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DEALERSHIPS, TRADING EXTERNALITIES, AND GENERAL EQUILIBRIUM

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DEALERSHIPS, TRADING EXTERNALITIES, AND
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ABSTRACT

A model of intermediated production-exchange equilibrium with constraints on trading capacity and "search" externalities is developed. The impact of specialization of agents into producers and dealers on the properties of equilibrium, in particular its uniqueness and welfare characteristics, is examined.
I. INTRODUCTION

In an important recent paper, Diamond [1982] has analyzed production-exchange equilibria in a model of search. Trading externalities in this model arise from the specification that the likelihood of consummating trade per-period is monotonically related to the proportion of agents having goods to trade. The probability of trading per-period in turn feeds back to determine agents' optimal production decisions. Diamond shows that this combination of the trading externality and the feedback effect can create a multiplicity of Nash equilibria, which are ordered in welfare and employment levels by the Pareto criterion. In Diamond [1982] and Diamond-Fudenberg [1982], the static and dynamic properties of such an environment are developed, and the role of macroeconomic policy ("aggregate demand management"), in facilitating transitions to Pareto-superior equilibria, is examined in detail. These models appear to have the intent of providing micro-foundations for Keynesian macroeconomic conclusions.

The Diamond [1982] story is an abstract model of an economy in which the problems of coordinating transactions, i.e., ensuring that two traders who meet have goods to exchange, are non-trivial owing to the time-consuming nature of the trade/matching process. Although both Diamond [1982] and Diamond and Yellin [1983] have emphasized that "transactions occur only at meetings between a buyer and a seller selected at random," we believe the crucial feature is one of two (or more) agents who have potential gains from trade not knowing if they currently do so or not. As a result, pairings among agents result in a probability of trade that is increasing in the proportion "employed," i.e., those who have already produced goods for trade. For tractability in modelling, in particular to avoid multi-good inventory problems, exchange is modelled through a convention that an agent
can only consume units of an homogeneous good produced by others.

In the Diamond [1982] model, the problem of trading externalities is coupled with a strong symmetry assumption, that the two trading agents are producers who, if they have goods, exchange goods at an exogenously given trading ratio of unity. As an abstraction of empirically relevant phenomena that these models might intend to capture, this is not entirely satisfactory. Many conceptions of trading processes will produce "localized" markets in which agents (in any period) trade with small subsets of the economy, rather than in a central (auctioneer-run) Walrasian market, e.g., the turnpike/location models of Townsend [1982], or plausible models in which commodity/agent characteristics are heterogeneous and known only "locally." However, the "real world" is also characterized by a great deal of specialization of agents, in particular into those who produce, and those who serve as dealers or intermediaries facilitating the trading process. The implications of such specialization for the qualitative properties of "search" equilibria with trading externalities, constitute the focus of this paper. The issues of multiplicity of (Pareto-ordered) equilibria and welfare optimal government intervention are of particular interest.

In a model in which dealers and producers coexist, and all trade is intermediated through dealers, an endogenous price-ratio emerges as an additional equilibrating mechanism. In the context of the Diamond [1982] set-up (which we also employ), in which producers exchange otherwise identical commodities, this price-ratio is simply a bid-ask spread. The bid-ask spread is the difference between the amount of the good given to the dealer and the fraction of the good received by the producer in return. At any time, the dealer only needs to have an inventory equalling a single unit of the good less the bid-ask spread. In addition to setting a bid-ask
spread, dealers choose the size of their market which, given the dealer's bounded trading capacity, determines the frequency with which the dealer calls on to trade with each of his customers. Dealers compete with each other (and potential entrants) by their choice of a bid-ask spread, and clientele size. Since we, like Diamond, assume that agents are ex ante identical, equilibrium has the additional feature that agents are indifferent between dealership and producer occupations, i.e., free entry.

The formulation above introduces a partial role for prices in mitigating trading externalities, which remain because a dealer's probability of consummating a trade with one of his producers in a "visit", is still assumed to depend on the production decisions of the latter. Dealers and producers are assumed to maximize their lifetime discounted expected utilities, taking as given the terms and production decisions of one another, as well as competitive constraints arising from potential entry. We ask if this partial introduction of prices serves, nevertheless, to diminish the impact of trading externalities, in the sense of the multiplicity of (Pareto-ordered) Nash equilibria, in comparison to the symmetric-trading model of Diamond [1982]. Sufficient conditions for existence of a unique equilibrium in our model are compared with those for a parametric version of Diamond's model, and these conditions are found to be very similar for the two models.

Our model also inherits the welfare properties of Diamond [1982]. It remains the case that the (Pareto-supreme) Nash equilibrium in our model is not ex-ante incentive-efficient (Myerson [1979]), and subsidization of trade can improve an equally-weighted sum of expected utilities, as in Diamond [1982]. Somewhat surprisingly, despite the greater complexity of our model, we are able to provide a fairly complete global characterization of such welfare-optimal interventions. We do not, however, pursue questions
relating to the dynamics of transitions across multiple equilibria, which have been analyzed in Diamond and Fudenberg [1982] for a simpler model.

The relationship of our work to several recent papers in the literature bears discussion. Diamond and Yellin [1983] have recently looked at the questions of equilibrium price-setting by firms in a search model with nominal money. In complete contrast to the assumptions (regarding dealers) in our model, their firms are assumed to have no price reputations, and the only tradeoffs on pricing policy thus arise from "Clower constraints," and the distribution of money holdings among consumers. The greater analytical complexity of their model also leads them to confine their attention to economies without feedback effects on production decisions, and with a given number of firms. Another type of extension of the Diamond [1982] model has been provided in the recent paper of Rubinstein and Wolinsky [1984], who have analyzed (perfect) sequential bargaining equilibria with two types of traders. However, neither endogenous specialization, nor endogenous levels of productive activity which in turn affect trading frequencies, is present in their model. Thus, the questions addressed in this paper are absent in both the analyses discussed above.

An early, and important, paper on search equilibrium with an endogenous number of "intermediaries" and price-setting was that of Mortensen [1976]. In his model, the primary motivation was that of establishing non-degenerate equilibrium price-distributions across labor exchanges, when potential employers and employees (rationally) randomly picked one exchange to list themselves with. Exchanges cleared at the reservation price of the side in short-supply, with employment equalling the minimum of labor supply and demand at the exchange. A theory of the equilibrium number of exchanges was developed by assuming that each exchange has increasing marginal costs
of listing as its clientele grows, and that transactions services are priced at expected marginal cost. Mortensen then showed that the number of exchanges resulting from a free-entry equilibrium with fixed costs -- which minimizes the total transaction costs -- exceeds the welfare-optimal number. The reason was a negative externality exerted by an additional exchange on the probability of matching at other exchanges.

Our model, with its additional feature of a positive trading externality arising from an increased number of dealers on producers, is quite distinct from Mortensen's work. As he noted: "Our model and others like it assume that any trader can search any of the exchanges at the same cost. This rules out costs that are functions of the geographical distance between traders." Our results are also strikingly different from his. We find that private Nash equilibria involve too few dealers, and a lower level of productivity, than that implied by an equally-weighted (incentive-compatible) welfare optimum. However, there may exist more complicated pricing schemes which -- if they were implementable within the context of the dealer-producer "relationship" -- may restore the constrained efficiency of equilibrium.

In Section II, we set out the basic structure of our model, and derive the equations characterizing Nash equilibria, as well as the comparable equations for the Diamond [1982] model. In Section III, the uniqueness properties of the two types of equilibria -- with or without specialized dealers -- are derived and contrasted. Following that, in section IV, the welfare properties of our equilibria are elaborated on. In Section V, which concludes the paper, we summarize our qualifications about the results, and suggest directions for future research.
II. DESCRIPTION OF DEALERSHIP EQUILIBRIUM

(a) Modelling Postulates

As in Diamond [1982], we assume that all agents are infinitely-lived, \textit{ex ante} identical, and risk-neutral, with preferences represented by the life-time utility function,

\[
W(x,c) = \sum_{t=1}^{\infty} \frac{1}{1+R}^t (x_t - c_t) 
\]

i.e., intertemporally additive with a discount factor \(1/(1+R)\), where \(x_t\) equals the quantity of the (homogeneous) good consumed and \(c_t\) is the cost/effort expended in producing goods, in period \(t\). Agents are assumed to face identically, independently distributed stochastic production opportunities, represented by the random variable \(\tilde{c}\) with distribution function \(G(c)\), which has unity in the interior of its support. We assume that \(G(c)\) is twice-differentiable. Faced with a realization \(c\) of \(\tilde{c}\), each agent can choose to either produce an unit of the good with effort/cost \(c\), or to forego the opportunity and wait for the next-period draw. In both Diamond [1982] and here, this decision turns out to be a "reservation cost" criterion \(c^*\).

Agents' consumption and production opportunities are constrained by two other rules. First, an agent who has already produced, and thus has an inventory of one unit -- Diamond [1982] terms such agents "employed" -- can not consume his own good, but must exchange it for goods from another agent who is also employed. Second, while an agent is employed, he can not engage in further production until he has exchanged his good and consumed the sales proceeds. (Immediate consumption is strictly optimal because the utility
function is linear and additively discounted.) Presumably, this constraint reflects the fact that "search" activity is time-consuming. Diamond [1982] assumes that if a fraction $e$ of agents is employed, then the probability of an employed agent meeting another one per-period is given by an increasing function $b(e)$ -- "random pairings" of agents motivates this assumption.³ In Section III, we shall consider the resulting equilibria in his model in more detail, for purposes of comparison; at present, it suffices to mention that multiple steady-state $(e,c^*)$ equilibria may arise.

We modify the Diamond [1982] trading process as follows. In our model, some proportion of agents become dealers, and this specialization permits them to meet with a greater number of agents per period than producers could.⁴ Each dealer has a "clientele" of $n$ producing agents who trade only through this dealer. This proportion of dealers in the population is $1/(n+1)$. Further, we assume that each dealer, in any period, randomly picks a client to visit, with probability $p = 1/n$. Both these assumptions, of each producer trading only through a single dealer, and dealers visiting in a memory-independent fashion, are made for the sake of tractability. The former assumption enormously simplifies the calculation of trading probabilities given production decisions, and the latter implies that each producer's optimal reservation cost $c^*$ -- production opportunities with $c < c^*$ are accepted -- is stationary.

The latter assumption is especially awkward since (i) dealers realize that agents who were visited earlier are more likely to have produced "by now," and (ii) it is difficult to reconcile with a model in which dealers know agents' identities (implied by the former assumption) and current physical location. These features make it non-trivial to provide an explicit Townsend [1982]-like visualization of the intermediated trading process. An
expanded model in which agents and/or commodities are heterogeneous, and "matching" is required on meeting to consummate a trade, may provide a better justification for these simplifying assumptions. However, even in simpler scenarios, like Mortensen [1976], such modelling has not proved to be very tractable, and much of the discussion of heterogeneity/matching issues there is essentially heuristic. We believe that exploring the qualitative properties of a simple formulation is logically prior to explicitly modelling such complexity.

Besides deciding on his clientele size \( n \), and thus the periodic visit probability \( p = 1/n \), each dealer also chooses his bid-ask spread \((1-q)\), i.e., each producer surrendering one unit of his production is given \( q \) units of the dealer's inventory, and the dealer promptly consumes \((1-q)\) units of the good surrendered. Given the terms \([p,q]\) offered by the dealer that a producer chooses to be affiliated with, the producer chooses the reservation cost \( c^* \) to maximize discounted expected utility. We look for a symmetric Nash equilibrium in which all producers choose the same \( c^* \), and all dealers the same \([p,q]\), taking the choice of one another as given. In particular, this implies for each dealer contemplating an alternative offer \([\hat{p},\hat{q}]\) that, given the \( c^* \) induced by \([p,q]\), producers must not be made worse off by accepting the terms \([\hat{p},\hat{q}]\). Dealers choose \([p,q]\) to maximize their expected discounted utility subject to this competitive constraint. A symmetric Nash equilibrium with free entry satisfies the further constraint that, given the \([p,q]\) which is optimal as above conditional on entry, and the correspondingly optimal \( c^* \), the discounted expected utilities of dealers and producers are the same.

Before proceeding to the detailed analytical formulation of the above conceptualization, it is worthwhile commenting further on the nature of
competition across dealers in our model (producers do not need to take any account of other producers' choices). Conditional on entry decisions by a set of dealers, and producers' choices to affiliate with them, in what sense is there competition among dealers, especially when informational or locational knowledge/adjacency reasons have been used to motivate both bounded trading capacities and dealer-producer affiliation? In essence, after affiliation, the issue of \textit{ex post} bargaining between dealers and producers might arise. However, this "imperfection" is ameliorated by the fact that other (producer) agents within a given location can also become dealers. Thus, if the terms offered by a dealer in location A do not match (in producer utility) terms offered by dealers in locations B, C, D, ..., then a producer in location A can potentially become a dealer and, by offering the terms offered by dealers in the other locations, capture the A-dealer's business. This is particularly true if going into the dealership occupation involves no sunk cost advantages, e.g., if the knowledge of local agents' needs or attributes has to be renewed every period. (If one takes the Townsend [1982] "turnpike" formulation literally, then again there are no sunk cost advantages for existing dealers, who simply give up being producers and "travel faster."") Note that the free entry condition ensures that a producer in location A will want to intervene as above.

(b) Maximizing Behavior and Competition

We now characterize the optimal actions of producers and competitive dealers in greater detail, using basic tools of dynamic programming. At the beginning of each period, a producer is either in the state of employment (e) or unemployment (u), where \{e,u\} constitute the state variables for his lifetime-discounted-expected-utility value functions, \(W_e\) and \(W_u\). During any given period, only one of two things may happen to a producer depending on
his/her initial state. If he is employed he will be visited by his dealer with probability p. This results in trade and consumption of q units at the end of the period, and transition to the unemployed state. If he is unemployed, he will be engaged in exploring productive activities, which is modeled as drawing an i.i.d. sample c from the distribution G(c). If \( c \leq c^* \), the producer's reservation cost criterion, then there will be production and thus transition to the employed state, at the end of the period. Thus, the dynamic programming value functions satisfy the following recursion relationships:

\[
W_u = \max_{c^*} \left\{ \frac{1}{c^* (1 + R)} \left[ \int_{0}^{c^*} [W_e - c]dG(c) + (1 - G(c^*))W_u \right] \right\}
\]  

(2)

which implies the first-order condition

\[
\partial W_u / \partial c^* = 0
\]

(3)

or, using equation (2), that

\[
c^* = W_e - W_u
\]

(4)

with second-order conditions that can be easily verified from the complete solution, which also satisfies

\[
W_e = \frac{1}{(1 + R)} \left\{ p(q + W_u) + (1 - p)W_e \right\}
\]

or, equivalently, that

\[
RW_e = p\{q + W_u - W_e\}
\]

Integrating in equation (2) by parts, and using equation (4), we obtain that

\[
RW_u = \int_{0}^{c^*} G(c)dc.
\]

(6)

Subtracting (6) from (5), and substituting from (4), we have
\[(R + p)c^* = pq - \int_0^{c^*} G(c) dc.\]  

(7)

Equation (5) itself can be simplified, using (4), to give

\[
W_e = \frac{p}{R} (q - c^*)
\]

(8)

Equations (4), (7) and either (6) or (8) fully specify \((c^*, W_e, W_u)\) given \([p, q]\), the competitively available "terms," that are determined through dealer utility maximization as follows.

The lifetime-discounted-expected-utility of a dealer offering terms \([p, q]\), and facing a producer reservation cost decision \(c^*\), is denoted as \(U_D(p, q; c^*)\). In each period, in a given random visit to one of his \(n\) clients (where \(p = 1/n\)), the dealer expects to make a trade with probability \(\Pi\), which represents his subjective expectation that the visited producer will have goods to sell, i.e., he is an "employed" person. Since the dealer makes these visits without foreknowledge of the agent's current state, and without memory as to when this (randomly picked) agent was last visited, his "rational" subjective probability of trade \(\Pi\) must satisfy

\[
\Pi = \sum_{i=1}^{\infty} p(1-p)^{i-1} \left\{ [1 - (1 - G(c^*))^{i-1}] \Pi + [1 - (1 - G(c^*))^{i}] (1 - \Pi) \right\}
\]

(9)

The basis for equation (9) requires some elaboration. The term \(p(1-p)^{i-1}\) represents the probability that this agent was last visited \(i\)-periods back. At the time of that visit, he/she had a good to trade with probability \(\Pi\), or did not have goods to trade with probability \((1 - \Pi)\); we are restricting ourselves to stationary states. If the last visit did result in a trade, then the current one will also if the person has had a 'good' production
draw in any of the last \((i - 1)\) periods, which has the probability \([1 - (1 - G(c^*))^{i-1}]\). If the last visit did not result in a trade, then the probability of a trade in the current visit is \([1 - (1 - G(c^*))^i]\). Finally, since the time of last visit is not known, the sum over \(i\) is taken, to arrive at the (stationary) probability of trade in a random visit.

Equation (9) can be simplified, using basic geometric series formulae, to yield

\[
\Pi = 1 - \frac{\Pi p}{[1 - (1 - p)(1 - G(c^*))]} - \frac{(1 - \Pi)p(1 - G(c^*))}{[1 - (1 - p)(1 - G(c^*))]} = \frac{[G(c^*) - \Pi pG(c^*)]}{[1 - (1 - p)(1 - G(c^*))]}
\]

or, on cross-multiplying and simplifying,

\[
\Pi = \frac{G(c^*)}{[G(c^*) + p]}
\]

(10b)

Given (10b), it is easy to calculate the dealer's expected utility to be

\[
U_D = \frac{(1 - q)\Pi}{R}
\]

\[
= \frac{(1 - q)G(c^*)}{R[G(c^*) + p]}
\]

(11)

Note that \(U_D\) is decreasing in \(p\), ceteris paribus, because of the lower likelihood of trade in each visit when the dealer has fewer clients and visits each more frequently (on average).

Armed with the above results, we are now in a position to complete the description of our model. It was noted before that each dealer, who is subject to competition from potential entrants (in every "location"), can
only maximize his utility subject to the competitive constraint that, given producers' (symmetric) production decisions, the reservation cost $c^*$, producers are at least as well off with this dealer's terms as with those of any other. The analytical formulation of this constraint is greatly simplified by noting the following key feature of the producers' utility maximizing equations.

From equations (6) and (4) it is clear that the optimized expected discount lifetime utilities produced by a set of terms $\{p,q\}$ are monotonically related to the induced reservation cost criterion $c^*$. Thus if all other dealers are offering terms $\{p,q\}$, and a single deviant dealer offers the terms $\{\hat{p},\hat{q}\}$, subject to the constraint that producers' expected discounted lifetime utilities are no lower with his terms, then he needs to ensure that the $c^*$ level induced by his terms is no lower than that induced by $\{p,q\}$. However, it would be sub-optimal for our deviant dealer to offer a set of terms $\{\tilde{p},\tilde{q}\}$ that induces a higher $c^*$ than that induced by $\{p,q\}$, but which results in the same level of expected utility as with $\{p,q\}$ for the $c^*$ induced by $\{p,q\}$. (Remember that dealers are taking producers' choices as given.) The reason is that the dealer could always find an alternative set of terms $\{\hat{p} = \tilde{p}, \hat{q} < \tilde{q}\}$ which induces the same $c^*$ as that induced by $\{p,q\}$ and thus keeps producer utility unaffected, but strictly increases the dealer's utility relative to offering $\{\tilde{p},\tilde{q}\}$ -- this fact is easily verified from equations (7) and (11). Thus, the dealers' competition constraint can be characterized as profit maximization at $(p,q)$ given choice over terms $\{\hat{p},\hat{q}\}$ which induce the same $c^*$ that $\{p,q\}$ induces.

The following definition formalizes our notion of a symmetric Nash equilibrium with free entry among dealers and producers, which arises from (potential) competition.
Definition: A Dealership Equilibrium with Free Entry (DEFE)

is a triple \( \{q,p,c^*\} \) and associated expected discounted utilities \( U_D(q,p,c^*) \), \( W_e(c^*;q,p) \), \( W_u(c^*;q,p) \), defined in equations (11), (8), (6), which satisfy the following conditions:

1. \( c^* \) maximizes each "unemployed" producer's expected discounted utility given \( \{q,p\} \), and thus satisfies equation (7),

2. \( \{q,p\} \) maximizes \( U_D(\hat{q},\hat{p};c^*) \) given \( c^* \), subject to the condition that

\[
W_u(c^*;\hat{q},\hat{p}) \geq W_u(c^*;q,p),
\]

i.e., that producers are at least as well off with \( \{\hat{q},\hat{p}\} \) as with \( \{q,p\} \), and

3. a dealer's lifetime discounted expected utility equals that of an "employed" producer (with good), i.e.,

\[
U_D(q,p;c^*) = W_e(c^*;q,p)
\]

(12)
given the equilibrium proportion of dealers.

(c) **Existence of Dealership Equilibrium**

As noted immediately prior to the definition of equilibrium above, competition among dealers implies that the equilibrium \( \{q,p,c^*\} \) must satisfy

\[
\{q,p\} = \arg \max_{\{\hat{q},\hat{p}\}} U_D(\hat{q},\hat{p};c^*)
\]

subject to

\[
c^* = \frac{\hat{p}q - \int_0^{c^*} G(c)dc}{R + \hat{p}} = \frac{pq - \int_0^{c^*} G(c)dc}{R + p}
\]

(7)
i.e., that the productive decision induced by \((\hat{q}, \hat{p})\) is the same as that induced by \((q, p)\). An interior solution to this dealer maximization problem (subject to competition) is readily characterized, by the equality of marginal rates of substitution over \(\{q, p\}\) among dealers and producers, given the productive decision \(c^*\); i.e., using equation (11) and (8),

\[
\frac{\partial U_D}{\partial p} = \frac{(1 - q)}{[G(c^*) + p]} = \frac{\partial W_e}{\partial p} = \frac{\partial W_e}{\partial q} = \frac{(q - c^*)}{p} = \frac{Rq + \int_0^{c^*} G(c)dc}{p(R + p)}
\]

where the last equality follows from equation (7). This is, of course, the necessary Lagrangean condition for the optimization problem (13) subject to (7), which is also sufficient here (given \(c^*\)), since the indifference curves in \(\{q, p\}\) space are linear for dealer utility \(U_D\) and strictly convex for producer utility \(W_e\), as is readily verified. Note that producer utilities increase in \(\{q, p\}\) while dealer utilities decline.

Writing equation (14a), representing condition 2 for a DEFE, in the form

\[
\frac{U_D}{G(c^*)} = \frac{W_e}{p^2}
\]

and combining with equation (12), the (free-entry) condition 3 for a DEFE, provides a partial characterization of equilibrium,

\[
p^2 = G(c^*).
\]

Using equations (11) and (8), equation (12) can be expressed as

\[
\frac{(1 - q)G(c^*)}{R[G(c^*) + p]} = \frac{p}{R}(q - c^*)
\]
Substituting for $G(c^*)$ in (12') using (15), we obtain

$$\frac{(1 - q)p^2}{R(p^2 + p)} = \frac{p}{R}(q - c^*)$$

which can be simplified to give

$$\frac{(1 - q)}{(1 + p)} = (q - c^*).$$  \hspace{1cm} (16a)

Our goal in further simplification is to substitute for both $p$ and $q$ in terms of (functions of) $c^*$ in (16a), using (15) and (7), to obtain an equation for equilibrium in $c^*$-space. As the first step, (16a) is simplified and rewritten as

$$[1 + c^*(1 + p)] = (2 + p)q.$$ \hspace{1cm} (16b)

Substituting for $q$ in (16b) from (7) we obtain

$$[1 + c^*(1 + p)] = \left(\frac{2}{p} + 1\right) [(R + p)c^* + \int_0^{c^*} G(c)dc]$$  \hspace{1cm} (16c)

which, on substitution for $p$ from (15), gives our characteristic equation

$$1 = c^* + \left(\frac{2}{\sqrt{G(c^*)}} \right)^{1/2} + 1[Rc^* + \int_0^{c^*} G(c)dc]$$ \hspace{1cm} (17)

The strategy for computing an equilibrium is thus exceedingly simple; solve equations (17), (15) and (16a) sequentially. Note that a valid $(0 < c^* < 1)$ solution with $0 < G(c^*) < 1$ results in $0 < p < 1$ by (15) and, since $c^* < q$ by (7), (16a) will indeed provide a $q$ satisfying $0 < q < 1,$
i.e., domain restrictions are not a problem. Thus, to prove existence of an equilibrium (DEFE), it is sufficient to find conditions such that (17) has a solution $c^* \in (0,1)$. The following result summarizes one such set of conditions:

**Proposition 1.** Suppose that the distribution function $G(c)$ is continuous, with support $[0,b]$ for $b > 1$, and that the right derivative (density) of $G(c)$ at $c = 0$ is strictly positive, $g(0) > 0$. Then a Dealership Equilibrium with Free Entry exists (with $c^* < 1$).

**Proof:** Simply note that in equation (17) the right-hand-side (RHS) exceeds unity at $c^* = 1$ and that as $c^* \to 0$,

$$\text{Limit}_{c^* \to 0} \text{RHS}(17) = \text{Limit}_{c^* \to 0} \frac{4(R + G(c^*))}{g(c^*) \sqrt{G(c^*)}} = 0$$

where the middle equality is obtained using L'Hopital's Rule and (17).

Thus, since RHS (17) is a continuous function that maps $[0,1]$ onto $[0,K]$ for some $K > 1$, there exists $c^*, 0 < c^* < 1$, such that RHS(17) equals 1, which is a DEFE. 

**Q.E.D.**

**Remark.** In characterizing equilibria above, we have ignored the trivial non-interior solution $\{q,p,c^*\} = \{-,-,0\}$ for which $w_u = w_c = u_D = 0$.

Before proceeding to analyze questions of uniqueness of equilibrium in Section III, we now set out a comparable characterization of equilibrium for a model of non-intermediated search.
(d) Equilibria without Intermediation

It is instructive to compare the equations above with those for an equilibrium in Diamond [1982], which has nonintermediated search with no (endogenous) specialization of agents. To do so, we consider a parameterized version of the Diamond model, as follows. Now all agents are producers, and "employed" (with good) producers conduct direct searches for other employed producers. If $e$ is the fraction of employed producers in the population, then we assume that the probability of any one employed producer meeting another employed producer in any period is given by $e/X$, where $X \geq 1$ is a scale factor that will allow us to simply compare Diamond's results with ours.

A steady-state equilibrium in Diamond's model is characterized by a pair $\{e, c^*\}$ such that, with a large number of agents, the per-period change in $e$, $\Delta e$, satisfies

$$\Delta e = (1 - e)G(c^*) - \frac{e^2}{X} = 0$$  \hfill (18)

Secondly, given $e$, $c^*$ is utility-maximizing for producers. The latter problem is solved by methods similar to those we used previously, to arrive at the solution $c^*(e)$ satisfying

$$\frac{e}{X}(1 - c^*) = R c^* + \int_0^{c^*} G(c)dc.$$  \hfill (19)

Note that (19) is very similar to (7), with $(e/X)$ replacing $p$, and $1$ replacing $q$, the payoff net of the bid-ask spread in our model. It should also be noted that the optimal solution for the producers satisfy conditions identi-
cal to equations (4) and (6) in our model; now the expected discounted lifetime utility functions $V_u$ and $V_e$ in the "unemployed" and "employed" states are given by \(^8\)

\[
RV_u = \int_0^{c^*} G(c)dc \tag{20a}
\]

\[
c^* = V_e - V_u. \tag{20b}
\]

Thus, as is intuitively clear (for Diamond's model), multiple equilibria are Pareto-ordered in welfare levels, as they are in our model, and welfare comparisons across (equilibria in) Diamond's model and ours can also be made in terms of the induced $c^*$ alone.

To characterize the Diamond-equilibria further, it is helpful to solve for $e$ in terms of $c^*$ in (18) to obtain

\[
e(c^*) = \frac{[-G(c^*) + \sqrt{G^2(c^*) + 4G(c^*)/X}]}{(2/X)} \tag{18'}
\]

Substituting from (18') in (19) gives us the characteristic equation for the equilibria of this model, i.e.,

\[
1 = c^* + \frac{2(Rc^* + \int_0^{c^*} G(c)dc)}{[-G(c^*) + \sqrt{G^2(c^*) + 4G(c^*)/X}] \tag{21}
\]

Using arguments similar to those used in Proposition 1 above, it is easy to establish the following existence result.

**Proposition 2** Suppose that $G(c)$ is continuous, with support $[0,b]$ for
b ≥ 1, and that the density g(0) is strictly positive. Then a Diamond equilibrium exists, with 0 < c* < 1.

Remark: Diamond [1982] shows existence of an equilibrium even without the assumption that g(0) > 0. Indeed, if the support of G(c) is [a,b] for 0 < a < 1, and b >> 1, then (21) necessarily has multiple solutions, including a "low level equilibrium trap" at c* near a. Since we choose to ignore the trivial equilibrium c* = 0, we do not wish to have our investigation of uniqueness below to hinge on equilibria arbitrarily close to the infimum of the support of G(c).

We now consider the issues of uniqueness of equilibrium, as well as welfare-efficiency, for the two models elaborated above.

III. INTERMEDIATION AND MULTIPLICITY OF EQUILIBRIA

As was noted in Section I, our reasons for being interested in an intermediated "version" of the Diamond [1982] model are twofold. On one hand, we seek to explore issues of optimality in the number and functioning of intermediaries, and thus extend the analysis of Mortensen [1976] to a scenario with trading externalities. This analysis is pursued in Section IV. On the other hand, we also wish to enquire if the additional price-setting (q ≠ 1), implied by specialization and intermediation, substantially alters the (sufficient) conditions required for an unique equilibrium, relative to Diamond's symmetric unintertmediated model. The motivation for this question arises from the observation that the unique Walrasian equilibrium in Diamond's set-up can be attained by a single dealer with unbounded trading capacity, who acts as a Stackelberg leader subject to the threat of potential entry, and sets q → 1, implying c* → 1, as the number of traders N → ∞. It is interesting to inquire if the augmentation of strategy spaces by the bid-
ask spread, as opposed to a drastically different equilibrium concept and trading capability, also improve on the "multiplicity problem."

Our results in this regard are (to us) surprising. We find that sufficient conditions for an unique equilibrium are nearly identical for our model and Diamond's, and that this "equivalence" prevails even for small trading externalities as $R \to 0$, i.e., delays and discounting are unimportant. These results attest to the robustness of the insight in Diamond [1982]. In section V, we comment on the relationship of our results to those in Fuchs and Laroque [1974], on continuity of (the unique Walrasian) equilibrium for "small" externalities, and briefly sketch an alternative model that may provide a tighter analog to their positive result.

Our work here also provides closure to the investigation of uniqueness conditions for Diamond's model, which was incomplete in prior work. These conditions do not appear to us to be exceedingly strong, and they do call into question some of the emphasis on Pareto-ordered multiple Nash equilibria as a source of "unsatisfactory" outcomes.⁹

Our results on the "uniqueness question" are presented successively in Propositions 3-5. First, we discuss ("minimally sufficient") conditions that turn out to imply uniqueness in both models, for any value of $R$. Second, we show that the weaker sufficient condition for uniqueness obtained as $R \to 0$ is also identical for the two models (Diamond's and ours). Both these results are based essentially on "contraction mapping" arguments, i.e., that the right-hand-sides of equations (17) and (21) are monotone increasing in $c^*$. In the last result, Proposition 5, we prove the sufficiency (in our model) of "intermediate" conditions, for "moderate" levels of $R$ relative to the (first) equilibrium $G(c^*)$, by using the prior knowledge that the first intersection in (17) is from below. We have been unable
to verify if these conditions are also sufficient for uniqueness of equilibrium in Diamond's model.

**Proposition 3** Suppose that the density \( g(c) \) satisfies \( g(c)c \leq 2G(c) \), for all \( c \in (c^*, 1] \), where \( c^* \) is the first solution to the characteristic equation for equilibrium, i.e., equation (17) or (21). Then \( c^* \) is the unique equilibrium.

**Proof:**

For our intermediated DEFE, we have upon differentiating in (17) that

\[
\frac{dRHS(17)}{dc^*} = \left[ 1 + R + G(c^*) \right] + \frac{2}{\sqrt{G(c^*)}} \left[ R + G(c^*) \right]
\]

\[
g(c^*)c^*[R + \frac{\int_0^{c^*} G(c)dc}{c^*}] - \frac{\int_0^{c^*} G(c)dc}{[G(c^*)]^{3/2}}
\]

Since \( G(c) \) is montone increasing, and has support \([0, b]\),

\[
G(c^*) \geq \frac{\int_0^{c^*} G(c)dc}{c^*}
\]

(23)

Hence, under the assumptions posited, \( dRHS(17)/dc^* > 0 \) for \( 1 \geq c^* > c^* \), and thus there is no other solution to (17).

For the unintermediated (Diamond) equilibrium solution, equation (21), we again obtain upon differentiating,

\[
\frac{dRHS(21)}{dc^*} = 1 + \frac{2[R + G(c^*)]}{\sqrt{G(c^*) + G^2(c^*) + 4G(c^*)/X}}
\]

\[
-2g(c^*)c^*[R + \int_0^{c^*} G(c)dc/c^*] \left\{ \frac{G(c^*) + 2/X}{\sqrt{G^2(c^*) + 4G(c^*)X}} \right\}
\]

\[
\frac{\left[ -G(c^*) + \sqrt{G^2(c^*) + 4G(c^*)X} \right]^2}{[-G(c^*) + \sqrt{G^2(c^*) + 4G(c^*)X} ]^2}
\]
It follows again that $d \text{RHS}(21)/dc^* > 0$ for $c^* \in [c^*, 1]$, by comparing the last two terms in (24), using (23), and noting that
\[
\frac{-G(c^*) + \sqrt{G^2(c^*) + 4G(c^*)/X}}{\sqrt{G^2(c^*) + 4G(c^*)/X}} > -2G(c^*) + \frac{2G^2(c^*) + 4G(c^*)/X}{\sqrt{G^2(c^*) + 4G(c^*)/X}}
\]
or, equivalently, that
\[
[G(c^*) \sqrt{G^2(c^*) + 4G(c^*)/X}] > G^2(c^*)
\]
which is self-evident. \text{Q.E.D.}

\textbf{Remarks:} (1) Note that the condition posited in Proposition 3 is satisfied globally, if there exists a number $M > 0$ such that for all $c \in [0, b]$, $M \leq g(c) \leq 2M$. Alternatively, if the second-derivative of $G(c)$ is negative, i.e., $g'(c) \leq 0$, then again we have that $g(c^*)c^* \leq G(c^*)$. Examples of distributions satisfying these conditions include the uniform, the negative exponential, and even the following "inverse exponential," $G(c) = (e^{Lc} - 1)$, $L > 0$, $e^{Lb} = 2$.

(2) The conditions identified in Proposition 3 are also "minimally sufficient," for arbitrary $R > 0$, without further knowledge of $G(c^*)$. This is self-evident in equation (24), and is obtained in (22) by evaluating only the terms proportional to $R$, i.e.,
\[
\frac{2R}{\sqrt{G(c^*)}} \left[ \frac{\sqrt{G(c^*)}}{2} + 1 - \frac{(g(c^*)c^*)}{2G(c^*)} \right]
\]
and noting that $\sqrt{G(c^*)}$ could be arbitrarily close to zero, for $R$ sufficiently large, i.e., large trading externalities.

For vanishingly small trading externalities, $R \to 0$, the following result
summarizes sufficient conditions for uniqueness in both models.

Proposition 4: For negligibly small \( R \) there is a unique DEFÉ/Diamond equilibrium, if \( g(c)c \leq 4G(c) \), for \( c \in [c^*, 1] \), where \( c^* \) represents the first solution to equation (17)/(21).

Proof: Ignoring the terms proportional to \( R \) in (22), and using equation (23), we obtain that

\[
\frac{dRHS(17)}{dc^*} \geq \sqrt{G(c^*)} \left[ \frac{1}{G(c^*)} + \sqrt{G(c^*)} + 2 - \frac{g(c^*)c^*}{G(c^*)} \right]
\]

Now, for any \( x \in (0,1) \), it is the case that\(^{10}\)

\[ Z(x) \equiv [x + 1/x] > 2 \]

Hence \( dRHS(17)/dc^* > 0 \), under the conditions posited, for \( 1 \geq c^* > c^* \), the unique equilibrium in the intermediated model.

For the unintermediated Diamond model, setting \( R = 0 \) in (24), and using (23), we obtain given \( g(c)c \leq 4G(c) \) that

\[
\frac{dRHS(21)}{dc^*} > \frac{2G(c^*)}{D^2} \left[ \{ G + 2/X - \sqrt{G^2 + 4G/X} \} + \{ -G + \sqrt{G^2 + 4G/X} \} - \{ -4G + \frac{(4G^2 + 8G/X)}{\sqrt{G^2 + 4G/X}} \} \right]
\]

where

\[ D \equiv [ -G(c^*) + \sqrt{G^2(c^*) + 4G(c^*)/X} ] \]

(25a)

Hence,

\[
\frac{dRHS(21)}{dc^*} > 0 \text{ provided}
\]

\[ (2/X + 4G) \sqrt{G^2 + 4G/X} > 4G^2 + 8G/X \]

(25c)
which always holds for $G > 0$, thus implying an unique equilibrium $c^*$. Q.E.D.

We now consider an intermediate sufficient condition for uniqueness, which holds for our model when $G(c^*)$ is not negligible relative to $R$, and is obtained by showing the convexity of RHS(17). This condition turns out in examples to be "intermediate" between those in Propositions 3 and 4, and it thus follows that our equilibria may be unique even though Diamond's are not, at least for large enough $X$ implying a low $c^*$ in the unintermediated model.

**Proposition 5:** Suppose that the first solution $c^*$ to equation (17) satisfies $[G(c^*)]^{3/2} > 2R$, and that $G(c)$ satisfies

\[
\left[ \frac{3}{2} \frac{g(c)}{G(c)} \right] \geq \left[ \frac{g'(c)}{g(c)} \right]
\]

for all $c \in [c^*, 1]$, where $g'(c)$ is the second derivative of $G(c)$. Then $c^*$ is the unique DEFE.

**Proof:**

Differentiating in (22), we obtain

\[
\frac{d^2 \text{RHS}(17)}{dc^*^2} = g(c^*) + \frac{2g(c^*)}{\sqrt{G(c^*)}} - \frac{2g(c^*)}{[G(c^*)]^{3/2}} [R + G(c^*)]
\]

\[+ \frac{3g^2(c^*)}{2[G(c^*)]^{5/2}} [Rc^* + \int_0^{c^*} G(c) dc]
\]

\[- \frac{g'(c^*)}{[G(c^*)]^{3/2}} [Rc^* + \int_0^{c^*} G(c) dc]
\]

\[= \frac{g(c^*)}{[G(c^*)]^{3/2}} \left\{ [G^{3/2} - 2R] + \left\{ \frac{3}{2} \frac{g}{G} - \frac{g'}{G} \right\} [Rc^* + \int_0^{c^*} G(c) dc] \right\}
\]

Hence, under the conditions posited, $d\text{RHS}(17)/dc^* > 0$ for $c > c^*$, the unique DEFE. Q.E.D.
We conclude this section with an illustration of the differences among the three uniqueness conditions developed above.

Example: Let the support of $G(c)$ be $[0,1]$ and let

$$G(c) = (1 - \alpha)c^a + \alpha c$$  \hspace{1cm} (26a)$$

$$a > 1, \alpha \in (0,1)$$  \hspace{1cm} (26b)$$

Then, the conditions for Proposition 3 are met, for $\alpha$ arbitrarily small, if $a \leq 2$, and those for Proposition 4 if $a \leq 4$. We now show that the conditions for Proposition 5 are satisfied -- given $[G(c^*)]^{3/2} \geq 2R$ -- provided $a \leq 3$. From (26a) we obtain that

$$g(c)/g'(c) = \frac{c}{(a-1)} + \frac{1}{[a(a-1)(1/\alpha - 1)c^{a-2}]}$$  \hspace{1cm} (26c)$$

and

$$G(c)/g(c) = \frac{c}{a} + \frac{(a-1)}{[a^2(1/\alpha - 1)c^{a-2} + a/c]}$$  \hspace{1cm} (26d)$$

Hence $\frac{2}{3} [G/g] \leq g/g'$, if $\frac{3}{2a} \leq \frac{1}{(a-1)}$, or

$$a \leq 3$$  \hspace{1cm} (26e)$$

which is the desired "intermediate" result.
IV. WELFARE, INTERVENTION AND EXTERNALITIES

Our goal in this section is to analyze the problem of a social planner who can order (or provide appropriate lump-sum subsidies for) agents to move into dealership versus producing occupations, but who can not dictate agents' c∗-choices. This is in the spirit of the literature on "incentive-compatible efficiency", e.g., Harris and Townsend [1981], in which it is recognized that the welfare-criterion must acknowledge that a subset of the economic data is privately known by agents, and not the mechanism designer. Here, the productive agent's draws c from the distribution G(c), where draws are independent across agents, plays such a role. In carrying out this analysis, our primary focus is on the relationship between the private equilibrium proportion of dealers versus that which is socially optimal. In Mortensen [1976], it is shown that the negative externality exercised by one dealer on others implies that in a (zero-profit) free entry equilibrium, there are too many dealers. Here, unlike in Mortensen's model, the number of dealers per se (and not just their pecuniary terms) matters to agents. As a result, we obtain the reverse result that the socially optimal proportion of dealers exceeds that which results in the (best) free-entry equilibrium.

Several subtle issues of formulation arise in carrying out the welfare analysis involved. Since dealers in our model have no private information or unobservable actions, and utility functions are linear (i.e., transferable), the allocation of income between producers and dealers is clearly immaterial to the equally-weighted social welfare function. This observation, together with the facts that (i) the private DEFE equates marginal rates of substitution between p and q for dealers and producers for a given c∗, and (ii) that the social opportunity cost of being a dealer is the producer utility
level ($W_e$), imply that the socially optimal choice also satisfies the allocative-efficiency condition $p^2 = G(c^*)$ of a DEFE,\textsuperscript{12} equation (15). In our detailed derivations below, we find that this feature can be shown to hold true only for the unrestricted social welfare problem which allows for $q > 1$, i.e., a production subsidy that is welfare-improving given the trading externalities involved. It is thus evident that if we allowed our dealers the capability for (i) paying $q > 1$, but (ii) recovering their costs with lump-sum charges (from both "employed" and "unemployed" producers), i.e., a two-part tariff, then the Pareto-supreme DEFE would also be socially optimal.\textsuperscript{13}

An additional issue that arises, with $q > 1$ being feasible (and indeed a characteristic of the unrestricted social optimum), is that of inventories for dealers, and these being state-variables in a much more complicated (utility maximization) problem than that we have solved. We tackle (evade) these whole set of interrelated issues by adapting the following "story." Each dealer is assumed to have many clients, so that (in equilibrium) his volume of trade does not fluctuate much from (properly measured) period to period. However, the identities of a given dealer's customers change stochastically over time. The notion is that our (or Diamond's) one commodity model is only an abstraction of a multicommodity model in which a particular agent's excess demand vector (taste) changes stochastically over time, whereas dealers specialize in goods. Given statistical stability of the distribution of excess demands over goods, however, a dealer's (or society's) aggregate trade vector is "stable" over time, in equilibrium. However, the absence of long-term trading "relationships" among dealers and specific clients implies that neither a production subsidy ($q > 1$), nor collection of lump-sum charges from producers, is economically feasible in a purely pri-
vate DEFE. On the other hand, the government can do all of (i) collect lump-sum taxes from producers, (ii) order/persuade dealers to provide production subsidies \(q>1\) by (iii) periodically paying dealers to replenish the predictable (purchasing power) inventory levels required for doing so, as well as, possibly (iv) paying a (one-time) lump-sum subsidy for the opportunity cost of not being a producer.\(^{14}\) Below we develop the details of this welfare analysis.

Consider a social planner dealing with \(N\) agents. He orders \(D\) to be dealers charging bid-ask spread \((1-q)\), and \((N-D)\) to be producers. Given the resulting production decision \(c^*\), the number of employed producers at any point is \(E\) which satisfies the steady-state equation (for large \(N\)):

\[
\frac{(N - E - D)}{(N - D)} G(c^*) - \frac{ED}{(N - D)(N - D)} = 0
\]

or

\[
E = \frac{(N - D)^2 G(c^*)}{(N - D)G(c^*) + D}
\]  

Equation (27) is readily reconciled with (10b), by noting that the fraction of "employed" producers is \(E/(N-D) \equiv \Pi\), and that the probability of a dealer visit per period is \(D/(N-D) \equiv P\).

The social planner maximizes the objective function

\[
\max_{\{c^*,D\}} Z(c^*,p,q(c^*,D))
\]

\[
= (N-E-D)\bar{w}_e + E\bar{w}_e + DU_D
\]  

where from equations (6), (7), (8) and (11) we have

\[
\bar{w}_u = \int_0^{c^*} \frac{G}{R} dc
\]  

\[ (28b) \]
\[ W_e = W_u + c^* \]  
\[ U_D = \frac{(1-q)G(c^*)(N-D)}{R[(N-D)G(c^*) + D]} \]  
\[ D - c^* \left[ R(N-D) + D \right] - (N-D) \int_0^{c^*} G(c)dc \]  
\[ (1-q) = \frac{D}{R(N-D)G(c^*) + D} \]  
\[ \frac{D}{N-D} = p \]

The expression for \( Z^*(c^*; D; q(c^*,D)) \) in (28a) can be rewritten, using (28b-e) and (27), as

\[ \max_{\{c^*,D\}} Z(c^*,D, q(c^*,D)) \]

\[ = \frac{D(N-D)}{R[(N-D)G(c^*) + D]} \left[ \int G(c)dc + (1-c^*)G(c^*) \right] \]  
\[ = 0 \]  

Assuming an interior solution exists, the first order conditions are

\[ \frac{\partial Z}{\partial D} = \frac{c^* \left[ \int G(c)dc + (1-c^*)G(c^*) \right]^*}{0} \]

\[ \frac{\partial Z}{\partial D} = \frac{\left[ \{(N-D)G(c^*) + D\}(N-2D) - (1-G(c^*))D(N-D) \right]}{R[(N-D)G(c^*) + D]^2} = 0 \]  

\[ \frac{\partial Z}{\partial c^*} = D(N-D) \left[ \{(N-D)G(c^*) + \right. \]

\[ D\left\{ (1-c^*)G(c^*) \right\} - (N-D)G(c^*) \left[ \int G(c)dc + \right. \]

\[ \left. \left(1-c^*\right)G(c^*) \right\} \]

\[ \frac{R[(N-D)G(c^*) + D]^2}{R[(N-D)G(c^*) + D]^2} = 0 \]

The first equation yields the condition

\[ G(c^*) = \frac{b^2}{(N-D)^2} = p^2 \]  

The second equation yields the condition
\[
(1 - c^*) \cdot \frac{(N-D)}{D} \cdot \int_0^{c^*} G(c)dc = (1 - c^*) \cdot \int_0^{c^*} G(c)dc = 0 
\]

Equation (31) is exactly the same as the productive efficiency condition (15) for DEFE. As mentioned earlier this condition appears in both the DEFE and the social planner's problem because in both cases (i) the marginal rates of substitution for producers and dealers are equated and (ii) the social opportunity cost of being a dealer is taken into account.

Equation (31) gives the value of \( p \) which maximizes the number of trades per period given \( c^* \). Combining the equations (31) and (32) gives the first-order optimality condition

\[
1 = c^* + \frac{1}{ \sqrt{G(c^*)} } \cdot \int_0^{c^*} G(c)dc 
\]

(33)

The following result gives a partial characterization of the social optimum.

**Proposition 6.** Suppose that the distribution function \( G(c) \) is continuous with support \([0, b]\) for \( b \geq 1 \), and the right derivative (density) of \( G(c) \) at \( c = 0 \) is strictly positive. Then there exists a solution to the social planner's problem. If \( 4G(c) \geq cg(c) \) for \( c \in [c^*, 1] \), where \( c^* \) is the first solution to (33), then \( c^* \) is the unique solution to equation (33).

**Proof:** (1) The existence proof is similar to that in Proposition 1, so it will not be repeated here.

(2) To show uniqueness, it is only necessary to show that the right-hand side of (33) is monotonically increasing if
4G(c*) ≥ c* g(c*). Differentiating the right-hand-side of (33) w.r.t. c* gives

\[
\frac{d\text{RHS}(33)}{dc^*} = 1 + \frac{G(c^*)}{\sqrt{G(c^*)}} - \frac{g(c^*)}{2G(c^*)\sqrt{G(c^*)}} \int_0^{c^*} G(c)dc
\]

\[
\geq 1 + \frac{G(c^*)}{\sqrt{G(c^*)}} - \frac{g(c^*)c^*}{2\sqrt{G(c^*)}}
\]

since \(c^* G(c^*) \geq \int_0^{c^*} G(c)dc\)

\[
= \frac{2\sqrt{G(c^*)} + 2G(c^*) - c^* g(c^*)}{2\sqrt{G(c^*)}}
\]

since \(\sqrt{G(c^*)} \geq G(c^*)\)

\[
\geq \frac{4G(c^*) - c^* g(c^*)}{2\sqrt{G(c^*)}} > 0
\]

Q.E.D.

We also obtain the following result, on the necessity of production subsidies (q > 1) in the social optimum.

Proposition 7. The social optimum always requires that q > 1 (i.e., production is subsidized).

Proof: From equation (7) we have

\[
c^*(R + p) + \int_0^{c^*} G(c)dc
\]

\[
q = \frac{c^*(R + p)}{p}
\]

Using equation (31) gives
\[ q = \frac{Rc^*}{\sqrt{G(c^*)}} + c^* + \frac{1}{\sqrt{G(c^*)}} \int_0^c G(c) dc. \]

Substituting in the social optimality condition (33) yields

\[ q = \frac{Rc^*}{\sqrt{G(c^*)}} + 1 > 1 \]

Q.E.D.

Given equations (31), (33) and Propositions 6 and 7, it is easy to compare the DEFE with the social optimum. It can be seen from inspection that the value given by the right-hand-side of (33) always lies below the value given by the right-hand-side of equation (17). When the right-hand-side of (33) is monotonically increasing it is clear that the social optimal \( c^* \) is greater than any \( c^* \) which solves (17). Since the equation for \( p \) is the same in both cases and \( p \) is increasing in \( c^* \), we know that the DEFE has too few dealers. We provide an illustration of these differences below.

Example: Let \( G(c) \) be uniformly distributed with support \([0,2]\) and let \( R = .075 \).

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V. CONCLUDING REMARKS

We have examined an intermediated version of the seminal general equilibrium search-theoretic model of Diamond [1982], and characterized its uniqueness and welfare properties. Many of the multiplicity of equilibria problems in the unintermediated Diamond model remain, and even the Pareto-supreme Nash equilibrium with free entry is inefficient (when only linear pricing can be used), in the precise sense that there are too few dealers, or intermediaries, and hence too low a level of productive activity, relative to the social optimum. Indeed, these conclusions and similarities persist even when the intertemporal discount rate (and hence the effective trading externality) is small; the social optimum productivity level (for an equally weighted welfare function) is independent of this discount factor.

Our results are "in contrast" to those of Fuchs and Laroque [1974], in a somewhat different context of production externalities across firms taking demand functions as given. This is highlighted by noting that there is a unique Walrasian equilibrium in Diamond's model ($q = 1$, $c^* = 1$), which however is not necessarily approached as a unique (intermediated or not) search equilibrium even as $R$, the discount factor, tends to zero.

The essential difference between our model and that of Fuchs-Laroque [1974] is, we think, that of strategy spaces. Our dealers take the productive choice ($c^*$-decision) of producers as given, rather than viewing it as being endogenous to their offers of (price and visit frequency) terms, and the resulting realignment of customers across dealers to equate the expected utilities obtained from each dealer. (It is this last adjustment that renders the characterization of competition among "Stackelberg-leader" dealers a difficult task.) Indeed, even with a single dealer with unbounded trading capacity, subject to a zero profit constraint, this Stackelberg
leader assumption vis-a-vis producers is essential to showing convergence to the unique Walrasian outcome. It would be interesting to study the question of (asymptotic) uniqueness of intermediated search equilibria with multiple Stackelberg-leader dealers, and to establish the relationship of the results obtained with those of Fuchs and Laroque [1974].
1. We prefer to think of Diamond's model in terms of these abstractions rather than as exogenously given random pairing. If it were costless to organize a central exchange, it is difficult to see why agents without goods to trade would be going there at all. Furthermore, prices and auctioneer mechanisms would come into being.

2. Thus, a Walrasian resolution, in which a single (social-welfare maximizing) dealer -- acting as a Stackelberg leader subject to potential competition -- brings about the Pareto-supreme production-exchange equilibrium, does not come about in our model.

3. It seems difficult to reconcile (completely) random pairings of unemployed and employed agents with the other assumption that only unemployed agents can produce. It seems preferable to think of a location-al model in which agents are a priori constrained to trade with nearby agents, who may or may not have goods currently.

4. We shall be emphasizing the differential visiting capacity even relative to Diamond's "employed" and thus currently unproductive agents. This may be viewed as specialization serving to improve the capacity to ascertain or verify producers' claimed characteristics. Alternatively, in a "turnpike" model as in Townsend [1982], in which two rows of agents move out at opposite directions and only agents "facing" each other can trade, dealers can be thought to have higher travelling velocity.
5. It is assumed that "registering" as a dealer requires an enforceable promise of never being out of inventory. Reputation considerations could plausibly eliminate the need to assume this exogenously.

6. The "unemployed" producer is also visited by his dealer with probability $p$, but the visit necessarily results in no trade. As in Diamond [1982], we are also assuming that trading is time-consuming for producers, even with intermediation and thus both production and trade in the same period is not feasible.

7. Fudenberg and Diamond [1982] consider non-steady state equilibria, and the role played by out-of-steady-state beliefs ("animal spirits") in the dynamics of transition to particular steady states.

8. The value functions $V_u$ and $V_e$ satisfy

$$V_e = \frac{1}{(1 + R)} \left[ \frac{e}{X} (V_u + 1) + \left( 1 - \frac{e}{X} \right) V_e \right] \quad \text{(i)}$$

and

$$V_u = \max \left[ \frac{1}{(1 + R)} \left[ (1 - G(c^*)) V_u + \int_0^{c^*} (V_e - c) dG(c) \right] \right] \quad \text{(ii)}$$

which has the first-order condition

$$c^* = V_e - V_u \quad \text{(iii)}$$

which on substitution in (i) gives us

$$RV_e = \frac{e}{X} (1 - c^*) \quad \text{(iv)}$$

and integration by parts in (ii), followed by substitution from (iii) gives

$$RV_u = \int_0^{c^*} G(c) dc \quad \text{(v)}$$
which, in combination with (iv) and (iii) yields

\[
\frac{e}{X} (1 - c^*) = R c^* + \int_0^{c^*} G(c) dc
\]  \hspace{1cm} (18)

9. In a complex multi-market environment that is being "proxied" by Diamond's model or ours, it is questionable if agents can coordinate on the Pareto-supreme equilibrium. Thus, in the presence of multiplicity, problems of "low-level traps" and "animal spirits" in transitions, seem germane.

10. Notice that \( dZ/dx = 1 - 1/x^2 = 0 \) at \( x = 1 \), and that \( d^2Z/dx^2 > 0 \), implying that the minimum is at \( x = 1 \), with \( Z(1) = 2 \).

11. Squaring both sides in (25c), we obtain

\[
\left( \frac{4}{X^2} + \frac{16G}{X} + 16G^2 \right) \left( \frac{G^2}{X} + \frac{4G}{X} \right) > \left( 16G^4 + \frac{64G^2}{X^2} + \frac{64G^3}{X} \right)
\]

which follows by termwise dominance.

12. However, the socially optimal \( c^* \) is unambiguously strictly higher than the \( c^* \)-level in the (Pareto-supreme) DEFE.

13. Note that two different lump-sum charges are involved, (i) to recover the cost of production subsidies (q-1), and (ii) that required to compensate dealers for their opportunity cost of not being producers. While
we have formally assumed that the second charge is not needed for the social problem, that is only semantics on the notion of "equal treatment." See Hayes [1984] for an alternative justification of two-part tariffs under competition, given risk-sharing externalities.

14. A rigorous defence of these assumptions is no doubt hard to make precise. The "most objectionable" part, production subsidies without stochastic inventories at the dealer level, would not be necessary had the restricted welfare problem with \( q \leq 1 \) been easy to solve!

15. This would also serve to verify their conjecture regarding the relevance of their results for "quantity competition à la Cournot," in a general equilibrium context.
REFERENCES


