HEDGING WITH STOCK INDEX FUTURES:
THEORY AND APPLICATION IN A NEW MARKET

by

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In early 1982 three exchanges were granted regulatory approval to begin trading futures contracts based on stock market indexes. All three markets have proved to be extremely successful, with open interest and daily trading volume rapidly growing to many thousands of contracts. The immediate success of stock index futures was to be expected. With over $1600 billion of traded equity outstanding and more than 25 million individuals in the U.S. who own shares directly (many more own stock through financial intermediaries) the cash market for stock is far larger than for any existing futures contract.¹

Stock prices are also highly volatile, so that the owners of stocks bear collectively an enormous amount of risk, only a portion of which can be diversified away by holding balanced portfolios. Systematic or market risk remains, due to the fact that all stocks tend to move with the market. In the past, strategies for reducing market risk have involved holding mainly low beta stocks, or selling stocks entirely to invest in fixed income securities. Stock index futures offer the possibility of directly managing the non-diversifiable risk of an equity position regardless of its composition. For example, by selling futures on the Standard and Poor's 500 Index, an investor can hedge against systematic risk by locking in a known return on the market component of his portfolio, without selling any of his stocks.

By allowing the investor to take a separate position in the market component and the nonmarket component of portfolio returns, stock index futures also widen the available range of active trading strategies. Both market timing, i.e., predicting the movement of the market as a whole and adjusting the portfolio's beta accordingly, and stock selection, i.e., trying to increase portfolio returns by buying stocks that are undervalued relative to the market, are made easier by use of index futures.²
Since any stock position involves some systematic risk, the list of potential hedging uses for stock index futures is very long. Institutional users could include "portfolio managers, corporations, new-issue underwriters, stock exchange specialists, block traders, over-the-counter market makers, and traders in stock options." Possible applications which have been proposed range from long term hedges of "indexed" portfolios to overnight hedges of inventories of one or a few stocks by OTC dealers and investment bankers. Since the futures contract can only offset systematic risk, it is essentially a cross-hedge, which will reduce but not eliminate the risk of price fluctuation in a stock position. Theoretical analysis has suggested that the maximum risk reduction is obtained by using a hedge ratio equal to the stock portfolio's beta.

In this paper we examine the performance of the new stock index contracts in hedging risk in various stock portfolios over holding periods of one day to three weeks. A major finding is that basis risk between the futures price and the underlying index, which has been ignored in much of the existing literature on index futures, has an important impact on the behavior of a hedged stock position. Because of day to day fluctuations in the basis, the ability of a futures hedge to reduce risk is strictly limited for positions that are held for only a few days. In some cases, hedging in the manner that is generally recommended leads to an increase in total risk. We also show that the risk minimizing hedge ratio is often well below the portfolio beta.

In the next section we discuss the index futures contracts and the underlying indexes. Section III derives the risk and return characteristics of a hedged portfolio when there is basis risk and uncertainty about dividends. The actual performance of the three futures contracts in hedging
several different stock portfolios during the period June 1982 through December 1982 is analyzed in Section IV. The last section summarizes our findings.

II. The Indexes and the Index Futures Contracts

There are currently three futures contracts trading on broad-based stock indexes, that is, indexes whose behavior is meant to reflect the movement of the whole stock market. The first index futures contract, based on the Value Line Composite Index, was introduced by the Kansas City Board of Trade on February 24, 1982. In April, the Chicago Mercantile Exchange began trading Standard and Poor's 500 Index futures, followed shortly by the New York Futures Exchange's contract on the NYSE Composite Index. All of the contracts are of similar design. There are four contract months a year: March, June, September and December. Each contract is nominally for an amount of cash equal to $500 times the futures price. At recent prices, this translates to about $45,000 for the NYSE, $80,000 for the S&P 500, and $90,000 for the Value Line contracts. In fact, at no point does the dollar value of the contract actually change hands. Only the profits and losses are transferred, which is done on a daily basis as the contracts are marked to market. The only difference that occurs at expiration is that the contracts are marked to the closing value of the spot index on that day instead of to a futures settlement price. After the final transfer of funds, the contracts expire.

The innovation of cash only delivery, necessary in this case because of the impossibility of assembling a deliverable portfolio of stocks which could duplicate any of the indexes on a small scale, is an important feature of these markets. Since the levels of the indexes at the close of each
day's trading are known unambiguously, the cash value of a futures contract can be calculated exactly. There is no problem of trying to determine what instrument represents the "cheapest delivery" against a futures contract, and no possibility of squeezes or other difficulties with delivery.

Both the Standard and Poor's 500 Index and the New York Stock Exchange index measure the market value of a portfolio of stocks, relative to a base period (the weekly average for 1941-3 and December 31, 1965, respectively). The 500 firms in the S&P index represent, with a few exceptions, the 500 largest publicly held U.S. corporations, while the NYSE index contains all stocks traded on the New York Stock Exchange, a total of 1524 companies as of March 31, 1982. Although the coverage is different, nearly all of the S&P 500 stocks are traded on the NYSE and both index portfolios are value-weighted, meaning that each stock enters the index with a weight proportional to its aggregate market value. S&P stocks represent about 75% of the value of the NYSE portfolio, which itself comprises over 85% of the total market value of all traded stock in the U.S. Either of these indexes is a good proxy for the "market" portfolio of modern asset pricing theory.

The Value Line index, on the other hand, does not represent the value of a stock portfolio. Rather, it is designed to measure the return on the average traded stock. It is the broadest of the three indexes, covering about 1700 companies. Nearly 90% are New York Stock Exchange issues, but the index also includes stocks traded on the American Exchange, on regional exchanges and over-the-counter. The stocks covered in the index account for more than 96% of the dollar trading volume of all U.S. equities.

What makes the Value Line index potentially much different from the
others is its method of computation. It is based on an equally weighted geometric average of returns. To compute the index, the geometric average of the price relatives \((P_{t+1}/P_t)\) for all of the included stocks is calculated. This average is then applied to the previous day's index to derive the latest figure.

Equal weighting allows small stocks to have a relatively larger impact on the Value Line than on other indexes. This might make Value Line futures a better contract than the others for hedging stocks of small firms. However, geometric averaging leads to the unfortunate result that it is impossible to construct a portfolio of stocks whose value behaves like the index. In particular, when there is dispersion across stocks, the geometric average of their returns will be less than the return on a portfolio of those stocks. Hence, returns on the Value Line index are likely to be below the other indexes.\(^4\)

III. Hedging with Stock Index Futures

Individual stocks and all stock portfolios, except for those specifically designed to have "zero beta" are subject to some measure of systematic risk. A single futures contract tied to the value of a broad-based market index can be used to hedge the systematic risk due to price fluctuation. There may also be systematic risk associated with fluctuations in the dividend payout, which is not captured in the index, as well as basis risk arising from the fact that the futures price will not normally track the cash index perfectly at all times. In this section, we will develop the theory of hedging stock portfolio risk with index futures. In the next, we will look at how the index futures contracts have actually performed in a variety of hedging applications.
Let us begin by defining the random variable returns on the portfolio to be hedged, $\tilde{R}_p$, the spot index, $\tilde{R}_I$, and the index futures contract, $\tilde{R}_F$, over a holding period of length $\tilde{T}$.

$$\tilde{R}_p = \frac{\tilde{V}_T - V_0 + \tilde{D}_p}{V_0},$$

where $V_0$ and $\tilde{V}_T$ denote the beginning and ending market values for the portfolio. $\tilde{D}_p$ represents the cumulative value as of $T$ of the dividends paid out on the portfolio during the period, assuming reinvestment at the riskless rate of interest from the date of payout until date $T$. The dividend payout is a random variable because the amount of the payout, its timing, and the reinvestment rate may all be uncertain as of time 0.

The return on the index portfolio is

$$\tilde{R}_I = \frac{\tilde{I}_T - I_0 + \tilde{D}_I}{I_0},$$

where variables are defined analogously to (1).

The rate of return on a futures contract is not a well-defined concept, since taking a futures position does not require an initial outlay of capital. For expository convenience we will define the rate of return on a futures contract as the change in the futures price divided by the initial level of the spot index:

$$\tilde{R}_F = \frac{\tilde{F}_T - F_0}{I_0}. $$

This can be expressed in terms of the "basis," which is the futures price
minus the spot price:

\[ R_F = \frac{\tilde{l}_T - l_0 + \tilde{D}_T}{l_0} - \frac{(F_T - \tilde{l}_T) - (F_0 - l_0)}{l_0} - \frac{\tilde{D}_T}{l_0} \]

\[ = \tilde{R}_1 + \frac{\tilde{B}_T - B_0}{l_0} - \frac{\tilde{D}_T}{l_0}, \text{ or} \]

\[ (4) \quad \tilde{R}_F = \tilde{R}_1 + \tilde{b} - \tilde{d}_1. \]

The rate of return on a stock index futures contract is equal to the total return on the underlying index portfolio, plus the change in the basis as a fraction of the initial index, minus the dividend yield on the index.

Now consider the return on a hedged portfolio, in which futures contracts on N "index units" have been sold short against the long portfolio of stocks. An index unit is defined to be $1 times the spot index. All currently traded stock index futures have contract sizes of 500 index units.

\[ \tilde{R}_H = \frac{(\tilde{V}_1 - V_0 + \tilde{D}_P) - N(F_T - F_0)}{V_0} \]

\[ = \tilde{R}_P - \frac{N}{V_0} \left( \frac{F_1 - F_0}{l_0} \right), \text{ or} \]

\[ (5) \quad \tilde{R}_H = \tilde{R}_P - h\tilde{R}_F, \]

where \( h \), the hedge ratio, is the current value of the amount of the index portfolio sold forward as a fraction of the current value of the stock portfolio being hedged. \( h \) determines the overall risk and return characteristics of the hedged position, which are given by

\[ (6a) \quad \tilde{R}_H = \tilde{R}_P - h\tilde{R}_F, \]
\( \sigma_H^2 = \sigma_p^2 + h^2 \sigma_F^2 - 2h \sigma_p \sigma_F. \)

Bars represent expectations, \( \sigma^2 \) with a single subscript denotes a variance and \( \sigma \) with two subscripts a covariance.

To find the risk minimizing hedge ratio, we set the derivative in (7) with respect to \( h \) equal to zero and obtain

\( h^* = \frac{\sigma_p \sigma_F}{\sigma_F^2}. \)

Substituting into (7) yields the variance of returns for the minimum risk hedge,

\( \sigma_{min}^2 = \sigma_p^2 (1 - \rho_{pF}^2) \)

where \( \rho_{pF} \) is the correlation coefficient between the returns on the stock portfolio and the futures contract. Only in the case of perfect correlation can risk be completely eliminated by hedging.

Now let us rewrite (8) and (9) using the definition of \( R_F \) from (4) to bring out the effects of the different components of risk in the futures price.

\( h^* = \frac{\sigma_{pl} + \sigma_{pb} - \sigma_{pd}}{\sigma_l^2 + \sigma_b^2 + \sigma_d^2 + 2\sigma_{lb} - 2\sigma_{ld} - 2\sigma_{bd}}. \)

From (10) it can be seen that increases in any of the following covariances lead to a higher minimum variance hedge ratio (assuming \( h^* \) is positive): covariance between returns on the portfolio and the index (\( \sigma_{pl} \)), covariance
between returns on the portfolio and the basis ($\sigma_{pb}$), covariance between total returns on the index portfolio and its dividend yield ($\sigma_{ld}$), and covariance between the basis and the index portfolio dividend yield ($\sigma_{bd}$). The risk minimizing hedge ratio is lowered by large values for the variances of the market's return, the basis or the market dividend, and by positive covariances between the portfolio's return and the market dividend yield ($\sigma_{pd}$) and between the return on the index and the basis ($\sigma_{1b}$). It is clear that the optimal hedge ratio is not simply a function of the relation between returns on the portfolio and the index. The behavior of both the basis and dividends must be taken into account.

In the special case in which dividends are nonstochastic and the hedge is to be held to expiration of the futures, so that the change in the basis is also nonrandom, most of the terms disappear from (10), leaving

\[ h^* = \frac{\sigma_{pl}}{\sigma_{l}} = \beta_p. \]

The risk minimizing hedge ratio in this case is the portfolio's beta coefficient with respect to the index. Material published by the futures exchanges and earlier academic discussion of the subject have suggested using the portfolio's beta as the appropriate hedge ratio. While dividends tend to be relatively stable over time, so that perhaps no major inaccuracy is introduced by treating them as being nonstochastic, the same is not true of the basis. Early experience shows that it is quite volatile over short periods of time, hence using beta as the hedge ratio is unlikely to be optimal, except when the position is to be held to expiration of the futures.\textsuperscript{6}
IV. Actual Hedging Performance of Stock Index Contracts

In the last section we observed that only a portion of the risk in a typical stock position can be hedged with stock index futures, and that the basis risk of the futures contract must be taken into account in calculating the appropriate hedge ratio. This section examines how effective the three futures contracts currently trading actually were at hedging five well known stock portfolios during the last seven months of 1982. The portfolios we use are the New York Stock Exchange Composite and the Standard and Poor's 500 index portfolios, i.e. the underlying portfolios for the two futures contracts, the American Stock Exchange and NASDAQ Over-the-counter index portfolios, both well diversified, but consisting of stocks in smaller companies than the first two, and finally the Dow Jones 30 Industrials index portfolio. The time period eliminates the very earliest trading in the futures contracts when the markets were just starting up. By the beginning of June 1982 all three markets were well established, with open interest exceeding 4000 contracts in each. The futures contracts we examine are those closest to expiration, since most futures trading occurs in the nearby months. We also tested the second position contracts (those with 3 to 6 months to expiration) and found the same general patterns of results as for the first position. The only substantive difference from the results reported here was that the longer maturity futures were a bit less effective in hedging.

There is a certain amount of ambiguity in defining an "optimal" hedge. Selling index futures against a stock portfolio affects both risk and expected return, according to equations (6) and (7). By altering the hedge ratio, $h$, an investor can achieve any combination of expected return and standard deviation of return along a portfolio possibility curve, as shown in
Figure 1. Choosing the best hedge ratio depends on the individual investor's subjective risk-return tradeoff, although in no case should a ratio greater than the risk minimizing value $h^*$ be picked. As was shown above, in the absence of basis risk and uncertainty about dividends $h^*$ should be equal to the beta coefficient of the stock portfolio with respect to the underlying index for the futures contract. Theoretically, the expected return on a portfolio whose systematic risk is fully hedged will be the riskless rate of interest (or more precisely, the return on a "zero beta" portfolio).

However, during 1982, stock index futures were priced on average well below the levels predicted by theory. This meant that selling futures to hedge systematic risk typically also led to an unattractively low expected return. On some occasions, however, futures were overpriced relative to their theoretical value and a hedged portfolio yielded positive excess returns. Thus a rational hedging strategy involves consideration of both the risk characteristics of a hedged portfolio and also the effect on expected return of relative over- or undervaluation of the futures contract with respect to its theoretical level.

The empirical results presented below focus primarily on risk reduction through hedging and contrast the effects of using a hedge ratio equal to the stock portfolio's beta with the ex post risk minimizing hedge. We consider six different holding periods, 1 day, 2 days, 3 days, 1 week, 2 weeks, and 3 weeks. The sample was subdivided into nonoverlapping periods of the given duration, leading to from 153 observations for overnight hedges down to 10 observations for 3 week (15 trading day) hedges. For ease in comparing results, mean returns and standard deviations are converted to annual rates by multiplying by $260/N$ and $\sqrt{260/N}$, respectively, where $N$ is the number of trading days in the holding period.
FIGURE 1

PORTFOLIO POSSIBILITY CURVE FOR HEDGED STOCK PORTFOLIO

Expected Return

\[ \bar{R}_P \]

\[ \bar{R}_{h^*} \]

minimum risk hedge

\[ h^* = \frac{\sigma_P}{\sigma_F^2} \]

unhedged

\[ h=0 \]

\[ \sigma_{h^*} \]

\[ \sigma_P \]

Standard Deviation
Although the rate of dividend payout on the stock portfolios we are considering varies from day to day, adjusting for this factor was beyond the scope of this paper. We have included dividends in the calculation of portfolio returns, but treated them as if they accrued continuously at a constant rate, computed from the indicated annual dividend rates on the individual stocks as of March 31, 1982. This eliminated one source of variability in the portfolio returns.

Table 1 shows the return and risk characteristics of the fifteen possible hedged positions over six time horizons. Each involves a purchase of one of the stock portfolios and a short sale of one of the futures contracts. The hedge ratio in every case was the beta coefficient of the cash portfolio with respect to the underlying index of the futures contract. Betas were estimated by regression using weekly data from the beginning of 1981 through the end of March 1982.

The second and third columns of the table show the annualized mean returns and standard deviations for the unhedged portfolios. These estimates vary slightly depending on the holding period used to compute them. We have reported figures calculated from weekly portfolio returns. Because of the major bull market during the second half of 1982, mean returns were 45 to 55% for the five portfolios over that period. Unhedged standard deviations were 20 to 24% annualized. For the hedged portfolios, at each duration we report the mean portfolio return and the standard deviation of returns on the hedged portfolio as a fraction of the unhedged value.

Consider first the two index portfolios. Naturally the beta of the NYSE composite or the S&P 500 with respect to itself is 1.00 and it will be very close to 1.00 with respect to the other index as well. Both index portfolios have slightly lower betas with respect to the Value Line index.
<table>
<thead>
<tr>
<th>Cash Portfolio</th>
<th>Unhedged</th>
<th>Futures Contract</th>
<th>Beta</th>
<th>1 day</th>
<th>2 days</th>
<th>3 days</th>
<th>1 week</th>
<th>2 weeks</th>
<th>3 weeks</th>
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<td>48.6</td>
<td>23.2</td>
<td>1.00</td>
<td>2.8</td>
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<td>-.7</td>
<td>.59</td>
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<td>S&amp;P 500</td>
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<td>13.9</td>
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<td>1.8</td>
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<td>S&amp;P 500</td>
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<td>.99</td>
<td>3.5</td>
<td>.72</td>
<td>.2</td>
<td>.50</td>
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<td>.39</td>
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<td>3.4</td>
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<td>AMEX</td>
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<td>-12.9</td>
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**NOTES:** Returns are in percent at annualized rates. Futures contracts were those closest to expiration. \( \sigma_u \) and \( \sigma_H \) are the standard deviation of returns for unhedged and hedged portfolios, respectively.
These portfolios should, in principle, allow the most effective hedges, since they contain only systematic risk. Even so, very short duration hedges do not perform very well. Hedging the NYSE composite portfolio overnight with NYSE futures reduced its risk by only 15% from the unhedged value. However, mean returns dropped from 48.6% to 2.8%. Surprisingly the Value line futures contract eliminated a much larger proportion of the risk in both index portfolios for very short durations than did their own futures contracts.

There are several features of interest in these results. Short duration hedges were not very effective, but risk reduction improved as the holding period extended to several weeks. Still hedge effectiveness seemed to be limited to reducing the standard deviation to 30 to 40 percent of the unhedged value. However, hedging caused a substantial drop in mean return even at the shortest duration. An interesting point is that the futures contracts showed no clear advantage vis-a-vis one another in hedging their own index portfolios. The Value Line contract provided as good or better risk reduction as the others for hedges of this duration, although because of its different construction, it is unlikely that it could track those indexes as well over long horizons.

One common feature of the results in this table and the next is that three week hedges seem to perform less well than two week hedges. We can think of no reason that this should be so. Indeed, since the basis cannot become arbitrarily large, basis risk should not increase directly with the length of the holding period as market risk does. Basis risk as a fraction of total risk ought to drop as the holding period grows longer, leading to better hedges. We are therefore inclined to explain the pattern as at least partly due to sampling error caused by the small number of two and
three week periods in our sample.

The next two stock portfolios are the American Stock Exchange composite index and the NASDAQ index of over-the-counter issues. Both are well diversified but are made up of small stocks which may not behave the same way as the large NYSE companies. Futures hedges for these portfolios using beta as the hedge ratio are not effective. This is especially true for short durations, where total risk is actually larger than for an unhedged portfolio. Even at the longer horizons, risk reduction amounts to a maximum of 30 to 40 percent. Further, in the case of the Amex portfolio, using the beta of 1.3 to 1.4 as a hedge ratio reduces the mean portfolio return from 46.1% to between -15 and -20%. As was expected, the larger weight on small stock returns in the Value Line index allowed it to hedge these portfolios more effectively than the other futures contracts, but residual risk was still substantial.

The last stock portfolio to consider is the Dow Jones portfolio of 30 industrial stocks. This turned out to be the portfolio for which hedging was most effective. In fact, it was more effective than for the underlying index portfolios for the NYSE and S&P 500 futures contracts. Approximately half of the risk was eliminated in an overnight hedge and risk reduction approached 80 percent for longer durations. Also, because the Dow outperformed the other indexes during this time period and hedge ratios of less than 1.0 were used, the mean returns on the hedged positions were relatively high. Finally, in contrast to the other portfolios, Value Line futures were markedly less effective than the other contracts at hedging the large firm stocks contained in the Dow.

One factor which may partly account for the apparent failure of index futures to track these portfolios over short time periods is infrequent
trading of some stocks. For less frequently traded issues, the "closing" price that is incorporated into the closing value of the index may actually represent a trade that took place hours before the close of the exchange. The equilibrium price of that stock may change before the market closes, but it will not be recorded in the index unless there is a trade. Thus the day's close may not actually be the true level of the index, while the futures contract, which trades continuously, might reflect this true index better. If the effect of infrequent trading is significant, the futures contracts could actually provide a better hedge of a portfolio's actual value than it seems, based on recorded prices. Fortunately, the effect of an hour or two difference in timing should be attenuated for hedges of longer duration.

Table 1 shows that the impact of basis risk in the futures contracts is significant. Even hedges of the underlying index portfolios themselves contained a substantial amount of residual returns variation. In Table 2 we report the risk and return characteristics of hedged positions in which the actual (ex post) risk minimizing hedge ratio was used. This $h^*$, from equation (3), is simply the "beta" of the cash portfolio with respect to the futures contract itself, instead of its underlying index. $h^*$ is equal to the covariance between return on the portfolio and returns on the futures contracts (as that term was defined in (3)) divided by the variance of futures returns, and is easily calculated from the sample data by regressing realized portfolio returns on futures returns. This is the constant hedge ratio for which the standard deviation of returns on a hedged position was a minimum over this time period. It represents the limit of risk reduction that was achievable with these contracts.

Columns 2 and 4 of the table show the standard deviation of the un-
### TABLE 2

HEDGE PERFORMANCE OF STOCK INDEX FUTURES OVER VARIOUS HORIZONS, USING EX POST RISK MINIMIZING HEDGE RATIO

<table>
<thead>
<tr>
<th>Cash Portfolio</th>
<th>Unhedged σ</th>
<th>Futures Contract</th>
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**NOTES:**

Returns are in annualized rates.
Future contracts were those nearest to expiration.
The risk-minimizing hedge ratio was computed as $\text{cov}[R_p, R_T]/\text{var}[R_F]$ based on data from the sample period.

$\sigma_u$ and $\sigma_{H}$ are the standard deviations of returns for unhedged and hedged portfolios, respectively.
hedged stock portfolio and its beta with respect to the underlying index of each futures contract. The remaining columns report the risk minimizing $h^*$ and the standard deviation of hedge returns as a fraction of the unhedged value for each duration.

There is no need to discuss the results individually, as all exhibit the same basic pattern. In nearly every case, the risk minimizing hedge ratio is well below the historical beta. It is smallest for overnight hedges and rises for longer duration, as the variance of systematic price movements increases relative to basis risk. Most portfolios show a substantial improvement in hedging effectiveness when the risk minimizing hedge ratio is employed. For example, at the 1 day horizon a hedge ratio of .56 instead of 1.00 lowered the standard deviation of the NYSE composite portfolio hedged by selling NYSE futures from .85 to .50 of the unhedged value. Another point, which is not visible in this table, is that by using a lower hedge ratio, a larger fraction of the positive returns earned by the stock portfolios remains in the hedged position. Thus these positions exhibited both lower risk and higher returns than hedges based on beta. (Of course, had the market been down over this period, the relationship between mean returns would have been reversed.)

The most extreme improvement in hedge performance was found for the Amex and NASDAQ portfolios. Using the risk minimizing hedge ratio (less than one third of the portfolio beta) turned a substantial increase in risk for a one day holding period into a modest risk reduction. The only portfolio which did not show much improvement was the Dow. Even for overnight hedges $h^*$ turned out to be very close to beta.

Figures 2a through 2e display some of these results in graphical form. We have selected the S&P 500 futures contract and plotted the hedge ratio
and remaining portfolio risk for both beta hedges and minimum risk hedges at all horizons for the five stock portfolios. These graphs make it easy to see the differences in risk reduction between the two strategies. In Figures 3a through 3e we have plotted the data from Table 2 in order to highlight the differences among the three futures contracts and also to facilitate interpolation to find risk minimizing hedge ratios for other hedge durations.

V. Summary and Conclusion

In this paper we have analyzed the ability of the new stock index futures contracts to hedge risk in stock portfolios. In interpreting the results it is necessary to bear in mind that the time period we have examined is relatively short - allowing as few as 10 nonoverlapping observations for three week hedges, and the markets are new. As we observe longer periods of time and as the investment community develops experience trading these instruments, the market's behavior may change.

In Section III we showed that hedging a stock portfolio with index futures will typically involve substantial basis risk arising from three sources. First, fluctuations in the value of a given stock portfolio are only imperfectly correlated with changes in a market index. Unsystematic price changes can not be hedged. Second, when the hedge is not held until the futures contract expires, additional basis risk arises due to changes in the price difference between the future and its underlying index over time. And third, the futures contract can only hedge against the risk of stock price movements. Risk arising from uncertain dividend payouts will remain. All three factors will play a role in determining hedging effectiveness and optimal hedge ratios.

In examining actual hedge performance of these contracts, several gen-
eral conclusions emerged. Because of basis risk, in general the hedge ratio which minimized the total risk for a hedged portfolio was well below its beta. This minimum risk hedge ratio tended to increase, and the unhedg-
eable risk to decrease, with longer hedge duration. We also found that hedging was more effective for portfolios of large stocks (the NYSE compo-
site, the S&P 500, and the Dow Jones Industrials) than small stock port-
folios (the American Stock Exchange and the NASDAQ composite portfolios). Hedged portfolio risk could be reduced to 20 to 30% of the unhedged value for the former, and only 40 to 60% for the latter. There also appeared to be some advantage to hedging large stocks with NYSE or S&P 500 futures and small stocks with the Value Line contract.
FOOTNOTES

1. By contrast, in 1981 the value of the entire U.S. annual harvest of wheat was under $10 billion (Source: The Statistical Abstract of the United States).


4. To see the effect of the different computation methods, consider the following example, using only two stocks. Firm A has 100 shares outstanding and Firm B has 200, day 1 prices are $100 for both stocks, and let $S_0$, $E_0$, and $V_0$ denote the initial levels of a value weighted S&P type index, an equally weight arithmetic index, and a Value Line type index respectively. Now suppose stock A's price drops to 50 while stock B goes up to 200. The new values for the three indexes are given by

\[
S_1 = S_0 \left( \frac{100}{100} \cdot \frac{50}{100} + \frac{100}{100} \cdot \frac{200}{100} \right) = 1.55 \, S_0 \\
E_1 = E_0 \left( \frac{1.50}{100} + \frac{200}{100} \right) = 1.25 \, E_0 \\
V_1 = V_0 = \sqrt{\frac{50}{100} \cdot \frac{200}{100}} = 1.0 \, V_0
\]

The value weighted index rises by 50%, the equally weighted index assigns the same weight to the small and large firm and rises 25%, while the geometric index shows no change.
This example is more extreme than is encountered in practice. Since all stocks tend to move together, the Value Line index actually tracks market movements very much like the other two over short time intervals.

5. The initial margin deposit on a futures contract does not count as an investment of capital since it can be posted in the form of interest bearing Treasury bills.

6. See, for example, Figlewski [1983].

7. See Figlewski [1983].

8. Hanson, et al. [1982] discuss this problem with respect to its impact on the behavior of the futures contract.
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Figlewski, S. "Why are the Prices for Stock Index Futures so Low?" Financial Analysts Journal, forthcoming 1983.


Kansas City Board of Trade. Value Line Composite Average, The Index behind the Futures, 1982.
Modest, D. and Sundaresan, M. "The Relationship between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence," 
FIGURE 2a

HEDGING PERFORMANCE OF S&P 500 FUTURES ON NYSE COMPOSITE INDEX PORTFOLIO

Legend

△ BETA
× RISK OF BETA HEDGING
□ MIN RISK HEDGE RATIO
⊗ MINIMUM RISK
HEDGING PERFORMANCE OF S&P 500 FUTURES ON S&P 500 INDEX PORTFOLIO

HEDGE DURATION

Legend

Δ BETA
× RISK OF BETA HEDGE
□ MIN RISK HEDGE RATIO
☒ MINIMUM RISK
FIGURE 2c

HEDGING PERFORMANCE OF S&P 500 FUTURES ON AMERICAN STOCK EXCHANGE COMPOSITE PORTFOLIO

Legend

△ BETA
× RISK OF BETA HEDGE
□ MIN RISK HEDGE RATIO
⊗ MINIMUM RISK

HEDGE DURATION
FIGURE 2d

HEDGING PERFORMANCE OF S&P 500 FUTURES ON NASDAQ OTC PORTFOLIO
FIGURE 2e
HEDGING PERFORMANCE OF S&P 500 FUTURES ON DOW JONES 30 INDUSTRIALS

Legend
△ BETA
× RISK OF BETA HEDGE
□ MIN RISK HEDGE RATIO
★ MINIMUM RISK
FIGURE 3a
MINIMUM RISK HEDGE RATIOS AND RISK LEVELS FOR NYSE COMPOSITE INDEX PORTFOLIO

Legend
△ NYSE HEDGE RATIO
× S&P 500 HEDGE RATIO
□ VALUE LINE HEDGE RATIO
● NYSE HEDGE RISK
● S&P 500 HEDGE RISK
★ VALUE LINE HEDGE RISK
Minimum risk hedge ratios and risk levels for S&P 500 index portfolio.
FIGURE 3c

MINIMUM RISK HEDGE RATIOS AND RISK LEVELS FOR AMERICAN EXCHANGE COMPOSITE PORTFOLIO

Legend

△ NYSE HEDGE RATIO
× S&P 500 HEDGE RATIO
□ VALUE LINE HEDGE RATIO
● NYSE HEDGE RISK
Ⅲ S&P 500 HEDGE RISK
☆ VALUE LINE HEDGE RISK
FIGURE 3d

MINIMUM RISK HEDGE RATIOS AND RISK LEVELS FOR NASDAQ OTC INDEX PORTFOLIO

Legend

△ NYSE HEDGE RATIO
× S&P 500 HEDGE RATIO
□ VALUE LINE HEDGE RATIO
■ NYSE HEDGE RISK
□ S&P 500 HEDGE RISK
× VALUE LINE HEDGE RISK
FIGURE 3c
MINIMUM RISK HEDGE RATIOS AND RISK LEVELS FOR DOW JONES 30 INDUSTRIALS

Legend
△ NYSE HEDGE RATIO
× S&P 500 HEDGE RATIO
□ VALUE LINE HEDGE RATIO
■ NYSE HEDGE RISK
× S&P 500 HEDGE RISK
♦ VALUE LINE HEDGE RISK