Research Program in Finance
WORKING PAPER SERIES

WORKING PAPER NO. 150
DEFINING CAPITAL-MARKET EFFICIENCY
by
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DEFINING CAPITAL-MARKET EFFICIENCY

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Finance Working Paper No. 150

April 1985
ABSTRACT

This paper proposes a new definition of the Efficient Markets Hypothesis (EMH), which is more formal and precise than those of Rubinstein (1975), Fama (1976), Jensen (1978), and Beaver (1981), and which fits well as a framework for interpreting the many tests of the EMH in the literature. Security markets are here considered "efficient with respect to information set φ" if and only if revealing φ to all agents would change neither equilibrium prices nor portfolios. In addition to other desirable features, this definition has the "subset property": efficiency with respect to φ implies efficiency with respect to any subset of φ. Furthermore, it is shown that efficiency with respect to φ does not imply the absence of private incentives to acquire φ.
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1. Introduction

Efficiency has many different connotations in economics and finance. This paper examines the concept called "market efficiency" in the finance literature. The concept was developed to describe and explain some observed statistical properties of capital-asset prices. The empirical market-efficiency literature is vast; for surveys, see Fama (1970), Jensen (1978), or look at any issue of any finance journal. The theoretical literature is comparatively small and, I will argue, lacking at a fundamental level: no unambiguous definition of market efficiency has yet been offered.

One contribution of this paper is to propose a "better" definition than previous ones, as well as to discuss characteristics that make a definition better. In a way it is meaningless to say that one definition is better than another, since it is relatively unimportant which terms we attach to which meaning, as long as we have a mutual understanding of what the meanings are. But the term "market efficiency" has been widely used with only an approximate mutual understanding of what it means. The prime characteristic sought here is that a definition make sense of the way the term has been used in the existing empirical and theoretical literatures.

A critical review of previous definitions is presented, unifying most of them into the evolution of a single idea. The mathematical concept of random walk is too strict to serve as a market-efficiency definition, although it has many of the right properties. Definitions in Fama (1970) and (1976), and Jensen (1978) are ambiguous or incomplete.¹ The "fully-revealing" definition in Grossman and Stiglitz (1980) is sufficient but not
necessary for what is meant by market efficiency in the finance literature.

The most promising definition is found in Beaver (1981), who extends Rubinstein (1975) by defining the market to be efficient with respect to some information if security prices would be unaffected by revealing that information to all participants. A subtle ambiguity is found in even this definition, but it is easily repaired. However, the definition is shown to lack the widely-accepted subset property: if the market is efficient with respect to some information, then it is efficient with respect to any subset of that information.

A new definition, "E-efficiency", is proposed, which requires that both prices and portfolios be unaffected by revealing the information. (Although this may appear to be a much stricter requirement, the set of cases where portfolios would change while prices would not is likely to have zero measure.) E-efficiency is then shown to have the subset property. Existing empirical tests of market efficiency can easily be interpreted as tests of E-efficiency, with some caveats. There is potential for tests involving the volume of security trading, though the difficulties are great. The extreme nature of E-efficiency is noted, along with the need for some concept of degree of efficiency.

It is often asserted that no individual should pay to acquire information that is already fully reflected in security prices, i.e. information with respect to which the market is efficient. This is shown to be false not only for E-efficiency, but also for the definitions of Fama, Jensen, and Beaver. Thus public information can have private value in security markets.
The remainder of the paper is organized as follows. Section 2 reviews existing definitions and identifies the properties to be sought in a definition. Section 3 critiques Beaver's definition in the context of a formal model with differential information. The proposed "E-efficiency" is defined, analyzed, and discussed in section 4. Section 5 proves that public information can have private value. Section 6 draws conclusions and mentions unresolved questions.

2. Existing definitions

A useful benchmark for comparing market-efficiency definitions is Fama's (1970) set of sufficient conditions:

"(i) there are no transactions costs in trading securities,

(ii) all available information is costlessly available to all market participants, and

(iii) all agree on the implications of current information for the current price and distributions of future prices of each security."

There seems to be general agreement that the market would be efficient under these circumstances, though they may not be necessary conditions.

2.1 Random walk, and the role of information sets

Fama discusses how the random walk model arose from early empirical work on security returns. The model states that returns are serially independent, and that their probability distributions are constant through time. To explain this, economic rationales such as the following were suggested: If there are enough well-informed investors in the marketplace, they will
destroy any serial correlation in prices by trying to profit from it. If a pattern of price movements reliably predicted a sharp rise in a stock's price, then those investors would buy the stock prior to the expected rise, thus pushing the price up earlier. (A similar story applies for sharp price declines.) This process of making the price move earlier continues until the move is virtually simultaneous with whatever had predicted it, thus eliminating any measurable serial correlation.

That story of the random walk being brought about by well-informed fast-reacting optimizing investors gave rise to the term "market efficiency", which has been used (by some authors\(^2\)) synonymously with random walk ever since. The argument is readily extended to imply zero correlation between any public information\(^3\) and subsequent security returns. The corresponding version of the random walk model is often called "random walk with respect to public information".

Some authors write of "market efficiency with respect to a [to-be-specified] information set", while others write of "market efficiency", unqualified. In the latter view, there is one particular information set with respect to which the market should be efficient -- all public information, for the reasons given above -- so there is no need to specify it every time. If an item of public information is not reflected in prices then the market is inefficient. If some non-public information is not reflected then it is not evidence of inefficiency.

Those who write of efficiency with respect to various information sets are taking a more flexible and agnostic view. Perhaps, for example, we
don't know at what time a given information-item becomes publicly known; in other words, we don't know exactly what the public information set is. The information-qualified designation of market efficiency gives us a term for expressing whether some information is fully reflected in prices at a given time, without needing to first decide whether it is public at that time. And, of course, the unqualified designation can be expressed in the qualified form: "market efficiency with respect to public information", a special case.

Roberts (1967) and subsequent authors distinguished three levels of market efficiency, by specifying three different information sets:

(i) Weak form -- the market is efficient with respect to the history of security prices.
(ii) Semi-strong form -- the market is efficient with respect to all public information.
(iii) Strong form -- the market is efficient with respect to all information known to anyone.

Thus the unqualified usage "market efficiency" is the same as semi-strong efficiency.

Notice that the three information sets are nested: \{history of security prices\} is a subset of \{all public information\}, which is in turn a subset of \{all information known\}. This is generally taken to imply that strong-form efficiency implies semi-strong-form efficiency, and that semi-strong implies weak. Let us call this the "subset property": market efficiency with respect to an information set implies market efficiency with respect to any subset of that set. That market efficiency has this property is taken for
granted in virtually all theoretical and empirical work on the subject.

Some of the authors reviewed below used the unqualified "market efficiency", but their contributions are discussed here in the qualified-with-respect-to-specified-information form. This amendment was made partly for simplicity, and partly because the subset property plays an important role later in the present paper, so that we will need to be able to vary the information set being considered. I believe the misrepresentation is minor.

It is well known that the random walk model as described above is too stringent, both empirically and theoretically. The probability distribution of future security returns is in fact measurably correlated with public information, and there are good theoretical reasons to expect that to be the case, even in an economy populated by well-informed fast-reacting optimizing investors. As surveyed in Merton (1980), changes in the variance of a stock's return can be predicted from its variance in the recent past and from option prices, both of which are public information. Clearly stock prices do not follow a random walk.

Nor should they. Even if all investors have homogeneous information and beliefs, there are still many factors that can affect the distribution of security returns in an equilibrium model, causing the distribution to vary stochastically through time. Risk aversion will be affected by random changes in investors' wealth levels, by birth of new investors, and by death of old ones; and changing risk aversion will change equilibrium expected returns. Furthermore, there is nothing to stop the exogenous sources of uncertainty (e.g., technology, politics) from varying through
time. For these reasons, even the martingale and martingale-with-drift models proposed by Samuelson (1965) do not provide an accurate characterization of market efficiency. Therefore, some other generalization of the random walk model seems needed. Brealey and Myers (1984, p. 269) imply that the appropriate relaxation is a submartingale model:

"But market efficiency does not imply that risks and expected returns cannot shift over time. The correct term for a wandering series with shifting expected changes and variability is a submartingale."

The mathematical definition of a submartingale is a stochastic process with nonnegative (though perhaps varying) drift. The statement that stock prices follow a submartingale claims only that expected returns are never negative. That is too general. If a piece of public information reliably predicted times when a stock price would double overnight, then the market would not be efficient (with respect to that information); yet the submartingale doesn't rule this out. Not only that, but it is possible for some securities to have negative expected returns in an efficient market: for example, in a Sharpe-Lintner CAPM setting, a put option typically has a negative beta, and can thus have a negative expected return if the interest rate happens to be low enough.

2.2 Equilibrium models

The problem is that informational efficiency can not be characterized solely in terms of the stochastic properties of security returns. The interaction of information with time-varying preferences and opportunities must be considered in an equilibrium model.
Fama (1970) recognized that even when prices "fully reflect" information, the probability distribution of future returns conditioned on that information will still depend on economic factors (such as preferences) embodied in an "expected return theory". If the market is efficient with respect to some information, then expected returns conditional on that information will be as predicted by the expected return theory.

Then arise the questions of where the equilibrium expected-return model comes from, and which one to use. Fama did not address these, so LeRoy (1976) appropriately criticized his "definition" as empty and tautological, since it did not in the end narrow down the range of probability distributions for security returns admissible in an efficient market. One could sidestep the problem by always qualifying the EMH by "with respect to such-and-such a model", so that a given market might be efficient with respect to one model and inefficient with respect to another. But surely in a given economy there is one "right" model: the description of the distribution of security returns that would occur at equilibrium given the actual preferences and trading opportunities of the investors involved.

That important insight was embodied in Fama's (1976) revised definition of market efficiency: the market is efficient with respect to an information set if the market uses all the (relevant) information in the set correctly when it determines security prices. This captures the key feature of efficiency -- that the information is reflected -- without having to say what the equilibrium pricing model is. Fama notes that empirical tests of efficiency generally require assumptions about the pricing model, making them joint tests of efficiency and those assumptions; but his definition effectively
separates the theoretical concept of efficiency from the problems of testing it. Indeed, if the EMH itself included the phrase "with respect to [such-and-such a] model", then EMH tests would never be described as joint tests.

Jensen (1978) presented an EMH definition which is similar to that of Fama (1970), and which shares the latter's main shortcoming: both say that expected returns conditional on the information should be "right", without narrowing down what equilibrium required returns should be. Jensen described abnormal returns as opportunities for making "economic profits" on the information, opportunities which should not exist if the information is freely available (public).

Fama's (1976) revised definition (above) involved "the market" as an entity in itself, in such phrases as "the market uses...all the information" and "probability density function...assessed by the market". These phrases can be easily and unambiguously understood if investors have homogeneous information and beliefs, but not otherwise. (Fama (1976) did not elaborate on this usage.) For empirical and theoretical reasons, it would be better to have a market-efficiency definition that could be applied to a world of disagreeing investors. Actual investors do seem to disagree, and permitting this heterogeneity gives more generality in a theoretical model.

Among the several market-efficiency definitions suggested by Rubinstein (1974) and (1975), the one "closest in spirit to the implicit definitions used in most of the empirical literature" is that prices would not change if
all private information were publicized. This explicitly recognizes heterogeneity of information and beliefs in an equilibrium setting, but applies only to one information set: all information known. So it can be taken to define only strong-form efficiency. Following a suggestion by William Sharpe, Beaver (1981) extended this idea to apply to any information set: the market is efficient with respect to an information set if revealing that information to all investors (while they keep the information they already have) would not change equilibrium prices. As a trivial example of this, suppose all investors already knew the information. Then it would already be reflected in prices, and the thought experiment of then revealing it to them would change nothing. Clearly Sharpe-Beaver (hereafter S-B) efficiency holds under Fama's (1970) set of sufficiency conditions, quoted earlier; and it avoids the ambiguities of its predecessors. But how well does it fit the empirical literature?

Market efficiency is usually tested by comparing returns from a trading strategy, based on the information in question, with theoretical returns in an equilibrium model. S-B efficiency, however, focuses on prices at one point in time rather than on returns over time. The link is roughly that if future abnormal returns were reliably predicted by some information today, then

(a) an empirical test would indicate inefficiency; and

(b) if the information were revealed to all investors today, then the attempts to profit from the predicted abnormal returns would cause current prices to change, indicating (by definition) S-B inefficiency of the original (actual) equilibrium.
So S-B efficiency seems to fit well. But a closer look shows two interesting discrepancies. One is an intertemporal issue: the release of information to investors today could have a delayed impact, leaving today's prices unchanged, then affecting prices a week later. (Note that the information-release is a thought experiment, and does not take place in the actual economy of which we are assessing the efficiency.) As it stands, S-B definition would call the market efficient today with respect to that information, whereas an empirical test would presumably pick up abnormal returns a week later, and conclude that the information had not been fully reflected. This discrepancy is not explored in the present paper; the main issues I pursue can be analyzed in a one-period model.

The second discrepancy concerns heterogeneity of beliefs. The usual equilibrium return models used as benchmarks in empirical tests assume homogeneous beliefs, whereas the benchmark for S-B efficiency allows heterogeneous beliefs. Investors may still have diverse information after the specified information set (with respect to which we are testing efficiency) is published. This residual heterogeneity could certainly influence the configuration of prices and returns, thus implying a difference between S-B efficiency and the as-usually-tested version of market efficiency. Conventional testing of returns conditional only on the specific information is not designed to pick up this difference. Limitations on discriminating power caused by the large amount of noise in security returns would make it hard to detect anyway, even if we could design an equilibrium model that would allow for heterogeneous information that may be unobservable to the empirical researcher. So perhaps this discrepancy is of no practical inter-
est. However, it is important to ask which concept we are trying to test (and why). The present paper takes the position that a homogeneous-beliefs benchmark is merely a convenient and tractable approximation to the heterogeneous-beliefs benchmark of S-B efficiency, and that what we are really trying to test is something more like S-B efficiency. Hence the detailed examination of the properties of S-B efficiency in section 3 below.

Grossman and Stiglitz (1980) reinterpret the market-efficiency idea, as expressed by "prices fully reflect the [specified] information", to mean that prices reveal the information (or at least a sufficient statistic for it). That is different. The idea that investors should be modeled as gleaning information from equilibrium prices is well explicated in Grossman (1981), and is adopted in the present paper. But that is not the only way for information to be widely available; information (besides prices) could, for example, be published in the Wall Street Journal. And if all investors have a given piece of information, it is generally agreed (e.g., by Fama's sufficient conditions) that prices will fully reflect it, so the market will be efficient with respect to it. Yet even in such a case the information may not be discernible from prices alone (see example in section 3 below), so that by the Grossman-Stiglitz definition the market would not be efficient with respect to it. Conversely of course, if prices do reveal some information, then in a rational-expectations model all investors will have the information, implying market efficiency with respect to it by all definitions considered here. Clearly the Grossman-Stiglitz concept of efficiency is sufficient but not necessary for those of Fama, Jensen, and Sharpe-Beaver.
2.3 Is semi-strong efficiency being assumed?

It might seem that the S-B definition has a built-in assumption that the market is always semi-strong-form efficient: if some information is public then everyone knows it, so revealing it to everyone will change nothing. And yet the most common type of market-efficiency test in the empirical literature is of the semi-strong form, using public information. Why would they be testing something that is by definition true? Lest this be seen as another discrepancy between S-B efficiency and conventional notions, the following discussion reconciles the two, turning up some interesting side-issues in the process.

Consider the possible reasons why semi-strong efficiency might fail to hold:

(a) There are transaction costs.
(b) Some agents are not optimizing.
(c) Somehow not everyone knows the information.
(d) Not everyone knows how best to use the information, what its implications are.

In the presence of transaction costs, there are two fundamentally different ways to go about defining market efficiency, and one is more strict than the other. The stricter way is to compare the empirical prices (or returns) with the prices (or returns) in a theoretical economy in which everyone knows the information in question and in which there are no transaction costs. The less strict way is to try to build the same transaction costs into the theoretical economy as there are in the actual (empirical) economy. The way to choose is a function of what question you want to ask, and that issue leads beyond the scope of this paper. The point here is that the price-oriented S-B definition of efficiency can only be equivalent
to a conventional return-distribution-oriented definition if either both are
defined the stricter way or both are defined the less strict way. If transac-
tion costs are preventing the semi-strong EMH from holding in a traditional
return-distribution definition, then the model being used for the theoretical
equilibrium return distribution must not have the same transaction costs as
the empirical economy. Typically this would be because zero transaction
costs are assumed -- the stricter type of definition. The corresponding
"stricter" S-B definition would ask whether prices would change if the
specified information were revealed to all and transaction costs were zero.
Notice that this amended definition no longer automatically holds in semi-
strong form. Notice also that under this definition it is possible for a
market to fail to be efficient with respect to no-information, which does not
seem sensible for a definition of informational efficiency. This casts doubt
on the corresponding return-oriented definition, and suggests that the
less-strict approach is the preferable way to define market efficiency in the
presence of transaction costs. Perhaps we need a separate concept of
transaction-cost efficiency. The remainder of this paper is concerned with
other issues, so zero transaction costs will be assumed hereafter in both
the actual and the benchmark economies.

If some agents in the actual economy are not optimizing (whatever that
means), the way to deal with it is the same as for transaction costs. There
will be two ways to define efficiency, one stricter than the other; the
price-oriented definition and the return-oriented definition will only be
equivalent if both took the same approach, etc. The remainder of this
paper assumes optimizing behavior.
If somehow not everyone knows the specified information, then of course it's not truly public and cannot be used to refute semi-strong efficiency. Or if you prefer a weaker definition of "public" that permits a few agents to be ignorant of some public information, then the S-B efficiency definition no longer assumes efficiency with respect to public information.

Finally, consider the issue of not knowing how to use information. Suppose that public announcement of firms' earnings is associated with abnormal subsequent returns, and although everyone knows the earnings figures costlessly and immediately, they do not know about the association with returns. Even though all agents are optimizing (in some sense), their faulty information-processing permits the abnormal returns to persist. A typical event study would detect the abnormal returns, showing semi-strong-form inefficiency using a traditional return-distribution-type definition. But if you call this "inefficiency with respect to earnings announcements", and imagine the ideal thought-experiment test of the S-B definition, you will conclude that S-B efficiency is not violated, since revealing the earnings figures to everyone will change nothing. The equivalence of the two definitions seems lost. But once again, it is just because the theoretical comparison-economy is different: the return-oriented definition was using a theoretical equilibrium where agents in effect knew about the associated returns, whereas the S-B definition used a theoretical equilibrium where agents did not know that. The S-B definition can be amended to correspond to the traditional one by requiring that prices not change if all are told the information and its association with future events. There is still the question of how complex an association of current information with future returns investors can be expected to perceive, especially in a world
of changing probability distributions. But the present paper will not examine the distinction between information and how to use it. Henceforth information is assumed to come with an accurate guide for users.


I choose the simplest setting rich enough to demonstrate the necessary ideas. The model is more explicit and detailed than that of Beaver, and uses different notation. It is a pure-exchange one-period economy. At \( t = 0 \) there is no consumption; investors (indexed by \( i \)) just trade their endowments \( z_{ik} \) of securities (indexed by \( k \)) to arrive at equilibrium security holdings \( z_{ik} \). At \( t = 1 \) each security \( k \) pays a liquidating (terminal) dividend \( p^1_k \), and the investors consume those payoffs. The amount of each security's payoff at \( t = 1 \) is uncertain as of \( t = 0 \), but is known to depend on the "state of nature" at \( t = 1 \). There is a finite set \( \Omega \) of possible states. A typical state is denoted \( \omega \); a typical set of states is \( \mathcal{S} \subseteq \Omega \).

Investors have homogeneous prior beliefs \( \pi(\omega) > 0 \) for the probability of state \( \omega \) occurring.\(^ 4\) Associated with each investor \( i \) is a partition \( \Xi_i \) of the state space \( \Omega \). Before trading at \( t = 0 \), investor \( i \) receives information about the true state \( \omega \) by being told which set of her partition contains \( \omega \) -- that set is called \( S_i(\omega) \). \( \Xi^* \) denotes the join (coarsest common refinement) of the \( \{\Xi_i\} \), and thus represents all information known to anyone. \( S^*(\omega) \) is the set in \( \Xi^* \) containing \( \omega \); so if investors pooled their information, they could narrow down the true state to being contained in \( S^*(\omega) \).

The vector of \( t = 0 \) equilibrium security prices is \( p^0(\omega) \), with typical
element $P^0_k(\omega)$, written as a function of $\omega$ although it will only depend on $\omega$ through dependence on $S^*(\omega)$. In other words, $t = 0$ prices are determined not so much by the future (unknown) true state as by the information available today about the future state. The $t = 1$ payoff of security $k$ is $P^1_k(\omega)$.

The characterization of equilibrium used here is one of rational expectations with learning-from-prices, as in Radner (1979). The standard construction that all uncertainty is represented or captured by the states $\{\omega\}$ implies that each investor knows what the pricing function $P^0(\omega)$ is. She can therefore narrow down the true state by observing the actual equilibrium prices. (All investors do this simultaneously in the determination of equilibrium. See Grossman (1981) for a discussion.) Let $\Xi^P$ be the partition of $\Omega$ generated by $P^0(\omega)$, so that for any two states $\omega$ and $\zeta$, $P^0(\zeta) = P^0(\omega) \iff \zeta \in S^P(\omega)$. Then we can represent the information that investor $i$ uses at equilibrium in the $t = 0$ trading round by the join of $\Xi_i$ and $\Xi^P$, denoted $\Xi_i^E = \Xi_i \cup \Xi^P$. Her posterior probabilities are then $\pi(\zeta \mid S_i^E(\omega))$ where $S_i^E(\omega) = S_i(\omega) \cap S^P(\omega)$:

$$
\pi(\zeta \mid S_i^E(\omega)) = \begin{cases} 
\frac{\pi(\zeta)}{\pi(S_i^E(\omega))} & \text{if } \zeta \in S_i^E(\omega) \\
0 & \text{if } \zeta \notin S_i^E(\omega)
\end{cases}
$$

$$
\pi(S_i^E(\omega)) = \sum_{\zeta \in S_i^E(\omega)} \pi(\zeta)
$$

$\omega$ is the state that is going to occur at $t = 1$, but at $t = 0$ investor $i$ only has it narrowed down to $S_i^E(\omega)$, and so perceives probabilities $\pi(\zeta \mid S_i^E(\omega))$ for the various states indexed by $\zeta$. She wants to maximize her
expected value of her state-independent strictly-concave utility function $U_i(c_i)$.\textsuperscript{5} She considers various possible portfolio vectors $Z_i$ before choosing the best one denoted $Z_i(w)$. Being ignorant of $w$, she anticipates various possible $t = 1$ states $\zeta$, in which her consumption would be $c_i(\zeta, Z_i) = \sum_k z_{ik} p_k(\zeta)$; but God knows investor $i$'s consumption will be $c_i(w, Z_i(w))$, or simply $c_i(w)$.\textsuperscript{6}

An equilibrium is a pair of functions $\{Z(\cdot), P^0(\cdot)\}$ on $\Omega$ such that

(a) markets clear: $\forall k, w$: $\sum_i z_{ik}(w) = \sum_i \hat{z}_{ik}$;

(b) portfolios are affordably optimal: $\forall i, w$: $Z_i(w)$ maximizes

$\sum_\zeta \pi(\zeta | S_i^E(w)U_i(c_i(\zeta, Z_i))$ subject to the budget constraint

$\sum_k z_{ik} p_k^0(w) \leq \sum_k \hat{z}_{ik} p_k^0(w)$; and

(c) prices reveal no more than the aggregate of investors' information:

$\Xi^*$ refines $\Xi^P$.

Market efficiency with respect to an information-partition $\Xi'$ will be
defined by comparing the actual equilibrium with equilibrium in an otherwise-identical economy where each investor receives $\Xi'$ as well as $\Xi_i$. This equilibrium is denoted $\{Z(\cdot), P^0(\cdot)\}$. The partition generated by $P^0(\cdot)$ is $\hat{\Xi}^P$, and $\hat{\Xi}^E_i = \Xi_i \lor \Xi' \lor \hat{\Xi}^P$. So in the above equilibrium conditions, a caret ($\hat{\cdot}$) is added to $z_{ik}, Z_i, S_i^E, c_i$, and $p_k^0$. 

Table 1: Notation

\( \Omega \)  
the space of possible states of nature at \( t = 1 \)

\( \omega \)  
a state, usually taken as the one that is going to occur at \( t = 1 \)

\( \xi \)  
a state; may or not not occur at \( t = 1 \)

\( i \)  
index for investors

\( k \)  
index for securities

\( \bar{Z} \)  
endowment matrix \( [\bar{z}_{ik}] \)

\( Z \)  
equilibrium portfolio holdings \( [z_{ik}] \)

\( Z_i \)  
vector of portfolio holdings of investor \( i \)

\( p^0 \)  
price vector \( [p^0_k] = [t = 0 \text{ price of security } k] \)

\( p^1 \)  
payoff vector \( [p^1_k] = [t = 1 \text{ payoff of security } k] \)

\( c_i \)  
\( t = 1 \) consumption of investor \( i \); \( c_i(\omega) = \sum_k z_{ik}(\omega)p^1_k(\omega) \)

\( \xi, Z_i \)  
\( c_i(\xi, Z_i) = \sum_k z_{ik}p^1_k(\xi) = \text{payoff of portfolio } Z_i \text{ in state } \xi, \text{ at } t = 1 \)

\( U_i(c_i) \)  
utility of consumption of investor \( i \)

\( U, U'' \)  
first and second derivatives of \( U \)

\( \Xi_i \)  
investor \( i \)'s endowed information-partition of \( \Omega \)

\( \Xi^* \)  
the join (coarsest common refinement) of the \( \{\Xi_i\} \): \( \Xi^* = \vee \Xi_i \)

\( \Xi^P \)  
the partition of \( \Omega \) generated by \( p^0(\omega) \)

\( \Xi^E \)  
the join of \( \Xi_i \) and \( \Xi^P \): \( \Xi^E = \Xi_i \vee \Xi^P \) (This is investor \( i \)'s information-partition at equilibrium.)

\( \Xi^I \)  
an information-partition of \( \Omega \), with respect to which efficiency is being assessed

\( S_i(\omega) \)  
the set in \( \Xi_i \) containing \( \omega \)

\( S^*(\omega), S^P(\omega), S^E(\omega), \text{ and } S^I(\omega) \)  
definitions parallel that of \( S_i(\omega) \)

\( \pi(\omega) \)  
unconditional probability for \( \omega \); homogeneously perceived

\( \pi(S) = \sum_{\omega \in S} \pi(\omega) = \text{unconditional probability for set } S \)

\( \pi(\omega \mid S) = \pi(\omega)/\pi(S) = \text{probability for } \omega \text{ conditional on } S \)

\( ^\wedge \)  
indicates variable in an imaginary economy where each investor \( i \) receives \( \Xi^I \) before trading, as well as \( \Xi_i \)
In articulating the Sharpe-Beaver (S-B) "no-price-change" notion of market efficiency, Beaver raised the important question of whether to define efficiency with respect to a given outcome ("signal") of an information variable, or to define efficiency with respect to the ex-ante variable ("information system"). In his notation, at time $t$ investor $i$ receives $y_{it}'$, which is one realization of the many possible signals from his information system $\eta_{it}$. So $y_{it}'$ is like our $S_i$, and $\eta_{it}$ is like our $\Xi_i$. Efficiency of the market at time $t$ will be assessed with respect to either an information system $\eta_t'$ or a signal $y_t'$ from $\eta_t$. So $y_t'$ is like our $S_t'$, and $\eta_t'$ is like our $\Xi_t'$.

Beaver's (1981, p. 28) definitions are:

"$y$-efficiency --

A securities market is efficient with respect to a signal $y_t'$ if and only if the configuration of security prices $\{P_{jt}\}$ is the same as it would be in an otherwise identical economy (i.e., with an identical configuration of preferences and endowments) except that every individual receives $y_t'$ as well as $y_{it}'$.

$\eta$-efficiency --

A securities market is efficient with respect to $\eta_t'$ if and only if $y$-efficiency holds for every signal $(y_t')$ from $\eta_t'$.

For example, suppose we are looking at efficiency of the market on September 2, with respect to a firm's announcement on September 1 of the amount of an upcoming dividend. Many (random) events before September 1 will help determine what amount will be announced. If one is ignorant of those events (e.g., take the view from the preceding January 1), then there are many possible amounts that may be announced. But from the
viewpoint of an ex-post event study, only one amount was announced—say $2. \( \gamma \)-efficiency asks whether security prices on September 2 fully reflected the fact that $2 was announced. \( \eta \)-efficiency asks not only that but also whether security prices on September 2 would have fully reflected the amount that would have been announced if preceding events had been different. To illustrate the distinction, think of the proverbial stopped clock that tells the right time twice a day. The question "Is the market efficient with respect to this dividend announcement?" parallels the question "Is this clock correct?" Either question can be intended two ways. We could be asking whether the clock is showing the correct time at this instant, or whether it is dependably correct through time. The former is like \( \gamma \)-efficiency, the latter like \( \eta \)-efficiency. Indeed, \( \eta \)-efficiency could be called "dependable \( \gamma \)-efficiency".

Beaver leaves open the question of which definition is preferable, but \( \eta \)-efficiency seems closer to conventional notions. \( \gamma \)-efficiency does not fit well with the idea of no economic profits on the information, in the sense that that would imply no incentive to gather or purchase the information. The decision to acquire information must necessarily be made ex-ante, but \( \gamma \)-efficiency is an ex-post concept. Also, \( \eta \)-efficiency seems to fit better with empirical tests of market efficiency, which assume some stationarity in the underlying structure, and then compute statistics from a large sample. Such procedures are designed to support conclusions about efficiency with respect to an information variable or system \((\eta)\) rather than merely about an individual outcome of that variable \((\gamma)\). And finally, \( \gamma \)-efficiency does not have the "subset property".

One indication of the widespread view that the EMH has the subset
property has already been mentioned: the belief that strong-form efficiency implies semi-strong-form efficiency, and that semi-strong in turn implies weak-form efficiency. Even more frequent evidence is found in discussions of EMH tests. Semi-strong efficiency, for example, is never tested using all public information, but rather using a small subset of public information, such as earnings announcements. Associated abnormal returns are taken to imply semi-strong-form inefficiency; this deduction (and many similar deductions in other EMH tests) clearly assumes the subset property. If "economic profits" can be made given some information, then being given more information besides that should not diminish that opportunity, other things equal. Returning to γ-efficiency, one can imagine a piece of good news and a piece of bad news that, together, cancel each other out but, separately, would affect prices -- a subset-property violation.

For these reasons, γ-efficiency is here set aside in favor of a closer look at η-efficiency.

While the definition of γ-efficiency is clear, that of η-efficiency is ambiguous. Using the present paper's notation, in γ-efficiency the phrase "a securities market" clearly refers to one particular realization of everything at t = 0: P_{0}(ω), Z(ω), and especially their determinants \{S_{i}(ω)\} across i, are being thought of as specific realizations. What is not explicit in η-efficiency is how those random variables are being varied when different signals from η'_{t} are being considered. Equilibrium prices are being determined by other information besides η'_{t} (namely, by \{η'_{it}\}); is the other information being held constant as we look across the possible signals from η'_{t}?
There appear to be two possible meanings for Beaver's n-efficiency, one from answering "yes" to the above, the other from answering "no". Next we argue that there is no unambiguous way to hold all other information constant, so that "no" is the only fruitful answer.

You can never hold everything else constant. If you change the length of one side of a rectangle, you can't hold constant both the length of the other side and the rectangle's area. To remove the ambiguity, you must specify which two parameters you are using to describe the range of all possible rectangles. Then the convention is the same as when taking the partial derivative of a function of several variables: the remaining specified parameters are held constant. One could argue that rectangles are naturally parametrized by the two side lengths, but such arrangements are far less clear in more complex models.

For example, take the well-known paradox that in the basic Black-Scholes (1973) world the price of a call option goes up if the interest rate goes up. This is odd because the call is a claim on a possible future cash flow, so one might think that it, like stocks themselves (which are also claims on possible future cash flows), would drop when interest rates rise. The trick is that in the Black-Scholes pricing formula one of the parameters is the price of the underlying stock; so the standard way to assess the relationship between call price and interest rate holds the stock price constant. If the stock price were replaced as a parameter by something like expected future dividends, then both the stock price and the call price would probably be decreasing functions of the interest rate.

Another paradox, from microeconomic theory, is that an individual's
Marshallian demand function (relating a good's price to quantity demanded by him) is always different from his Hicksian demand function (relating the same things), even though his Marshallian demand always equals his Hicksian demand. In any given scenario he only has one demand figure for the good in question, but the same range of all possible scenarios is being parametrized two different ways: the Marshallian parametrization specifies prices and budget (which then determine the utility level), while the Hicksian specifies prices and utility level (which determine the budget). So each holds different things fixed when relating price to quantity demanded, thus giving different demand curves from the same array of demands by running across the array in different directions.

So "ceteris paribus" is always ambiguous, unless there is a natural or specified parametrization. There is none for η-efficiency, because the range of imaginable real-world information variables is so large. Consider the following illustration.

In a world with large numbers of securities and investors, let there be an industry comprising four firms, A, B, C, and D. All four make their quarterly earnings announcements on the same day T, but some of this information leaks out before that, and we are interested in assessing market efficiency on day T-1 with respect to total industry earnings that will be published on day T. At T-1 suppose five of the investors have somehow received the following information on the upcoming earnings announcements:
<table>
<thead>
<tr>
<th>Investor</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E_A + E_B + E_C + E_D$</td>
</tr>
<tr>
<td>2</td>
<td>$E_A$</td>
</tr>
<tr>
<td>3</td>
<td>$E_B$</td>
</tr>
<tr>
<td>4</td>
<td>$E_C$</td>
</tr>
<tr>
<td>5</td>
<td>$E_D$</td>
</tr>
</tbody>
</table>

So for example, investor 5 knows only what earnings firm D is going to announce, while investor 1 knows what industry earnings will be but not its breakdown.

In a given security-market realization, to assess $\gamma$-efficiency at $T-1$ with respect to industry earnings at $T$, we imagine publishing the industry earnings figure a day early to see if prices change. Fine. But to assess $\eta$-efficiency in the all-else-constant sense, we must imagine the test in all possible realizations in which all information except that industry's total earnings is held constant. The point here is simply that you cannot hold constant the information of investors 2 through 5 while changing that of investor 1. The five signals are not independent. And this example is the tip of the iceberg -- there are so many interdependent signals in the world that you can never change just one.

In theory you could get rid of this ambiguity by setting up a parameter system to describe the space of possible information outcomes. It would be a complicated task. And then assessments of market efficiency with respect to the same variable would give different answers depending on the choice of parameter system. A statement of market efficiency (or
inefficiency) would have to name both the relevant information item and the choice of parameter system -- a substantial departure from any previous usage of the term "market efficiency". If you can make economic profit from some information then that won't change when a modeler changes the parameters he uses to describe the same information.

So we seem forced to allow all information to vary, interpreting $\eta$-efficiency thus:

A securities market is efficient with respect to $\eta^t$ if and only if $\gamma$-efficiency holds with respect to the signal from $\eta^t$, in every possible realization of that securities market at time $t$.

Or, in our notation:

A securities market is $\eta$-efficient with respect to $\Xi^t$ if and only if $\hat{P}^0(w) = P^0(w)$ for all $w \in \Omega$.

Furthermore, since existence and uniqueness of equilibrium in such models as this is by no means guaranteed, the definition is given more precisely as:

An equilibrium is $\gamma$-efficient with respect to $\Xi^t$ if and only if there exists an equilibrium in the benchmark economy ($\Xi^t$ published) such that $\hat{P}^0(w) = P^0(w)$ for all $w \in \Omega$.

This interpretation of S-B efficiency is adopted for the remainder of this paper.
It turns out that S-B efficiency, even as interpreted above, does not have the feature that efficiency with respect to an information set implies efficiency with respect to each of its subsets. That is proved by the counterexample below. (\(\Xi_1\) is an "information subset" of \(\Xi_2\) if and only if \(\Xi_2\) refines \(\Xi_1\).)

The full counterexample is based on a less complex example in which the release of information causes trading but no price changes. Beliefs and information are homogeneous (across investors) throughout. There are two investors, one more risk-averse than the other; and two securities. Each investor is endowed with half a share of each security. One way to describe the securities and the information is that one security is riskier than the other, but at first no one knows which is which, so both investors simply hold their endowments. The information then released tells which is riskier, and the investors trade accordingly, each increasing her/his welfare, while the numbers are chosen so that the price ratio is always 1.

Here is a simple but technically incorrect tableau of payoffs and probabilities:
<table>
<thead>
<tr>
<th></th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security-1 payoff</td>
<td>$P_1^1(w)$</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Security-2 payoff</td>
<td>$P_2^1(w)$</td>
<td>3</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>No-information probabilities</td>
<td>$\pi(w)$</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>Probabilities given signal a</td>
<td>$\pi(w \mid a)$</td>
<td>.4</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>Probabilities given signal b</td>
<td>$\pi(w \mid b)$</td>
<td>.1</td>
<td>.4</td>
<td>.4</td>
</tr>
<tr>
<td>$[\pi(a) = \pi(b) = 0.5]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

That is incorrect because $a$ and $b$ cannot be represented by a partition of $\Omega$, as our model requires. Here is the correct version:

<table>
<thead>
<tr>
<th></th>
<th>$w_{1a}$</th>
<th>$w_{1b}$</th>
<th>$w_{2a}$</th>
<th>$w_{2b}$</th>
<th>$w_{3a}$</th>
<th>$w_{3b}$</th>
<th>$w_{4a}$</th>
<th>$w_{4b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1^1(w)$</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$P_2^1(w)$</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$\pi(w)$</td>
<td>.2</td>
<td>.05</td>
<td>.05</td>
<td>.2</td>
<td>.05</td>
<td>.2</td>
<td>.2</td>
<td>.05</td>
</tr>
<tr>
<td>$\pi(w \mid a)$</td>
<td>.4</td>
<td>0</td>
<td>.1</td>
<td>0</td>
<td>.1</td>
<td>0</td>
<td>.4</td>
<td>0</td>
</tr>
<tr>
<td>$\pi(w \mid b)$</td>
<td>0</td>
<td>.1</td>
<td>0</td>
<td>.4</td>
<td>0</td>
<td>.4</td>
<td>0</td>
<td>.1</td>
</tr>
</tbody>
</table>

Utility functions for $t = 1$ consumption are:

$$U_1(c) = -(8 - c)^2, \text{ for } c < 7;$$

$$U_2(c) = -(12 - c)^2, \text{ for } c < 7.$$
[\text{c}_i(w) < 7 \text{ for all } i, w \text{ in this example.}] \text{ The coefficient of absolute risk aversion (-}U''/U') \text{ is } 1/(8 - c) \text{ for investor 1 and } 1/(12 - c) \text{ for investor 2, so investor 1 is more risk-averse.}

Under no-information, it is clear by symmetry that no trading occurs and that the price ratio \( P^0_2/P^0_1 = 1 \). If signal a is announced, the equilibrium equations are easily solved to give \( P^0_2/P^0_1 = 1 \), \( z_{11} = z_{22} = 0.38 \), and \( z_{12} = z_{21} = 0.62 \); here security 1 is riskier, and is held more heavily by the less risk-averse investor, #2. If b is announced then security 2 is riskier, \( P^0_2/P^0_1 = 1 \), \( z_{11} = z_{22} = 0.62 \) and \( z_{12} = z_{21} = 0.38 \).

Thus the no-information economy is S-B-efficient with respect to the \{a, b\} information-partition, even though no-one knows the information. Prices (and conditional expected returns) are "right", although portfolio holdings do not reflect the information.

To examine the subset property we need two information-partitions, one of which refines the other. We modify the above example by introducing another partition, denoted \{c, d\}, which has as refinement a partition \{a, a', b, b'\} that is in effect equivalent to \{a, b\} above. Recall that under no-information, \( \pi(a) = \pi(b) = .5 \); signal c can be thought of as indicating that \( \pi(a \mid c) = 2/3 \) and \( \pi(b \mid c) = 1/3 \); similarly \( \pi(a \mid d) = 1/3 \) and \( \pi(b \mid d) = 2/3 \). The simple-but-incorrect version of the tableau is:
<table>
<thead>
<tr>
<th></th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1^1(\omega)$</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$p_2^1(\omega)$</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>$\pi(\omega)$</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>$\pi(\omega</td>
<td>a)$</td>
<td>.4</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>$\pi(\omega</td>
<td>b)$</td>
<td>.1</td>
<td>.4</td>
<td>.4</td>
</tr>
<tr>
<td>$\pi(\omega</td>
<td>c)$</td>
<td>.3</td>
<td>.2</td>
<td>.2</td>
</tr>
<tr>
<td>$\pi(\omega</td>
<td>d)$</td>
<td>.2</td>
<td>.3</td>
<td>.3</td>
</tr>
</tbody>
</table>

$[\pi(a) = \pi(b) = \pi(c) = \pi(d) = .5]$

To represent the above correctly, with \{a, b\} refining \{c, d\}, the state space is expanded and a partition \{a, a', b, b'\} is defined, where a and a' are the same except that a is associated with c while a' is associated with d. Likewise, b goes with d, and b' with c:
<table>
<thead>
<tr>
<th></th>
<th>(w_{1ac})</th>
<th>(w_{1b'c})</th>
<th>(w_{1a'd})</th>
<th>(w_{1bd})</th>
<th>(w_{2ac})</th>
<th>(w_{2b'c})</th>
<th>(w_{2a'd})</th>
<th>(w_{2bd})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1^1(\omega))</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>(P_2^1(\omega))</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(\pi(\omega))</td>
<td>0.1333</td>
<td>0.0167</td>
<td>0.0667</td>
<td>0.0333</td>
<td>0.0333</td>
<td>0.0667</td>
<td>0.0167</td>
<td>0.1333</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>a))</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>b'))</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>a'))</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>b))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>c))</td>
<td>0.2667</td>
<td>0.0333</td>
<td>0</td>
<td>0</td>
<td>0.0667</td>
<td>0.1333</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>d))</td>
<td>0</td>
<td>0</td>
<td>0.1333</td>
<td>0.0667</td>
<td>0</td>
<td>0</td>
<td>0.0333</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(w_{3ac})</th>
<th>(w_{3b'c})</th>
<th>(w_{3a'd})</th>
<th>(w_{3bd})</th>
<th>(w_{4ac})</th>
<th>(w_{4b'c})</th>
<th>(w_{4a'd})</th>
<th>(w_{4bd})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_1^1(\omega))</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>(P_3^1(\omega))</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(\pi(\omega))</td>
<td>0.0333</td>
<td>0.0667</td>
<td>0.0167</td>
<td>0.1333</td>
<td>0.1333</td>
<td>0.0167</td>
<td>0.0667</td>
<td>0.0333</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>a))</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>b'))</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>a'))</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>b))</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>c))</td>
<td>0.0667</td>
<td>0.1333</td>
<td>0</td>
<td>0</td>
<td>0.2667</td>
<td>0.0333</td>
<td>0</td>
</tr>
<tr>
<td>(\pi(\omega</td>
<td>d))</td>
<td>0</td>
<td>0</td>
<td>0.0333</td>
<td>0.2667</td>
<td>0</td>
<td>0</td>
<td>0.1333</td>
</tr>
</tbody>
</table>
Here \( \pi(c) = \pi(d) = .5 \), while \( \pi(a) = \pi(b) = 1/3 \), \( \pi(a') = \pi(b') = 1/6 \), \( \pi(a \mid c) = 2/3 \), \( \pi(a' \mid c) = 0 \), \( \pi(b' \mid c) = 1/3 \), etc.

The equilibrium prices and portfolios are most conveniently calculated by applying the equilibrium equations to the numbers in the previous "incorrect" tableau. Equilibria under no-information, signal a (and \( a' \)) and signal b (and \( b' \)), are the same as before. If signal c is announced, we get \( P^0_2/P^0_1 = 1 \), \( z_{11} = z_{22} = 0.46 \), and \( z_{12} = z_{21} = 0.54 \); if d then \( P^0_2/P^0_1 = 1 \), \( z_{11} = z_{22} = 0.54 \), and \( z_{12} = z_{21} = 0.46 \).

Here the subset property is being obeyed: the no-information economy is S-B-efficient both with respect to the \{a, a', b, b'\} partition and with respect to the (information-subset) \{c, d\} partition. But we can perturb a utility function to upset the latter while preserving the former. Consider the various possible levels of investor 1's consumption at \( t = 1 \), under the three different information settings (no-information, \{a, b\} and \{c, d\}):

<table>
<thead>
<tr>
<th>Investor-1 Consumption</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( w_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-information</td>
<td>2</td>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Signal a</td>
<td>2.24</td>
<td>1.76</td>
<td>6.48</td>
<td>5.52</td>
</tr>
<tr>
<td>Signal b</td>
<td>1.76</td>
<td>2.24</td>
<td>5.52</td>
<td>6.48</td>
</tr>
<tr>
<td>Signal c</td>
<td>2.08</td>
<td>1.92</td>
<td>6.16</td>
<td>5.84</td>
</tr>
<tr>
<td>Signal d</td>
<td>1.92</td>
<td>2.08</td>
<td>5.84</td>
<td>6.16</td>
</tr>
</tbody>
</table>
His utility function is $U_1(c) = -(8 - c)^2$. If we change $U_1(c)$ in a small region around $c = 2.08$, we can change the equilibria under signals $c$ and $d$ while preserving the equilibria under $a$, $b$, and no-information. We must maintain $U'_1 > 0$, but that still leaves plenty of scope for altering $U_1$ in the interval $2 < c < 2.24$. The old $U_1$ has $U'_1(2.08) = 11.84$. If the new $U_1$ has $U'_1(2.08) = 11.85$, then $P^0_2/P^0_1 = 1$ will no longer be the equilibrium price ratio under signal $c$: investor 1 would demand more of security 1 (and less of security 1) than before. So the new signal-$c$ equilibrium $P^0_2/P^0_1$ will be $> 1$; likewise signal $d$ will have $P^0_2/P^0_1 < 1$.

It is this final example, with the perturbed utility function $U_1(c)$, which shows that S-B efficiency does not have the subset property. The price of security 2 in terms of security 1, $P^0_2/P^0_1$, is 1 under no-information, and 1 under any of signals $\{a, a', b, b'\}$. So the no-information economy is S-B-efficient with respect to $\{a, a', b, b'\}$. But under signal $c$, $P^0_2/P^0_1 > 1$, and under $d$ it is $< 1$, so the no-information economy is S-B-inefficient with respect to $\{c, d\}$. Finally, $\{c, d\}$ is an information-subset of $\{a, a', b, b'\}$ because the latter refines the former, since $c = (a, b')$ and $d = (a', b)$. Therefore, S-B efficiency with respect to an information set does not imply S-B efficiency with respect to any subset.

There are several ways one could react to this shortcoming. One could forego the subset property; or one could change the efficiency definition in some fashion. The latter course is taken below.
4. **Proposed definition: "E-efficiency"**

It turns out that all examples of S-B efficiency's violating the subset principle involve an information set that would affect portfolios but not prices, like \{a, a', b, b'\} in the above example. So one way to salvage the subset property would be to deem it inefficiency whenever prices or portfolios change on publication of the information. That is the view of E-efficiency:

**Definition**

An equilibrium is E-efficient with respect to \(\Xi'\) if and only if there exists an equilibrium in the benchmark economy (\(\Xi'\) published) such that \(\hat{P}^0(\omega) = P^0(\omega)\) and \(\hat{Z}(\omega) = Z(\omega)\) for all \(\omega \in \Omega\).\(^8\)

We next examine the properties of E-efficiency, arguing that although it may seem a radical departure, it is nonetheless the most useful formal definition for understanding what is conventionally meant by market efficiency.

**Theorem 1** (subset property)

If an equilibrium is E-efficient with respect to \(\Xi'\), and \(\Xi'\) refines \(\Xi''\), then it is E-efficient with respect to \(\Xi''\).

**Proof**

What follows is based on the fact that if an individual knows that her decision, after receiving some extra information, would be the same across all possible outcomes of that information (and if she knows what that decision
would be), then she would make the same choice without that extra information.

Given that the actual-round equilibrium \( \{Z(\cdot), P^0(\cdot)\} \) satisfies both the actual-round equilibrium conditions and the imaginary-round-with-\( \Xi' \) equilibrium conditions, we will show that it also satisfies the imaginary-round-with-\( \Xi'' \) equilibrium conditions.

For the benchmark economy with \( \Xi'' \) (and not \( \Xi' \)) published:

(a) \( Z(\cdot) \) clears all markets.
(b) Take any \( w_i \), i. Budgets and prices are the same in all three rounds being considered, so investor \( i \) chooses \( Z_i \) from the same affordable set in all three optimizations. We want to show that \( Z_i(w) \) maximizes

\[
\sum_{\xi} P_i(\xi) \pi_i(\xi | S^E_i(w)) U_i(c_i(\xi, Z_i))
\]

where \( \tilde{S}^E_i(w) = S_i(w) \cap S''(w) \cap S^P(w) \). The partition \( \{\tilde{S}^E_i(w)\} \) is denoted \( \tilde{\Xi}^E_i \).

Now \( \Xi' \) refines \( \Xi'' \), so \( \tilde{\Xi}' \) refines \( \tilde{\Xi}^E_i \). So \( \tilde{S}^E_i(w) \) is the union of \( n \) sets in the partition \( \tilde{\Xi}^E \), where \( n \geq 1 \).

Suppose \( n = 2 \). Then there is some \( w_2 \in \tilde{S}^E_i(w) \) such that

\[
\tilde{S}^E_i(w) = S_i(w) \cup \tilde{S}^E_i(w_2).
\]
From the actual-round equilibrium, we know that $Z_i(w)$ maximizes

$$\Sigma_{\xi} \pi(\xi | S_i^E(w)) U_i(c_i(\xi, Z_i))$$

and $Z_i(w_2)$ maximizes

$$\Sigma_{\xi} \pi(\xi | S_i^E(w_2)) U_i(c_i(\xi, Z_i)).$$

But $w_2 \in S_i^E(w)$ implies $w_2 \in S_i^E(w)$, so that $S_i^E(w_2) = S_i^E(w)$. Furthermore, budgets and prices are the same for $w_2$ as for $w$. Therefore $Z_i(w) = Z_i(w_2)$ since they both solve the same maximization problem and $U_i$ is strictly concave.

Now $\Sigma_{\xi} \pi(\xi | S_i^E(w)) U_i(c_i(\xi, Z_i)) =$

$$\frac{\pi(S_i^E(w))}{\pi(S_i^E(w))} \Sigma_{\xi} \pi(\xi | S_i^E(w)) U_i(c_i(\xi, Z_i)) + \frac{\pi(S_i^E(w_2))}{\pi(S_i^E(w))} \Sigma_{\xi} \pi(\xi | S_i^E(w_2)) U_i(c_i(\xi, Z_i)).$$

$Z_i(w)$ maximizes each of two parts of the above sum separately, so it also maximizes the sum. The adaptations necessary if $n \neq 2$ are straightforward.

(c) $\Xi^*$ refines $\Xi^P$, so clearly the corresponding condition holds for the with-$\Xi^*$ economy: $\Xi^* \vee \Xi^*$ refines $\Xi^P$. 
Therefore the equilibrium \( \{Z(\cdot), P^0(\cdot)\} \) does not change on publication of \( \Xi'' \), and is thus \( E \)-efficient with respect to \( \Xi'' \).

Q.E.D.

Notice that Theorem 1 proves the claim that the only way to construct an example where \( S-B \) efficiency fails the subset test, is with an information set whose publication changes portfolios but not prices. For this to happen, the new information must affect each investor's portfolio demand function so that at the previously-prevailing prices, aggregate new purchase orders equal aggregate new sale orders for each security.\(^9\) Such a perfect balance is not robust to small perturbations in utility functions, as illustrated by the counterexample in section 3 above. So the set of instances where portfolios would change while prices hold constant is small, and likely to have zero measure under any reasonable measure definition.

Therefore \( E \)-efficiency and \( S-B \) efficiency are in practice very similar. \( E \)-efficiency implies \( S-B \) efficiency, and the converse would also hold but for a few exceptional cases. If we grant that \( S-B \) efficiency, with its focus on prices (and hence on returns), is appropriate for interpreting the existing return-oriented empirical literature, then we must grant that \( E \)-efficiency is likewise appropriate. Empirical tests never prove market efficiency. They either reject it or fail to reject it. \( E \)-efficiency is (slightly) stronger than \( S-B \) efficiency, so any test that rejects the latter also rejects the former, and anything that tests the latter tests the former. The extra strength of \( E \)-efficiency merely opens opportunities for further tests, that might not have been relevant for testing \( S-B \) efficiency. These would be tests of portfolio revision, and thus of trading volume.
Theories of what determines trading volume are far less developed than
theories of what determines security returns, but the methodologies and
pitfalls of testing would be quite parallel. To find evidence of inefficiency,
a researcher would, for example, look for public information that predicted
abnormal volume at some future date. The idea is that in a perfectly
efficient capital market, trading on new information should occur promptly,
and volume should return to "normal" levels after that. Of course the
problems here are of what determines normal volume, and of whether the
information in question is correlated with the determinants of volume. If it
were, then some degree of predictability of volume fluctuations would be
consistent with E-efficiency.¹ This parallels the problem of testing effi-
ciency by correlating future returns with an information variable that might
be a "yield-surrogate", as discussed in Ball (1978). Volume tests are likely
to be less dependable (less powerful) than yield tests, however, because it
would be difficult to acquire and use data on who is buying and who is
selling. If public information predicts a too-high yield, then unambiguous-
ly, someone should have been buying who wasn't; if it predicts a too-high
trading volume, that could be explained by existence of some investors who
know they should do some trading but need some more (subsequent) informa-
tion to know who should buy and who should sell.

E-efficiency may seem to be too exacting a standard to those who are
used to thinking of capital-market efficiency in terms of returns and prices.
All it takes to make the market E-inefficient is one uninformed small inves-
tor holding the wrong portfolio. Market inefficiency is traditionally asso-
ciated with profit opportunities, and yet surely if everyone except one small
investor is rational and well-informed then there would be no profit oppor-
tunity. The standard rationale for why the market would tend to be efficient when many investors make poor portfolio decisions, is that there are many intelligent investors who would eliminate any inefficiencies by trying to profit from them, pushing prices into line. In this light E-efficiency looks very different from earlier notions.

But we are just being confused by a question of degree. We are used to thinking of each individual investor as a price-taker -- such a microscopically small part of the market that his actions do not affect prices. But each one's actions do affect prices, by an amount that is closely linked to the size of his portfolio as a proportion of all assets. Except for a measure-zero set, if the economy's assets are allocated slightly "wrongly", then prices (and hence future returns) will be slightly "wrong". Even if the intelligent investors' superior information is costless, and even if they are numerous and act competitively, they will not in general drive prices all the way to where they would be if everyone were well informed. It may be hard to detect such a small aberration, but that is an empirical issue.

Therefore S-B efficiency is not significantly more exacting than return- or price-oriented definitions such as S-B efficiency.

For various reasons it may be desirable to devise a notion of "market near-efficiency", to enable us to measure, discuss, and analyze how close the market is to being perfectly efficient with respect to some given information. The present paper does not attempt this; E-efficiency is an extreme ideal. While E-efficiency does not in general obviate private incentives to acquire information (see section 5 below), it will do so in many
cases; if that information is costly, then for those cases the equilibrium cannot become E-efficient given rational investors, as pointed out by Grossman and Stiglitz (1980). Near-efficiency might then be a useful concept.

5. The private value of public information

Market efficiency has always been closely associated with questions of information-value. As Beja (1976, p. 7) mentions, "... it is generally accepted that if markets are efficient with respect to some information investors need not use ... this information in the determination of their optimal market position." If that were true, then no investor would be willing to pay to acquire the information. Grossman and Stiglitz (1980) point out a consequence of this, that the market can never be efficient with respect to information that is costly.

The truth of these propositions will depend on which market-efficiency definition is used. As mentioned as section 2.2 above, Grossman and Stiglitz adopt the unorthodox interpretation that market efficiency means that equilibrium prices reveal the information. If some information will be revealed by prices then in a rational-expectations model no investor would pay to acquire it -- it is available to all for free. (This does not mean, however, that the information is not used; it may be very useful -- see Grossman (1976) for an example.)

The verdict is not so clear for E-efficiency. The question of whether an investor should ever pay to acquire information with respect to which the market is E-efficient, can be asked in two distinct ways. Notice first
that E-efficiency says that at equilibrium every investor has a sufficient statistic for the information, since revealing it would change neither portfolio nor prices. So if a too-high cost makes an investor choose not to acquire some relevant information, then the market can not become E-efficient with respect to it and the question can not be asked about that information.

But the question can be more tractably interpreted in a partial-equilibrium analysis. Suppose the market is E-efficient with respect to Ξ', except for one microscopic investor. That is, if Ξ' were revealed to all other investors, the configuration of equilibrium prices and portfolios would be unaffected. Ignore the tiny effect he may have on prices and the similarly-tiny effect he may have on other investors' portfolios. Then we can ask whether he might benefit from acquiring Ξ' even at a positive cost.

The answer is yes, he might. As Sharpe (1981, p. 538) points out:

"Even in a perfectly efficient market there is work to be done. Investors do differ in circumstances, portfolios should be tailored to accommodate such differences, and successful performance of this task generally requires estimation of the sources and magnitude of risk and return."

For example, consider a CAPM world where everyone is equally well informed except you, who are poorly informed. The two-fund separation theorem implies that you should not pay for information on stock selection; you already know you should hold the market portfolio. But information that helps you estimate the mean and variance of the market portfolio's return is valuable to you, even if everyone else knows it. And it is not hard to
construct examples where such information would not be revealed by prices. (In this connection, it is interesting to note that almost all market-efficiency tests study stock-selection strategies, and are not concerned with information on the mean and variance of the market.)

Therefore E-efficiency with respect to $\Xi'$ does not imply that a (microscopic) investor should not pay to acquire $\Xi'$. And that applies also to S-B efficiency and Fama's and Jensen's earlier notions, since they too would deem the market efficient if everyone (except perhaps a microscopic price-taker) knew the information.

6. Conclusions and open questions

A new formal definition of capital-market efficiency is proposed: the market is "E-efficient" with respect to an information set if and only if revealing that information to all investors would change neither equilibrium prices nor portfolios. In addition to providing a framework suitable for interpreting the existing empirical market-efficiency literature, E-efficiency captures the spirit of previous theoretical definitions, as in Fama (1970) and (1976), Jensen (1978) and Beaver (1981), while avoiding some of their drawbacks. E-efficiency can be defined relative to any chosen information set, even when information and beliefs differ across investors. And it has the widely-accepted subset property: E-efficiency with respect to an information set implies E-efficiency with respect to any subset. However, neither E-efficiency nor any conventional notion of market efficiency has the property of implying zero private value of the information in question.
A precise explicit definition of market efficiency also provides the basis necessary for the analysis of many important questions: Is this informational efficiency necessary or sufficient for productive or allocative efficiency? How should degrees of near-efficiency be defined when information is costly? Can trading volume be used to test market efficiency? How should efficiency be defined in an intertemporal setting? How important is it to allow for heterogeneous information in security-return models used as benchmarks for empirical tests? Even partial answers would contribute significantly to our understanding of what is probably the most widely studied concept in finance.
FOOTNOTES

* Financial support from the Social Sciences and Humanities Research Council of Canada; the University of California, Berkeley, School of Business Administration; and the Berkeley Program in Finance is gratefully acknowledged. For valuable discussion, I thank participants in finance seminars at Massachusetts Institute of Technology, University of California at Berkeley, and Stanford University. Suggestions from Fischer Black, Robert C. Merton, and James Ohlson were particularly helpful.

1. Several of these shortcomings have been pointed out before, such as in LeRoy (1976).

2. For example, Malkiel (1981).

3. "Public" information can be (and has been) defined in several non-equivalent ways, such as:
   (a) known to all investors;
   (b) known to "most" investors;
   (c) available at "low" cost to all investors.
   The present paper minimizes this confusion by making little or no significant use of the term.

4. The results are easily generalized to heterogeneous beliefs, but keeping them homogeneous makes for simpler notation.
5. State-independent utility is assumed for simplicity. The results generalize to state-dependence by replacing $U_i(c_i(\xi, Z_i))$ with $U_i(\xi, Z_i)$.

6. Readers who are atheists may simply assume the existence of God in this model.

7. Another counterexample (in the given information and security regimes) is:

$$U_1(c) = \log(c)$$
$$U_2(c) = -(8.02613 - c)^2.$$

8. "E" is for equilibrium, since not only equilibrium prices but also equilibrium holdings are considered. Rubinstein (1975, p. 823) suggested imposing such a condition on portfolios.

9. Technically, the benchmark for both S-B and E-efficiency is an equilibrium resulting from publishing $\Xi^t$ and trading from the original endowments of the actual economy. But Latham (1984) proves that equivalent definitions (to both S-B and E-efficiency) can be formed by letting the benchmark economy's endowments be the actual economy's equilibrium holdings. So you can think of assessing efficiency by publishing $\Xi^t$ immediately after the actual round of trading and allowing re-trading.

10. For example, the costs and inconvenience of immediate trading may cause a lagged and sluggish portfolio-revision response to new information, even if all investors get the information immediately.
REFERENCES


Latham, Mark, 1984, Essays on information and equilibrium in security markets, Ph.D. dissertation, Sloan School of Management, Massachusetts Institute of Technology.


