) for all \(i\);

negative unanimity: \(c^i \gtrless c''^i\) for all \(i\).

As will become clear, the two unanimity cases require distinct treatments.
There are no reasons generally to expect the analysis to be symmetric, since typically $\bar{\theta}_i^m \neq \bar{\theta}_i^n$. It is also worthwhile noting that the two equilibrium allocations $\{c_i^m\}_i$ and $\{c_i^n\}_i$ might be Pareto noncomparable, in which case there is neither positive nor negative unanimity. Further, when $c_i^m \sim c_i^n$ for all $i$, then, and only then, is there both negative and positive unanimity; this implication of indifference is simply a matter of convenience.

Conditions sufficient for unanimity were first developed by Eke&kern and Wilson [74] (E-W henceforth). Let

$$\Delta Y \equiv Y^n - Y^1$$

be the contemplated change in firms' production plans. Then the following four conditions are sufficient for either positive or negative unanimity:

(a) $y_{j^*}^n = y_j$ for all $j$ but one, $j^*$ say ("analysis of a specific firm")

(b) $\theta_{ij^*}^i \geq 0$ for all $i$ ("endowed ownership positions in the firm subject to analysis are not short")

(c) $\Delta Y$ (or $\Delta y_{j^*}$) is sufficiently small ("the contemplated change of production plan is small")

(d) $C(Y^n) = C(Y^1)$ ("spanning equivalence" or simply "spanning").

From E-W one can also infer that positive (negative) unanimity obtains if and only if

$$\Delta y_{j^*} v^1 \geq 0 \quad (\leq 0).$$

Hence, using existing implicit prices, positive (negative) unanimity is equivalent to a non-decrease (non-increase) in the firm's value. (Of course, mild regularity conditions have been imposed on individuals' preferences.)

Subsequent analysis investigates in some detail the significance of each of conditions (a) through (d) for both kinds of unanimity. This leads to a number of relaxations of the conditions sufficient for unanimity.

Almost all insights developed are based on a straightforward Lemma that
needs no proof.

**Lemma:** Let $c' \in C(Y')$ and $c'' \in C(Y'')$ be two optimal plans, and assume that $C(Y') \subseteq C(Y'')$.

A. If $c' \succ c''$, then $c' \triangleright v'' \succ c'' \triangleright v''$.

B. If $c' \triangleright v'' \prec c'' \triangleright v''$, then $c'' \succ c'$.

The Lemma has a straightforward interpretation. Part A simply means that, if a plan is preferable to a chosen plan and is feasible in that it is spanned, then the plan is not affordable. Part B is almost (but not quite) the converse: a plan that is affordable (with slack) and feasible but is not chosen cannot be optimal. The two observations are obvious indeed, and they are also independent of the ex post condition.

Parts A and B imply each other: if condition 'X' is sufficient for a certain conclusion, then 'not X' is necessary for the opposite (complementary) conclusion. This simple fact will be useful. Specifically, any conditions sufficient for positive (negative) unanimity directly yield conclusions necessary for negative (positive) unanimity. It will therefore be appropriate to view the problem at hand in two parts: the conditions sufficient for positive unanimity, and the conditions sufficient for negative unanimity.

An important quantity in the analysis that follows is the change in an individual's endowed consumption plans due to a change in production. Let this vector be denoted by $\tilde{c}_i'' - c'_i = \Delta \tilde{c}_i$ where thus $\tilde{\theta}_i'' = \tilde{\theta}_i'' = \tilde{c}_i''$. While $\tilde{\theta}_i = \tilde{\theta}_i'$ by the ex post condition, this obviously does not imply that $\Delta \tilde{c}_i = 0$; $\Delta \tilde{c}_i$ is generally zero (for some $i$) if, and only if, $\Delta Y$ is a zero matrix. The following result is now an immediate consequence of the Lemma and the budget-constraints $c'_i \triangleright v'' = c'_i \triangleright v''$. 


Proposition I: Suppose the following two conditions are satisfied:

(i) \( C(Y'') \supseteq C(Y') \);

(ii) \( 0 < \Delta c_i v'' \) all \( i \in I \),

or, equivalently,

\( 0 < \theta_I \Delta Y v'' \) all \( i \in I \).

Then there is ex post positive unanimity.

Hence, given \( C(Y'') \supseteq C(Y') \), it also follows that \( \Delta c_i v'' \leq 0 \), all \( i \in I \), is a necessary condition for ex post negative unanimity.

The following corollary obtains as a special case.\(^4\)

Corollary: There is ex post positive unanimity if \( \theta_i^t \geq 0 \), all \( i \in I \), \( Y''v'' \geq Y'v'' \), and \( C(Y'') \supseteq C(Y') \).

Several useful observations follow from the results. First, the ex post condition implies that \( \theta_I \Delta Y v'' \) is equivalent to \( \Delta c_i v'' \), and the latter quantity measures the change in the value of endowments using the implicit prices that would prevail under the contemplated plans. Hence, if the values of endowments (measured by \( v'' \)) increase for all individuals, and there is no reduction in the set of feasible mixes of consumption patterns, i.e., \( \hat{C}(Y'') \supseteq C(Y') \), then everybody is (strictly) better off under the contemplated plans. The conditions ensure that the previously optimal plan, \( c_i' \), remains feasible and affordable, and thus individuals will be no worse off. An important conclusion therefore emerges: conditions (c) "small change" and (d) exact "spanning equivalence" of E-W are unnecessarily stringent for ex post positive unanimity. Second, Proposition I, taken in conjunction with the Corollary, stresses that there is no need to confine the analysis to one particular firm. Further, with positive
share endowments, individuals are better off when firms' production plans increase in value (and where the values considered are those of the contemplated economy). E-W's condition (a) can therefore be relaxed, at least for positive unanimity. Condition (b), however, can be relaxed only to the extent one allows for an evaluation of changes in all individuals' endowed plans, rather than an evaluation of changes in firms' plans. Third, there appears to be no particular role for the "small change" condition (d) in positive unanimity. Neither $\theta_i \Delta Yv'' > 0$ nor $C(Y') \subseteq C(Y'')$ can be weakened simply because the contemplated change is "small" rather than "large." The absolute magnitudes (or "scale") of the quantities $\Delta Y$ in condition (ii) are irrelevant, and similarly, the idea that $C(Y')$ is a subspace of $C(Y'')$ does not particularly depend on the absolute magnitudes of the elements in $\Delta Y$.

Conditions necessary for ex post positive unanimity will of course be considered in the context of conditions sufficient for ex post negative unanimity. Nevertheless, it is worth recognizing already at this point that the spanning condition (i) cannot be weakened. This follows more or less immediately: the elimination of certain mixes of consumption patterns has a potentially adverse impact on the risk-sharing arrangement. A closely related problem is that condition (ii) is simply not meaningful unless condition (i) is met; $\theta_i' \Delta Yv''$, and its sign, is the same for all $v''$ consistent with $(Y'', P'')$ if and only if condition (i) is satisfied. Not surprisingly, it follows that conditions sufficient for negative unanimity (or conditions necessary for positive unanimity) cannot be inferred directly from Proposition 1.

To develop conditions for ex post negative unanimity, one simply reverses the spanning condition and applies the Lemma again. The double-prime and single-prime notations in the two parts A and B must be interchanged, but there are no other complications. Let $\Delta v = v'' - v'$ denote the change in the
implicit price system.

**Proposition II:** Suppose the following two conditions are satisfied:

1. \( C(Y'') \subseteq C(Y') \);
2. \( (c_i' - c_i'') \Delta v + \delta_i \Delta Yv'' < 0, \) all \( i \in I \),
   or, equivalently,
   \( (\tilde{c}_i'' - c_i'') \Delta v + \delta_i \Delta Yv' < 0, \) all \( i \in I \).

Then there is ex post negative unanimity.

Hence, given \( C(Y'') \subseteq C(Y') \), the opposite of condition (ii) for all \( i \in I \) is a necessary condition for ex post positive unanimity.

**Proof:** The ex post condition implies that \( \delta_i \Delta Yv'' \) equals \( c_i'' v'' - c_i' v'' \). The first inequality of condition (ii) is therefore equivalent to

\[
(c_i' - c_i'') \Delta v + (c_i'' - c_i') v'' < 0.
\]

The latter simplifies to

\[
c_i' v' - c_i' v' < 0,
\]

and the proposition follows from the Lemma. Verifying that the two expressions of condition (i) are equivalent is trivial.

A comparison of the conditions for positive and negative ex post unanimity reveals that the sign of \( \delta_i \Delta Yv' \) is not an adequate or relevant indicator for either type of unanimity, even if the sign is the same for all individuals and, additionally, \( C(Y') = C(Y'') \). This stands in contrast to \( \delta_i \Delta Yv'' \) which is a useful indicator for positive unanimity when the signs are positive. However, it is easy to show by example that spanning equivalence and \( \delta_i \Delta Yv' < 0 \) (or \( \delta_i \Delta Yv' < 0 \)) do not imply the requisite condition for negative unanimity, i.e., the condition \( (c_i' - c_i'') \Delta v + \delta_i \Delta Yv'' < 0 \) might not be met. As long as the term \( (c_i' - c_i'') \Delta v \) exceeds or equals the decline of the value of endowments, i.e., \( (c_i' - c_i'') \Delta v \geq - \)
\((\vec{c}_i'' - c_i')v'' = -\theta_i'\Delta Yv''\), then the individual is not necessarily worse off under the contemplated plans. The missing element in the analysis is gains (loss) from exchange, as is most clearly seen from the second expression in (ii). The gain from exchange is measured there by \((\vec{c}_i'' - c_i')\Delta v\), and if this gain combined with gain on value of endowments, \(\theta_i'\Delta Yv' = \Delta \vec{c}_i'v'\), is negative, then the individual is worse off. In sharp contrast, under conditions of positive unanimity all individuals benefit sufficiently from exchange as long as the endowments increase in value (using \(v''\) implicit prices). (It is not implied, however, that \(\vec{c}_i''\) is preferable to \(c_i'\), and exchange is essential even when \(\vec{c}_i''v'' > c_i'v''\).) The positive and negative unanimity cases are therefore asymmetric, and it is not a matter of just reversing the inequality sign and use the \(v'\) implicit prices rather than \(v''\) implicit prices. The negative unanimity case needs "correction factors" which vary across individuals. This further implies that there is no direct way one can obtain a corollary similar to the one that followed Proposition 1. I.e., \(\theta_i' > 0\) and \(\Delta Yv' \leq 0\) is not sufficient for negative unanimity. With \(\Delta v\) generally being non-zero, the only obvious exception occurs when individuals are identical and do not trade.

However, note that the analysis is symmetric in case the ex ante "competitive markets" assumption holds: with \(v' = v''\) presumed, the relevant welfare indicator is \(\theta_i'\Delta Yv' = \Delta \vec{c}_i'v' = \Delta \vec{c}_i''v''\) for both positive and negative unanimity. The correction factors appearing in the expressions of (ii), Proposition II, vanish. (To be sure, the assumption \(v'' = v'\) makes the ex post condition \(\theta_i'' = \theta_i'\) redundant for unanimity).\(^6\)

It is not difficult to show that there are cases such that everybody is better off in the contemplated economy even though \(\theta_i'\Delta Yv'' < 0\) for all \(i \in I\). (No particular conditions on \(C(Y')\) relative to \(C(Y'')\) are needed.) It follows that \(1\Delta Yv'' = \sum_i(c_i'' - c_i')v'' \geq 0\) is not a necessary condition for positive
unanimity. In a sense, the implicit prices used for that particular inequality are "incorrect." The proper statements are that

$$[(\sum c''_i - c'_i)]v' = 1\Delta Yv' \geq 0 \quad (C(Y') \supseteq C(Y''))$$

is a necessary condition for positive unanimity, and

$$[(\sum c''_i - c'_i)]v'' = 1\Delta Yv'' \leq 0 \quad (C(Y'') \supseteq C(Y'))$$

is a necessary condition for negative unanimity. Neither conclusion is surprising, since $1\Delta Yv' \leq 0$ ($1\Delta Yv'' \geq 0$) is a sufficient condition for the existing (contemplated) economy to be weakly more efficient than the contemplated (existing) one when $C(Y') \supseteq C(Y'') (C(Y') \subseteq C(Y''))$. Hence, measures of changes in the values of aggregate outputs provide conditions necessary for positive or negative unanimity. These conditions are not particularly informative, for they reflect aggregates and thus do not account for potential redistributive effects.

A closer look at Proposition II indicates that sharper results are possible if E-W's assumption (c) -- that of a small change in $\Delta Y$ -- is invoked. Put simply, the idea is that $\Delta Yv'' < 0$ implies $(c''_i - c'_i)\Delta v + \Delta Yv'' < 0$, because the first term is small relative to the second. Somewhat more formally, in a standard equilibrium setting in which $1\Delta Y11 = 0$ (where $1\Delta Y11 = \max_{j,s} \{ \Delta Y_{js} \}$, say), the magnitudes of $1\Delta c''_i - c''_i$ and $1\Delta v11$ should be of the same or lower order as $1\Delta Y11$; thus, $1\Delta (c''_i - c'_i)\Delta v11$ should be no greater than $1\Delta Y11^2$, and the quantity is therefore insignificant relative to $1\Delta Y11^2$, which is only of the order of $1\Delta Y11$.

Although the above analysis is appealing and basically sound, it is nevertheless not quite complete. There is no reason to assume that $1\Delta c'_i - c''_i$ and $1\Delta v11$ are small unless $Y'$ and $Y''$ span identical spaces. Thus, via examples, it is easily shown that if $C(Y') \neq C(Y'')$ one must generally have $c''_i(\Delta Y) \neq c'_i(0) (=c'_i)$ even as $1\Delta Y11 \to 0$. A "small change" $\Delta Y$ is effectively not small because
the spaces spanned are distinct, and this introduces discontinuities in the sequence of equilibrium holdings and prices. Thus, condition (d) of E-W is of no use unless C(Y') = C(Y'') as ||ΔY|| → 0.

It follows that both conditions (c) ("small change") and (d) ("spanning") of E-W are necessary if either positive or negative unanimity is to be derived for any preferences/beliefs. However, conditions (a) ("specific firm analysis") and (b) ("non-negative endowments") can be modified in an obvious fashion in view of Propositions I and II. The general result is therefore stated as follows.

Proposition III: Suppose that C(Y'') = C(Y') and that the signs of $θ_i^i ΔYv_i$ are the same for all $i \in I$. Then, as $ΔY → 0$, there is either ex post positive or ex post negative unanimity. With no restrictions on preferences/beliefs, the three conditions are also necessary.

Using differential calculus, it is readily shown that the sufficiency part amounts to

$$dEU_i = dc_i v' = θ_i ΔYv'$$

where $dEU_i$ is the infinitesimal change in individual $i$'s expected utility, and $r_i$ denotes "proportional to". (The proportionality is implied by the fact that $EU_i$ and $dEU_i$ are arbitrary up to a positive scale factor). The interesting aspect is thus that $dv$ is of no consequence in the analysis. This is appealing as a "practical matter" since $v''$ is basically unpredictable; any firm that contemplates a change must anticipate not only the implications of its own actions, but also all other firms' production responses and the implications of those if $v''$ is to be properly anticipated.

Not much can generally be said about $θ_i ΔYv'$ on the basis of knowledge of $ΔYv'$ alone. However, the Corollary is now applicable for not only positive unanimity but also negative unanimity (after the inequality signs of firms'
values have been reversed). In this context, note that $\Delta Y^i$ effectively can be put equal to $\Delta Y^u$ since $\Delta Y^\Delta$ is negligible relative to $\Delta Y^v$. As a further implication, a necessary condition for negative (positive) unanimity is the decrease (increase) in the value of aggregate output, and, with small $\Delta Y$ and exact spanning, these evaluations can always be based on the existing implicit price system.

Finally, it should be affirmed that the unanimity requirement that certain quantities be of the same sign for all individuals implicitly imposes restrictions on the characteristics of individuals. The observation is immediate since prices and allocations are endogenous variables in an equilibrium setting. The problem cannot be avoided via prior constraints on security holdings (such as non-negativity); this point has been emphasized by Nielsen [76]. Hence, while ex post unanimity permits some "limited" diversity in individuals' characteristics, diversity cannot be extended arbitrarily beyond mild regularity conditions.

IV. SUMMARY

The contribution of this paper is twofold. First, a complete rather than a partial treatment of ex post stockholder unanimity is provided; necessary and sufficient conditions are considered. Previous work, by Ekrn and Wilson [74] and others, considered only sufficient conditions, but not the weakest sufficient conditions possible: it is not necessary to focus on a specific firm (in which individuals have a positive endowment). The more general analysis focuses directly on individuals' consumption-patterns rather than on a firm's payoff patterns. The analysis is dichotomized into positive and negative unanimity, because gains from exchange make the two cases quite different. More precisely, the "small production change" and "spanning" requirements in the standard analysis have roles only in the case of negative unanimity. The two
conditions effectively force the gains from exchange to become of second-order, negligible, magnitude. The analysis need now not distinguish between positive and negative unanimity when the gains from exchange are negligible. In particular, there is positive (negative) unanimity to the extent all individuals' endowments have increased (decreased) in value.

Second, the analysis is much simplified in comparison with previous work. The techniques used to develop the results are elementary, and individuals' preferences are captured by the basic principle of revealed preference. Furthermore, as is almost always the case in capital market models, it is easier to consider consumption patterns and their (implicit) prices than to apply cumbersome transformations that identify security holdings and security prices; that implicit prices are generally not unique poses no particular problems.
FOOTNOTES

1. All of the analysis and results could be modified to accommodate two periods and an input good used in the production processes. Such a routine generalization poses only notational complications. Because this leads to no additional insights, the simplest possible setting is selected.

2. The analysis would not change if zero net supply securities (\(\Sigma_i \theta_{ij} = 0\)) are introduced.

3. Strictly speaking, given that condition (c) considers a sequence of plans, condition (d) requires that \(C(Y') = C(Y' + \Delta Y)\) for all sufficiently small \(\Delta Y\), \(Y'\) fixed. Ekern and Wilson [74] and Leland's [73] concept of spanning can be expressed as: there are matrices \(T(\Delta Y)\), dependent on \(\Delta Y\), such that \(\Delta Y = T(\Delta Y)Y'\) for all \(\Delta Y\) sufficiently small. (In the limit write \(dY = TY'\).) It is readily shown that the latter is implied by the prior definition of "spanning." The converse is not true; it is only the case that \(C(Y') \subseteq C(Y'')\). However, as \(\Delta Y\) and \(T(\Delta Y)\) approach zero it will indeed be the case that \(C(Y') = C(Y'')\). Thus, for small changes, the definition given here is equivalent to that of others.

4. The Corollary, but not the Proposition, is also found in Ohlson and Kunkel [81].

5. Note the following: if \(C(Y'') \supseteq C(Y')\) and \(v''_1\) and \(v''_2\) are any two possibly distinct but valid implicit price systems (i.e., \(Y''v''_1 = Y''v''_2\)), then, and only then, is \(Y'v''_1 = Y'v''_2\) (and \(\Delta Yv''_1 = \Delta Yv''_2\)). The observation is immediate, since there exists a matrix \(T\) such that \(TY'' = Y'\) if and only if \(C(Y'') \supseteq C(Y')\).
6. Hence, the general result of ex ante positive unanimity obtains if 

\[ v'' = v' \equiv v', \ C(Y'') \subseteq C(Y') \] 

and 

\[ \tilde{\theta}_i \tilde{Y}''v \geq \tilde{\theta}_i \tilde{Y}'v \] 

where \( \tilde{\theta}_i'' = \tilde{\theta}_i' \) but, possibly, \( \tilde{\theta}_i' \neq \tilde{\theta}_i' \). A special case occurs when \( \tilde{\theta}_i' \geq 0 \) and \( \tilde{Y}'v \geq \tilde{Y}'v \), which corresponds to the Corollary. Reversing the spanning condition, and assuming that \( \Delta Yv \leq 0 \), negative unanimity obtains.

7. To illustrate the possibility of a discontinuity when \( C(Y'') \neq C(Y') \) as \( \Delta Y \rightarrow 0 \), note that if 

\[ Y'' = Y' + \Delta Y = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \varepsilon \\ 0 \\ 0 \end{bmatrix}, \]

then it makes a big difference whether \( \varepsilon = 0 \) or \( \varepsilon \neq 0 \): \( C(Y'') \supseteq C(Y') \) for all \( \varepsilon > 0 \) and markets are, in fact, complete. If \( \varepsilon = 0 \) markets are incomplete and with \( S = 2 \) there is no trading ever (i.e., \( \tilde{\theta}_i' = \tilde{\theta}_i'' \) for all \( \tilde{\theta}_i' \)). Thus, there is obviously a discontinuity point around \( \Delta Y = 0 \). Further note that the problem remains even if \( C(Y'') \) is neither a superset nor a subset relative to \( C(Y') \). This is not difficult to show.

8. The proof is as follows: differentiating the budget-constraints implies that 

\[ d\tilde{\theta}_i P' + \tilde{\theta}_i dP = \tilde{\theta}_i dP \]

Thus, given the ex post condition, 

\[ d\tilde{\theta}_i P' = 0 \quad \text{or} \quad d\tilde{\theta}_i Y'v' = 0. \]

Differentiating \( EU_i \) implies that 

\[ dEU_i = dc_i u_i', \]

where \( u_i' = (u_i', \ldots, u_i', \ldots, u_i') \) and \( u_i' = \partial EU_i / \partial c_i \) evaluated at \( c_i' \). Further, \( dc_i = d\tilde{\theta}_i Y' + \tilde{\theta}_i dY \) and hence 

\[ dEU_i = d\tilde{\theta}_i Y'u_i' + \tilde{\theta}_i dYu_i'. \]
But $Y'u' \propto Y'v'$ in view of standard equilibrium conditions, and $dY'u' = TY'u' \propto TY'v' = dYv'$ since spanning implies that there will always exist a matrix $T$ such that $dY = TY'$ (see footnote 3). It now follows that $d\theta_i Y'u' = d\theta_i Y'v' = 0$ and

$$dEU_i = \partial_i Y'u' \propto \partial_i Yv' = d\bar{c}_i v'.$$
REFERENCES


