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OPTIMAL INSURANCE OF THE COMMON FORM UNDER MORAL HAZARD

BY

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Optimal Insurance of the Common Form
Under Moral Hazard

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OPTIMAL INSURANCE OF THE COMMON FORM

INTRODUCTION

The optimal form of insurance under moral hazard\(^1\) has been studied within the framework of the principal-agent problem, particularly by Spence & Zeckhauser [1971], Marshall [1976], who considered important special cases in medical insurance, Harris & Raviv [1979], Shavell [1979a & b], and Mirrlees [1979]. The optimal form without moral hazard has been analyzed by Arrow [1974], Raviv [1979], Blazenko [1985], who commented on Raviv’s work, and Schlesinger [1981]. Much of this work has centered on finding the second-best Pareto-optimal form of contract in a very general but unspecified class of contracts or functions. This paper also uses the principal-agent framework, but focuses on the optimal properties and parameters of the particular second-best contractual form most commonly used in insurance markets under moral hazard.

In most insurance situations the insured (she) can take some action (e.g., spending to control losses) for which one of these conditions holds: (a) the action is costly to her (and/or she dislikes more of it), and it causes a first-order, stochastically-dominant improvement in the probability distribution over the frequency and/or severity of loss; or (b) the action yields her money (and/or she likes more of it), and it causes such a first-order degradation in the distribution. In many cases it is too expensive for the insurer (he) to obtain reliable information on her action, because it is obscured by the effect on the outcome of the stochastic state-of-nature (which he also cannot observe). This precludes a first-best contract, which specifies the premium, the coverage, her action, and a Ross penalty she pays if the specified action is not taken.

Consider situation (a). Suppose that the first-best insurance contract were offered

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\(^1\) Moral hazard here refers to the insurer’s (principal’s) inability to verify the insured’s (agent’s) action to control losses when this action affects the insurer’s wealth under the contract. Others define it as the difference between the insured’s action to control loss when the action is verifiable and when it is not. The association of asymmetric information with “moral hazard” is misleading. The insured (she) is assumed to be a von Neumann-Morgenstern expected-utility maximizer. The insurer (he) is assumed to know both her preferences and her beliefs about stochastic losses. Hence, under any given contract, he can calculate her action in advance. This means there is no asymmetric information, just asymmetric observability of the action he knows is occurring.
without verification of her spending, and that the coverage proportion did not exceed 100%. In general, she would take less action than specified in the first-best contract. From the insurer's perspective, this would worsen the probability distribution over benefits that would be paid by him. If she spent more, then a contract specifying more expenditure would have made him better off in the first-best situation, contradicting the initial contract being first-best. An analogous statement not restricted to the common form also holds.

In practice and theory, severe moral hazard of this sort leads the parties to a second-best contract, which is a function of the loss alone. Generally, both parties could be made better off if they could verify the insured's expenditure and thus implement a first-best contract. The optimal second-best contract usually differs from the first-best in: premium, coverage, and induced expenditure of the insured. Each parameter of the optimal second-best contract could be higher or lower than that of the first-best, depending on the parties' risk aversion and the technology of loss reduction via expenditure. A higher or lower value for each parameter could induce more or less spending. Again, an analogous statement with the contract not restricted to the common form also holds.

Most of the papers approach moral hazard through the first-order conditions but do not consider the second-order conditions. They also neglect to specify either a space of functions to maximize over or a norm for the space. These oversights prevent a rigorous definition for the derivatives used. An exception is the work of Clarke & Darrough [1980] which rigorously describes a space and norm for the principal-agent problem.

The second-order conditions accompanying the first-order approach were first treated by Mirrlees [1975]. Grossman and Hart [1983] developed an innovative and rigorous approach to the general principal-agent problem. Rogerson [1985] found a class of sufficient conditions under which the first-order approach is valid.

Raviv's [1979] enlightening analysis of insurance contracts was greatly simplified by its treating only situations in which no insured's action influenced losses, i.e., situations without moral hazard. This simplification yielded sharper results than had been obtained previously. He urged further study of insurance situations in which the insured privately influences losses, i.e., situations with moral hazard. Raviv also warned of the
mathematical difficulties in this analysis. This paper does such analysis; however, it is restricted to a particular form of contract.

Under this contractual form, there are three parameters agreed upon: premium, coverage proportion, and deductible. Before the loss is observed, the insured pays the insurer a premium; after the loss is observed by all, he pays her the benefit equal to the coverage proportion of that part of the loss, if any, which exceeds the deductible.

Limiting our consideration to contracts of this form reduces an infinite-dimensional analysis to a three-dimensional analysis. This simplification allows us to avoid the mathematical problems already mentioned and the consequent special assumptions that are very restrictive.

This simplification also allows more specific and hopefully more operational results than would be available in a very general space. In particular, it facilitates comparative statics analyses of both her optimal behavior (choice of spending to control losses) and his optimal behavior (choice of contract to offer) in light of what her behavior will be. These analyses uncover a partition of the fundamental forces at work in this nested maximization behavior that she and he perform. These forces link both the contract and the model's exogenous circumstances to the parties' expected utility.

The indirect fundamental forces act through intermediate steps. The direct ones do not. The first indirect force is the reaction force, which occurs through the insured's choice of spending. The second is the moving trigger force, which occurs through the division of the states-of-nature between values causing uncovered losses (losses not above the deductible, thus not triggering benefits) and values causing covered losses (above the deductible, thus triggering benefits).

Some insight from this modest analysis may help with problems of greater generality. The results may also be useful to the insurance industry since most of its contracts are of the form treated here. But this work does not exhaust the analysis of such contracts. It merely formulates a model for them and proves some basic results. Two issues are left to subsequent papers: default; and useful conditions, if any, under which the optimal contract offers the insured an expected utility greater than her alternative utility. Since there may be such conditions, we examine both Case A, in which her alternative does not bind the
solution to the optimal contract, and Case B, in which it does.  

The remainder of this paper has seven sections and four appendices. The model is specified in Section I; the first appendix lists the notation. Sections II and III describe the optimization problems of the insured and of the insurer, respectively, and contain existence and uniqueness results. The cost of moral hazard for Case B is expressed as the difference between his second-best ex-ante expected utility and his first-best.

A comparative statics analysis of the insured's problem appears in Section IV. This analysis is done for shifts under Case A in: each of the three contractual parameters, pure uncertainty over losses, optimism over losses (second-order stochastic dominance), and risk aversion. This analysis is repeated for compensated shifts under Case B, in which the premium changes with the shift to maintain her expected utility equal to her alternative.

Section V characterizes the insurer's optimal contract in both cases. A comparative statics analysis of the insurer's problem is done for Case B in Section VI. It provides the signs of changes in the optimal coverage proportion and deductible resulting from compensated shifts in: pure uncertainty, optimism over losses, her alternative utility, and his risk aversion. The comparative statics analyses and characterizations of the optimal contract are found in terms of the model's exogenous circumstances. Section VII contains conclusions and a chart comparing results under Case B for the first-best contracts (without moral hazard) with those for the second-best contracts (with moral hazard).

I: THE MODEL

Consider a single-period model at the end of which a person or firm, known as the insured (she), suffers a loss, \( x \). At the beginning of the period, another person or firm, known as the insurer (he), offers a contract \( I(.) \), to share \( x \) in exchange for a premium. If she agrees to \( I(.) \), she then chooses and spends an amount, \( \sigma \), to control losses. Following this, an exogenous and stochastic state-of-nature, \( \theta \), occurs, immediately resulting in an \( x \)-value. He then pays any amount due her under \( I(.) \).

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2 Thanks to William P. Rogerson for his helpful advice on the difficulties of determining if the insured's alternative utility is always binding on the optimal contract.
This \( x \) is uncertain because it is a function of both \( \alpha \) and the stochastic \( \theta \); i.e., \( x = X(\alpha, \theta) \). All uncertainty is in \( \theta \) and neither party can observe it. Further, he can never verify \( \alpha \). Therefore, any \( I(.) \) he offers must be a function of \( x \), since it is the only variable that is mutually verifiable. The moral hazard of this model is due to the masking of \( \alpha \) from the insurer by \( \theta \), i.e., by the unobservability of \( \theta \) and the nonverifiability of \( \alpha \).

Both parties know the loss-reduction technology, \( X(., .) \); and he knows that they share a common subjective probability distribution, \( H \), over \( \theta \). However, the model is robust to a different \( H \) for her that is known to him. He also knows both her utility function, \( U(.) \), over wealth, \( w \), and the value of her alternative utility, \( U \).

The form of \( I(.) \) over \( x \) that we treat here is the most common one used in insurance markets. Under \( I(.) \), she pays a premium, \( p \), to him before \( \theta \) occurs (i.e., \textit{ex-ante}). After it occurs (i.e., \textit{ex-post}) he pays her the benefit owed, which is a coverage proportion, \( c \), times that part of \( x \), if any, that exceeds a deductible, \( d \). This part is called the covered loss and is written \( X - d \). For any \( \theta \) that occurs, \( x = X(\alpha, \theta) \) is realized, which under \( I(.) = (p, c, d) \) implies that her change in wealth is \( w \), where:

\[
w = p - x - X(\alpha, \theta) \text{ if } X(\alpha, \theta) \leq d, \text{ and } w = -p - x - X(\alpha, \theta) + c[X(\alpha, \theta) - d] \text{ otherwise.}
\]

For brevity, we write equivalently \( w = -p - x + (cD - 1)(X(\alpha, \theta) - dD) - dD \), where \( D = 0 \) if \( \theta \in \Theta'(\alpha, d) \), and \( D = 1 \) otherwise. This \( \Theta' \) is defined in the next paragraph. The proportion of \( X \), that is not to be recovered in benefits (and thus changes her wealth) is the copayment proportion, and it is written \( cD - 1 \). For any \( \theta \) which occurs, his change in wealth is \( \pi \), where:

\[
\pi = p \text{ if } X(\alpha, \theta) \leq d, \text{ and } \pi = p - c[X(\alpha, \theta) - d] \text{ otherwise.}
\]

The \( \theta \)-level that causes the deductible level of loss, \( d \), for any given \( \alpha \), is defined as \( \Theta'(\alpha, d) \in \mathbb{R} \) such that \( X[\alpha, \Theta'(\alpha, d)] = d \) for all \( \alpha \) and \( d \). Functions evaluated at \( \Theta' \) will be denoted with an apostrophe mark, \( ' \). For example, \( x' = X[\alpha, \Theta'(\alpha, d)] = d \) and

\[
U' = U(w') = U(-p - x + (cD - 1)(X(\alpha, \Theta'(\alpha, d)) - dD) - dD) = U(-p - x - d) \text{ since } D[\Theta'(\alpha, d)] = 0.
\]

This \( U' \) is her utility under \( I(.) \) when the loss equals exactly \( d \). When \( (p, c, d) \) is understood, \( w \) may be written as \( w = w(\alpha, \theta) \) and \( \pi \) as \( \pi = \pi(\alpha, \theta) \).

The total derivative of a function, with respect to a variable, will be denoted by subscripting the function with the variable of differentiation, and the partial derivative by
subscripting with the number of the function's argument that is the variable of
differentiation. Second derivatives will have two subscripts. Expected values are denoted
by an E; e.g., Eθ, Ew, EX, EU, & EV. For example, EX_uU_w is her expected marginal
utility, weighted by the marginal loss reduction per α (Note E applies to the product of
X_uU_w.); and EUD is the expected utility over θ-values only in the covered-loss range,
i.e., for θ > θ'(a,d) or equivalently for X > d. We write EUD = θ∫U_w(α,θ)dH(θ).

For any θ-value, X is constant in α or X falls in α (i.e., any rise in the insured's
expenditures reduces loss or has no effect) at a rate which does not rise in magnitude as α
rises; i.e., X_α ≤ 0 ≤ X_αα. But we assume X_α < 0 for positive measure, i.e., for a set
of θ that has positive probability of occurring. In other words, the insured's expenditures
control losses by inducing a weak first-order, stochastically-dominant change in the
probability distribution over losses, which is strong for some set of possible θ. Similarly,
X is constant in θ or X rises in θ, and X_θ > 0 for positive measure.

She and he have preferences only over changes in their own wealths, w and π; and
they are von Neumann-Morgenstern expected-utility maximizers. Her utility, U, rises in w
at a decreasing rate; i.e., she is risk-averse and U_w > 0 > U_ww. His utility, V, rises in
π at a nonincreasing rate; i.e., he is not risk-loving and V_π > 0 > V_ππ. He will offer an
I* only if the EV under it exceeds his EV from not contracting, V = EV(0). To ensure
differentiability of EU_α, let H be any cumulative distribution function except one with a
point mass at θ* = θ'(a*,d). See the Notational Appendix for a list of symbols.

II: THE INSURED'S PROBLEM

Given any I(.), the insured's problem is to calculate the value of α that maximizes her EU
over her change in wealth, w, as per (1) below. Thus α-value is written as ̄α or α*. We
also write α* = A(p,c,d) = argmax EU over α ∈ (p,c,d). Functions evaluated at α = α*
will be denoted by "*", e.g., EU* = EUw(α*,θ) & dH* = dH(θ'(α*,d)). After
calculating α*, she compares her EU* with her alternative (to contracting) utility, U.

If EU* > U, as per (2), she accepts I(.), pays him p, and spends α*. Then a
θ-value occurs which immediately yields X* = X(α*,θ). This in turn causes him to fulfill
I(.) by paying her \( c(X^*-d)D \), which leads to her utility of \( U[(\alpha^*, \theta)] = U[-p - \alpha^* - X^* + c(X^*-d)D] \). If \( EU^* < U \), she rejects \( I(.) \) and has utility \( U \). Note that \( U \) must be at least the EU from self-insuring, i.e., from \( I = (0,0,d) \), which is \( EU[X(A(0,0,d),\theta) - A(0,0,d)] \). Thus \( EU^* \geq U \) is a self-imposed constraint on her acceptance of \( I(.) \); it does not constrain her choice set, which consists of all \( \alpha \geq 0 \).

Differentiating (1) by \( \alpha \) yields (3) and differentiating (3) by \( \alpha \) yields (4). This uncovers three fundamental forces at work in the interrelation of his and her behavior.

These forces link shifts in the model’s exogenous circumstance or the contract to changes in \( EU \) and \( EV \). The first two forces are indirect because they act through intermediate steps. They often are mixed in the same term. The first is the reaction force which acts through her choice of \( \alpha^* \). It is based on \( A_\tau \) in Case A and \( B_\tau \) in Case B, for shift parameter \( \tau \). The second is the moving trigger force, \( T \), which changes \( \theta' = \theta'(\alpha,d) \), the \( \theta \)-value yielding \( X(\alpha,\theta) = d \), i.e., triggering benefits, given such a shift and \( \alpha^* \). This value bisects the possible \( \theta \) into those causing an uncovered loss, \( \theta \leq \theta' \) (i.e., \( \theta \) such that \( X(\alpha,\theta) = d \)), and those causing a covered loss, \( \theta > \theta' \) (\( \theta \) such that \( X(\alpha,\theta) > d \)). The third force links the shifts directly to \( EU \) and \( EV \), not through intermediate steps.

\[\begin{align*}
(1) \quad \text{MAX} & \quad \alpha \geq 0 \quad \text{EU} \quad \text{where} \quad EU = EU[w(\alpha,\theta)] = \\
& \quad e'(\alpha,d) + \int U[-p - \alpha - X(\alpha,\theta)]dH(\theta) + \int U[-p - \alpha + (c-1)(X(\alpha,\theta) - d)]dH(\theta) \quad \text{subject to} \\
& \quad \quad \theta'(\alpha,d) \quad \theta'

(2) \quad EU^* \geq U.

(3) \quad EU' = EW' \quad \alpha \quad = - \int (X_\alpha + 1)U' \quad w \quad dH - \int (1-c)X_\alpha + 1)U' \quad w \quad dH = E(cD-1)X_\alpha \quad w \quad - EU'

(4) \quad EU'' = EW'' \quad \alpha \quad + EW'' \quad \alpha \quad = cX \quad T.
\]
pairs with respect to any variable sum to zero in both EU and EV. But such pairs in the second derivatives of EU and EV do not, in general. The moving trigger force, \( T = X_\alpha^2 E_U^x dH^{xx}/X_\alpha^x \), appears in (4) and is the sum of the derivatives of the limits in (3).

Note that \( w_{\alpha} = (cD-1)X_\alpha - 1 \) and \( w_{\alpha\alpha} = (cD-1)X_{\alpha\alpha} \leq 0 \) for \( c < 1 \) by our sign assumptions.

Our model’s assumptions imply that \( T < 0 \). Most shifts in exogenous circumstances of the model change \( \theta \). This changes the solution to her problem (behavior), \( \alpha^x \) the ARGMAX of (1), since it changes the border between \( \theta \) with a copayment proportion of one (uncovered losses), and such of \( 1-c \) (covered losses). Thus it changes the weights between the two integrals in (1), and in (3). Similarly it his affects behavior.

The local optimization conditions for the insured’s spending to prevent losses, \( \alpha^x \), are expressed below where \( EU^x_{\alpha} \) is described in (3) above.

\[
(5.1) \quad \alpha^x \geq 0 \quad \text{and} \quad EU^x_{\alpha} = 0 > EU_{\alpha\alpha} \quad \text{OR} \quad (5.2) \quad \alpha^x = 0 > EU_{\alpha}.
\]

In (5.2) the nonnegativity of \( \alpha \) is binding, and in (5.1) it is not. The solution of (5.2) involves pure risk-sharing with no incentive for any \( \alpha \). That is, for all \( \alpha \geq 0 \), her rise in expected utility from the loss reduction caused by \( \alpha \), \( E(cD-1)X_\alpha E_U \), is less than the magnitude of the fall in her expected utility from the spending of \( \alpha \) itself, \( -EU_{\alpha} \). Thus \( \alpha = 0 \). That solution is analyzed by Raviv [1979] and Blazenko [1985] without restricting the form of \( I(.) \). The solution to (5.1) has risk-sharing and the incentive for nonnegative \( \alpha \), because there is an \( \alpha \geq 0 \) such that \( E(cD-1)X_\alpha E_U = -EU_{\alpha} \). We treat (5.1) in this paper.

**Lemma 1** There exists an optimal spending level, \( \alpha^x \), to control losses if the coverage proportion does not exceed one. It is unique if either \( Ew_{\alpha}^x E_{UW} < \alpha X_{\alpha}^x T \) for all possible \( \alpha \) or \( dH^{xx} = 0 \).

**Proof** Our model assumes \( 0 < x < +\alpha \) and \( X_\alpha < 0 < U_{\alpha} \). Hence \( \lim X_\alpha = 0 \) as \( \alpha \to +\infty \) since \( X \) is bounded below and is monotonically decreasing in \( \alpha \). Then (3) \( \Rightarrow \lim EU_\alpha = \lim -EU_{\alpha} \) as \( \alpha \to +\infty \). Because \( U_{\alpha} > 0 \) for all \( \alpha \) and \( \theta \), we know that \( \lim -EU_{\alpha} < 0 \). Hence \( \lim EU_\alpha < 0 \), and thus, there exists some finite \( \alpha^0 \in (0, +\infty) \) such that \( EU_\alpha(\alpha, \theta) < 0 \) for all \( \alpha > \alpha^0 \). Since our model specifies \( \alpha \geq 0 \), \( \alpha^x \) is limited onto \( [0, \alpha^0] \). Because
EU is continuous, a finite maximum is achieved. The hypothesis of \( c < 1 \) implies that \( \omega_{\alpha} < 0 \), which, with the model's sign assumptions, \( \rightarrow \) only the third term of (4) can be positive. Hence \( EU_{\alpha} < 0 \) if the final hypothesis holds. Q.E.D.

III: INSURER'S PROBLEM

The insurer's problem is to calculate, as per (6), the \( I = (p, c, d) \) that maximizes his ex-ante EV over his change in wealth, \( \pi(c, \theta) \), given he knows: \( U(\cdot), U, \) and that the insured has the same \( H \) over \( \theta \) as he does. If he offers an \( I(\cdot) \) such that (2) does not hold, then: she does not accept \( I(\cdot) \); \( \alpha = 0 \); and he obtains the utility from not contracting, \( V(0) \). Hence (6) is subject to (2). Case A is (2) not binding the solution, \( I^* \), to (6); Case B is (2) binding it.

Any \( I(\cdot) \) for which (6) & (2) hold is optimal for the insurer and is written \( I^* \) in Cases A & B. Such \( I(\cdot) \) are Pareto-optimal by definition in Case B. This also holds in Case A if \( I^* \) is unique. But he will offer such an \( I(\cdot) \) if and only if his ex-ante utility under \( I(\cdot) \) exceeds \( V(0) \). In Section V, the \( I^* \) for each of Cases A & B is characterized by the first- and second-order conditions for (6). This characterization will be in terms of \( A_p, A_c, \& A_d \) for Case A, and \( B_c \& B_d \) for Case B. These \( A_i \) and \( B_i \) will be signed in terms of \( X, H, \) and \( U \) in Section IV, and the terms will be grouped into the effects of the three fundamental forces, described in Section II.

\[
\max_{p, c, d} \text{EV} \quad \text{where EV} = \int V(p) d\theta + \int V[p - c[X(A(p, c, d), \theta) - d]] dH(\theta) \quad \text{subject to} \quad A(p, c, d, d)
\]

In (6) we see a typical nested maximization that makes the principal-agent problem difficult to analyze. In Case A, he maximizes EV over \( p, c, \) and \( d \), where EV contains \( p, c, \) and \( d \), both explicitly and in \( A(p, c, d) \), which is her ARGMAX of EU over \( \alpha \). Thus maximizing over \( i = p, c, \) or \( d \) involves a direct effect on EV (written as \( Ev_2(\alpha(i), i) \) in Section VI), and a mixing of a reaction effect on EV, via \( \alpha_1(i) \) in Section VI, with a moving trigger effect. In Case B, there is also the premium compensation effect on EV, caused by the change in \( p = P(c, d) \) preserving \( EU^* = U \), as described in Section IV.
RESTRICTION 1 Consideration is restricted to those exogenous circumstances for which
there exist some $I(\cdot)$ and $\alpha$ such that $EU \geq U$ and $EV > V(0)$ without the
insurer exceeding the maximum probability of his default, $s \in [0,1)$, that
he is willing to incur.

RESULT 1 If both parties accept a fixed arbitrary $I^\ast = (p^\ast,c^\ast,d^\ast)$, then the following are
positive: loss coverage proportion, $c^\ast$; premium, $p^\ast$; and the probability of a
covered loss occurring ($X > d$). Without loss of generality, negative
deductibles ($d < 0$) may be excluded.

RESULT 2 Bounded initial wealths of the parties imply that a finite optimum contract exists.

The proofs of Results 1 and 2 are in the Appendix for Section III. Restriction 2 implies
$\pi^\ast$ is regular because there is no restriction on the choice of $(p,c,d)$ in $R^3$ or of
$(p(c,d),c,d)$ in $R^2$.

RESTRICTION 2 In Sections V & VI, consideration is restricted to exogenous circumstances
for which there exists an $\pi^\ast$ and $\alpha^\ast$ such that $s$ is not binding.

Consider a typical $\pi^\ast$, among the common form treated here, that is offered by a
risk-averse insurer who can never verify $\alpha$ or $\theta$. Steven Shavell [1979b] showed that each
party will bear some of the risk (hence $0 \neq c^\ast \neq 1$ whether we restrict the form of the
contract or not). The insured will spend up to the level at which her expected utility of
the marginal cost per unit $\alpha$, $-EUW\pi$, equals her expected utility over the consequent
marginal reduction in her loss net of benefits, $E(cD-1)X\pi W\pi$.

This is not socially optimal, as it does not induce $\alpha$ up to the point at which its cost,
$EU\pi$, equals its benefits (to her & him), e.g., in Case B, up to $cEXW\pi W +$
$\lambda E(c^D-1)X\pi W\pi$, where $\lambda$ depends on $EU\pi$ and $U$. The first-best $\pi^\ast$ does induce such $\alpha^\ast$,
but the second-best $\pi^\ast$ induces, instead, $\alpha^\ast$ such that $E\pi = 0$. In Case B, this
difference in $\alpha^\ast$ causes the difference between his first- and second-best EV$\pi$. The $\alpha^\ast$ and
$\pi^\ast$ for each is in Section VII. This difference in EV$\pi$ is the cost of moral hazard.
Consider a regular $\alpha^*$ for any initial problem, (1), the insured faces: $I(.)$ and exogenous circumstances $U, U, X, & H$. This $I(.)$ may not be an $I^*$. Each problem here is a modification via the shift parameter, $\tau$, of the initial problem in one of $p, c, \text{or } d$; or in an aspect of one of $U, X, \text{or } H$. Let $\alpha(\tau): R \rightarrow (0, +\infty)$ be the $\alpha^*$ for her new problem. For each new one, $\omega(\alpha(\tau), \tau) = w(\alpha^*, \theta) | \tau|$; and she maximizes $E u$ over $\alpha(\tau)$ with $\theta' = \phi(\alpha(\tau), \tau) = \theta'((\alpha(\tau), d)| \tau)$. The marginal $\alpha$ per $\tau$, $\alpha^*_\tau$, is the subject of this section. By the construction of $\alpha$, this $\alpha^*_\tau$ is the reaction force to any shift via $\tau$. Further, $X(\alpha(\tau), \phi(\alpha(\tau), \tau)| \tau) = d$ for all $\tau$ and $\phi(\alpha(0), 0) = \theta'(\alpha, d)$.

Also by construction at $\tau = 0$, $\alpha(\tau)$ is a regular maximum, $\omega = \omega$, and $u = U$. Hence $E u(0, 0) = E u[w(\alpha^*, \theta)] = E u^*$ and $E u_d[\alpha(0), 0] = E u_d^* = 0 > E u_d[\alpha(0), 0] = E u_d^*$. The model's continuity insures that $E u^*$ twice continuously differentiable (i.e., is $C^2$) in $\alpha$, and since $\tau$ is used continuously, that $E u^*$ is $C^2$ in $\tau$. The implicit function theorem implies there is a $C^2$ function, which by construction is $\alpha(\cdot)$, such that $0 = E u_{d^2}[\alpha(\tau), \tau]$ for all $\tau$ in some open neighborhood of $\tau = 0$. Hence $0 = E u_{d^2} = \delta_{\tau} E u_{d^2} + E u_{\beta_d}$ for all $\tau$, $\delta_{\tau}(\tau) = E u_{d^2}[\alpha(\tau), \tau] / - E u_{d^2}[\alpha(\tau), \tau]$. At $\tau = 0$, we have $u_{d\alpha} = u_{d\alpha}$, $\delta_{\tau}(0)$, where $E u$ means $E u(0, 0)$. From (7), we see that the reaction force, $\delta_{\tau}(0)$, is partly the moving trigger force, $T$, because $E u_{d\alpha}$ contains $T$, as per (4). Since we assume that $U_w > 0 > U_{ww}$, the sign of $\delta_{\tau}$ can be determined from (8) primarily in terms of $w_d$. For shifts in $p, c, \text{and } d$, we obtain (9), but compute $E u_{d^2}$ directly for the other shifts.

(7) $\delta_{\tau} = E u_{d^2}[\alpha(\tau), \tau] / -E u_{d^2}$

for any shift parameter, $\tau$.

(8) SIGN $\delta_{\tau}(0) = \text{SIGN } E u_{d^2}[\alpha(0), 0]$

(9) $E u_{d^2} = E u_{d^2} U + E u_{d^2} U_{\alpha^*} w + \delta_{\tau} E u_{d^2}$

with the $i$ in (1) replaced with $i + \tau$ for $i = p, c, d$.

Partial differentiation by $\tau$ is represented by "_\tau" , e.g., in $E u_{d^2}$, and is with respect to the $\tau$ that are not in $\alpha(\cdot)$. For each shift, $E u_{d^2}$ contains $\phi_{\alpha}$ and we use $\phi_{\alpha} = -X^{\tau}_{d^2}/X^{\tau}$ in computing $\phi_{\alpha}$ from $X(\alpha(\tau), \phi(\alpha(\tau), \tau)| \tau) = d_{\tau}$ at $\tau = 0$. After signing these
six shifts under Case A, we also sign the last five of them under Case B.

Standard Conditions (10) helps sign the shifts. They are $\omega_d$ being either positive or negative, and is equivalent to her marginal loss reduction per spending being more or less than the reciprocal of her copayment proportion, $1/(cD-1)$. Each of these two mutually exclusive conditions will be used to imply a separate sign relation. Note $\mu^2 \geq \mu^1$ if $\mu^3 \leq \mu^1$ means both that $\mu^1 > \mu^2$ if $\mu^3 < \mu^1$, and that $\mu^2 < \mu^1$ if $\mu^3 > \mu^1$. Single sign conditions are (10a), (10d), etc. Result 16 shows that $c^* < 1$ in Case A.

Conditions (10) $E \omega \frac{d}{d \omega} \geq E \omega \frac{d}{d \omega} D \quad (10') E \omega \frac{d}{d \omega} \frac{D}{D \omega} \geq 0$

CASE A: INSURED'S UNCOMPENSATED SHIFTS

In the Appendix for Section IV, $a_\tau(0) = E \omega \frac{d}{d \omega} D / E \omega$, is computed for the three contractual shifts in (11). The other three shifts are computed with Results 6, 7, & 8, but are listed immediately below. We write $A_\tau(p,c,d,\tau)$ to mean $a_\tau$ where each $p$ in (1) becomes $p + \tau$, and $a(0) = A(p,c,d,0) = A(p,c,d)$. Let $\gamma$ denote evaluation at $\theta = ES$, and let $\gamma$ and $\Gamma$ be her absolute and relative risk aversions, over covered losses ($\gamma = -U_x \omega D / U_x \omega$ & $\Gamma = -U_x \omega D / U_x \omega$). The sign of these reaction forces, $A\tau$, for $\tau = d$ and the $\tau$ in (12), & (13) depend on the moving trigger force, $T$, since it appears in their numerators.

(11) $A_p = \frac{E \omega \frac{d}{d \omega} \frac{x_d \omega}{-E \omega}}{E \omega} \quad A_c = \frac{E(x_d \omega - \frac{d}{d \omega} \gamma)U_x \omega}{-E \omega} \quad A_d = \frac{-c(E \omega \frac{d}{d \omega} \frac{D + T}{D \omega})}{-E \omega}$

(12) For a mean-preserving & mean-regressing rise in pure certainty

$A_\tau = [(E(cD-1)(E\theta) + x_d \theta \gamma)U_x \omega - cMT_x \theta(E\theta) / U_x \omega]

(13) For a second-order, stochastically-dominant rise in the insured's optimism, via $X$

changing to $X(\tau, \theta) = \tau Z(\theta)$ where $Z > 0$ for positive measure and < 0 nowhere,

$A_\tau = [(E(1 \cdot cD)Z_x \theta \omega - c^2 \theta T / U_x \omega

(14) For a shift in the insured's absolute risk aversion or alternative utility,
\[ A^*_\tau = \left( E_{\omega} \left[ E_{\omega} \left( x_{\omega} - \omega \right) \right] + E_{\omega} \left[ E_{\omega} \left( x_{\omega} - \omega \right) \right] \right) / E_{\omega} \]

The Insured's Uncompensated Shifts in Contract: Our model's sign assumptions and (9) imply that \( \alpha^* \) will change as per the signs below if the coverage proportion, \( c \), does not exceed 100%. Note that Result 17 shows \( A_p > 0 > A_q \) & \( A_c > 0 \) and \( c^* < 1 \) for \( I^* \) in Case A.

**RESULT 3**

\( \alpha^* \) rises as \( p \) rises if (10) holds.

\( \alpha^* \) falls

**RESULT 4.1**

\( \alpha^* \) falls as \( c \) rises if (10>) & (10') hold.

**RESULT 5.1**

\( \alpha^* \) rises as \( d \) rises if (10<) holds.

**RESULT 4.2**

\( \alpha^* \) rises as \( c \) rises if (10) holds & \( E_{\omega} U_{D} > -E_{\omega} X_{\omega} U_{D} \).

\( \alpha^* \) falls

**RESULT 5.2**

\( \alpha^* \) rises as \( d \) rises if \( T > -E_{\omega} U_{D} \).

\( \alpha^* \) falls

The Insured's Uncompensated Shift in Pure Certainty is a mean-preserving, mean-regressing contraction of \( E \). It is \( EU(X(\alpha^*) - \alpha^*) + (cD-1) \left[ X(\alpha^*) - (1-\theta)X + \theta F \right] - dD \) and \( X(\alpha^*) - (1-\theta)X + \theta F = 0 \). Note \( \theta_2 = \theta - F \). Here \( E_{\omega}(\alpha^*) - cD \) where \( U_{D} = (cD-1)(X_{D}U_{w} + X_{w}U_{D}) \) and \( x_{D} > 0 \) if (10<) holds and \( x_{D} \geq 0 \) (i.e., \( \alpha \) does not become less efficient in reducing \( X \) as \( \theta \) rises). This \( U_{D} \) is the copayment proportion times her marginal utility weighted by the sum of two effects. This first is the cross effect between \( \alpha \) and \( \theta \) on loss reduction. The second is her absolute risk aversion weighted by the product of 

**RESULT 6**

The insured's optimal spending to control losses, \( \alpha^* \), rises (falls) as pure certainty rises if the change in her marginal utility per \( \alpha \) as the state-of-nature,
\( \theta \), rises, \( \frac{\partial x}{\partial \theta} \), weighted by the mean of \( \theta \), minus \( \theta \), is positive (negative), and the deductible is more (less) than the average loss.

The Insured's Uncompensated Shift in Optimism is a shift of second-order stochastic dominance over losses. This could result from a change in \( H(\cdot) \) or in \( X(\cdot, \cdot) \). The shift is described by (6), with \( X \) replaced by \( X - \tau \) and \( \theta' \) replaced by \( \phi \) such that \( X(\sigma, \omega) - \tau(\phi) = d \). Let \( Z(\theta); (0, +\infty) \rightarrow (0, +\infty) \) such that \( \sum \phi z(x, \theta) > 0 \) for some \( 0 \leq \phi < \phi^* \) and \( \sum \phi z(x, \theta) > 0 \) for all such \( \theta \). Then \( \text{E}[w_\theta(x, \theta); (0, +\infty)] = \text{E}[1-cD]w_\theta x \theta \)

\( \alpha' = Z/\alpha \). Hence (3) \( \rightarrow \) Result 7 and also \( \alpha' > 0 \) if \( \text{E}[1-cD]w_\theta x \theta \)

\( \geq \alpha' x \), which depends on her absolute risk aversion, \( \gamma \), as in Result 5.2.

RESULT 7 Insured's optimal spending, \( \alpha' \), to control losses, rises with a second-order, stochastically-dominant shift in the distribution over losses, if (10\#) holds when it is weighted by her copayment proportion times the shift, i.e., by \( (1-cD)Z \).

The Insured's Uncompensated Shift in Risk Aversion is parameterized by a convex combination of her initial risk-averse utility function, \( U \), and a risk-neutral one, \( U^0 \), which is the average affine equivalent of \( EU \). This \( U^0 \) has the same slope as the average of \( EU \), and has the same value at \( E\theta \) as \( EU \) does, i.e., \( U^0 = (EU)_0 + U^0 \). Hence \( EU(\tau_\theta) = (1-\tau)EU^0 + \tau\text{E}U^0 \). Since \( \phi = 0 \) here, we have \( EU(\tau_\theta) = EU^0 + \text{E}w_\theta^x \text{E}w_\theta x \theta \). In this section, \( \alpha' \) is regular, \( \rightarrow \text{E}w_\theta x = 0 \), hence (7) \( \rightarrow \text{SIGN } \alpha'(0) > 0 \) if (10) holds, \( w_\theta x \geq \text{E}w_\theta x \), and either \( \text{E}w_\theta x \geq w_\theta^x \) or \( \text{E}w_\theta x \geq \text{E}w_\theta x \).

RESULT 8 Insurer's optimal spending, \( \alpha' \), to control losses rises/falls as her risk aversion falls, if: (10\#) holds; her expected wealth is more/less than her wealth at the average state-of-nature, \( E\theta \); the same for her marginal wealth per \( \alpha \); and the latter is positive at \( E\theta \).

CASE 3: INSURED'S COMPENSATED SHIFTS

Given any coverage proportion, \( c \), and deductible, \( d \), that we consider (i.e., any \( c > 0 \) s
d as per Result 1), let \( P(\ldots) \) be the function equal to the premium value, \( p \), which keeps (2) holding with equality. In determining this \( P(\ldots) \) over \( c \) and \( d \), we have her choosing \( \alpha^* \) for the \( I = (P(c,d), c, d) \) she faces. Formally, \( P(c,d) : (0, +\infty)^2 \times R \to (0, +\infty) \) such that \( EU^* \{ P(c,d), c, d \} = 0 \). Let \( B(c,d) = A(P(c,d), c, d) \). The signs of the same three shifts depend on \( T \) as did in Case A.

The constructions of \( P \) and \( B \) imply that \( EU[P - B - X(B, \theta) + c(X(B, \theta) - d)] = 0 \) for all \( c \) and \( d \). Hence \( EU^*_d = 0 = EU^*_d \). Since \( \alpha^* \) is regular, we substitute 0 for \( EU^*_d \) in these equalities, yielding the premium compensation forces from \( c \) and \( d \), respectively:

\[
(15.c) \quad P_c = E(X_d)^{\frac{\theta}{\alpha}} \frac{\mathcal{E}^{\frac{\theta}{\alpha}} -EU^*_w}{EU^*_w} > 0 \quad \text{and} \quad (15.d) \quad P_d = -cEU^*_w / EU^*_w < 0.
\]

These inequalities stem from the model's sign assumptions. Letting \( M = EU^*_w / EU^*_c \) and \( N = E X_d^\alpha / EU^*_c \), we have \( P_c = N - d \) and \( P_d = -cM. \)

We use the same procedures in finding each shift as we did in the corresponding uncompensated shifts via (7), (8), and (9), but with \( p \) changed to \( P[c,d,T] \), and \( A(p,c,d,T) \) changed to \( B(c,d,T) \). Again \( \phi_\alpha = -X_d^{\frac{\theta}{\alpha}} / X_d^{\frac{\theta}{\alpha}} \) helps us find \( \phi_\gamma \). We obtain expressions for the partial derivatives of \( \alpha \) with respect to each of the five compensated shifts at \( T = 0 \) in (16) - (20). See the Appendix for Section IV for the substitution from (15c) & (15d) and of \( \gamma \) and \( \Gamma \). Since \( \alpha^* \) is regular, \( EU^*_d < 0 \); and we can sign these \( B_i \).

\[
(16) \quad B_c = \left[ (\alpha^{\frac{\theta}{\alpha}} D + \phi_\gamma ((N - d)) \gamma - \Gamma ) \right] EU^*_w - EU^*_w
\]

\[
(17) \quad B_d = cEU^*_d (M - D) EU^*_w / EU^*_d
\]

(18) For a rise in pure certainty,

\[
B_T = [E(cD - 1)(E - \theta)(X^\alpha_\theta - \phi_\gamma (EU^*_w / EU^*_w) - \gamma )] EU^*_w / EU^*_d
\]

(19) For any second-order, stochastically-dominant rise in optimism, via \( X \) changing to \( X(B, \theta) - T Z(\theta) \) for \( \Theta = \int^\Theta Z(\theta) dH(\theta) > 0 \) for some \( \Theta \) where \( \Theta < 0 \) never,
$$B_{r} = E[(1-cD)(1 - \sqrt{E_{U}\omega_{u}^{w}} - cZ')]/ -E\omega_{d}$$

(20) For a fall in the insured's risk aversion,

$$B_{r} = \{(E^{r} - u^{r})/E_{u}^{r} + u^{r} - E_{u}^{r}E_{u}^{r}\omega_{u}^{w}/ -E\omega_{d}\}$$

**Insured's Compensated Shift Results**

**RESULT 9**  \(B_{r} < 0\) if (10a) & (10b) hold and \(d > \frac{EXu^{w}}{Ew^{w}}\).

**RESULT 10**  \(B_{r} < 0\) if \(M > \frac{Ew_{m}^{w}u^{w}}{Ew_{m}^{w}u^{w}}\).

**RESULT 11** For a rise in pure certainty, \(B_{r} > 0\) if \(\theta' \geq 0\) and \(E(cD-1)(E\theta - \theta)'(X_{d}^{w} - X_{d}^{w})\omega_{u}^{w} > 0\) where \(Y = \frac{E_{u}^{w}}{E_{u}^{w}} - \theta\).

**RESULT 12** For any second-order, stochastically-dominant rise in optimism,

\[B_{r} > 0\] if \(E(1-cD)(1 - \sqrt{E_{U}\omega_{u}^{w}}) \omega_{u}^{w} > 0\).

**RESULT 13** For a fall in the insured's risk aversion, \(B_{r} > 0\) if (10) holds and \(Ew' > w^{2}\).

**V: CHARACTERIZATION OF OPTIMAL CONTRACTS**

**CASE A: WHEN THE INSURED'S ALTERNATIVE IS NONBINDING**

The model assumes that his maximum default possibility, \(\delta\), does not bind the solution of (6). Hence she solves (6) over \(p, c, d\) unconstrained by (2) or \(\delta\), and \(d\) is the \(A(p,c,d)\) defined in Section II. Thus the first-order conditions are (21), (22), & (23). Then (21) & (23) \(\rightarrow\) (24c), and (21) & (22) \(\rightarrow\) (24d). The second-order conditions for Cases A & B are in the Appendix for Section V. They contain a moving trigger effect, \(\omega\), where \(\omega = X_{d}^{w}u_{u}^{w}dE'X_{d}^{w}/X_{d}^{w}\). It is analogous to the \(T\) in Sections II & IV.

The first-order conditions partition the effects on \(EV\) of raising each of \(i = p, c, d\) alone at \(\omega\) into the two terms following the second equals sign in each of (21), (22), & (23), respectively. The first is the direct effect. The second is \(c\) times the reaction
effect, which is the reaction force, $A_i$, times his marginal utility over the covered losses, $V_{D}^{\pi}$, weighted by the marginal loss reduction per spending, $X_{\alpha}^{\pi}$. Let $m = EV_{D}^{\pi}/EV_{D}^{\pi}$ and $n = EX^{\pi}V_{D}^{\pi}/EV_{D}^{\pi}$.

\[
(21) \quad 0 = EV_{D}^{\pi} = EV_{D}^{\pi} - CA_{D}^{\pi}EX_{D}^{\pi}V_{D}^{\pi} \rightarrow \quad \frac{-EX_{D}^{\pi}V_{D}^{\pi}}{EV_{D}^{\pi}} = \frac{EV_{D}^{\pi}}{EV_{D}^{\pi}/A_{D}^{\pi}}
\]

\[
(22) \quad 0 = EV_{C}^{\pi} = -E(X_d)EV_{D}^{\pi} - CA_{D}^{\pi}EX_{D}^{\pi}V_{D}^{\pi} \rightarrow \quad \frac{-EX_{D}^{\pi}V_{D}^{\pi}}{EV_{D}^{\pi}} = E(X_d)EV_{D}^{\pi}/A_{C}^{\pi}
\]

\[
(23) \quad 0 = EV_{d}^{\pi} = cEV_{D}^{\pi} - CA_{D}^{\pi}EX_{D}^{\pi}V_{D}^{\pi} \rightarrow \quad \frac{-EX_{D}^{\pi}V_{D}^{\pi}}{EV_{D}^{\pi}} = \frac{cEV_{D}^{\pi}}{A_{d}^{\pi}}
\]

\[
(24c) \quad c = \frac{A_{d}^{\pi}/m_{A_{D}^{\pi}}}{(24d)}
\]

**RESULT 14** The insured's optimal spending against losses, $c$, in Case A under the optimal contract, falls, falls, and rises, as the premium, deductible, and coverage proportion rise, respectively, i.e., $A_{p}^{\pi} < 0$, $A_{d}^{\pi} < 0$, and $A_{C}^{\pi} > 0$.

**Proof** Our sign assumptions and the second equalities of (21), (22), & (23). Q.E.D.

If we again assume $V_{D}^{\pi} = 0 = X_{D}^{\pi}$, then $m = p$ and $n = EXD/p$ in (24c), (24d), (25c), & (25d). Thus, at the level of analysis in which $A(p, c, d)$ is a black box to the insurer, $c^{\pi}$ and $d^{\pi}$ are each inversely proportional to the probability of a covered loss occurring, $p$, i.e., of a positive benefit.

**RESULT 15** The coverage proportion, $c^{\pi}$, of a risk-neutral insurer's optimal contract, $T^{\pi}$, in Case A is the reciprocal of the probability of a covered loss occurring, $1/p$, times the ratio of the reaction force for the deductible, $d$, to that for the premium, $p$. The $d^{\pi}$ is $1/p$ times the sum of the expected covered loss and the ratio of the reaction force for $c$ to that for $p$.

Substituting the expressions for $A_{p}$, $A_{C}$, & $A_{d}$, from (11) into (24c) and (24d) yields: (25c) which is $c^{\pi}$ implicitly in terms of the model's exogenous circumstances; and
(25d), which is \( d^* \) in terms of \( c^* \) and those circumstances. Let \( \tilde{n} = n/m \).

\[
(25c) \quad \tilde{d} = (E(1+X_q^*)(m+D)) \tilde{U}_{w} - T \iota / (1+\tilde{n}) E X_q^* U_w^D
\]

\[
(25d) \quad \tilde{d} = E[(\tilde{X} - \tilde{n}) X_q^* U_w^D / T + \tilde{n}]
\]

**CASE B: WHEN THE INSURED'S ALTERNATIVE IS BINDING**

Here \( \alpha = B(c,d) \) and \( p = P(c,d) \), as per Case B of Section IV. The model assumes that an insured's decision is not binding the solution of (8) subject to (2). Hence, we have the unconstrained and implicit first-order conditions over \( c \) and \( d \) in (23c) and (23d). Both the reaction force, \( B_1 \), and the premium compensation force, \( P_1 \), contain both \( c \) & \( d \) for \( i = c, d \). Note

\[
E[(X^*-d^*)D]_d = (B_0 X_d^* - 1) D. \quad \text{The Appendix to Section V lists the second-order conditions.}
\]

\[
(28c) \quad 0 = \tilde{E} V_c = \tilde{p} \tilde{E} V_D - E(X^*-d^*) V_D - \tilde{c} \tilde{E} X_q^* V_D \quad \Rightarrow \quad \tilde{c} = \frac{\tilde{E}(X^*+cX_q^*) V_D - \tilde{c} \tilde{E} X_q^* V_D}{\tilde{E} V_D}
\]

\[
(28d) \quad 0 = \tilde{E} V_d = \tilde{p} \tilde{E} V_c + \tilde{c} \tilde{E} V_D - \tilde{c} \tilde{E} X_q^* V_D \quad \Rightarrow \quad \tilde{c} = \frac{\tilde{p} \tilde{E} V_D}{\tilde{E}(X_d^* - 1) V_D}
\]

**RESULT 17** In Case A the optimal coverage proportion, \( c^* \), equals the compensated premium for the deductible, \( d^* \), times the ratio of the insurer's marginal utility over all losses, \( EU_q^* \), to that which both is only over covered losses, and is weighted by the marginal covered loss per \( d^* \).

These conditions, (28c) & (28d), partition the effect of changing each of \( i = c \) & \( d \) into three terms following the second equals sign. The first is the premium compensation effect, which is \( EV_D^* \), weighted by the premium compensation force, \( P_1 \). The second is the effect of the direct force, which is \( EV_D^* \), weighted the partial derivative of \( w \) per \( i \) exclusive of the reaction force, \( B_1 \). The third is the effect of \( B_1 \), which is \( EU_D^* \), weighted by \( cX_q^* \) over covered losses. From (23c) & (23d) we isolate \( B_c \) & \( B_d \). This implies at \( \bar{D} = \bar{X}^* \), that \( B_c \geq 0 \) if \( E(X^*-d^*) V_D / EU_D^* \geq E(X^*-d^*) U_D^* / EU_D^* \) and \( B_d \geq 0 \) if \( EV_D^* / EU_D^* \geq EU_D^* / EU_D^* \), i.e.,
RESULT 18 In Case B the reaction force for the coverage proportion is positive if the ratio of the insured's (her) expected marginal utility to the same weighted by covered losses, exceeds such a ratio for the insurer (he). The reaction force for the deductible rises if the ratio of his expected utility over covered losses to that over all losses, exceeds such a ratio for her.

Substituting from (16), (17), (15c), & (15d) into (28c) & (28d), yields: \( c^* \) in terms of the model's exogenous circumstances in (29c); and \( d^* \) in terms of \( c^* \) and those circumstances in (29d). Let \( Y = E(D-M)\hat{V}_w^* \) and \( S = \hat{E}X_d\hat{V}_d^*D \).

\[
(29c) \quad \hat{c} = \frac{-G + \sqrt{G^2 - 4F}}{2F} \quad \text{for} \quad F = k\hat{E}_d^{\hat{X}_d}(M-1)\hat{V}_w^*D, \quad G = k(E(1+\hat{X}_d)(D-M)\hat{V}_w^*T), \quad J = M-M
\]

\[
(29d) \quad \hat{d} = \frac{\ln(1-N + c\hat{k}\hat{E}_d^{\hat{X}_d}(\hat{X}_D-N)\hat{Y} - \hat{X}_D)}{[m - \hat{M} + \hat{c}\hat{k}\hat{E}_d^{\hat{X}_d}(M-D)\hat{V}_w^*]} \quad \text{for} \quad k = s/\hat{E}_d^{\hat{X}_d}
\]

Assume that he is risk-neutral for all possible \( \omega \) under \( I^* \), i.e., \( \hat{V}_{\omega}^{\omega*} = 0 \). (See Babbel and Economides [1985] for conditions under which risk neutrality is consistent with finance theory). Further assume that the marginal loss reduction from spending is constant in \( \omega \) and \( \alpha \), i.e., \( X_{d\omega} = 0 = X_{d\alpha}^* \). Note that, \( X_{d\omega} = 0 \) may imply a Kuhn-Tucker boundary solution for \( \alpha^* \) if \( X \) is bounded. But this would preclude \( \rho > 0 \) under any \( I^* \), and thus Result 1 precludes such solutions.

The \( V_w^* \) terms cancel in (29d) & (29c). The \( E\hat{V}_D^* \) terms become \( E\hat{D} \), which is \( E\hat{D}^* \)

\( \equiv \rho, m = \rho, n = E\hat{D}/\rho, \) and \( s = X_{d\rho}^* \). This implies Result 19. Note that Result 1 and (15d) applied to (30c), \( \Rightarrow B_{dx}^* - 1 < 0 \), which is Lemma 2.

\[
(30c) \quad \hat{c} = \frac{\hat{F}_d}{(\hat{B}_{dx}^* - 1)\rho} \quad \quad (30d) \quad \hat{d} = (E\hat{D} - \hat{F}_c - (\hat{F}_d \hat{B}_{dx}^*)/(\hat{B}_{dx}^* - 1)) / \rho
\]

**Lemma 2** The insured's expected covered loss per the deductible is negative under Case B.

**RESULT 19** Assume in Case B that a risk-neutral insurer faces constant marginal loss reduction per spending. The optimal commission proportion, \( c^* \), is the ratio of
the compensated premium force from the deductible, \( d \), to the product of the probability of a covered loss times the marginal covered loss per \( d \).

**COROLLARY** Assume the hypothesis of Result 19. The coverage proportion, \( c^* \), of an optimal contract is more (less) than one if the premium compensation force, \( p^*_d \), is more (less) than her covered loss per the deductible.

Note that this covered loss per \( d \) is \( Ew_d \delta = \rho(X^2 - d^2)d = \beta_d X_d^2 - 1 \). It represents a reaction effect, \( \beta_d X_d^2 \), and a direct effect, \(-1\).

**VI: INSURER’S COMPARATIVE STATICS**

This section examines under Case 3 how \( I^2 \) changes in response to compensated shifts, each in one of several exogenous parameters of his problem. Again assume that \( V^x_{\infty} = 0 = X^x_0 \) = \( X^x_0 \). Section 5 discusses these assumptions, and Section IV explains this type of shift. The four shifts treated are: a mean-preserving, mean-regressing addition of pure certainty; a second-order, stochastically-dominant improvement in the distribution over losses; a fall in her risk aversion; and a rise in her alternative utility. The first three are the same as those in Case 3 of Section IV, but are applied to (6), instead of (1).

The optimum \( c \) and \( d \) for any given \( \tau \) are written \( c(\tau) \& d(\tau) \). For each shift, we define \( Ev(c(\tau), d(\tau), \tau) = Ev(\beta(c(\tau), d(\tau), 0), \theta, \tau) \) and \( P(c(\tau), d(\tau), \tau) = P(\beta(c(\tau), d(\tau), 0), \theta, \tau) \) for the \( P \& B \) of Section IV. Let \( Ev \equiv Ev(c(0), d(0), 0) \) in \( (20) \) - \( (22) \) below. We sign the change in \( I^2 \) from \( \tau \) rising at the margin, i.e., sign \( c'(0) \& d'(0) \).

They are expressed in terms of the first and second derivatives of her spending to control loss with respect to \( c \), \( d \), and \( \tau \), i.e., \( B_i \& B_j \& B_k \) for: \( i = c, d, \tau \); and \( jk = c, d, d \).

By construction and Restriction 2, \([c(0), d(0)]\) is a regular \( I^2 \) and \( Ev = Ev(\beta(c(0), d(0), 0), \theta, 0) \). The former implies the following hold at \( \tau = 0 \): \( Ev_c = 0 = Ev_d \), \( Ev_{cc} < 0 > Ev_{dd} \), and \( L = Ev_{cc} Ev_{dd} - (Ev_{cd})^2 > 0 \). We use the same type of arguments as in Section IV on \( Ev \). We find for \( i = c, d \) that there are \( c^2 \) functions, \( c(.) \& d(.) \), defined above, such that for all \( \tau \) in some open neighborhood of \( \tau \)
= 0: 0 = Ev_{1\tau} [c(\tau), d(\tau), \tau] = Ev_{1c} c_\tau + Ev_{1d} d_\tau + Ev_{13}; and Ev_{11} < 0 < L. Solving 0 = Ev_{c\tau} and 0 = Ev_{d\tau} for c_\tau and d_\tau yields c_\tau(\tau) = (Ev_{d3} Ev_{cd} - Ev_{c3} Ev_{dd})/L and d_\tau(\tau) = (Ev_{c3} Ev_{cd} - Ev_{d3} Ev_{cc})/L. Since L > 0, above, we have

(32c) \text{SIGN } c_\tau(0) = \text{SIGN} (Ev_{d3} Ev_{cd} - Ev_{c3} Ev_{dd}), \quad \text{and}

(32d) \text{SIGN } d_\tau(0) = \text{SIGN} (Ev_{c3} Ev_{cd} - Ev_{d3} Ev_{cc}).

Here c_\tau(0) > 0 < d_\tau(0) if each of the first three Ev_{jk} terms is positive or if the first two are negative and the third is positive, since Ev_{cc} < 0 > Ev_{dd}. Two other combinations of signs will make c_\tau(0) < 0. Similarly for d_\tau(0) > 0 and d_\tau(0) < 0. For brevity in each of the four shifts below, we will show only the necessary sign conditions for c_\tau(0) > 0 < d_\tau(0) via the combination of the first three Ev_{jk} terms being positive. The other combinations are obtained by the same types of steps used below.

An important condition in signing these shifts is (36), i.e., at \tau = 0, the derivative of the reaction forces, \hat{P}_{ij} (the cross partials of P(c^\tau, d^\tau, \tau)) exceed the ratio of the same partial of the compensated premium, P(c^\tau, d^\tau, \tau), divided by the product of the marginal benefit per action, c^\tau X_{\alpha}^\tau, times the probability of a covered loss occurring, \rho. To the insurer, (33) means that each partial of the reaction force is more than a constant times the corresponding cross partial of the compensated premium. Another important condition is (34), which is that B_{c}(c, d) is less than the ratio of minus \rho times the product of the marginal loss per \theta, to cX_{\alpha}^\tau times the density at X = d. The moving trigger effect for the insurer is \tau = X_{\alpha}^\tau dH^\theta / X_{\alpha}^\tau. It is analogous to T in Section II.

(33) \hat{P}_{ij} > \hat{P}_{ij} /cX_{\alpha} \rho \quad \text{for } ij = c3, cd, \text{ & } ds. \quad (34) \hat{S}_{c} < -\rho/c_{\tau}

In every shift below, the first term of each expression for Ev_{ij} is the premium compensation effect. The last is the moving trigger effect. Parts of the other terms with B_{i} or B_{ij}, multiplicatively, are reaction effects, while parts without are direct effects.

**INSURER'S PURE CERTAINTY SHIFT**

Our shift for the mean-preserving, mean-regressing contraction adds pure certainty to the
parties' probability distribution. It is described by \( Ev(c,\tau,\tau) \) = the EV in (6) with \( X \) replaced by \( XIB(c,\tau,\tau) \), \( (1-\tau)\phi(B(c,\tau,\tau),\tau) + \tau \Theta \), \( \Theta' \) replaced by \( \phi \), as well as \( \alpha \) and \( p \) replaced by \( B(c,\tau,\tau) \) and \( F(c,\tau,\tau) \), respectively.

Differentiating this new EV yields (35), (36), & (37), which with (32c), (32d), & Lemma 2, allow us to sign \( c_\tau \) and \( d_\tau \) in Result 19.

(35) \[
Ev_{c_\tau} = \frac{\pi}{\pi c_\tau} - \frac{\pi}{\pi} (B_3 + c_\tau B_3) X^\alpha + B_3 X (\Xi - \phi) \frac{\pi}{\pi} d_\tau
\]

(36) \[
Ev_{d_\tau} = \frac{\pi}{\pi} X B_3 - \frac{\pi}{\pi} \frac{\pi}{\pi} \frac{\pi}{\pi} X^\alpha (B_3 - 1) + \frac{\pi}{\pi} B_3 d_\tau
\]

(37) \[
Ev_{d_\tau} = \frac{\pi}{\pi} d_\tau - \frac{\pi}{\pi} \frac{\pi}{\pi} \frac{\pi}{\pi} X^\alpha + \frac{\pi}{\pi} X (X_\alpha - 1) (\Xi - \phi) \frac{\pi}{\pi} d_\tau
\]

**RESULT 20** The insurer's optimal contract parameters, \( c_*^x \) and \( d_*^x \), each rise at the margin, as pure certainty is added if: (36) & (37) hold, the deductible is at least the average loss, \( B_c^x > 0 > B_d^x \), and \( 0 < B_d^x \).

**INSURER'S OPTIMISM SHIFT**

The shift for second-order stochastically-dominant changes in the distribution over losses is the \( Ev(c,\tau,\tau) \) defined by the EV in (6) with \( \alpha, p, \) and \( X \) changed to \( B(c,\tau,\tau) \), \( F(c,\tau,\tau) \), and \( XIB(c,\tau,\tau) \), \( \Theta' \) = \( \tau Z(\theta) \), respectively. This \( \tau \) and \( Z \) are defined in the optimism shift of Section IV. Differentiating this EV yields (38), (39), & (40), which sign \( c_\tau(0) \) and \( d_\tau(0) \) in Result 21 from (32c), (32d), & Lemma 2.

(38) \[
Ev_{c_\tau} = \frac{\pi}{\pi} \frac{\pi}{\pi} c_\tau - (\frac{\pi}{\pi} + c_\tau B_3) X^\alpha + \frac{\pi}{\pi} Z \frac{\pi}{\pi} + (\frac{\pi}{\pi} \frac{\pi}{\pi} \frac{\pi}{\pi}) B_c^x
\]

(39) \[
Ev_{d_\tau} = \frac{\pi}{\pi} \frac{\pi}{\pi} \frac{\pi}{\pi} + (1-\frac{\pi}{\pi} \frac{\pi}{\pi}) X^\alpha - c_\tau B_3 X^\alpha + \frac{\pi}{\pi} (1 \frac{\pi}{\pi} B_d^x)
\]

(40) \[
Ev_{d_\tau} = \frac{\pi}{\pi} \frac{\pi}{\pi} \frac{\pi}{\pi} - c_\tau B_3 X^\alpha + \frac{\pi}{\pi} (1 \frac{\pi}{\pi} B_d^x) (Z^\alpha X^\alpha - B_d^x)
\]

**RESULT 21** The insurer's optimal contract parameters, \( c_*^x \) and \( d_*^x \), each rise at the margin, with a stochastically-dominant shift in the parties' probability distribution, if:

(33) & (34) hold; \( B_c^x > 0 \); and \( 0 < B_d^x < Z^x X_d^x \).
TWO SHIFTS IN INSURER’S PREFERENCES

Any shift in the insured’s preferences will affect EV only via its effect on B and P. Hence, any such shift can be described by the $Ev(c(\tau), d(\tau), \tau)$ in (6) with $c$ and $p$ changed to $B(c, d, \tau)$ and $P(c, d, \tau)$, respectively. Differentiating this $Ev$ yields (41), (42), & (43), which signs $c_{\tau}(0)$ and $d_{\tau}(0)$ from (32c), (32d), Result 13, and Lemma 2.

\begin{align*}
(41) \quad Ev_{c_3} &= \frac{\pi}{\pi} \frac{F_{c_3}}{\pi} - \frac{\pi}{\pi} \frac{B_{c_3}}{\pi} - \frac{\pi}{\pi} \frac{C_{c_3}}{\pi} \frac{\pi}{\pi} \frac{a}{\pi} - \frac{\pi}{\pi} \frac{B_{c_3}}{\pi} \tau \\
(42) \quad Ev_{cd} &= \frac{\pi}{\pi} \frac{F_{cd}}{\pi} + (1-\frac{\pi}{\pi} \frac{B_{cd}}{\pi}) \frac{\pi}{\pi} \frac{G_{cd}}{\pi} + \frac{\pi}{\pi} \frac{B_{cd}}{\pi} \frac{\pi}{\pi} \frac{a}{\pi} + \frac{\pi}{\pi} \frac{B_{cd}}{\pi} \tau \\
(43) \quad Ev_{d_3} &= \frac{\pi}{\pi} \frac{F_{d_3}}{\pi} - \frac{\pi}{\pi} \frac{C_{d_3}}{\pi} \frac{\pi}{\pi} \frac{a}{\pi} + \frac{\pi}{\pi} \frac{B_{d_3}}{\pi} \tau \tau
\end{align*}

RESULT 22 Let $\tau$ be the shift parameter raising the insured’s alternative utility, $U$, or lowering risk aversion as it rises. His optimal contract parameters, $a^*$ and $d^*$, rise at the margin if: (33) holds and $B_c < 0$.

VII: CONCLUSIONS

The purpose of this paper has been to analyze the rational behavior of insurers and insureds under moral hazard. Our results do not hinge on asymmetric information because there is none. Rather, they derive from moral hazard caused by asymmetric observability—the insurer’s inability to monitor the actions of the insured. The properties of optimal contract parameters offered by the insurer, and her spending to control losses have been determined for the particular second-best contractual form mostly commonly available.

Most of the results depend on assumptions no more restrictive than nondecreasing returns-to-scale in the insured’s spending to control losses, and risk aversion. The optimal contractual parameters are found in terms of the parties’ marginal utility and the loss-reduction technology. Insurer’s risk neutrality and constant returns to spending on loss control greatly simplify results. For instance, the optimal commission proportion and deductible are each inversely proportional to the probability of a covered loss occurring.

This risk-neutral case is the one most consistent with the notions of firm-value
maximization in finance theory, because in many insurance situations, the risks underwritten are nonsystematic in nature. Of course, to the extent that the insurer is risk-averse over total (including diversifiable) risks, our more complex results become appropriate.

Apart from technical results (e.g., existence & uniqueness), our main findings specify optimal contract parameters. They also show how shifts in these parameters, probability distribution over losses, and preferences induce changes in the insured's spending; and how shifts in the latter two areas change the optimal contract. The effects of these shifts on her spending depend on (10), while those on the optimal contract depend on (36) & (37).

The complicated interaction between the insurer and insured is found to have three fundamental forces. The first is the reaction force. This indirect force links shifts in the exogenous circumstances to effects on the parties' expected utilities, EU & EV, via changes in the insured's spending induced by the shifts. The second is also indirect and it is called the moving trigger force. It is a force which changes EU and EV, via the change in the state-of-nature, $\Theta = \Theta'$, that causes the deductible level of loss, i.e., separates the $\Theta$ that cause covered losses (i.e., losses above the deductible) from those that cause uncovered losses (losses not above the deductible). The reaction force and the moving trigger force intertwine in many of their effects on EU and EV. The direct force links such shifts to $EU$ & $EV$ that do not occur via $A_T$, $B_T$, or $\Theta'$.

Below on the left are the Case B second-best (under moral hazard) results which are easily capsulized and for which comparison with the analogous first-best (not under moral hazard) is helpful. Equivalent Case B first-best results are on the right. If $I^*$ under Case A, (25c) & (25d), does not equal the corresponding $I^*$ under Case B, (29c) & (29d), then both parties are at least as well off under the $I^*$ in Case A and (2) is not binding on (6). In general this would mean that Case A describes $I^*$, and it offers the insured more than her alternative utility. See footnote 2 in Section I. If the $I^*$ in Cases A and B are equal, then (2) holds with equality and, except by coincidence, binds (6).

\[
\begin{align*}
\text{SECOND-BEST} & \quad \text{FIRST-BEST} \\
\frac{\pi^*}{d \pi} &= \frac{E^*}{E(\frac{\pi^*}{d \pi} - 1) \pi} & \frac{\pi^*}{d \pi} &= \frac{E^*}{E(\frac{\pi^*}{d \pi} - 1) \pi}
\end{align*}
\]
\[ d = \{E[X + cX + cX']/\pi] - EV \cdot \pi \} / EV \cdot \pi \]

Insured picks \( c \) so \( 0 = EU(w) \)

where \( w = -P - \alpha + (c-d)X \), and \( P(c,d,\alpha) \) adjusts to all \( c, d, \) \( \alpha \) so that \( EU = \bar{U} \) is preserved.

Compensated Comparative Statistics

\[ B_c = \frac{E[(X - P + \Gamma)\alpha]/\pi}{-EU\alpha} \]

\[ B_d = -cE[(P + cD)\alpha]/\pi] / -EU\alpha \]

Pure certainty shift:

\[ B_\tau = [E(cD - 1)(E \cdot \theta)(\alpha\theta - \alpha\theta') - E_3 \cdot \pi \cdot \alpha\theta - cE(\theta - \theta')] / -EU\alpha \]

Second-order stochastic dominance shift:

\[ B_\tau = \frac{[(1-cD)P + E(cD)]\alpha}{-EU\alpha} \]

\[ B_\tau = \frac{E(P - cxD + cD \cdot EV \cdot \pi + (z'X \cdot V_{EXT} + d')/\alpha)}{-EV\alpha} \]

NOTATIONAL APPENDIX

Optimal values of choice variables (i.e., \( \alpha \) and \( I = (p, c, d) \)) is denoted by \( \ast \). Evaluation of a function at \( \theta = \theta' \) and \( \theta = \theta' \) is denoted by \( \ast \) and \( \ast \), respectively. The insurer is \( \alpha \) and the insured is \( \theta \). Derivatives are denoted by subscripted numbers for the partial derivatives with respect to the numbered argument, and by variables for total derivatives with respect to the variable. All functions listed are \( C^2 \).

\( \alpha \) = money spent by her to control losses. Only she can observe \( \alpha \). \( \alpha \in (0, +\infty) \).

\( \alpha(\tau) \) = optimal \( \alpha \) given some extent of a comparative statics shift via \( \tau \). \( \alpha \) is understood.

\( A(p, c, d, \alpha) = \alpha \ast \) given she has agreed to some \( I = (p, c, d) \). \( A(p, c, d) : R^3 \rightarrow (0, +\infty) \).

\( B(c, d) = A(p, c, d, c, d) \) where the \( \alpha \ast \) under \( I = (p, c, d, c, d) \) provides her ex-ante alternative utility, \( U \).

\( c \) = the contract's coverage proportion. That part of the realized loss in excess of \( d \) that is paid to her in benefits.

\( d \) = the contractual deductible. The amount of loss that triggers benefits.

\( \delta \) = the maximum probability of default that the insurer will allow in an \( I(\cdot) \) he offers.

\( D \) = a characteristic function which keeps benefits nonnegative. \( D = 0 \) if \( X(\alpha, \theta) \leq d \) and \( D = 1 \) otherwise, i.e., \( D = 0 \) if \( \theta = \theta(\alpha, d) \) and \( D = 1 \) otherwise.

\( E \) = expectation of the variable that follows, i.e., the integral of that variable over \( \theta \).

\( \phi = \theta \) the function that equal to \( \theta \) for each value of a comparative statics shift via \( \tau \) where \( d \) is understood. \( \phi(\alpha(\tau), \tau) : (0, +\infty) \times R \rightarrow R \).
H(θ) is the cumulative probability distribution over the exogenous state-of-nature, θ.
I(p,c,d) is the insurance contract in which she pays, ex-ante, p in exchange for him paying her, ex-post, c times that part of any loss in excess of d. She spends α, losses x, & gets I(x) = -p + c(x-d)D from him. He retains π = x - I(x).

M = EU_w^D/EU_w^x, which is the ratio of her expected utility over covered losses to her expected utility over all losses.
N = EU_w^D/EU_w, which is M with the numerator weighted by the loss.

p = insurance premium under I. p ∈ R, but Result 1 shows p > 0.
P(c,d) = level of p for which the α" = A(p,c,d) provides her ex-ante alternative utility, U, i.e., for which EU" = 0, given c and d.

π = his change in wealth. π = -I(x) = -p - c(X(α,θ) - d)D.

R = the real number line.
ρ = ED = θ'α, the probability of a covered loss, i.e., of a positive benefit.
Q = EX_w^D, which is his marginal utility for wealth over the loss range of benefits.
X = EX_w^x, weighted by the marginal loss reduction from her spending.

T = X_w^x/EX_w^x. It is the moving trigger force for the insured.
Θ = the exogenous and stochastic state-of-nature. θ ∈ \{0, +∞\}

θ' = the function that equals the θ-value for which X(α,θ) = d, given α and d. θ'(α,d):

\begin{align*}
\theta'(α, d) & \equiv (0, +∞) \\
\Rightarrow & \theta(0, +∞) \\
\theta'(α, d) & \equiv (0, +∞) \times (0, +∞) \rightarrow (0, +∞). \quad X_θ < 0 \leq X_θ.
\end{align*}

Y = E(D-M)V_w^x.

Z is the function used to form the second-order stochastic dominance shifts. Z(θ):

\begin{align*}
(0, +∞) & \rightarrow (0, +∞) \text{ such that } \int Z(θ)dH(θ) > 0 \text{ for some } θ'^2 & < 0 \text{ never}.
\end{align*}

APPENDIX TO SECTION III

Proof of Result 1: Assume "c" = 0. Since he accepts I", the model's assumption on his alternative → EV(π") > V(0) where π" = p" - c"(X(A(I"),θ)) - d"D. But then the model's assumption of V(π") < 0 → by Jensen's Inequality that V(π") = EV(π"). Hence V(π") > V(0) and the strict monotonicity of V → π" > 0. This → EW" < EW" where w" = X" - A(I") - p" + c"(X" - d")D, and w" = X" - A(I") + 0 is her w from self-insurance, (0,0,d), when she spends A(I").

Consider the slope of w in θ, wθ. Since c" = 0 < Xθ, it follows that wθ < wθ ∀ θ.

Then from the more convenient perspective of higher (i.e., worse) θ to lower (i.e., better) θ, we find that w" is everywhere at least as steep as w" is in θ. This relative steepness and EW" < EW" imply w" exhibits second-order stochastic dominance over w" in θ.

Hardar & Russell (Proposition 2, 1969) showed that such dominance implies EU(w") < EU(w") for all U that are rising and concave in θ (from high θ to low θ). Thus she would self-insure, i.e., would prefer (0,0,d) to any I" with c" = 0. Hence c" > 0.

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If \( p'' < 0 \), then \( \text{EV}[-p'' \cdot c''(X'\cdot-d')] \leq \text{EV}(0) \) by construction of \( D \), and the insurer will not accept such an \( I \). Hence \( p'' > 0 \). Since \( X \geq 0 \) by definition, it follows that \( d < 0 \) implies payment to the insured of at least \( c(0-d) \). Therefore \( p \) may be reduced to \( p + cd \) for the old \( d \), and then \( d \) raised to \( 0 \) without changing the wealth of either party under any possible \( \theta \). Thus consideration of \( d < 0 \) may be excluded without loss of generality.

If there is no chance of \( X^\theta > d \), then \( D = 0 \) always and his EU under such \( I \), \( \text{EU}[-p -\alpha^x + (cD-1)(X^\theta-dD) -dD] = \text{EU}[-p -\alpha^x -X^\theta] \), is less than his EU from self-insurance \( \text{EU}(-\alpha^x -X^\theta) \). But our model assumes EU from self-insurance is no more than \( U \). Hence the insured will not accept such an \( I \).

Q.E.D.

**Proof Result 2** To show that the choice set of \( I = (p,c,d) \) or \( (P(c,d),c,d) \) is compact we will show below that the set of possible optimum \( I^\theta \) is bounded. Since \( I(.) \) can have any value in \( R^3 \) or \( R^2 \), it can have the value of its bounds; and thus boundedness implies \( I(.) \) is defined on a compact set. Since the model's assumptions assure that EV is continuous, this boundedness is sufficient to imply the existence of a finite optimum.

**Show that the Set of Possible \( I^\theta \) is Bounded** Let \( w_0 \) and \( \pi_0 \) be his and her bounded initial wealths. She must pay \( p \) ex-ante, hence \( p < w_0 \). But \( p > 0 \) from Result 1 then implies \( p \) is bounded. Let \( \theta_S = \text{MAX} \theta \in \Theta \) such that \( \theta \) does not default, i.e., \( \pi_0 + p - c(X(\alpha^x, \theta) -dD) = 0 \). If \( d = x_\theta = X(\alpha^x, \theta_S) \), then \( \pi_0 = p > 0 \) for all \( \theta \) and we have \( \theta_S = \text{MAX} \theta \in \Theta \); but this contradicts that \( X^\theta > d \) with positive probability from Result 1. Hence \( D = 1 \) here. Then \( c < (\pi_0 + p)/(x_\theta - dD) \), which with \( c > 0 \) from Result 1, implies \( c \) is bounded. Since \( X^\theta > d \) with some positive probability, \( d \) is bounded above. Result 1 also showed that we can disregard \( d < 0 \), hence the \( d \) we consider is bounded. Q.E.D.

**APPENDIX TO SECTION IV**

**Computations for Results 3 - 5.2** For a shift in \( p \), we have \( \omega[\alpha(\tau), \tau] = -(p + \tau) - \alpha + (cD-1)(X-dD) -dD \& X[\alpha(\tau), \phi(\alpha(\tau), \tau)] \). Hence \( \omega_1 = -1 + (cD-1)X_\alpha, \omega_2 = -1, \omega_3 = 0, \& \phi_2 = 0 \), which via (9) \( \Rightarrow \text{EU}_{\alpha_2}[\alpha(0), 0] = -\text{EU}_{\alpha_2}U_{v_0}^{\alpha_2} \). Then (8) \( \Rightarrow \) Result 3: SIGN \( \alpha_\tau = \text{SIGN} \omega_2^\alpha \). For a shift in \( c \), we have \( \omega[\alpha(\tau), \tau] = -(p + (c+\tau)D -1)(X-dD) -dD \& X[\alpha, \phi(\alpha(\tau))] = d \). Hence \( \omega_1 = -1 + ((c+\tau)D -1)X_\alpha, \omega_2 = (X-dD), \omega_3 = X_\alpha D, \& \phi_2 = 0 \), which via (9) \( \Rightarrow \text{EU}_{\alpha_2}[\alpha(0), 0] = \text{EU}_{\alpha_2}U_{v_0}^{\alpha_2} + \text{EU}_{\alpha_2}(X^\theta - dD)U_{v_0}^{\alpha_2} = \mathbb{E}[X_\alpha - \omega_2^\alpha]U_{v_0}^{\alpha_2} \).
Then (9) \(\rightarrow\) Results 4.1 and 4.2. For a shift in \(d\), we have \(\omega(d,\tau) = -p - \alpha - x + (X - d - \tau)D\) & \(X\alpha, \alpha(\alpha, \tau) = d + \tau\). Hence \(\omega_1 = -1 + (c_2 - 1)X \alpha, \omega_2 = -c_2, \omega_2 = 0,\) & \(\Phi_2 = 1/X_\theta\), which via (9) \(\rightarrow\) \(E_\alpha \{\sigma(0), \sigma\} = -c_2 [E_{\alpha\alpha} \{\alpha(0), \sigma\} + T]\). Then (8) \(\rightarrow\) Result 5.1 & 5.2 because: \(U_{w\omega} < 0 < U_{w}, \) & \(X_\theta > 0\).

Intermediate steps for the following equations where \(E_{\alpha\alpha} = E_{\alpha\alpha} \{\alpha(0), \sigma\}\)

(16): \(E_{\alpha\alpha} = -E((X - d)D - P_{c\alpha})\{\omega_{\alpha\omega} + E\alpha\omegaD, \phi = 0\}.

(17): \(E_{\alpha\alpha} = [CT + E(P_{c\alpha}D)] - E_{\alpha\omega}D, \phi = 1/X_\theta\).

(18): \(E_{\alpha\alpha} = E(c_2 - 1)(X\theta - \theta)X_{\alpha\omega}^\theta - E(P_3) - (c_2 - 1)(X\theta - \theta)X_{\alpha\omega}^\theta - cT(\phi - E_\theta)\) where \(P_3 = E(c_2 - 1)(X\theta - \theta)X_{\alpha\omega}^\theta / E_{\alpha\omega}^\theta, \phi = \phi - E_\theta\).

(19): \(E_{\alpha\alpha} = X(1 - c_2)(X\theta - \theta)X_{\alpha\omega}^\theta - cT = X(1 - c_2)(X\theta - \theta)X_{\alpha\omega}^\theta - cT\) since \(P_3 = E((1 - c_2)X_{\alpha\omega}^\theta / E_{\alpha\omega}^\theta, \phi = X_\theta, \phi = 0, \) and \(E_{\alpha\alpha} = -P_{3\alpha\omega}E_{\alpha\omega}^\theta\).

APPENDIX TO SECTION V

Terms that vanish if \(V_{\alpha\alpha}^\omega = 0 = X_{\alpha\alpha} X_{\alpha\alpha}

Second-Order Conditions for Case A:

\(0 > E_{dd}^p = c^p \beta / X_\alpha^\omega + E_\alpha^p \beta - c^p D^\omega / X_\alpha^\omega,\) and

\(0 > E_{cc}^p E_{dd}^p - (E_{cd}^p)^2\) where for \(\beta = B^\alpha X_{\alpha}^{-1}\),

\(E_{cc}^p = c^p X_{\alpha}^\omega - E_\alpha^p c^p -(2c^p + c^\alpha X_{\alpha}^\omega) D^\omega / X_\alpha^\omega,\) and

\(E_{cd}^p = c^p X_\alpha^\omega - E_\alpha^p -(2c^p + c^\alpha X_{\alpha}^\omega) D^\omega / X_\alpha^\omega + X_\alpha^\omega (X - d + c^\alpha X_{\alpha}^\omega) D^\omega / X_\alpha^\omega.

Second-Order Conditions for Case B:

\(0 < E_{pp}^p = -c^p X_\alpha^\omega - c^p = E_\alpha^p X_{\alpha}^\omega - E_{pp}^p X_{\alpha}^\omega D = -c^p X_{\alpha}^\omega - E_{pp}^p X_{\alpha}^\omega D\) and

\(0 > E_{pp}^p E_{cc}^p - (E_{pc}^p)^2\) where

\(E_{cc}^p = c^p X_{\alpha}^\omega - E_\alpha^p (2c^p + c^\alpha X_{\alpha}^\omega) D^\omega / X_\alpha^\omega,\) and

\(E_{pc}^p = c^p X_{\alpha}^\omega - E_\alpha^p (2c^p + c^\alpha X_{\alpha}^\omega) D^\omega / X_\alpha^\omega + E_{pc}^p X_{\alpha}^\omega D = -c^p X_{\alpha}^\omega - E_{pc}^p X_{\alpha}^\omega D\).

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Let $\eta = A_{d}^{x}X_{d} - 1$. 

$$E_{pp}^{x}E_{pc}^{x}E_{pd}^{x}$$

where

$$E_{dd}^{x} = \eta^{2} - \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha}$$

$$E_{cc}^{x}E_{cd}^{x} < 0$$

$$E_{cd}^{x} = -\frac{\pi}{\alpha} + \frac{\pi}{\alpha} + \frac{\pi}{\alpha} + \frac{\pi}{\alpha} + \frac{\pi}{\alpha} + \frac{\pi}{\alpha}$$

and

$$E_{dd}^{x} = \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha} - \frac{\pi^{2}}{\alpha}$$

REFERENCES


