FINANCE WORKING PAPER NO. 138

Why Are Prices for Stock Index Futures so Low?

by

Stephen Figlewski

Research Program in Finance Working Papers are preliminary in nature; their purpose is to stimulate discussion and comment. Therefore, they should not be cited or quoted in any publication without the permission of the author. Single copies of a paper may be requested from the Institute of Business and Economic Research.
RESEARCH PROGRAM IN FINANCE AT THE
WALTER A. HAAS SCHOOL OF BUSINESS,
UNIVERSITY OF CALIFORNIA, BERKELEY

The Research Program in Finance in the Walter A. Haas School of Business at the University of California has as its purpose the conduct and encouragement of research in finance, investments, banking, securities markets, and financial institutions. The present reprint and working paper series were established in 1971 in conjunction with a grant from the Dean Witter Foundation.

INSTITUTE OF BUSINESS AND ECONOMIC RESEARCH
Carl Shapiro, Director

The Institute of Business and Economic Research is an organized research unit at the University of California, Berkeley, whose mission is to promote research by faculty and graduate students in the fields of business and economics. The Institute carries out its mission by organizing programs and activities that enrich the research environment, administering extramural research awards, publishing working papers, and making direct grants for research.
WHY ARE PRICES FOR STOCK INDEX FUTURES SO LOW?

by

Stephen Figlewski*

February 1983

Finance Series
WORKING PAPER NO. 138

* Visiting Associate Professor, Graduate School of Business Administration, University of California, Berkeley, California 94720.

I would like to thank the Interactive Data Corporation which provided data and computer support for this research, and the participants at the Q group Fall 1982 Research Seminar for very helpful discussion of these ideas.
The most significant development in futures trading since the introduction of financial futures occurred last year when three exchanges began trading contracts based on stock market indexes. The first index futures contract, based on the Value Line Composite Index, was introduced by the Kansas City Board of Trade on February 24, 1982. In April, the Chicago Mercantile Exchange began trading Standard and Poor's 500 Index futures, followed shortly by the New York Futures Exchange's contract on the NYSE Composite Index. All three contracts have proved to be extremely successful, with open interest and daily trading volume rapidly growing to many thousands of contracts.

The literature on futures markets contains several different theories about how a futures contract should be priced in relation to the cash price for the underlying commodity or security. A puzzle created by the new stock index futures markets is that prices did not seem to be consistent with any of the existing theories. Let us look at a specific example to illustrate the problem.

On September 30, 1982, the New York Stock Exchange composite index stood at 69.18, the dividend yield on the index portfolio was about 6%, money market securities were paying about 10%, and the December NYSE futures contract had exactly three months left to expiration. What should the futures price have been?

One theory holds that in an efficient futures market, speculation should bid the price to the market's expectation of the spot price on the expiration date. To generate a ballpark estimate for the appropriate futures price under this theory, suppose stocks were priced at that time to return an expected 18% annualized.¹ This would consist of the 6% dividend yield plus a 12% capital gain. The latter implies the market's forecast for the
NYSE composite on December 30 should have been about 3% above the September 30 level, or around 71.25. If the December futures price were significantly below 71.25, speculators should buy futures, expecting the price to rise over time, while at a higher price, speculators would sell contracts short.

This argument depends on speculators being willing to take risky positions when they see a profit opportunity. But when the underlying commodity can be stored easily and also sold short, as stocks and most other financial instruments can be, a more powerful pricing relationship based on the possibility of arbitrage between the cash and futures markets exists. Suppose one could buy the NYSE index portfolio at 69.18 and sell futures against it at 71.25. This futures hedge would eliminate the risk from market movements and would lock in a rate of return of 18% for 3 months. But since the position could be financed by borrowing in the money market at 10%, a riskless arbitrage profit of 8% would be created. This cannot be an equilibrium. To avoid arbitrage, futures must be priced so that a hedged stock portfolio yields the same return as other riskless investments, or 10% in this case. Subtracting the 6% dividend yield, the futures price should be about 4% above the spot index on an annual basis. The December future with three months to expiration should therefore have been around 69.87, 1% above the cash index. If the price were higher, buying the hedged portfolio just described would yield an arbitrage profit, while at a futures price much lower than 69.87 the arbitrage trade could be reversed by selling short the index portfolio, investing the proceeds in the money market, and covering the short position by buying index futures.

Because violation of an arbitrage condition implies the existence of unlimited riskless profit, before trading began most people expected stock in-
dex futures to be priced according to the above "carrying cost" relationship. The futures price should be higher than the current level of the index by an amount equal to the interest rate on risk free securities minus the dividend yield on the index portfolio.²

This has not been the case. On September 30, the December NYSE futures contract closed at 68.45, well below the value predicted by either theory, and even at a discount to the spot index itself. Nor was this an unusual situation. Throughout their brief trading history, index futures prices have been systematically lower than the levels predicted by theory.

Table 1 presents statistics on the extent of the discount for NYSE Composite index futures and S&P 500 futures during the 7 months from June 1, 1982 through December 30, 1982. We computed the theoretical futures prices for the 1st position (0-3 months to expiration) and 2nd position (3-6 months to expiration) contracts. Most of the trading activity occurs in the nearest two contracts.

In calculating theoretical futures prices, it was necessary to adjust for the fact that the dividend payout on the index portfolio is not a smooth stream, but varies widely from day to day. Consider taking a long position at time t in the index portfolio of stocks, hedged by a short position in index futures with a maturity date T. For simplicity, assume it is possible to trade the cash portfolio and futures contracts in units of $1 times the index. The initial cost of this hedged portfolio is just 1ₜ, the current level of the index, and the futures price at which it is sold forward is Fₜ. Over the period t + 1 ≤ w ≤ T, assume the portfolio pays daily dividends D(w), which are reinvested at the riskless rate of interest.

At time T this portfolio will be worth
(1) \[ V_T = (\tilde{I}_T + \sum_{w=t+1}^{T} D(w)e^{r(T-w)}) - (\tilde{F}_T - F_t), \]

where the first term is the value of the cash portfolio and the second is the profit or loss on the futures hedge. (We have ignored the effects of marking the futures contract to market, essentially treating it as a forward contract). Since the futures price must be equal to the index at expiration, \( \tilde{F}_T = \tilde{I}_T \), and since the hedged portfolio must just earn the riskless rate, we also have \( V_T = I_t e^{r(T-t)} \). Making these substitutions in (1) and solving for the theoretical value of the initial futures price \( F^e_t \) gives,

(2) \[ F^e_t = I_t e^{r(T-t)} - \sum_{w=t+1}^{T} D(w)e^{r(T-w)} \]

This is the theoretical relationship used in constructing Table 1. We have taken the interest rate on 3 month bank certificates of deposit, converted to a daily rate, as the riskless interest rate \( r \).

For all four contracts over the whole sample, and over both halves of the sample, the actual futures price was on average below the theoretical level. Discounts on NYSE futures were somewhat larger than on S&P 500 futures, and second position contracts were discounted more than the near contracts. However, this latter effect does not hold when the discounts are expressed as annualized rates of return, i.e., in terms of the difference between the rate of return on the hedged portfolio and the riskless interest rate. Expressed in this way, it is clear how large the futures discounts really were, especially in the first half of the sample.

How can this violation of a pricing relationship based on arbitrage be accounted for? A number of possible explanations have been proposed. First, it is clear that the arbitrage argument depends upon a theoretical
world of perfect markets. In real world markets with transactions costs, time delays in executing trades, and restrictions on short sales, the actual arbitrage trades required to enforce the carrying cost relationship are not easily accomplished. This means there should be a band around the theoretical price within which the actual futures price can fluctuate fairly freely without inducing arbitrage trading.

Since the arbitrage is difficult, many people have argued that the futures price is actually reflecting the market's expectations about the future spot index after all. In their view, futures sell at a discount because of investors' pessimistic expectations. This is not a very plausible explanation, however, because it requires market expectations reflected in the futures price to be very different from those embodied in the current level of the index. On September 30, if investors really believed the NYSE composite index would fall to 68.45 by the end of December, they were anticipating an annualized rate of return on stocks over that period of less than 2%. That could not possibly have been an equilibrium situation when money market securities were yielding 10%. Assuming expectations in the stock market and the index futures market should be about the same, we would have expected to find the December futures price somewhere between 69.87 and 71.25, at the upper arbitrage bound, rather than the lower one.

Modest and Sundaresan have suggested that the futures discount may be explained by the fact that short sellers seldom obtain full use of the proceeds of their short sales. Since a short seller must pay the dividends on the shares he has borrowed, if the proceeds of the short sale are not available to be invested in the money market, the short stock-long futures arbitrage becomes profitable only when the futures price is below the current spot index by an amount greater than the dividend yield on the index port-
folio. The December NYSE futures price could have been as low as 68.14 on September 30 before an arbitrageur who could not get the proceeds of his short sale would have been willing to trade.

This point, though correct, only explains why arbitrage might not force the futures discount to disappear once it developed. It gives no insight into why the discount should be there in the first place. Why were futures traders willing to sell contracts on September 30 at a price four percent (16 percent annualized) below the market’s expectation of the spot price in December? Why didn’t investors who held diversified stock portfolios in that date increase their rate of return by more than 8% by selling out and buying a combination of stock index futures and money market securities?4

Cornell and French point to a difference in the way stock and futures returns are taxed.5 For index futures all paper profits are taxed as if they had been realized by the end of the fiscal year. Returns on a stock portfolio, however, are taxed as short term gains if realized within one year, but if the holding period is extended beyond one year, the tax rate drops to 40% of the short term rate. This means that a stock portfolio offers a tax related "timing option" that the futures contract does not possess.

Once an investor buys a portfolio of stocks, if its value goes down within a year, he can sell and deduct the loss at short term rates, while if it goes up he has the option to extend his holding period to take advantage of the long term gains rates. Thus for the taxable investor the equilibrium relationship is for stock index futures to be priced below their theoretical level by an amount equal to the value of the timing option.

It is difficult to judge how much of the futures discount this phenomenon can account for. Valuing the timing option is a complicated problem,
which depends upon unknown parameters such as the average holding period and taxable basis for the market stock portfolio, and the average investor's marginal tax rate. Several factors suggest that the effect may not be large at this time. One of the most telling is that major investors simply do not mention the timing option as influencing their investment decisions in this market. In any case, there exists a huge volume of stock held by tax exempt investors and by taxable investors whose holding periods are already greater than one year. If the effect of the timing option on futures prices were the main cause of the futures discount, these investors should be writing timing options by selling their stock and buying index futures and risk free securities to increase their returns.

There is another set of reasons based on investors' expectations and preferences to explain why they might be willing to sell stock index futures at prices that are "too low" and to buy stocks at prices that are "too high." Investors will have different motives depending on what position they have in the cash market. Take first the case of an investor who owns no stock and believes the market is likely to drop. Because of the problems of selling short actual shares, he will prefer to take a short futures position, even if he must accept a lower price to do so. On the other hand, an investor with bullish expectations can easily buy either stocks or stock index futures. But if he wants to hold a stock portfolio that is different from the index portfolio, he may be willing to pay a premium for actual shares, rather than buy futures even though they appear under-priced.

The bulk of the investor population holds stocks long. Most have carefully selected portfolios of stocks which they believe have positive "alphas" and will outperform the market. For them, futures contracts on
the market portfolio are not a good substitute for the positions they are currently holding. They will not necessarily want to sell their stocks at market prices which they feel are below their true values just because index futures are also underpriced. Only if the discount on futures becomes significantly greater than the expected alpha on their stocks will that be worthwhile. But, if they become pessimistic about the market, they may be willing to sell index futures at a discount to hedge their portfolios because they expect to do better than the market even when it drops.

All of these arguments are what might be called "equilibrium" arguments. That is, they try to explain the existence of a discount on stock index futures in the context of a market in which all investors are behaving optimally, given their expectations and the institutional constraints of the marketplace, such as restrictions on short selling. In combination they give a set of persuasive reasons why there should be downward pressure on the futures price relative to the spot index. Still, it seems unlikely that these arguments are sufficient to explain the full amount of the discounts we have seen in this market. The profit opportunities for institutional investors are simply too large for these prices to represent an equilibrium relationship.

I believe what we are seeing in the stock index futures market is actually a situation of disequilibrium. Not short run disequilibrium in the sense that prices do not adjust to equate supply and demand -- there is no question of that -- but long run disequilibrium in that the actions of investors already in the market create profitable investment opportunities for outside investors who, for one reason or another, are slow to take them up. Under this view, the discount on index futures is largely a transitory phenomenon caused by unfamiliarity with the new markets and institutional
inertia in developing systems to take advantage of the opportunities they present.

Table 1 contains evidence to support this view. For all four contracts the futures discount was smaller in the second half of the sample than in the first. The standard deviations were also smaller, implying that actual futures prices did not deviate as far from their theoretical levels in the later period as they did at the beginning. To get more direct evidence on this question we regressed the discounts on a time trend. The number of days to expiration for the futures contract was also included as an explanatory variable, to adjust for the fact that the discount must go to zero as a contract approaches maturity. The results are reported in Table 2. The positive coefficients on the time trend for all four contracts show that the futures discount has been decreasing over time, after taking account of the effects of the time to expiration and the high degree of serial correlation in the relationship. This is what we expect of a market which is slowly coming into equilibrium.

In conversation, institutional investors reveal a variety of problems they have in beginning to trade stock index futures. First there is a general attitude of caution bred by lack of familiarity with futures markets. In particular, the daily marking to market of futures contracts makes them unlike stocks, or even options which they resemble in other ways, and requires a carefully designed trading program to avoid liquidity problems. There is also uncertainty regarding the legal aspects of trading futures. Portfolio managers worry that they could be accused of imprudence and bad judgment if they were to take an early loss trading index futures. Even mundane matters such as integrating accounting procedures for futures trading into daily operations require time to implement. Finally, the ques-
tions about why stock index futures are priced the way they are do not inspire confidence in the mind of a cautious fund manager that a given futures trading strategy will necessarily work out the way it is supposed to.

Such reluctance to get involved in the stock index futures market cannot persist indefinitely in the face of easily implemented, low risk profit opportunities for institutional investors. My own, purely personal, expectation is that within a year there will be enough large investors who have integrated stock index futures into their overall equity investment programs that futures discounts will have almost entirely disappeared.
FOOTNOTES

1. Ibbotson and Sinquefield in "Historical Returns on Principal Types of Investments," CRSP Working Paper No. 71, report that from 1925 to 1980, stock returns averaged 8.9% above yields on Treasury bills, so 18% is a reasonable figure based on historical relationships.

2. This pricing relationship does not apply to Value Line futures. Because the Value Line index is based on a geometric average of stock returns, it is not possible to construct a stock portfolio whose value behaves like the index. The arbitrage is not possible in this case.


4. The 8% figure comes about because money market securities were paying 4% more than the dividend yield, and buying the index portfolio at the futures price of 68.45 would have added 4% annualized to the capital gain.

TABLE 1
PERCENT DISCOUNT OF ACTUAL STOCK INDEX FUTURES PRICES FROM THEORETICAL VALUES

<table>
<thead>
<tr>
<th></th>
<th>6/1/82-</th>
<th>6/1/82-</th>
<th>9/15/82-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WHOLE SAMPLE</td>
<td>FIRST HALF</td>
<td>SECOND HALF</td>
</tr>
<tr>
<td></td>
<td>12/20/82</td>
<td>9/14/82</td>
<td>12/30/82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ACTUAL DISCOUNT</th>
<th>MEAN</th>
<th>DEVIATION</th>
<th>MEAN</th>
<th>DEVIATION</th>
<th>MEAN</th>
<th>DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYSE 1st POSITION</td>
<td>-0.34</td>
<td>1.13</td>
<td>-0.55</td>
<td>1.32</td>
<td>-0.13</td>
<td>0.85</td>
</tr>
<tr>
<td>NYSE 2nd POSITION</td>
<td>-1.26</td>
<td>1.55</td>
<td>-1.83</td>
<td>1.88</td>
<td>-0.70</td>
<td>1.02</td>
</tr>
<tr>
<td>S&amp;P 500 1st POSITION</td>
<td>-0.18</td>
<td>1.02</td>
<td>-0.26</td>
<td>1.11</td>
<td>-0.11</td>
<td>0.92</td>
</tr>
<tr>
<td>S&amp;P 500 2nd POSITION</td>
<td>-1.13</td>
<td>1.33</td>
<td>-1.53</td>
<td>1.48</td>
<td>-0.73</td>
<td>1.03</td>
</tr>
</tbody>
</table>

| DISCOUNT OF HEDGE       |        |           |        |           |        |           |
| PORTFOLIO RETURN        |        |           |        |           |        |           |
| FROM RISKLESS RATE      |        |           |        |           |        |           |
|                         | MEAN   | DEVIATION | MEAN   | DEVIATION | MEAN   | DEVIATION |
| NYSE 1st POSITION       | -4.79  | 12.61     | -6.72  | 13.73     | -2.77  | 11.07     |
| NYSE 2nd POSITION       | -3.40  | 4.11      | -4.75  | 4.59      | -2.06  | 3.05      |
| S&P 500 1st POSITION    | -3.14  | 12.89     | -6.14  | 15.16     | -0.19  | 9.37      |
| S&P 500 2nd POSITION    | -3.19  | 3.82      | -4.54  | 4.41      | -1.87  | 2.54      |

NOTES:  
The actual percent discount is defined as $100 \left( \frac{F_t - F^e_t}{F^e_t} \right)$, where $F_t$ is the actual futures price at date $t$ and $F^e_t$ is the theoretical value. 
The discount of the hedge portfolio return from the riskless rate is defined as $100\left(\left(\frac{F_t - F^e_t}{I_t + 1}\right)^{365/T} - 1\right)$ where $T$ is the number of days to expiration. To avoid extreme values, annualized rates were not computed for 1st position contracts with 3 or fewer days to expiration.
### TABLE 2

**REGRESSIONS OF PERCENT DISCOUNT ON TIME AND DAYS TO EXPIRATION**

<table>
<thead>
<tr>
<th>FUTURES CONTRACT</th>
<th>CONSTANT</th>
<th>TIME x 10^{-2}</th>
<th>TIME x 10^{-2}</th>
<th>$R^2$</th>
<th>rho</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYSE 1st POSITION</td>
<td>-0.11</td>
<td>0.37</td>
<td>-1.65</td>
<td>0.31</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.93)</td>
<td>(2.35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NYSE 2nd POSITION</td>
<td>-1.78</td>
<td>1.80</td>
<td>-0.90</td>
<td>0.57</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>(1.59)</td>
<td>(3.45)</td>
<td>(1.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 1st POSITION</td>
<td>-0.16</td>
<td>0.51</td>
<td>-1.33</td>
<td>0.35</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(1.49)</td>
<td>(2.21)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P 500 2nd POSITION</td>
<td>-1.36</td>
<td>1.62</td>
<td>-1.08</td>
<td>0.60</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(1.80)</td>
<td>(3.66)</td>
<td>(1.61)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTES:**
- $t$-statistics are in parentheses.
- The dependent variable in all regressions is the percent discount, $100(F_t - F^e_t)/F^e_t$.
- The number of observations was 148 for NYSE 1st position, and 149 for the other contracts.
- All regressions were estimated with a correction for positive serial correlation.