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SHORT-TERM MOVEMENTS OF LONG-TERM REAL INTEREST RATES: EVIDENCE FROM THE U.K. INDEXED BOND MARKET
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SHORT-TERM MOVEMENTS OF LONG-TERM REAL INTEREST RATES: EVIDENCE FROM THE U.K. INDEXED BOND MARKET

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ABSTRACT

The central government now issues both nominal and inflation indexed long-term bonds in the United Kingdom. The difference in their yields provides one measure of the long-term expected rate of inflation. The evidence suggests that higher long-term, expected, real yields are associated with forecasts of higher income, with tighter monetary policy, and with positive aggregate supply shocks. Changes in the short-term growth rate of the monetary base, which presumably capture the so-called liquidity effect on short-term interest rates, do not perceptibly alter long-term real rates. Long-term real rates also appear to be unaffected by the rate of expected inflation. Comparison with nominal interest rate equation estimates reveals that conclusions about the effect of all variables are extremely sensitive to the choice of a proxy for expected long-term inflation.
I. Introduction: Review and Preview

Identifying the determinants of long-term real interest rates has been hampered by the difficulty of measuring the very series whose movements we wish to explain. Neither financial markets nor surveys have provided estimates over time of long-term real rates directly or, through forecasts of long-term inflation, indirectly.\(^1\) Econometric forecasts of long-term inflation are necessarily very sensitive to the assumed values over the long-term of the driving variables, like the money supply. Unconditional long-term forecasts obtained using time-series models seem equally unsatisfactory since those methods essentially predict that the average inflation rate over the long term will be the same as that observed over the sample period.

A new source of information about financial market expectations of long-term inflation now exists. Since 1981, the central government of the United Kingdom has issued both nominal and indexed long-term bonds.\(^2\) Here we use the additional information provided by the yields on these real bonds to obtain a measure of expected inflation and then investigate what factors drive these real rates over time. In the remainder of this section we briefly review the recent empirical literature on U.K. interest rates. The next section lays out a model of real interest rates. Section III presents the results of estimating the real rate model. Sections IV and V investigate the effects of taxes and alternative expected inflation measures. Section VI offers concluding remarks.

Although the movements of interest rates and especially the relation between nominal rates and (expected) inflation "have been the subject of a great deal of recent research in the United States over the last decade, hardly any work has been done in the United Kingdom until very recently..."(Foster (1979)). This relative lack of attention to U.K.
interest rates is probably due to the fact that the Bank of England traditionally has virtually pegged short-term interest rates. "Changes in MLR (Minimum Lending Rate offered by the Bank of England to clearing banks) were administrative in nature..." ... "Of greater operational significance in enforcing its desired level of short-term interest rates was the Bank of England's practice of quoting the price at which it would buy or sell Treasury and other eligible bills..." (Barclays Bank, p. 1). Estimates of the relation between nominal short interest rates and other factors based on pre-1980 data then seem as likely to capture Bank of England policy rules as private sector behavior. In the 1980s the Bank of England has completely abandoned quoting bill prices, having decided to permit "market forces a greater role in determining the structure of short-term interest rates, and permit greater flexibility."\(^3\) Nevertheless, some attempts have been made to explain the movement of nominal interest rates in the U.K. before this regime switch.

Ball (1965) examines the movement of the annual average yield on consols over the 1921-61 period. Though he concludes that movements in money (relative to income) are negatively associated with movements in the nominal long rate, he finds no evidence "that the rate of inflation affected the bond rate" (p. 91). As he notes, deriving a measure of the average inflation expected over the infinite future is problematic. As proxies for the desired measure, Ball uses actual year-over-year inflation and a geometrically-declining weighted average (with decay rate 0.5) of past inflation. Neither proxy significantly affects nominal rates over the whole sample or over its post-war portion. From 1921 to 1961, the price level grew at an annual compound rate of 2 percent. Year-to-year inflation
rates, however, ranged from minus 20 percent (1921-1922) to plus 20 percent (1950-1951) during this sample period. Since the horizon of the expected inflation rate relevant for consols is very long, expected inflation proxies with such large weights on recent observations of inflation are likely to be dominated by measurement errors. Obtaining near-zero estimated coefficients on these measures of expected inflation then is not surprising. Nor is it compelling evidence that long nominal rates are immune to expected long-term inflation.

The other empirical studies of U.K. interest rates focus on short-term interest rates, where more reliable proxies for the inflation rate expected over the relevant term are usually thought to be obtainable. Demery and Duck (1978) and Foster (1979) use the Carlson and Parkin (1975) survey-based proxy for expected (6 month) inflation in short-term (3 month) nominal interest rate equations. Both articles obtain estimates of the reaction of nominal interest rates to expected inflation that are significant but considerably below one. Demery and Duck's point estimates of the long-run response of short-term rates average about 0.65; Foster's average less than 0.50. The Carlson-Parkin data may approximate market expectations of inflation over the upcoming six months with considerable error (see Evans and Gulamani (1984)). Second, data limitations prevent matching the term to maturity of interest rates (3 months) to that of the expected inflation rate proxy (6 months). Together, these factors may produce downward bias in the expected inflation coefficient estimate that is sizable. Even with a perfect expected inflation measure, we would not expect its coefficient to be unity in these specifications. Levi and Makin (1978) demonstrate that the reaction of short-term interest rates to short-
term expected inflation is likely to be smaller in the short-run than in the long-run. Even if short real rates are impervious to inflation in the long run, until that long run arrives, nominal short rates would not rise one-for-one with expected inflation. Each of these articles also presents evidence that demand pressure (as measured by job vacancies) raises rates and that the liquidity effect associated with faster money growth reduces real rates. Such reductions are due to the acceleration and not the level of money growth. Since far-forward short-term interest rates are likely to be much less (if at all) affected by current deviations of output from equilibrium or by the acceleration of money growth, the effects of these latter two forces on long-term real rates are likely to be attenuated.

Symons (1983) foresees the Carlson-Parkin data in favor of a forecasting equation for inflation. His ex-ante, or expected, real short rate is constructed by subtracting forecasted inflation from the nominal interest rate. Symons then tests only whether actual inflation affects his expected real rates, not whether expected does. Given accelerations in money growth, he finds that his expected real short rate is not affected by actual inflation. To the extent that Symons' inflation forecasting equation provides a proxy that more accurately captures movement in expected inflation than actual inflation (the presumption underlying the construction of his expected real rate), however, we want to know the response of expected real rates to expected inflation. Since he finds a "substantial" difference between actual and his expected inflation measure, these results tell us little about the response of expected real rates to expected inflation.
Here we examine the movements of long-term yields, which can be viewed as a weighted average of forward short rates. Since the dynamics alluded to by Levi and Makin are likely to be more complete the longer the horizon, the response of long-term yields to long-term factors is likely to be larger. Compared to estimates from static specifications for short rates, we expect to find larger coefficient estimates for long-term phenomena and smaller ones for transient phenomena in equations that explain long-term yields. Expected long-term inflation is an example of the former; accelerations of money growth and cyclical output movements are examples of the latter.

II. A Model of Real Interest Rates

The model in this section seeks to incorporate some of the major measurable forces that drive long-term interest rates. The implied reduced forms for the other endogenous variables (real output and prices) may less adequately capture the dynamics and other important factors driving those variables. The aggregate demand side of the economy is composed of expenditure (1) and portfolio balance (2) functions. The aggregate supply side has been condensed into a price function (3). Equation (4) defines the nominal interest rate as the real rate plus the (expected) inflation rate. The presumed sign of the derivative of the left-hand-side variable precedes each of the respective right-hand-side variables.

\[
(1) \quad Q = E(+X, -r, +MBP, -GMB, -RPOIL) \\
(2) \quad MBP = L(+Q, -i) \\
(3) \quad P = P(+\Phi, +Q, +RPOIL) \\
(4) \quad i = r + \Phi
\]
The expenditure function specifies the demand for output (Q) as being positively related to exogenous demand (X), negatively related to the expected real interest rate (r), positively related to the real monetary base (MBP) through a real balance effect, negatively related to growth in the nominal monetary base (GMB), and negatively related to the relative price of petroleum (RPOIL). This specification embodies the liquidity effect on interest rates of changes in money growth rates. Over the short period examined here, changes in GMB approximate changes in the money base relative to its own trend growth rate. To the extent that financial markets clear faster than the output market, it is necessary to distinguish between the longer-run expenditure function and its steeper short-run counterpart. Inclusion of GMB captures this phenomenon by allowing the longer run expenditure function to shift downward temporarily in response to an acceleration in base growth. We have not included a government deficit variable in the expenditure function because of measurement problems. Generating month-by-month estimates of the expected, long-term, weighted, natural real output, public sector deficit, which may well influence long real rates, is problematic. Point estimates on such a proxy would almost certainly be so dominated by measurement error that little would be gained by including one.

The inclusion of the relative price of energy term allows for the possibility that business expenditure falls in response to the reduced profitability associated with higher real input costs (see Wilcox (1983)). Crystal (1984) suggests another avenue for such real supply side forces to affect expenditure on U.K. goods. A rise in the price of petroleum "caused an appreciation of sterling and a rise in the relative price of British
manufactured goods. As a result, British manufactured goods became uncompetitive and production contracted sharply." (p. 37)

Equation (2) posits that the demand for the monetary base in real terms depends on real income and on the nominal interest rate (i). The aggregate supply function (3) relates the price level to the expected price level (p_e) through nominal wages, to output, and to the relative price of oil. Increases in real energy costs may lower real wages, increase costs to firms, and lower equilibrium output; the net effect on real rates of these changes is ambiguous. The quasi-reduced form for real interest rates then is:

\[ (5) \quad r = r(\dot{p}, \dot{X}, \dot{MBP}, \dot{GMB}, \dot{RPOIL}) \]

III. Determinants of Real Rates

The U.K. began issuing indexed bonds in 1981. Until March, 1982 only pension funds were allowed to hold these real bonds. The first issue carried an initial coupon of two percent and matures in 1996. Since this maturity has the longest history, we use its yield as our measure of the expected long-term real yield. Both coupon payments and principal are indexed, one-for-one, with the U.K. equivalent of the CPI, the retail price index (RPI). To obtain a measure of expected inflation embedded in the prices of these assets, we subtract the real yield from the nominal yield on U.K. government bonds maturing in 1997. The arithmetic spread between nominal and real yields then consists of expected inflation and an inflation risk premium. This measure of expected inflation may be biased downward (or upward) by a positive (or negative) inflation risk premium.
There may well be such a premium. As long as it is either constant or uncorrelated over time with the other variables, however, its omission will not bias our coefficient estimates. It might be presumed that, to the extent there is an inflation risk premium, it is positive and rises with maturity. For maturities between 6 and 36 years, however, the nominal-real yield differential was virtually constant at 7 percent in late 1984. If expected inflation as of that date was the same for horizons between 6 and 36 years, the term structure of the inflation risk premium was also flat. This does not deny that the inflation risk premium may vary through time (as opposed to across maturities at a given time). A flat inflation risk premium term structure seems most likely to suggest relatively small inflation risk premia. We take it to be zero in what follows.

Figure 1 plots the nominal \( (i) \), real \( (r) \), and (the derived) expected inflation \( (p) \) rate data over the 1981:7 - 1984:5 sample period.\(^6\) Figure 1 points out the difficulty of using movements in long nominal rates to glean information about long real rates. The movements in the real yields are small relative to those of nominal yields, generating an expected inflation series which closely mimics the nominal yields (correlation = 0.99).\(^7\) The full-sample correlation between the levels of \( i \) and \( r \) is not strong and is negative (-0.36). On average, a 100 basis point rise in \( i \) is associated with a 6 basis point decline in \( r \) and, concomitantly, a 106 basis point rise in \( p \).\(^8\)

It is sometimes argued that, over short periods, changes in nominal rates signal changes in real rates, even though over longer periods, their levels may be unrelated. This does not hold true for these long yields. First differences in long nominal rates are associated largely and
FIGURE 1
Long-Term Nominal Interest, Real Interest, and Expected Inflation Rates (monthly, 1981:7–1984:5)
consistently with first differences in expected inflation (correlation = 0.93) and move less reliably with first differences in real rates (correlation = 0.54). On average, a 100 basis point month-to-month change in nominal yields consists of approximately an 80 basis point move in expected inflation, while real yields move only 20 basis points. There is a second barrier to using nominal rates as indicators of real rates. In addition to the relation between the two series being loose, Figure 1 hints that it may also be unstable. Before 1983, i fell 400 basis points while real rates meandered. After that, real rates rose fairly steadily while nominal rates drifted.

The reduced form for nominal yields \((5)\) suggests a role for exogenous spending, \(X\). As our proxy for this variable we use the index of leading economic indicators for the U.K. The series was detrended by dividing it by an exponential trend with a 2 percent annual growth rate, which is approximately the long run real output growth rate in the U.K. To remove the noise in its month-to-month movements, we used a three month moving average (based on the current and two previous months) of the detrended series. This series is referred to as ILI. MBP is the seasonally adjusted monetary base, deflated by the seasonally adjusted RPI and by the 2 percent trend growth variable. GMB is the annualized growth rate of the nominal, seasonally-adjusted monetary base over the last three months. The supply shock proxy, RPOIL, is the U.S. dollar price of Saudi crude oil, deflated by the dollar/pound exchange rate and by the seasonally adjusted RPI. The resulting variable is the real, pound price of petroleum and has been similarly smoothed by taking its three month moving average. The last regressor is D823ON, a dummy variable which takes on the value one starting
in March 1982 and is zero otherwise. This variable allows for the fact that prior to that date only pension funds could hold indexed bonds.

Our model posits six relevant exogenous variables. As a check on the appropriateness of the assumption of their exogeneity we have conducted bivariate Granger causality tests. For each explanatory variable except D8230N, we tested whether the first four lags of the dependent variable, r, were significantly related to the current value of each explanatory variable, given the presence of its first four lags and a constant term. In no case were lagged real yields jointly significant. Thus each of these variables satisfies at least this minimal exogeneity criterion. We then reversed the procedure, regressing r on its first four own lags, a constant, and each of the first four lags of the independent variables in turn. The lags of ILI, GMB, and RPOIL each proved to be jointly significant. MBP and p were insignificant.

The results of estimating truncated and complete versions of (5) are presented in Table 1. Using a maximum likelihood technique that allows for first-order autoregressive errors (AR1), row 1 shows the relation of real yields to expected inflation and D8230N. Contrary to what is usually reported in bivariate U.K. or U.S. short-term interest rate regressions, long real rates are unaffected by expected inflation. As this market was opened up to allow agents other than pension funds to hold indexed bonds, we would expect real yields to fall and the coefficient on the associated dummy variable to be negative. The coefficient on D8230N is -0.344, but insignificant. As is often the case, this abbreviated specification requires a very large estimated autocorrelation correction coefficient, \( \hat{\rho} \) (0.936).
<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Estimation Technique</th>
<th>Constant</th>
<th>( p )</th>
<th>ILI</th>
<th>MBP</th>
<th>GMB</th>
<th>RPOIL</th>
<th>D823ON</th>
<th>( \rho )</th>
<th>( R^2 )</th>
<th>S.E.E.</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981:7-84:5</td>
<td>AR1</td>
<td>3.44</td>
<td>0.006</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.344</td>
<td>0.936</td>
<td>.6802</td>
<td>.1947</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.50)</td>
<td>(0.09)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-1.73)</td>
<td>(14.26)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>AR1</td>
<td>4.13</td>
<td>-0.074</td>
<td>0.216</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.291</td>
<td>0.674</td>
<td>.6851</td>
<td>.1943</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7.01)</td>
<td>(-1.44)</td>
<td>(2.05)</td>
<td></td>
<td></td>
<td></td>
<td>(-1.46)</td>
<td>(4.83)</td>
<td></td>
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<td>3</td>
<td>AR1</td>
<td>4.07</td>
<td>-0.046</td>
<td>0.161</td>
<td>-0.086</td>
<td>0.005</td>
<td>-</td>
<td>-0.544</td>
<td>0.615</td>
<td>.7320</td>
<td>.1857</td>
<td>1.90</td>
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<tr>
<td></td>
<td></td>
<td>(7.10)</td>
<td>(-0.89)</td>
<td>(1.66)</td>
<td>(-2.23)</td>
<td>(0.52)</td>
<td></td>
<td>(-2.45)</td>
<td>(4.10)</td>
<td></td>
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<tr>
<td>4</td>
<td>AR1</td>
<td>3.94</td>
<td>-0.032</td>
<td>-</td>
<td>-0.133</td>
<td>0.002</td>
<td>-8.65</td>
<td>-0.556</td>
<td>0.329</td>
<td>.7810</td>
<td>.1664</td>
<td>1.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8.11)</td>
<td>(-0.75)</td>
<td></td>
<td>(-4.14)</td>
<td>(0.25)</td>
<td>(-4.26)</td>
<td>(-2.94)</td>
<td>(1.67)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>AR1</td>
<td>3.99</td>
<td>-0.038</td>
<td>-</td>
<td>-0.130</td>
<td>-</td>
<td>-8.59</td>
<td>-0.562</td>
<td>0.314</td>
<td>.7804</td>
<td>.1638</td>
<td>1.80</td>
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<tr>
<td></td>
<td></td>
<td>(9.20)</td>
<td>(-1.02)</td>
<td></td>
<td>(-4.55)</td>
<td></td>
<td>(-3.03)</td>
<td>(1.62)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>AR1</td>
<td>3.49</td>
<td>0.007</td>
<td>0.131</td>
<td>-0.128</td>
<td>0.004</td>
<td>-8.13</td>
<td>-0.460</td>
<td>0.209</td>
<td>.8160</td>
<td>.1551</td>
<td>1.85</td>
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<td></td>
<td></td>
<td>(7.24)</td>
<td>(0.17)</td>
<td>(2.41)</td>
<td>(-4.44)</td>
<td>(0.59)</td>
<td>(-4.76)</td>
<td>(-2.60)</td>
<td>(1.05)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>7</td>
<td>OLS</td>
<td>3.49</td>
<td>0.008</td>
<td>0.125</td>
<td>-0.134</td>
<td>0.004</td>
<td>-8.11</td>
<td>-0.473</td>
<td>-</td>
<td>.8090</td>
<td>.1579</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7.43)</td>
<td>(0.21)</td>
<td>(2.62)</td>
<td>(-5.01)</td>
<td>(0.54)</td>
<td>(-5.55)</td>
<td>(-2.80)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 1982:3-84:5</td>
<td>AR1</td>
<td>3.53</td>
<td>-0.051</td>
<td>0.116</td>
<td>-0.123</td>
<td>-0.024</td>
<td>-6.27</td>
<td>-</td>
<td>-0.139</td>
<td>.8296</td>
<td>.1590</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6.76)</td>
<td>(-0.90)</td>
<td>(2.46)</td>
<td>(-2.50)</td>
<td>(-1.67)</td>
<td>(-4.47)</td>
<td>(-0.58)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>OLS</td>
<td>3.36</td>
<td>-0.031</td>
<td>0.129</td>
<td>-0.124</td>
<td>-0.019</td>
<td>-6.98</td>
<td>-</td>
<td>-</td>
<td>.8281</td>
<td>.1596</td>
<td>1.88</td>
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<tr>
<td></td>
<td></td>
<td>(6.22)</td>
<td>(-0.53)</td>
<td>(2.50)</td>
<td>(-2.34)</td>
<td>(-1.27)</td>
<td>(-4.24)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rows 2 through 7 add various combinations of the remaining variables suggested by the model. Regardless of which specification we look at, expected inflation never affects real yields by either an economically or statistically significant amount. Increases in the index of leading economic indicators (ILI) drive real rates up. Tighter overall monetary policy (lower MBP) raises rates as well but the liquidity effect (GMB), though perhaps important at short maturities, has no discernible impact on these long real rates. The supply shock proxy, RPOIL, is consistently and strongly negative. These estimates also suggest that the opening up of the market in March, 1982 to non-pension-fund demanders reduced real rates by about one-half a percentage point (50 basis points).

It is tempting to interpret the coefficients on the constant as estimates of the steady-state, long-run real rate since the variables (except p) are entered as deviations from their respective means. Over a sample period this short, however, these variables may well have exceeded or fallen below their steady-state values on average. If money had been unsustainably tight, for example, the coefficient on the constant would be higher than the steady-state long-run real rate.

As more complete versions of the model are estimated, the residual autocorrelation coefficients and their significance levels drop noticeably. Row 7 has been estimated with ordinary least squares (OLS) since the complete specification (row 6) indicates an insignificant amount of residual autocorrelation. This produces little change in the estimates. Rows 8 and 9 omit the portion of the sample when indexed bonds could only be held by pension funds. The estimates in these rows are very similar to
those in rows 6 and 7. This supports handling the March, 1982 change in regulations with an intercept shift.

IV. Tax Effects

Whether there are income tax rate effects on U.S. interest rates has been a matter of dispute ever since the theoretical argument was put forth by Darby (1975), Feldstein (1976), and Tanzi (1976). Table 2 presents the results of testing for tax effects using non-nested specification tests like those used by Peek (1982) in tests for tax effects on U.S. short-term interest rates. We may interpret the specification of the model to this point as being appropriate if either taxes are ignored or if the marginal pound of investment income and expense is tax-free. If that marginal pound faces a constant, proportional income tax rate, $t$, the real after-tax interest rate, $r_{a.t.}$, is now determined by the model.

\[(6) \quad r_{a.t.} = r(p, \bar{z})\]

where $\bar{z}$ is the vector of explanatory variables other than expected inflation. The real after-tax return on indexed bonds is $r(1-t)$ since only real capital gains are taxed in the U.K. Dividing through (6) by $(1-t)$ gives

\[(7) \quad r = r\left(\frac{p}{1-t}, \frac{\bar{z}}{1-t}\right)\]
<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Coefficient (in alternative model) on Fitted Value from Model that Embodies Taxes</th>
<th>Coefficient (in alternative model) on Fitted Value from Model that Embodies No Taxes</th>
<th>Taxes Favored?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1981:7-84:5</td>
<td>1.35 (0.62)</td>
<td>-0.35 (-0.16)</td>
<td>yes</td>
</tr>
<tr>
<td>2. 1981:7-84:5</td>
<td>-0.71 (-0.48)</td>
<td>2.11 (1.46)</td>
<td>no</td>
</tr>
<tr>
<td>(t = 0 prior to 1982:3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. 1982:3-84:5</td>
<td>-5.33 (-1.12)</td>
<td>6.29 (1.34)</td>
<td>no</td>
</tr>
</tbody>
</table>
The expected inflation measure used so far, however, is derived on the assumption of no tax effects. The real after-tax yield on indexed bonds is assumed equal to that on nominal bonds

(8) \( r(1-t) = i(1-t) - p \)

This means that \( p \) can no longer be obtained by subtracting real from nominal yields. Equations 7 and 8 imply

(9) \( p = (i-r)(1-t) \)

and

(10) \( r = r(i-r, \frac{Z}{1-t}) \)

where \( i-r \) is the original expected inflation measure. Note that expected inflation is given by (9), but the after-tax specification (10) still uses \( i-r \), the original measure.

The results of estimating the specifications adjusted for and unadjusted for taxes are listed in Table 2. The test statistics are generated using the specification test procedure suggested by Davidson and MacKinnon (1981). The income tax rate series we use is taken from Buiter and Miller (1983). The tax-unadjusted (5) and tax-adjusted (10) specifications are each estimated. The fitted values from each are then added to the list of independent variables in the alternative specification and the models are re-estimated. The test statistics presented are the estimated coefficients and associated standard errors for the fitted values. In row 1, the coefficient in the tax-unadjusted specifications of the fitted values from the tax-adjusted model is 1.35 (t=0.62). The
estimated coefficient when the fitted value variable from the tax-unadjusted model is added to the tax-adjusted model is -0.35(t= -0.16). These test coefficients sum almost precisely to unity. They need not be. Nor need they lie between zero and one, though that might seem intuitive. These results from row 1 weakly favor adjusting for income tax rate effects. The tax-unadjusted model's fitted values add nothing to the alternative. The coefficients in both tests in row 1 simultaneously differ insignificantly from zero and from one, however.

Row 2 uses the same tax rate series but sets the pre-1982:3 rates equal to zero, since only tax-exempt pension funds could hold index-linked bonds then. This specification of the test favors the tax-unadjusted model. Thus, this row provides equally weak support for this alternative model. Row 3 drops the pre-1982:3 observations altogether. The results again provide no strong support for either model against the other. This evidence is eminently inconclusive. Two reasons may be given. Over this short span of time ordinary income tax rates varied very little and unearned income tax rates were unchanged until mid-1984. Second, the pattern of effective tax rates expected over the life of the bond, and not the current realized tax rate, may be the relevant tax rate. As time passes, additional data may come to support one model relative to the other. Given the evidence summarized in table 2 and that the conclusions differ immaterially, we use the tax-unadjusted specification in what follows.
V. Alternative Measures of Expected Inflation

The measure of long term expected inflation used in Figure 1, \( p \), is replotted in Figure 2 along with two alternative proxies, the actual inflation rate over the last twelve months \( (p_{12}) \) and that over the last three months \( (p_3) \). All three decline sharply from late 1981 until late 1982 and have trendless movements thereafter. Not surprisingly actual, shorter-term inflation measures are more volatile than \( p \). Somewhat surprisingly, these alternative measures do move fairly closely with \( p \); the correlation between \( p \) and \( p_{12} \) is 0.94, the correlation between \( p \) and \( p_3 \) is 0.72. These correlations may imply little about causation, however. Bivariate Granger tests (using four lags due to the brevity of the sample) do not reject the hypothesis that \( p \) is unrelated to \( p_{12} \) or to \( p_3 \). Nor can the converse hypotheses be rejected. Nor would we necessarily expect a tight relation between them. Nevertheless, it will prove interesting to examine how these other measures perform in long-term interest rate equations.

Table 3 shows the effect of alternative expected inflation measures on real and nominal interest rate equation estimates. Row 1 uses \( p \), the difference between real and nominal yields, as the expected inflation measure and therefore is identical to row 7 of Table 1. Rows 2, 3, and 4 use as alternative expected inflation measures, \( p_{12} \), \( p_3 \), and \( i \), the nominal interest rate. This last measure is included on the argument that nominal rates predict inflation well. \( p_{12} \) and \( p_3 \) are considered not because of a prior belief that they predict long-term inflation well, but to examine the sensitivity of the estimates to different expected inflation proxies.
FIGURE 2
Alternative Measures of Expected Inflation
(monthly, 1981:7–1984:5)
### TABLE 3

The Effect of Alternative Measures of Expected Inflation on Real and Nominal Interest Rate Equation Estimates

Monthly, 1981:7-84:5

Estimation Techniques: OLS

(t-statistics in parentheses)

<table>
<thead>
<tr>
<th>COEFFICIENT ON</th>
<th>Constant</th>
<th>p</th>
<th>p12</th>
<th>p3</th>
<th>i</th>
<th>ILI</th>
<th>MBP</th>
<th>GMB</th>
<th>RPOIL</th>
<th>DB23ON</th>
<th>R^2</th>
<th>S.E.E.</th>
<th>D.W.</th>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dependent variable: r</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>0.008</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.125</td>
<td>-0.134</td>
<td>0.004</td>
<td>-8.11</td>
<td>-0.474</td>
<td>.8090</td>
<td>.1579</td>
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<td>(0.21)</td>
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<td></td>
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<td>(-5.01)</td>
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<td>(-5.55)</td>
<td>(-2.80)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>(0.95)</td>
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<tr>
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<td>-</td>
<td>0.086</td>
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<tr>
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<td></td>
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<td></td>
<td>(2.18)</td>
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<tr>
<td>4.</td>
<td>2.82</td>
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<td>-</td>
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<td>-0.137</td>
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<td>(1.35)</td>
<td>(3.31)</td>
<td>(-5.61)</td>
<td>(1.24)</td>
<td>(-6.08)</td>
<td>(-1.99)</td>
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</tbody>
</table>

dependent variable: i

|                |         |   |     |     |   |     |     |     |       |        |       |       |      |
| dependent variable: i |
| 5. | 3.49   | 1.008 | - | - | - | - | 0.125 | -0.134 | 0.004 | -8.11 | -0.474 | .9942 | .1579 | 1.49 |
|    | (7.43) | (24.82) |   |   |   |   | (2.62) | (-5.01) | (0.54) | (-5.55) | (-2.80) |       |       |      |
| 6. | 10.03  | - | 0.433 | - | - | - | 0.156 | 0.124 | -0.053 | 3.39 | -0.793 | .9350 | .5266 | 1.46 |
|    | (10.59)|   | (5.47) |   |   |   | (0.87) | (1.50) | (-2.49) | (0.74) | (-1.32) |       |       |      |
| 7. | 12.90  | - | - | 0.197 | - | - | -0.276 | 0.148 | -0.064 | 14.90 | -2.004 | .9053 | .6355 | 1.26 |
|    | (18.17)|   | (3.43) |   |   |   | (-1.57) | (1.47) | (-2.48) | (2.30) | (-3.27) |       |       |      |
Judgments about robustness to these differences in specification have to be tempered by the limited period for which data are available.

The alternative expected inflation proxies deliver real rate equation estimates which are remarkably similar. The estimated expected inflation coefficients are each virtually zero, ranging from -0.033 to 0.052. Only for \( p_3 \) is there a statistically significant effect. Conclusions about significance and even the size of the other variables' effects are also all but invariant to the choice of proxy. This is surprising given the relatively large correlation between \( p \) and \( r(-0.51) \) and between \( p \) and the other explanatory variables (\( \text{ILI}(-0.62), \text{MBP}(0.46), \text{GMB}(-0.60), \) and \( \text{RPOIL}(0.43) \)). Also notable is the fact that the model based on \( p_3 \), which it might be argued would be a poor proxy for long-term expected inflation, fits best. Its standard error of 0.1427 is nearly 10 percent smaller than that based on \( p \). Row 4 illustrates that, though nominal interest rates may be very highly correlated with expected inflation (correlation = 0.98), other variables may still be significantly related to real rate movements.

Rows 5 through 7 use the nominal interest rate as the dependent variable. Row 5 reproduces row 1's coefficients (except 1 is added to the \( p \) coefficient). It appears here to facilitate comparison with rows 6 and 7. In rows 2 and 3 real rates appear virtually immune to \( p_{12} \) and \( p_3 \). At the same time, nominal rates do not react one-for-one to them; their respective coefficients are 0.433 and 0.197. And no longer are the estimated coefficients on the remaining variables robust to which proxy is used. The entire pattern of significance (and sometimes sign) is reversed when \( p_{12} \) and \( p_3 \) are substituted for \( p \). The overall fits also deteriorate
dramatically, standard errors of the estimate (S.E.E.) rising by a factor of three or four.

Rows 1 through 4 suggest that we can assess the effects on real rates of variables other than expected inflation by choosing any of the four proxies used there or by omitting a proxy altogether since the coefficients are very close to zero and generally insignificant. To do this, however, requires a measure of r. Without a direct measure of r or p for this maturity, we must effectively use i as the dependent variable. The coefficients and significance of the variables other than expected inflation are then extremely sensitive to the proxy chosen, as we see from rows 5 through 7. This is ironic. Given an indexed bond market, we would not need to generate p to assess effects on real rates since they would be directly observable. Real rate equations could be estimated directly. Without an indexed market, however, we require the (unobtainable, implicit, market) forecast of inflation to obtain accurate estimates of the real interest rate equation. These last rows then indicate that assessing the effects of any of these factors on long real rates may be extremely problematic without an indexed market or some other way of generating a long-term expected inflation proxy.

VI. Concluding Remarks

Assessing the level, movement, and determinants of long-term real interest rates has been difficult because of the lack of either direct measures of the rates themselves or widely accepted proxies for long-term inflation. The operation of the indexed government bond market in the U.K. in recent years provides a new source of information on both real rates and
expected inflation. The evidence to date suggests that real rates are unaffected by expected inflation but are affected by forecasts of demand, by monetary policy, and by supply shocks. It also suggests that expected long-term inflation may be quite volatile.

The simultaneous operation of real and nominal long-term bond markets also allows us to assess the robustness of conclusions about real rates based on nominal interest rate equation estimates. More sophisticated measures of expected long-term inflation than the alternatives used here can probably be constructed. Section V, however, suggests that conclusions about expected, long-term real rates may prove extremely sensitive to the choice of expected inflation proxy. Though the correlation between $p$ and $p_{12}$ is quite high (0.94), for example, the conclusions based on these proxies are drastically different.
# DATA APPENDIX

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition and Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPI</td>
<td>Retail price index, seasonally adjusted. International Monetary Fund, International Financial Statistics.</td>
</tr>
</tbody>
</table>
REFERENCES

Ball, R.J. "Some Econometric Analysis of the Long Term Rate of Interest in the United Kingdom, 1921-61," Manchester School of Economic and Social Studies (January 1965), pp. 45-96.


1. For simplicity, unless otherwise noted, interest rates refer to long-term, expected real interest rates. The same shorthand applies to inflation. Cargill and Meyer (1980) point out the advantages and difficulties of matching interest rate and expected inflation maturities.

2. By 1984, about one-third of the flow of new central government debt issued was indexed and about ten percent of the outstanding stock was indexed.

3. Barclays Bank apparently took this quote from the Bank of England. It does not cite the source. This change in policy seems the natural result of focussing more on monetary aggregate growth rates and less on market yields.

4. $p_e$ in (3) refers to the price level expected a short time into the future. A true reduced form would have solved the actual price level out of MBP and RPOIL, replacing it with $p_e$. I do not think the results are materially affected. Since no measure of $p_e$ is available, the alternative is to use a constructed $p_e$ series. I feel that the formulation based on actual P serves as well as any based on constructed $p_e$ is likely to. The movements in the other components of MBP and RPOIL dilute the impact of this measurement error to some extent.

5. The indexing is not completely neutral, however. Coupon and principal are indexed to the RPI eight months earlier. There is a two month RPI reporting lag. Secondly, there is a perceived need to ascertain accrued interest between semi-annual coupon payments for day-to-day trading purposes. "The eight-month lag is, therefore, due to the need to determine accrued interest." (Rutterford (1983)).

6. We restrict the sample period to a July, 1981 starting date to allow the market to become established.

7. Later we compare this expected inflation measure with actual inflation over this same period.

8. Regressing r on i (and a constant) gives a coefficient estimate of -0.06 (t = -2.19).

9. Regressing first differences of r on those of i (with no constant term) gives a coefficient estimate of -0.18 (t = -3.23).

10. Other variants of this liquidity effect proxy, like the growth rates of either the real monetary base or real M1 over three or twelve months or the growth rate of nominal M1 over the past six months relative to its growth rate over the last three years, produced similar results. The specification in the text produced a somewhat better overall fit.
11. The sample mean of each of these explanatory variables (except for p) has been subtracted.

12. For the 1981 observations the rate is 0.310. For 1982, it is 0.324. For 1983, it is 0.332. We use the same value for 1984 as for 1983. This rate is the marginal direct tax rate of a married couple.