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PRICING DEPOSIT INSURANCE:
THE EFFECTS OF MISMEASUREMENT

BY

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Pricing Deposit Insurance: The Effects of Mismeasurement

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I have received useful comments from Jack Beebe, Elizabeth Chen, Eitan Gurel, Greg Hawkins, and James Hoag. Valuable research assistance was provided by Joseph Bachar and Tom Iben. My debt to Robert Merton's pioneering work on deposit insurance will be apparent to the reader. I am grateful to the Federal Reserve Bank of San Francisco for financial support as a Visiting Scholar during the period in which this research was carried out. This paper was presented at a seminar at the Federal Reserve Bank of San Francisco and at the Finance seminar at the University of California, Berkeley. I thank the participants for their comments. I alone am responsible for any errors and the opinions expressed are my own and not necessarily representative of the views of the Federal Reserve System. If any errors remain, they might be attributable to the well-known difficulty in teaching new tricks to older dogs.
Pricing Deposit Insurance: The Effects of MisMeasurement

Abstract

Building on Merton’s 1978 paper on deposit insurance with surveillance costs, a model for analyzing the deposit insurance premium is derived. In this model, insurance premia are assessed at each audit date and all auditing costs are paid by the insured bank. Auditing costs that increase as the bank approaches insolvency and that do so more rapidly for a bank with riskier assets are considered. This deposit insurance structure leads to minimum auditing costs and positive bank capital in the competitive equilibrium. However, the insurance premium depends on two parameters that are subject to measurement error, the riskiness of the bank’s assets and the value of those assets. Modest errors in measuring these parameters will overwhelm the desirable properties of risk-adjusted deposit insurance premia. The analysis in the paper shows that the effect of a proportional error in measuring asset value is substantially greater than the effect of the same proportional error in measuring asset risk.
1. Introduction

The premium on fair deposit insurance must cover the insurer’s liability. Either the insurer’s loss distribution must be controlled to correspond to the premium or the premium set to be consistent with the liability. Many policy analysts are critical of the flat rate premiums on federal deposit insurance because different depository institutions impose different potential losses on the insurers.¹

Congress mandated studies of the deposit insurance system in the Garn-St Germain Depository Institutions Act of 1982. These studies were to include “the feasibility of basing deposit insurance premiums on the risk imposed by either the insured institution or the category or size of the depository institution rather than the present flat rate system.”

The Federal Deposit Insurance Corporation (FDIC), the Federal Home Loan Bank Board (FHLBB), and the National Credit Union Administration (NCUA) sent reports (FDIC [1983], FHLBB [1983], NCUA [1983]) to Congress in Spring 1983. The FHLBB and FDIC reports support some form of risk-related premiums while the NCUA report opposes them.² The difficulty in accurately measuring the insurer’s liability is noted in all three reports.

To my knowledge, no one has analyzed this measurement problem in a formal model of deposit insurance pricing. The purpose of this paper is to provide such an analysis. Building on Merton’s [1978] paper, a model for evaluating the deposit insurance premium is derived. In this model, insurance premia are assessed at each audit date and auditing costs are paid by the audited bank. Auditing costs that increase as the bank approaches insolvency and that do so more rapidly for a bank with riskier assets are considered. This deposit insurance structure leads to minimum auditing costs and positive bank capital in the competitive equilibrium. However, the insurance premium depends on two parameters that are subject to measurement error, the volatility rate (i.e. riskiness) of the bank’s assets and the value of those assets. Modest errors in measuring the parameters will overwhelm the desirable properties of risk-adjusted insurance. The analysis in this paper shows that the effect of a proportional error in measuring asset values is substantially greater than the same proportional error in measuring volatility.

¹See Mayer [1965].
²For a thoughtful discussion of the case against risk-related premiums, see Horvitz [1983]. Horvitz’ conclusions, in many ways, anticipate the results reported in this paper.
2. Perpetual Deposit Insurance with Insurer-Paid Audit Costs

Option-theoretic analysis of deposit insurance was pioneered in two papers by Merton [1977, 1978]. His 1978 model of deposit insurance with surveillance costs provides the starting point for the analysis in this paper. The assumptions underlying his model of deposit insurance are:

1. A bank is defined as an institution that holds financial assets and finances their purchase by equity and by issuing deposits that are fully insured.
2. The banking industry is competitive with constant returns to scale.
3. Deposits are insured by an agency of the government.
4. Trading in securities takes place continuously in time.
5. Securities are priced to satisfy the security market-line equation in the continuous-time version of the capital asset pricing model.\(^3\)
6. An exchange market exists where some investors and institutions (including the insured banks and the insurer) can borrow and lend at the same rate of interest, \(r\), which is assumed to be constant through time.
7. Some investors face a transaction cost for lending in the exchange market so, if the return on deposits, \(R\), is greater than \(r\) minus the transaction cost per unit time for a given investor, that investor will hold deposits. If not, the investor will lend directly through the exchange market.
8. The dynamics for aggregate deposits for a given bank, \(D\), are non-stochastic and described by \(\frac{dD}{dt} = gD\) where the percentage growth in deposits, \(g\), is a known constant. The dynamics for the value of a bank's assets, \(V\), can be described by a diffusion-type stochastic process with stochastic differential equation:

\[
dV = [\alpha V - RD]dt + \sigma Vdz
= [\alpha V - (R - g)D]dt + \sigma Vdz, \quad V > 0
= 0, \quad V = 0
\]

where \(\alpha\) is the instantaneous expected rate of return on the assets per unit time; \(\sigma^2\) is the instantaneous variance of the return per unit time and is assumed constant; \(dz\) is a standard Gauss-Wiener process.

9. The insurer charges the bank a one-time premium to insure all the deposits of the bank in perpetuity provided the bank is solvent. The solvency of the bank is determined by audit. If the bank is solvent when audited it continues operations. If it is found to be insolvent, the insurer liquidates the bank and pays off depositors. The insurer uses a random-time audit procedure where the event of an audit is Poisson-distributed with characteristic parameter \(\lambda\). Successive audit times are independently and identically distributed. There is a cost per audit to the insurer that is assumed to be a first-degree homogeneous function \(C(V, D)\).

\(^3\)The assumptions used here are consistent with, but not identical to, Merton’s.
\(^4\) See Merton [1973].
Using standard option pricing methods\(^5\) and adding the assumptions that the growth rate in aggregate deposits equals the total return on deposits (i.e. \(g = R\)) and that the audit costs per dollar of deposit are constant (i.e. \(C(V, D) = KD\)), Merton shows that the equilibrium premium per dollar of deposits charged by an insurer operating on a no subsidy-no excess profits basis is

\[
p^*_1(x) = 1 - \frac{(k' - 1)}{(\delta^* + k')} x^{-\delta^*}, \quad x \geq 1
\]

where \(p^*_1\) is the equilibrium value of the insurer’s liability per dollar of deposits for \(x \geq 1\), \(x \equiv \frac{V}{D}\), \(\delta^* \equiv \frac{2\mu^*}{\sigma^2}\)

\[
= \frac{2\lambda K}{\sigma^2}, \quad k^* \equiv \frac{1}{2}(1 - \delta^* + ((1 + \delta^*2 + \gamma)^{1/2}) \quad \text{and} \quad \gamma \equiv \frac{8\lambda}{\sigma^2}.
\]

The equilibrium value of equity per dollar of deposits for this case is

\[
f^*_1(x) = x - \frac{(k' - 1)}{(\delta^* + k')} x^{-\delta^*}, \quad x \geq 1.
\]

where \(f^*_1\) is the equilibrium value of equity per dollar of deposits when \(x \geq 1\). The equilibrium deposit rate is

\(R^* = r - \mu^*\) where \(\mu^* = \lambda K\) so the bank pays less than the riskless rate for its deposits.

Because the insurer pays all audit costs and receives compensation for these costs as part of the deposit insurance premium, \(p^*_1\) can be thought of as having two components: the present value of the perpetual deposit guarantee plus the present value of the future audit costs.

3. Single-Audit Deposit Insurance with Pay-as-you-go Audit Costs

The audit cost component of Merton’s deposit insurance premium causes unusual comparative statics for the parameters of that model: an increase in the asset value-to-deposit ratio results in an increase in the deposit insurance premium and an increase in volatility results in a decrease in equity value. For the purposes of this paper, it is useful to put audit costs on a pay-as-you-go basis in which the insured bank pays the audit costs at the time of the audit except when the bank is found to be insolvent. In the latter case, the insurer absorbs the audit cost along with the difference between total asset value and aggregate depositor claims.

The premium given by equation (1) is for a perpetual deposit guarantee. For audit costs of less than 50 basis points per dollar of deposits and asset return variances of .01 or greater, the deposit insurance

\(^5\) See Black and Scholes [1973] and Merton [1976].
premium for a perpetual guarantee is rather insensitive to the current asset value-to-deposit ratio (the elasticity is less than 1). Again the comparative statics of parameter mismeasurement are more easily interpreted for a deposit guarantee that covers the period between the current premium assessment and the date at which the next audit occurs.

Finally, it seems reasonable to assume that surveillance costs are higher for banks that are near insolvency than they are for more solvent banks and higher for a bank with riskier assets. These assumptions are consistent with the argument made by Black, Miller, and Posner [1978] (p.386) "Because the incentives for the bank to hurt the government are greater when the bank's capital is low relative to the risk of its assets, the cost of the appropriate level of supervision of such a bank will be high..." To capture these effects, the audit cost per dollar of deposits is assumed to be a decreasing, non-negative function of the asset value-to-deposit ratio and the elasticity of the audit cost function with respect to \( x \) is assumed to depend (inversely) on the volatility of the bank's assets. The following Cobb-Douglas function for total audit costs is tractable and has reasonable properties:

\[
C(V, D) = K_0 V^{-\beta} D^{1-\beta}, \quad x \geq 1
\]

\[
= K_0, \quad x < 1
\]

where \( \beta \equiv \frac{2K_1}{\sigma^2} \) and \( K_1 \geq 0 \). The resulting function for audit costs per dollar of deposits is

\[
c(x) = K_0 x^{-\beta}, \quad x \geq 1
\]

\[
= K_0, \quad x \leq 1.
\]

---

6There does not appear to be reliable evidence on the total cost of bank examinations; let alone information that would allow one to relate these costs to the condition of the bank at the time of examination. However, the FDIC report to Congress alludes to such cost increases and states "...the additional [supervisory] costs incurred when a bank receives a high risk-rating should be borne by that bank" (FDIC [1983], p. II-21). A rough estimate of the direct cost to the FDIC of its examinations in 1980 is that those costs were between $0.0002 and $0.0003 per dollar of deposits in the examined banks. However, as Benston [1973] and others have observed, banks also incur examinations costs. As far as I can determine, there are no published estimates of these costs. Note, however, that the examination costs incurred by the bank that is being examined mean that some audit costs are currently on a pay-as-you-go basis.

7This function implies an audit cost of \( K_0 \) for an insolvent bank and an audit cost that approaches zero as \( x \to \infty \). With some additional notational complexity, the audit cost function could be defined so that the minimum audit cost would be reached at a finite asset value-to-deposit ratio.
3.1. The Cost of Deposit Insurance

For pay-as-you-go audit costs as given by equation (3) and a deposit guarantee that extends only until the next audit, the insurer's liability per dollar of deposits \( p \) must satisfy the differential equations (4a) and (4b)\(^8\)

\[
\begin{align*}
\frac{1}{2} \sigma^2 x^2 p_1'' + \mu x p_1' - (\mu + \lambda) p_1 &= 0, \quad x \geq 1 \\
\frac{1}{2} \sigma^2 x^2 p_2'' + \mu x p_2' - (\mu + \lambda) p_2 + \lambda \left[ K_0 + 1 - x \right] &= 0, \quad x \leq 1
\end{align*}
\]

where \( \mu \equiv r - g \equiv r - R \), \( p(x) = p_1 \) when \( x \geq 1 \) and \( p(x) = p_2 \) when \( x \leq 1 \), and primes denote derivatives.

The boundary conditions for these differential equations are

\[
\begin{align*}
p_1(1) &= p_2(1) \quad \text{(4c)} \\
p_1'(1) &= p_2'(1) \quad \text{(4d)} \\
p_2(0) &= \frac{\lambda + \lambda K_0}{(\lambda + \mu)} \quad \text{(4e)} \\
\lim_{x \to \infty} p_1 &\text{ is bounded.} \quad \text{(4f)}
\end{align*}
\]

The solution for \( p_1 \) using equations (4a) and (4b) and the boundary conditions (4c-f) is \(^9\)

\[
p_1(x) = \left[ 1 + \frac{(\lambda K_0 - \mu)}{(\lambda + \mu)} k \right] x^k, \quad x \geq 1
\]

where \( \delta \equiv \frac{2\mu}{\sigma^2}, \quad k \equiv \frac{1}{2} \left( 1 - \delta + (1 + \delta)^2 + \chi \right) > 1, \quad \ell \equiv 1 - \delta - k < 1, \) and \( \chi \equiv \frac{3\lambda}{\sigma^2}. \)

3.2. The Value of the Bank's Equity

Similarly, the value of the bank’s equity per dollar of deposits must satisfy differential equations (6a) and (6b)

\[
\begin{align*}
\frac{1}{2} \sigma^2 x^2 f_1'' + \mu x f_1' - \mu f_1 - \lambda K_0 x^{-\delta} - \lambda p_1 &= 0, \quad x \geq 1 \\
\frac{1}{2} \sigma^2 x^2 f_2'' + \mu x f_2' - (\mu + \lambda) f_2 &= 0, \quad x \leq 1
\end{align*}
\]

\(^8\) See the Appendix to this paper.

\(^9\) These boundary conditions are the same as those given in Merton [1978].
The boundary conditions for these differential equations are\textsuperscript{10}

\begin{align}
  f_1(1) &= f_2(1) \\
  f_1'(1) &= f_2'(1) \\
  f_2(0) &= 0 \\
  \lim_{x \to \infty} \frac{f_1'}{x} &= 1. 
\end{align}

The solution to (6) for $f_1$ is

\begin{equation}
  f_1(x) = \left[ \frac{\lambda K_0 (\beta + k)}{(\mu - K_1)(\beta + 1)} - \frac{(\lambda + \lambda K_0)}{(\lambda + \mu)} \right] k \frac{x^{-\delta}}{\delta + k} + \left[ \frac{\nu K_0}{(\mu - K_1)(\beta + 1)} \right] x^{-\beta} + p_1(x), \quad x \geq 1 
\end{equation}

3.3. The Competitive Equilibrium

Assume that the insurer charges a fair premium as defined by equation (5). With no barriers to entry, the following no profit condition is necessary for an interior and sustaining equilibrium

\begin{equation}
  \pi(x) = f_1(x) - p_1(x) + 1 - x = 0
\end{equation}

where $\pi$ is the excess profit per dollar of deposits from investing in banking.\textsuperscript{11} Substituting $p_1$ and $f_1$ from equations (5) and (7) into the profit function, we obtain the equilibrium condition

\begin{equation}
  \pi(x) = 1 + \left[ \frac{\lambda K_0 (\beta + k)}{(\mu - K_1)(\beta + 1)} - \frac{(\lambda + \lambda K_0)}{(\lambda + \mu)} \right] k \frac{x^{-\delta}}{\delta + k} - \frac{\lambda K_0}{(\mu - K_1)(\beta + 1)} x^{-\beta} = 0. \tag{9}
\end{equation}

\textsuperscript{10} Although with pay-as-you-go audit costs the bank has a liability for future audit costs as well as a liability for the deposits, the value of these liabilities, measured in units of per dollars of deposits, is bounded. Therefore, condition (6f) follows.

\textsuperscript{11} Equation (8) requires the value of the bank's equity to cover the deposit premium plus the net cash flow (the cost of the assets less the deposits available for investment) needed to purchase the bank's assets. See Merton [1978] p.448.
3.3.1. Constant Audit Costs

For the constant audit cost case (i.e. \( K_1 = 0 \) so that \( \beta = 0 \)), equation (9) simplifies to

\[
\pi(x) = 1 - \frac{\lambda K_0}{\mu} + \frac{\lambda (\lambda K_0 - \mu)}{\mu (\lambda + \mu)} \left( \frac{k}{(\delta + k)} \right) x^{-\delta} = 0.
\]

A necessary and sufficient condition for this equation to hold for all \( x \) is \( \mu^* = \lambda K_0 \). The equilibrium value of the deposit spread with constant audit costs, pay-as-you-go auditing, and a single-audit guarantee is identical to the equilibrium spread determined in Merton’s model. The bank pays less than the riskless rate for its deposits by an amount sufficient to cover the expected audit cost per dollar of deposits.

Substituting \( \mu^* \) into equation (5) gives the equilibrium deposit insurance premium for the alternative model with constant audit costs.

\[
p_1^*(x) = \left( \frac{1}{k^* - l^*} \right) x^{l^*}, \quad x \geq 1
\]

where \( \delta^* \), \( k^* \), and \( l^* \) are the equilibrium values of \( \delta \), \( k \), and \( l \) respectively.

The implicit audit costs in the deposit insurance premium with insurer-paid audit costs induce the unusual comparative static results of that model. As noted earlier, the insurance premium given by equation (1) is increasing in the asset value-to-deposit ratio. Furthermore, from equation (2), it can be shown that a decrease in the volatility rate results in an increase in equity value. Merton notes that the latter result "...provides an attractive 'stabilizing' side effect to this structure..." Having paid the insurance premium given by equation (1), a bank that altered the volatility rate for its assets would want to reduce that rate.

Comparative statics analysis of equation (10) shows that with pay-as-you-go auditing and constant audit costs, a decrease in the volatility rate results in a decrease in equity value. It is the fact that the audit costs are sunk in Merton’s model, not the favorable deposit spread per se, that induces the stabilizing effect.
in that model. Furthermore, in the pay-as-you-go model, the insurance premium decreases with increases in the asset value-to-deposit ratio.\footnote{These comparative statics hold for pay-as-you-go auditing with a perpetual guarantee as well as for the single-audit guarantee. The equilibrium deposit insurance premium for the constant audit cost, perpetual guarantee case is 
\[ p_1^*(x) = \frac{1}{(x + k^*)^{-\delta}}. \]}

3.3.2. Audit Costs that Decrease with \( x \)

Turning to the case of audit costs that decrease with \( x \) (i.e. \( \beta_1 > 0 \) and \( \beta_2 > 0 \)), it is clear from equation (9) that the zero-profit deposit spread depends on the asset value-to-deposit ratio. As one would expect, this spread decreases as the asset value-to-deposit ratio increases. The following ratio of partial derivatives must be evaluated to demonstrate this analytically

\[
\frac{d\delta^-}{dx} = \frac{\frac{\partial \pi^-}{\partial x}}{\frac{\partial \pi^-}{\partial \delta}}.
\]

It can be shown that \( \frac{\partial \pi^-}{\partial x} \) is positive for all \( x \) and that \( \frac{\partial \pi^-}{\partial \delta} \) is positive at \( x = 1 \). The zero-profit deposit spread decreases as \( x \) increases from \( x = 1 \). Numerical evaluation of the model using a variety of parameter values that are consistent with zero-profit shows that this result holds for larger values of \( x \) as well.

As noted earlier, the audit cost function given by equation (3) (with \( K_1 > 0 \)) asymptotically approaches zero as \( x \) increases. While useful for notational simplicity, it may be more realistic to consider a non-zero minimum audit cost in the numerical examples that follow. Results are reported for the following decreasing cost function

\[
c(x) = 0.001 + 0.01 x^{-\frac{25}{\sigma^2}}
\]

but similar results were obtained using other values for the cost function parameters. I have also considered a constant audit cost of 0.001 (i.e. 10 basis points per dollar of deposits); the decreasing cost function reaches 10 basis points at \( x = 1.1 \) for \( \sigma^2 = 0.01 \) and at \( x = 1.05 \) for \( \sigma^2 = 0.005 \).
Table 1 gives the zero-profit deposit insurance premium and the zero-profit deposit spread for constant and decreasing audit costs and for two asset volatility rates ($\sigma^2 = .005, \sigma^2 = .01$).\textsuperscript{13}

It may be of some interest to compare the premia from the model with the FDIC and FSLIC statutory premium of $1/12$ of one percent. For an asset volatility rate of $\sigma^2 = .005$ and constant audit costs of .001, the model premium is equal to $0.00083$ at an asset value-to-deposits ratio of $1.19$, for $\sigma^2 = .003$ the required ratio is $1.13$, and for $\sigma^2 = .01$, it is $1.31$.\textsuperscript{14}

<table>
<thead>
<tr>
<th></th>
<th>$c(x) = .001$</th>
<th>$c(x) = .0001 + .01x^{-\frac{2\sigma^2}{\mu^2}}$</th>
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<tbody>
<tr>
<td>$\sigma^2 = .005$</td>
<td>$\sigma^2 = .01$</td>
<td>$\sigma^2 = .005$</td>
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<tr>
<td>$\mu^2$ (S)</td>
<td>$\mu^2$ (bp)</td>
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<tr>
<td>1.0</td>
<td>.025 10</td>
<td>.035 10</td>
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<td>1.05</td>
<td>.010 10</td>
<td>.018 10</td>
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<td>1.1</td>
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<td>.010 10</td>
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<tr>
<td>1.2</td>
<td>.0007 10</td>
<td>.003 10</td>
</tr>
</tbody>
</table>

The market equilibrium deposit spread will be the lowest zero-profit spread. The audit cost function given by equation (3) (with $K_1$ and therefore $\beta$ positive) would, in principle, produce the minimum deposit spread at an infinite asset value-to-deposit ratio (i.e. for an all equity bank). As a practical matter, the incentive to increase the asset value-to-deposit ratio would stop when the change in the deposit spread was insignificantly different from zero. At the cost of some notational complexity, an audit cost function that reaches (rather than approaches) a positive minimum can be employed with similar results.

The incentive for a bank to be perceived as having a strictly positive asset value-to-deposit ratio at each audit date is an attractive feature of the single-audit model with decreasing audit costs. Furthermore, with

\textsuperscript{13}For a given value of $\sigma^2$ in the range $1$ have considered, $k$ and $l$ will be approximately constant for all values of $c(x) \leq .01$. Therefore, $p_1$ will vary little for audit costs of .01 or less per dollar of deposits. All of the Tables were produced using $\lambda = 1$ or an expected audit frequency of one per year.

\textsuperscript{14}Merton (1977) notes that $\sigma^2 = .003$ "...would correspond to a one-year term where the volatility of the bank assets is similar to those historically observed from holding long-term U.S. government bonds."
audit costs that increase as a bank approaches insolvency, a combination of correctly priced deposit insurance and fully allocated auditing costs would eliminate the incentive to shift risk to the insurer and simultaneously provide a positive capital cushion for the period between audits.

In this formulation of the deposit insurance problem, the insurer has two targets, the allocation of risk and the allocation of auditing costs, and two instruments, the insurance premium and the audit cost function. The proper allocation of risk has no implications for bank capital. Proper allocation of the auditing costs, if they do increase as the bank approaches insolvency, leads to minimum auditing costs in equilibrium. The minimum auditing costs come at the expense of additional capital in the bank. In the context of this model and more generally for banks with access to efficient capital markets for subordinated debt and equity, properly priced deposits and capital are perfect substitutes so this is a socially desirable trade-off. Competition in deposit markets forces the minimum audit cost result. In the example given in Table 1, a bank with an asset variance rate of 0.01 would need an asset value-to-deposit ratio of 1.2 in order to offer the same deposit spread as a bank with an asset variance of 0.005 and an asset value-to-deposit ratio of 1.05.

The difficulty is to correctly price the deposit insurance. This is the problem to which I shall now turn. In doing so, I shall take the viewpoint that the difficulty is in measuring the parameters of the insurance pricing function and not in choosing the proper function. This is not to imply that the choice of the pricing function is unimportant. The point is that parameter mismeasurement will occur with any pricing model. The deposit insurance structure I have developed provides a useful framework for analyzing the effects of that mismeasurement.

4. Parameter Mismeasurement

The parameters of the deposit insurance premium function that I shall treat as being subject to mismeasurement are the asset volatility rate ($\sigma^2$) and the market value of the assets ($V$). The problem of obtaining good estimates of return volatilities for the underlying asset has been discussed extensively.\(^{15}\)

Mismeasurement of asset values is not a problem in many option pricing contexts since the optioned assets are traded securities with observable market values. The difference here is that many bank assets are non-traded securities. Mismeasurement of their value by an external examiner becomes an important

\(^{15}\) See Smith [1976].
consideration. Specific auditing practices used by the insurer, for example the use of book values rather than market values, can introduce error in valuing the insurer's liability.

Mismeasurement of $V$ affects the deposit insurance premium in two ways. The auditor can report an incorrect current asset value. The second source of error is the failure to properly identify an economically insolvent bank. In the long-run, the second type of error is the more important.

To allow for both types of mismeasurement, I shall assume that the true asset value-to-deposit ratio $(x)$ is proportional to the measured asset value-to-deposit ratio $(\hat{x})$

$$x = \phi \hat{x}$$

with the proportionality factor $\phi$ constant across all values of $x$.

Furthermore, I assume that $\phi$ is less than or equal to 1. One justification for this assumption is that the bank owners would challenge mismeasurement for $\phi > 1$. In this circumstance, it would be in their interest to reveal information on the true value of the assets. Conversely, the bank owners would have no incentive to correct mismeasurement for $\phi < 1$. Furthermore, banks with access to the exchange market would substitute exchange market borrowing for deposits if the mismeasurement of $x$ led to overpricing of deposit insurance. In short, the insurer faces moral hazard and adverse selection with respect to errors in asset valuation.

The objective in this section of the paper is to compare the effect of an error in asset volatility measurement with an error in measuring the value of the bank’s assets or, equivalently, an error in measuring $x$. How much should the equilibrium price of deposit insurance change if the standard deviation of asset returns were to be 0.105 instead of 0.1? How much should it change if the true value of the assets were to be 0.95 times the measured value? The proposed comparison is between the elasticity of $p^*$ with respect to $\phi$ and the elasticity with respect to $\sigma$.\footnote{For a given value of $\hat{x}$ with $x = \phi \hat{x}$, the percentage change in $\phi$ measured at $\phi = 1$ is equal to the percentage error in the measured asset value-to-deposit ration.} To make this comparison, it is necessary to derive the equilibrium deposit insurance cost function when $x = \phi \hat{x}$.
4.1. Asset Value Errors and the Cost of Deposit Insurance

Thus far the assumption regarding insolvency has been that insolvency is declared when an audit takes place and the asset value-to-deposit ratio is less than one. If the measured asset value-to-deposit ratio is different from the true ratio and insolvency is based on the measured ratio, the correct deposit insurance cost function should be based on the insolvency condition $x \leq \phi$. In this case, the analysis of the equilibrium deposit insurance premium proceeds as before except that the continuity conditions, (4c), (4d), (6c), and (6d), become

$$p_1(\phi) = p_2(\phi) \tag{4c'}$$

$$p'_1(\phi) = p'_2(\phi) \tag{4d'}$$

$$f_1(\phi) = f_2(\phi) \tag{6c'}$$

$$f'_1(\phi) = f'_2(\phi) \tag{6d'}$$

Using these alternative boundary conditions, the equilibrium deposit insurance premium is

$$p^*_1(x) = \left[ (1 - k^*) \phi + \frac{(\lambda + \lambda K_0)}{(\lambda + \mu)} k^* \right] x^{i'} \phi^{i'}, \quad x \geq \phi$$

or

$$p^*_1(\tilde{x}) = \left[ (1 - k^*) \phi + \frac{(\lambda + \lambda K_0)}{(\lambda + \mu)} k^* \right] \tilde{x}^{i'}, \quad \tilde{x} \geq 1. \tag{11}$$

Assuming that audit costs are based on the measured value of the asset value-to-deposit ratio, the equilibrium condition for this case is

$$\pi(\tilde{x}) = 1 + \left[ \frac{\lambda K_0 (\beta + k)}{(\mu - K_1)(\beta + 1)} - \frac{(\lambda + \lambda K_0)}{(\lambda + \mu)} k \right] \tilde{x}^{-\delta} = \frac{\lambda K_0}{(\mu - K_1)(\beta + 1)} \tilde{x}^{-\beta} = 0. \tag{12}$$
4.1.1. The Elasticity of the Deposit Insurance Premium

For a given measured value of the asset value-to-deposits ratio, the elasticity of the premium with respect to $\phi$ evaluated at $\phi = 1$ is

$$
\varepsilon_\phi = \frac{(1 - k^*)}{1 + \left(\frac{\lambda K_0 - \mu}{\lambda + \mu}\right) k^*}
$$

Since the equilibrium deposit spread is a function of $\sigma$, a general closed form solution for the elasticity with respect to $\sigma$ is quite complicated and I shall use numerical analysis to evaluate it. The solution for this elasticity given constant audit costs is

$$
\varepsilon_\sigma = \frac{2\delta^*(1 + \delta^*) + \gamma}{((1 + \delta^*)^2 + \gamma)^{\frac{1}{2}}} \left(\frac{1}{(k^* - 1)^2} + \frac{2\delta^*(1 + \delta^*) + \gamma}{2((1 + \delta^*)^2 + \gamma)^{\frac{1}{2}}} \right) \ln(\hat{x}).
$$

Table 2 gives point elasticities of the deposit insurance premium with respect to $\phi$ and $\sigma$ for the constant cost case and Table 3 comparable arc elasticities (for a 5 per cent change in the two parameters) for cases with decreasing audit costs.

From these tables it is clear that the pricing error induced by the mismeasurement of the value of the bank’s assets will be substantially greater than the error induced by the same proportional error in measuring asset volatility. Furthermore, the error induced by the mismeasurement of asset values increases, both in absolute terms and relative to $\varepsilon_\sigma$, as the bank approaches insolvency. For a bank with a measured asset value-to-deposits ratio of 1.05, a 1 per cent error in measuring the value of the bank’s assets will lead to an error in pricing deposit insurance that is greater than 10 percent.

Finally, note that a modest mismeasurement of asset values will offset the effect of decreasing audit costs on the optimal asset value-to-deposit ratio. For example, with $c(\hat{x}) = 0.001 + 0.01 \hat{x}^{-25}$, an error of between 1 and 2 per cent in measuring the asset value-to-deposit ratio is sufficient to make the optimal deposit spread zero for all values of $\hat{x}$. 
### Table 2
Deposit Insurance Premium Elasticities with constant audit costs

\[ c(\hat{x}) = .001, \phi = 1.0, \lambda = 1.0 \]

<table>
<thead>
<tr>
<th>(\hat{x})</th>
<th>(e_\phi)</th>
<th>(e_\sigma)</th>
<th>(\frac{e_\phi}{e_\sigma})</th>
<th>(\sigma^2 = .005)</th>
<th>(\sigma^2 = .01)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>19.3</td>
<td>1.0</td>
<td>19.3</td>
<td>13.6</td>
<td>1.0</td>
</tr>
<tr>
<td>1.05</td>
<td>19.3</td>
<td>2.0</td>
<td>9.7</td>
<td>13.6</td>
<td>1.7</td>
</tr>
<tr>
<td>1.1</td>
<td>19.3</td>
<td>2.9</td>
<td>6.7</td>
<td>13.6</td>
<td>2.4</td>
</tr>
<tr>
<td>1.2</td>
<td>19.3</td>
<td>4.7</td>
<td>4.1</td>
<td>13.6</td>
<td>3.6</td>
</tr>
</tbody>
</table>

### Table 3
Deposit Insurance Premium Arc Elasticities for audit costs that decrease with \(x\)

5 percent change in parameters

\[ c(\hat{x}) = .0001 + .01\hat{x}\sigma^2, \phi = 1.0, \lambda = 1.0 \]

<table>
<thead>
<tr>
<th>(\hat{x})</th>
<th>(e_\phi)</th>
<th>(e_\sigma)</th>
<th>(\frac{e_\phi}{e_\sigma})</th>
<th>(\sigma^2 = .005)</th>
<th>(\sigma^2 = .01)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5.5</td>
<td>0.4</td>
<td>13.1</td>
<td>4.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1.05</td>
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<td>0.9</td>
<td>6.1</td>
<td>4.5</td>
<td>0.8</td>
</tr>
<tr>
<td>1.1</td>
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<td>1.3</td>
<td>4.1</td>
<td>4.5</td>
<td>1.2</td>
</tr>
<tr>
<td>1.2</td>
<td>5.5</td>
<td>2.2</td>
<td>2.5</td>
<td>4.5</td>
<td>1.7</td>
</tr>
</tbody>
</table>
5. Conclusions

A deposit insurance structure with audit costs that increase as the bank approaches insolvency has the attractive feature of inducing optimal asset value-to-deposit ratios that are strictly greater than one. Unfortunately, this property is overwhelmed by small errors in value measurement.

The problem of parameter mismeasurement in pricing deposit insurance has implications for deposit insurance reform. A major emphasis in that reform should be in improving the measurement of asset values by the examiners and, in particular, in setting minimum net worth requirements that reflect economic net worth or that protect the insurer against mismeasurement of net worth. A step in the right direction would be the abandonment of solvency rules that are based on book values.

Prior to deposit deregulation, Black, Miller, and Posner [1978] suggested that capital requirements are not a costly form of bank regulation, at least for banks with access to efficient capital markets for equity and subordinated debt. This argument has been strengthened by deposit deregulation. The remaining economic advantage of financing bank assets with deposit liabilities rather than with non-deposit liabilities is any subsidy provided by the deposit insurance system. This subsidy is what the proponents of risk-related premia hoped to eliminate or at least reduce substantially. The suggestion made here is that, given the inherent parameter measurement problems, larger and enforceable minimum net worth standards are a more promising approach.
REFERENCES


Appendix

The derivation of the differential equations for the cost of deposit insurance and for the value of a bank's equity for a single-audit guarantee with pay-as-you-go auditing costs parallels Merton's [1978] derivation for perpetual insurance with insurer paid audit costs. The interested reader is referred to that paper (p.441-8) for the details of the derivation.

Merton shows that the return on the insurer's liability over a short time interval depends on whether an audit takes place during the interval. If $P(V,D)$ is the value of the insurer's liability and $dR_p$ is the rate of return on that liability, the return on the liability may be written as

$$P\;dR_p = L[P(V,D)]\;dt + \sigma V\frac{\partial P}{\partial V}dz, \text{ if no audit occurs}$$

(1a)

$$= L[P(V,D)]\;dt + \sigma V\frac{\partial P}{\partial V}dz + C(V,D), \text{ if an audit occurs and } V > D$$

(1b)

$$= L[P(V,D)]\;dt + \sigma V\frac{\partial P}{\partial V}dz + C(V,D) + D - V - P, \text{ if an audit occurs and } V < D.$$ (1c)

where $L$ is an operator defined by

$$L \equiv \frac{1}{2}\sigma^2 V^2 \frac{\partial^2}{\partial V^2} + [\alpha V - (R - g)L] \frac{\partial}{\partial V} + gD\frac{\partial}{\partial D}.$$ 

In this formulation of the problem, the audit cost is incurred by the insurer whether the bank is solvent or not. For pay-as-you-go auditing, the insurer only incurs audit costs when the bank is found to be insolvent. Therefore, in the alternative formulation, the audit cost term appears only in equation (1c). A second innovation in this paper is the assumption of a single-audit guarantee. This means that when an audit occurs the insurer's liability will either cease (for $V < D$) or will be repriced and the insurer will receive a cash flow equal to the value of the liability at that date (for $V > D$). Therefore, to value the single-audit guarantee, the value of the liability at the time of the next audit should be subtracted from equation (1b). The resulting system of equations for the return on the insurer's liability is
\[ P \, dR_p = L[P(V, D)] \, dt + \sigma \, V \frac{\partial P}{\partial V} \, dz, \text{ if no audit occurs} \]  
\[ = L[P(V, D)] \, dt + \sigma \, V \frac{\partial P}{\partial V} \, d - P, \text{ if an audit occurs and } V > D \]  
\[ = L[P(V, D)] \, dt + \sigma \, V \frac{\partial P}{\partial V} \, dz + C(V, D) + D - V - P, \text{ if an audit occurs and } V < D. \]  

Following the procedure used by Merton, equation system (1') leads to the differential equations (4a) and (4b) given in the text:

\[ \frac{1}{2} \sigma^2 x^2 p_1'' + \mu x p_1' - (\mu + \lambda) p_1 = 0, \quad x \geq 1 \]  
\[ \frac{1}{2} \sigma^2 x^2 p_2'' + \mu x p_2' - (\mu + \lambda) p_2 + \lambda [c(x) + 1 - x] = 0, \quad x \leq 1 \]  

The comparable equations (see Merton [1978] p.443) for a perpetual guarantee with insurer paid audit costs are:

\[ \frac{1}{2} \sigma^2 x^2 p_1'' + \mu x p_1' - \mu p_1 + \lambda c(x) = 0, \quad x \geq 1 \]  
\[ \frac{1}{2} \sigma^2 x^2 p_2'' + \mu x p_2' - (\mu + \lambda) p_2 + \lambda [c(x) + 1 - x] = 0, \quad x \leq 1 \]  

For the single-audit guarantee with pay-as-you-go audit costs, the periodic audit costs and the new deposit insurance premium are negative cash flows for the equity of a solvent bank. Given these modifications, the derivation leads to differential equations (6a) and (6b) in the text:

\[ \frac{1}{2} \sigma^2 x^2 f_1'' + \mu x f_1' - \mu f_1 - \lambda (c(x) + p_1) = 0, \quad x \geq 1 \]  
\[ \frac{1}{2} \sigma^2 x^2 f_2'' + \mu x f_2' - (\mu + \lambda) f_2 = 0, \quad x \leq 1. \]  

The comparable equations for the perpetual guarantee with insurer paid audit costs are (see Merton [1978] p.447):

\[ \frac{1}{2} \sigma^2 x^2 f_1'' + \mu x f_1' - \mu f_1 = 0, \quad x > 1 \]  
\[ \frac{1}{2} \sigma^2 x^2 f_2'' + \mu x f_2' - (\mu + \lambda) f_2 = 0, \quad x \leq 1. \]

---

1For notational simplicity I have written the total deposit return as \( R \) rather than \( R + \sigma^2 \) and throughout the derivation I have assumed the deposit growth rate is \( g = R \). The equations from the Merton paper are modified to account for these notational differences.