THE BRENNAN AND SCHWARTZ TWO FACTOR MODEL
OF THE TERM STRUCTURE OF INTEREST;
EMPIRICAL EXTENSION

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Introduction

In a series of papers by Michael Brennan and Eduardo Schwartz [1979, 1980, 1982, 1983], a two factor model of the term structure of interest has been developed and tested, using data from Canada and the U.S. The model focuses upon the movements in the short and long term rates of interest over time. In the most recent specification and estimation of their model [1982, 1983], these authors begin with conjectures about the stochastic evolution of the "short" and "long" (i.e., instantaneous and consol) rates of interest, r and l, respectively. These are assumed to follow a stochastic process of the type

\[ dr = (a_1 + b_1(l-r)) dt + r \sigma_1 dz_1 \]  
\[ dl = \ell (a_2 + b_2 l + c_2 l) dt + l \sigma_2 dz_2 \]

(1)

(2)

where t denotes calendar time, and dz_1 and dz_2 are increments to a standard Wiener process, so that E[dz_1] = E[dz_2] = 0, E[dz_1 dz_2] = \rho dt and E[dz_1^2] = E[dz_2^2] = dt; \rho is the instantaneous correlation between the processes.

In the formulation given above, the scale of the unanticipated change in each of the interest rates is presumed to be proportional to the current value of that rate, an hypothesis which is not rejected by their empirical tests. The coefficient b_1 reflects the essence of expectations based theories of the term structure, wherein long rates are based upon expectations about future short interest rates. The authors note that if such expectations are rational, the short rate will have a tendency to regress towards the current value of the long rate so that b_1 > 0. Their system of equations also allows the unanticipated changes in the two interest rates to be correlated with each other. For other aspects of the motivation and derivation of their system of equations, the reader is referred to the aforementioned set of papers.

For empirical purposes the system was replaced by the discrete approximation:

\[ \frac{r_t - r_{t-1}}{r_{t-1}} = a_1 + b_1 \frac{l_{t-1} - r_{t-1}}{r_{t-1}} + \epsilon_1 t \]

\[ \frac{l_t - l_{t-1}}{l_{t-1}} = a_2 + b_2 l_{t-1} + c_2 l_{t-1} + \epsilon_2 \]

(3)

(4)

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These equations were estimated by the authors using yields to maturities on U.S. government obligations. (Their earlier two papers estimated a pair of equations that differed slightly from (3,4) using yields to maturity on Canadian and U.S. government securities.) Thus, while their short rates were yields to maturity on pure discount instruments, their long rates were proxied by yields to maturity (internal rates of return) on long-term government bonds (with terms to maturity ranging from 13.5 to 40 years). The latter yields to maturity are, in some sense, comprised of weighted combinations of spot (pure discount bond equivalent) rates spanning the entire maturity spectrum. As is well known, these implicit weights change with changes in the term structure. Lower levels of interest across the term structure tend to impute higher relative weights on the longer-term spot rates in determining yields to maturity on bonds than do higher levels of interest. In checking the “durations” (see Bierwag, Kaufman and Khang [1978]) of the underlying long-term bonds used in the previous studies, we found they ranged from less than 9 years to greater than 22 years. This wide range of fluctuation could potentially impart some bias to the estimations reported in the previous papers. It should be stressed at this point, however, that the primary interest of the authors in these papers was not in the stochastic process (3, 4), per se, but in the predictive ability of the bond pricing model which they derived from it. As our focus in this paper is on the stochastic process itself, we must take further steps to remove this potential source of bias.

To see how use of yields to maturity could render biased estimates, consider the following scenario. Suppose we have a spot rate term structure that is increasing monotonically over all maturities, as shown by curve A in Figure 1 below. Next, suppose an upward parallel shift occurs in the term structure, represented by curve B. If the earlier short and long rates were given by S and L, the newer ones would be given by points somewhere near to S' and L'. Note that L' has not increased by the full amount of shift in the term structure. This is because the yield to maturity on a long-term bond will be increasingly weighted towards

![Figure 1](image-url)
shorter-term spot rates as the levels of interest rates rise. Under these conditions, two sources of bias emerge. First, use of yields to maturities would not capture the full movement in the long rates relative to the short rates, potentially impacting the correlations among the error terms of (3) and (4). Second, the shift to an effectively shorter-term interest rate acts to close the gap between the long and the short rates, rendering the estimated speed of gap closure biased. While the direction of bias is clear under the above scenario, opposite biases could result from different scenarios. The magnitude of the bias and its inconsistency across different samples would be most pronounced if there were both a slope to the term structure (either positive or negative) that predominated over time and a secular trend in interest rate levels (upward or downward) over the time periods sampled.

In the reestimations that follow, we attempt to control for this source of statistical ambiguity by utilizing spot rates of interest. In opting for the theoretical advantages of focusing on spot rates, which need to be estimated, a tradeoff has been made; the exactness attained by using actual yields to maturity is sacrificed. As these yields to maturity are not independent of the particular instrument with which they are associated, this sacrifice is likely to be small. The paper is organized into five sections. In the second section, we describe the data sets and term structure estimation techniques that were employed in our study. These data are used in reestimating the Brennan and Schwartz system of equations (hereafter MBES), and the results are compared. Next, the system of equations is transformed into a single equation focusing on movements in the short-term interest rate. Estimations of this equation are reported for both the United States and Great Britain. In the fourth section, out-of-sample tests are done using the model developed in section three as a forecasting model. The model is found to possess some forecasting value. The fifth and final section provides a summary of our findings.

Data and Reestimation

To reestimate the basic stochastic process (3,4), term structure data were required for the United States. Our data series spanned before and beyond the 1958-1979 time period sampled by the Brennan and Schwartz papers, but only data covering the same time intervals were used in the comparisons. The remaining data were used in the development and testing of an extended model presented later. Our testing of the extended model also used a parallel series of British data for corroboration. Although all of these data are described below, results stemming from use of the British and extended U.S. data series are not discussed until sections 3 and 4.

The basic raw data used in estimating the term structures of interest consisted of monthly government securities ask prices and bond characteristics (e.g., maturity date, coupon rate, special tax treatment, etc.), covering the period from March of 1947 through January of 1982 for the United States and 1955 through October of 1978 for Great Britain. For the United States the basic data were obtained from the same source used by Brennan and Schwartz: CRSP government monthly bond data files. The term structures were estimated using the tax-adjusted cubic spline fitting technique developed by McCulloch [1975]. The source of the data on British government securities (gilt's) for 1955 to 1976 was quotations from gilt brokers, and for 1976 to 1978, quotations published in the Financial Times. Stephen Schaefer at the London Business School collected the British data, and using a linear programming technique he developed [1981] that explicitly takes into account the tax treatment of the various cash payments, generated term structure estimates for each of the 286 months that comprise the period of study. Schaefer has argued that his linear programming technique produces better estimates of the term structure for Britain, while the McCulloch technique seems better suited for the United States, due to differences in the tax systems of each country. Both programs give the zero coupon equivalent yields (or spot interest rates) for various maturities. The longest spot rates that were obtainable throughout all periods in both countries were 13-year rates. Accordingly, in future references to the "long rate," the 13-year rate was used as its proxy.1

1 Livingston (1982) has shown that although yield to maturity curves are virtually flat for longer maturities, this does not imply that the spot rate yield curves are necessarily flat; thus, some distortion may have
Consequently, the long rate possessed a constant 13-year duration that approximated the mean duration of the bonds MBES used in determining the yields to maturity proxies for their long rate.

The "short rate" used in the reestimated (hereafter REST) equations were the same as those employed by MBES—effective annual yields on Treasury Bills having a maturity close to but not exceeding 30 days. This allows for direct comparisons to be made with the earlier studies, where all differences devolve from using different long rates.

New estimations (hereafter NEST) were also performed using one-year spot rates of interest obtained from the term structure calculations. Thus, the NEST used short and long interest rates that differed from MBES, while sharing in common with REST the long rates. While neither the 30-day nor one-year spot rates conform exactly with the instantaneous short rates elicited by the continuous time equations posited by MBES, and therefore cannot (strictly speaking) be used for an instantaneous hedge in their arbitrage-based bond pricing equations, the former more closely approximate this objective. In other respects, focus on the one-year spot rate adds some interest [no pun intended]. Clearly this rate likely has more impact upon the value of most bonds whose terms to maturities extend beyond a few months into the future. Indeed, in some of the discrete-time single-factor generalized duration measures that have been derived by Babbel [1983], Nelson and Schaefer [1983] and Maloney and Yawitz [1983], fluctuations in the one-year rate are the focal point of the models.

Several points of contrast and comparison between the MBES and REST equations are of interest. First, the correlations between the REST residuals of equations (3) and (4), as well as the standard errors of the regressions σ₁ and σ₂ are of generally similar magnitudes for the overall period and the two subperiods. Before proceeding with further comparisons of statistics shown in Table 1, a comment is in order on findings not included there regarding serial correlations. While MBES reported (sometimes significant) negative serial correlations in their residuals, we found serial correlations insignificantly different from zero (and of alternating signs across periods). MBES suggested their negative serial correlations were evidence of a possible error in variables problem, a conjecture that receives support from our contrasting findings.

One item of particular note in the REST equations is their low explanatory power. Unfortunately, we are unable to compare our results directly with those of MBES, as they did not report the coefficients of determination devolving from their system of equations. The equation focusing on movements in the short rate (3) exhibited a corrected r-square value that was rather low — .065 — over the full 21-year period. This figure was also low, but positive² in each of the subperiods, but deteriorating markedly over time. The equation focusing on movements in the long rate (4) exhibited abysmally lower explanatory power (r-square of .017) over the full period, and even negative over the first subperiod.

An examination of the estimated coefficients and their t-statistics reveals several comparisons of note. First, we observe a much lower t-value for α₁ over the full period, and note that this coefficient no longer carries the same sign across subperiods. From this evidence it would be difficult to maintain that the percentage changes in the short rate of interest are consistently related to their levels. We also observe a higher t-value over the full period for β₁ than that reported earlier, and note that the estimated coefficients are 28 to 50 percent lower, depending on the time span considered. These lower estimated coefficients are indicative of a slower rate of closure in the gap between long and short rates than that shown by MBES. We also note that although this estimated rate of closure differs considerably across subperiods, it appears to be less volatile than that indicated by earlier estimations.

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² It will be recalled that r-square values produced under an Aitken-Zellner type of estimation, when corrected for degrees of freedom, are occasionally negative.
Turning to the second equation of the system, we note higher and statistically more significant estimated intercepts $a_2$ overall and across both subperiods. The $t$-values associated with the $b_2$ and $c_2$ coefficients were also modestly higher than those estimated earlier, and the absolute magnitudes of the coefficients were also higher than in MBES; additionally, the ratios of $c_2$ to $b_2$ were more negative than before, serving to counterbalance the higher positive intercept values. Little importance can be imputed to these latter comparisons, however, because the overall explanatory power of the equations is so low.

Table 1

<table>
<thead>
<tr>
<th>Period</th>
<th>$a_1$</th>
<th>$b_1$</th>
<th>$a_2$</th>
<th>$b_2$</th>
<th>$c_2$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\rho$</th>
<th>$\frac{CRSQ_{eq.1}}{\text{CRSQ}_{eq.4}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 1958</td>
<td>-0.0887</td>
<td>0.1102</td>
<td>0.0089</td>
<td>0.00358</td>
<td>-0.0037</td>
<td>1.133</td>
<td>0.0298</td>
<td>0.2063</td>
<td>N/A</td>
</tr>
<tr>
<td>Dec 1979</td>
<td>(-1.6863)</td>
<td>(3.6611)</td>
<td>(1.2913)</td>
<td>(2.1059)</td>
<td>(-1.8500)</td>
<td>0.005</td>
<td>0.1141</td>
<td>0.0323</td>
<td>0.2034</td>
</tr>
<tr>
<td>Reestimation</td>
<td>0.0115</td>
<td>0.0771</td>
<td>0.0146</td>
<td>0.00454</td>
<td>-0.0052</td>
<td>1.141</td>
<td>0.0323</td>
<td>0.2034</td>
<td>0.017</td>
</tr>
<tr>
<td>New Estimation</td>
<td>0.0010</td>
<td>0.0658</td>
<td>0.0104</td>
<td>0.00325</td>
<td>-0.0040</td>
<td>0.0770</td>
<td>0.0327</td>
<td>0.5718</td>
<td>0.014</td>
</tr>
<tr>
<td>Dec 1958</td>
<td>-1.809</td>
<td>0.1882</td>
<td>0.0151</td>
<td>0.00468</td>
<td>-0.0662</td>
<td>1.286</td>
<td>0.0233</td>
<td>0.0519</td>
<td>N/A</td>
</tr>
<tr>
<td>June 1969</td>
<td>(-2.3992)</td>
<td>(3.9208)</td>
<td>(0.7550)</td>
<td>(1.2649)</td>
<td>(-0.9234)</td>
<td>0.104</td>
<td>0.1311</td>
<td>0.0834</td>
<td>0.002</td>
</tr>
<tr>
<td>Reestimation</td>
<td>0.0176</td>
<td>0.0980</td>
<td>0.0263</td>
<td>0.00699</td>
<td>-0.0100</td>
<td>1.311</td>
<td>0.0313</td>
<td>0.0824</td>
<td>0.002</td>
</tr>
<tr>
<td>New Estimation</td>
<td>0.0284</td>
<td>0.1160</td>
<td>0.0220</td>
<td>0.00162</td>
<td>-0.0053</td>
<td>0.0715</td>
<td>0.0320</td>
<td>0.5418</td>
<td>0.033</td>
</tr>
<tr>
<td>July 1969</td>
<td>-0.0135</td>
<td>0.0377</td>
<td>0.0319</td>
<td>0.00444</td>
<td>-0.0074</td>
<td>0.0914</td>
<td>0.0349</td>
<td>0.3923</td>
<td>N/A</td>
</tr>
<tr>
<td>Dec 1979</td>
<td>(-0.1634)</td>
<td>(1.0217)</td>
<td>(1.4434)</td>
<td>(1.9389)</td>
<td>(-1.8974)</td>
<td>0.002</td>
<td>0.0918</td>
<td>0.3755</td>
<td>0.033</td>
</tr>
<tr>
<td>Reestimation</td>
<td>-0.0029</td>
<td>0.0270</td>
<td>0.0466</td>
<td>0.00560</td>
<td>-0.0102</td>
<td>0.0918</td>
<td>0.0330</td>
<td>0.3755</td>
<td>0.033</td>
</tr>
<tr>
<td>New Estimation</td>
<td>0.0251</td>
<td>0.0409</td>
<td>0.0379</td>
<td>0.00545</td>
<td>-0.0095</td>
<td>0.0824</td>
<td>0.0335</td>
<td>0.5849</td>
<td>0.001</td>
</tr>
</tbody>
</table>

In comparing the MBES and NEST estimations, where one-year short rates were used, many of the prior points of departure are reconfirmed. There are some striking differences, however. We note that the correlations $\rho$ over time between the NEST residuals of equations (3) and (4) were substantially higher (.57 vs. .21) and within a much (7.9 times) narrower range across subperiods (.54-.58 vs. .05-.39) than the MBES correlations. Another finding in a similar vein is that the standard errors of the NEST regressions ($\sigma_1$ and $\sigma_2$) were much more stable across time periods than those of MBES. While $\sigma_1$ of MBES ranged from .0914 to .1286, the NEST range was from .0715 to .0824, a spread 3.5 times narrower. Further, the levels of the errors appear to have undergone substantial reductions. Turning to $\sigma_2$, we note that the NEST levels, while averaging slightly higher than those of MBES, also fell within a much (7.7 times) narrower range across time periods (.0320-.0335 vs. .0233-.0349). Thus, in both of these cases the NEST data sets of spot interest rates produced the more stable patterns of regression statistics. These more stable estimations were associated with even lower r-square values than under MBES. Clearly most of the correlation in these two equations stems from the contemporaneous fluctuations in their residuals, a finding that will be exploited in the reformulation of their model presented in Section 3.
In the remainder of this paper, references to the short rate of interest will be solely directed to the one-year spot rate.

Model Reformulation, Extension, and Estimation

Having observed a virtual absence of explanatory power in our second equation, except with respect to how its errors are correlated with those of the first equation, it is reasonable for us to recast the equations by combining those elements exhibiting the most promise, while discarding those shown to have negligible merit. As more than 99 percent of the fluctuation in the long rate has been shown to pass through to the error term, $e_{2t}$, which in turn is highly correlated with $e_{1t}$, this suggests approaching the reformulation of equation (3) in the context of a problem in omitted variables. In this case, the omitted variable to be included as part of (3) would be the percentage change in the long rate, and a new, much smaller error term would be produced.

It is also apparent from the new estimation of (3) that the $a_1$ coefficient on the reciprocal of the short rate variable, with an overall $t$-statistic of .03, lacked statistical importance. Hence, this variable was dropped from further inclusion in our reformulated model.\(^3\) It was replaced by a constant term to capture any normal spread (liquidity premium) that might prevail over time between the long and short rates, as well as control for a secular trend in interest rates.

We also included the percentage change in the long rate, lagged by one month, as an explanatory variable, in the event that a portion of the fluctuation in the short rate was not related concurrently with fluctuations in the long rate. Several justifications could be proffered for the inclusion of this lagged variable. For example, adaptive expectations mechanisms or partial adjustment models are commonly used to rationalize the importance of lags; alternatively, Frankel [1982] has supplied a simple macroeconomic model of the adjustment in the price level and interest rate that could elicit such a lag, and Cornell [1983] provides a number of scenarios connected with money supply announcements that evoke or are consistent with a lag between the time of a change in the long rate and the full change in the short rate.

The final form of the model to be estimated was therefore comprised of four variables plus a constant, and is given below.

$$\frac{r_t - r_{t-1}}{r_{t-1}} = \lambda_0 + \lambda_1 \frac{l_t - l_{t-1}}{l_{t-1}} + \lambda_2 \frac{l_{t-1} - l_{t-2}}{l_{t-2}} + \lambda_3 \frac{l_{t-1} - r_{t-1}}{r_{t-1}} + \nu_t$$  \( (5) \)

In words, the equation specifies that percentage changes in the short rate are composed of two parts — anticipated and unanticipated movements. The anticipated movement is given by the first, third and fourth explanatory terms. Aside from the constant, the other two terms are both comprised of variables that are observable beforehand — the percentage change in the long rate during the previous period and the gap between the long and short rate at the beginning of the period. In some sense these two terms together can be considered as a time-subscripted intercept term in the regression equation, where a positive coefficient on the lagged change in the long rate indicates some delay in the adjustment of the short rate to movement in the long rate, and a positive coefficient on the reversion term is consistent with the expectations hypothesis. A negative constant term would be consistent with a positive liquidity premium in the long rate, provided that other things do not enter into this constant term (such as a secular trend in interest rates over time). The unanticipated portion of changes in the short rate is related to the contemporaneous changes in the long rate, and is given by the second term.\(^4\)

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\(^3\) Before this variable was dropped altogether, it was further tested along with the new variables (the contemporaneous and lagged percentage changes in the long rate) but found to contribute negligibly to the model.

\(^4\) It should be noted that the discrete-time model given by eq. (5) above does not have a continuous time counterpart analogous to the MBES eqs. (1) and (2), both because it does not focus on the instantaneous rate of interest and because of the presence of the term reflecting prior movement in the long rate.
To test the extended model, the data for the United States and United Kingdom were divided into three subperiods of 94-95 months each of exactly corresponding time periods. The remaining U.S. data were also used in a period ending before the U.K. data began and in a period beginning after the U.K. data were exhausted. Equation (5) was then estimated by ordinary least squares using these data. The results are reported in Table 2.

<table>
<thead>
<tr>
<th>Period</th>
<th>( \Delta r/r_{-1} = \lambda_0 + \lambda_1(\Delta l/l_{-1}) + \lambda_2(\Delta l_{-2}/l_{-2}) + \lambda_3(l_{-3}/r_{-1} - 1) + \nu_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_0 )</td>
</tr>
<tr>
<td><strong>United States</strong></td>
<td></td>
</tr>
<tr>
<td>Mar 1955-</td>
<td>-.0188</td>
</tr>
<tr>
<td>Oct 1978</td>
<td>(-3.004)</td>
</tr>
<tr>
<td>Apr 1947-</td>
<td>-.0440</td>
</tr>
<tr>
<td>Feb 1955</td>
<td>(-2.039)</td>
</tr>
<tr>
<td>Mar 1955-</td>
<td>-.0268</td>
</tr>
<tr>
<td>Jan 1963</td>
<td>(-1.744)</td>
</tr>
<tr>
<td>Feb 1963-</td>
<td>-.0102</td>
</tr>
<tr>
<td>Dec 1970</td>
<td>(-1.420)</td>
</tr>
<tr>
<td>Jan 1971</td>
<td>-.0157</td>
</tr>
<tr>
<td>Oct 1978</td>
<td>(-1.575)</td>
</tr>
<tr>
<td>Nov 1978</td>
<td>.0084</td>
</tr>
<tr>
<td>Jan 1982</td>
<td>(.768)</td>
</tr>
<tr>
<td><strong>United Kingdom</strong></td>
<td></td>
</tr>
<tr>
<td>Mar 1955-</td>
<td>-.0159</td>
</tr>
<tr>
<td>Oct 1978</td>
<td>(-2.219)</td>
</tr>
<tr>
<td>Mar 1955-</td>
<td>-.0208</td>
</tr>
<tr>
<td>Jan 1963</td>
<td>(-1.461)</td>
</tr>
<tr>
<td>Feb 1963-</td>
<td>-.0045</td>
</tr>
<tr>
<td>Dec 1970</td>
<td>(-0.669)</td>
</tr>
<tr>
<td>Jan 1971</td>
<td>-.0467</td>
</tr>
<tr>
<td>Oct 1978</td>
<td>(-2.203)</td>
</tr>
</tbody>
</table>

The estimations for the United States tend to provide support for this model. For the overall period the coefficients all have the expected signs and are statistically significant well beyond the 99 percent level. Further, most of the coefficients retain a high level of statistical significance across subperiods. For the United Kingdom, the data tend to provide less support for this extended model, although over the entire period, the coefficients are all statistically significant at the 95 percent level (using one-tailed tests) and carry the anticipated signs. During some of the subperiods, however, the decline in statistical significance is marked.

An examination of the intercept terms indicates that they are all negative, except for the United States during the final subperiod. While the low t-statistic associated with this intercept term does not lend much confidence in its sign, the positive sign may well have derived from the upward trend in interest rates during that period, a trend which could have swamped any of

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5 Nelson and Schaefer (1983) have noted that this regression procedure provides only a first-order approximation to the true values of the coefficients. However, Schaefer (1980) has provided evidence that such approximations may be quite good in this context.
the tendency for this term to be negative from a liquidity preference effect.

Turning to the coefficients on the term representing the contemporaneous change in the long rate, we note that for the United States the magnitudes are in excess of one in every subperiod but the first, and that their t-statistics are very large (excepting the first subperiod). When these coefficients are added to those of their lagged counterparts, it is evident that the short rate fluctuates in the same direction but about twice as far as the long rate. A rather surprising result is that the coefficients associated with the lagged changes in the long rate were marked by such high t-statistics. This is one indication of an absence of serious multicollinearity; indeed, in checking for possible collinearity between the lagged and unlagged changes in the long rate, we found the correlation to be virtually nil, indicating no cause for concern. Reestimating the equation using generalized least squares procedures to correct for moving average and autocorrelation patterns (however slight) in the residuals only served to strengthen the t-values associated with this lagged variable, while rendering the coefficient estimates virtually unchanged. Further tests for multicollinearity did not give rise to concern except during the early 1970s, where changes in the lagged variables were negatively correlated. Perhaps this accounts for the drop in the t-values.

In the case of Great Britain, the general pattern is somewhat different. The short rate tends to fluctuate in the same direction as the long rate but, unlike in the United States, the distance is about the same when the lagged response is included. The relationship also appears to be considerably weaker.

The coefficients on the term controlling for reversion of the short rate toward the current long rate all possess the appropriate signs and are statistically significant above the 95 percent level of confidence, regardless of the subperiod considered. An analysis of the beta statistics reveals that each of the lagged variables, which together with the intercept term account for the anticipated portion of movements in the short rate, is responsible for roughly one half as much of the fluctuation in the dependent variable as that associated with the contemporaneous change in the long rate (except during the 1947-55 period during which their effects were even larger), a rather sizable amount.

Testing was conducted to determine if the coefficients given by these separate regressions were stable over time. Although casual observation of some rather wide swings in their values gave cause for concern, Chow tests did not allow us to reject the hypothesis that the estimated coefficients were stable over time. It is likely that the failure of these tests to detect instability is related more to the relatively poor explanatory power of these models than to the absence of such instability. One technique available to help us determine whether the (casually) observed instability in the coefficients was due to outlying observations caused by factors outside the model is the method of robust regression, where we no longer attempt to fit a line that minimizes the sum of squared residuals but rather use a different penalty function on the residuals. In using this technique, we found sizable changes in the estimated coefficients from those obtained earlier, but the pattern of instability in these coefficients remained. We did encounter some disturbing multicollinearity for Great Britain. Fortunately, for purposes of statistical estimation, the correlation coefficients exhibited changing signs across periods, rendering the estimations of each subperiod somewhat suspect, while leaving the whole period estimations less affected, as much of the intraperiod multicollinearity was canceled over time.

Given the rather poor explanatory power of the extended model for the United Kingdom, and noting that most of what little explanatory power it had was derived from the

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6 Beta coefficients measure the change in the explained variable (in standard-deviation units) for unit changes in each explanatory variable (in standard-deviation units) holding other variables constant. By controlling for units of measurement among the independent variables, these beta coefficients are more directly comparable in terms of conveying the relative impact of each variable upon the explained variable.

7 For the United States, the estimated coefficient $\lambda_1$ exhibited swings in values over time ranging from 0.38 to 2.61, when measured by least squares techniques with 24 months of data. For the United Kingdom, the range was from -1.5 to 1.80.
contemporaneous (and unpredictable) change in the long rate, we had little cause to be optimis-
tic about the viability of the model as a forecasting tool. This was especially true in light of the
insolubility of the coefficients over time. However, in the case of the United States, where the
fits were much better, there was some cause for hope. The results of our tests on the forecasting
value of this model are given in the section that follows.

Out-of-Sample Forecasts

In this section we present an analysis of the forecasting ability of the extended short-rate
equation. The methodology and benchmarks used in assessing the forecasting ability of our
equation have become standard with the passage of time (see, for example, Meese and Rogoff
[1983]).

Our first task was to estimate coefficients for equation (5). The procedure used was a roll-
ing regression technique where 60 consecutive months of data were used to generate
coefficients appropriate for forecasting changes in the short rate over the ensuing (i.e., 61st)
month. These coefficients were then applied to the data observable at the beginning of the
forecasting period (i.e., the spread between the long and short rates and the percentage change
in the long rate over the previous month) to give a forecast of the percentage change in the
short rate. The coefficient associated with the contemporaneous change in the long rate was
multiplied by zero in all cases, as the magnitude and direction of change in the long rate could
not be foreseen in our model. The resulting predicted rate of change was applied to the current
short rate to provide a forecast of the future short rate. The procedure was then repeated with
a focus on the next (62nd) month by utilizing data from the 2nd through 61st months to esti-
mate the coefficients vector and continue as before. The process was repeated again and again
until our entire data set was exhausted.\footnote{It is noted that the intercept term had the least statistical signif-
ificance from the in-sample tests. Thus, in the out-of-sample forecasts, the intercept term was dropped from
the model whenever during the 60 months preceding the forecast it produced t-values less than 1.5.}

In assessing the merit of the extended model (Model 1), the forecast error was compared
with four benchmarks. First, the error was compared with that which would have resulted if
the contemporaneous percentage change in the long rate had been correctly foreseen (recall
that a change of zero was assumed), but where the coefficients applied to all variables were at
the same out-of-sample levels as before (Model 2).\footnote{In other words, Model 2 was exactly the same as
Model 1 except that forecasts of Model 2 take advantage of perfect foresight about the contemporaneous
change in the long rate, while forecasts of Model 1 utilize the expectation of zero contemporaneous
change in the long rate.} Second, the error was compared with that resulting from a naïve random-walk model, where
the predicted future short rate was given by:

$$\hat{r}_t = r_{t-1}. \tag{6}$$

Third, the error was compared with that resulting from a simple submartingale model, where
the predicted future short rate was given by:

$$\hat{r}_t = \hat{\alpha} + \hat{\beta} r_{t-1}, \tag{7}$$

and where the coefficients were estimated using data from the prior 60 months, as in (5).
Finally, the error was evaluated relative to that associated with the forecast implicit in the for-
ward interest rates of the term structure at each point in time. During the final 1977-1982 sub-
period, when 90-day Treasury Bill Futures were in existence, it was possible to develop a fifth
benchmark. One-year forward rates of interest were inferred from the prices of four nonover-
lapping 90-day futures contracts spanning a year. Because the maturities of these futures con-
tracts did not correspond exactly to the timing intervals needed to provide for inferred forward
rates that coincided exactly to the forecast periods, interpolations were undertaken to achieve
the desired time patterns.\footnote{The CRSP Government Bond data files contain month ending price and yield quotations while interest
futures contracts are for delivery on the Thursday following the third Monday of the delivery month. For
the period under consideration, most of the delivery months were in March, June, September, and De-}
In Table 3 below the test results of our forecasting model for U.S. interest rates are reported. The data are presented for six subperiods of five years, each of which relates to approximately 60 forecasts for each of the models. The results for the entire period of 30 years precede those for the subperiods. Two techniques for measuring the forecast error were employed. First, we used the model to generate a forecast of the expected percentage change in the short rate over the ensuing month, where this percentage change was computed in accordance with equation (5). The actual percentage change that then occurred was compared with the forecasted percentage change from our forecasting model, together with those forecasted percentage changes from the random walk and submartingale models (the former of which always provides a zero change forecast), as well as the forecasted percentage changes implied by the forward rate, to give us the errors associated with each of the models. Second, we used each of the models to generate an interest rate forecast, and subtracting these forecasts from the actual interest rate that was subsequently observed, obtained the errors in terms of basis points. Thus, while the first measure focuses on percentage errors relative to the prior interest rate level, the latter is concerned only with total basis points errors.

These two error measures were then compared along two dimensions. Both the mean errors and the root mean square errors were computed. A glance at the mean error can indicate the general direction of bias over time, whereas the root mean square error is a better indication of the severity of the forecasting errors incurred. In the column furthest to the right, the model ranking highest among the forecasting models is identified.

Before proceeding to an analysis of the United States estimation results, a couple of comments are in order regarding our tests for the United Kingdom. As expected from analyzing the earlier regressions, the relationships set forth in equation (5) were so loosely fitted that little hope was held out for a viable forecasting model to result from it. These pessimistic expectations were fulfilled, as the random walk and submartingale models out performed the extended model in three of the four subperiods. To conserve space, these results are not presented here. Henceforth, all discussion will relate to forecasting results for the United States.

One of the primary results is readily apparent in the last column of Table 3. The extended model ("Model 1") produced the best forecasts, as evidenced by the lowest root mean squared errors, both for the overall period and for five of the six subperiods. This result held true whether the focus was on forecasting percentage changes or actual levels of the interest rate over time. Another result of note is that the extended model was associated with the lowest bias or mean error over the extended period, with the forward rate model its nearest

cember, although other delivery months were available during part of the period. Nonetheless, it was impossible to get futures forecasts that corresponded precisely to the forecast horizon without interpolation. In making my interpolations, the second and third 3-month rates implicit in the second and third of a series of four futures contracts were each given one-quarter weights, while the first and fourth contracts shared the remaining weight, with greater weight given to that contract which did not quite span the forecast period, and lesser weight to the contract which overlapped the forecast period. Clearly this is less than ideal, but was necessitated by data availability.

11 It would also be appropriate to present the mean absolute errors to cover problems that might arise if interest rate changes are drawn from a stable Pareto distribution with infinite variance. However, we know of no evidence that would support such a supposition, and so in the interests of conserving space, these mean absolute errors are not reported. Nonetheless, these statistics were computed and they reinforced the patterns observed in Table 3, with an exception to be noted later.

12 These results, as well as any others alluded to but not presented in this paper, are available directly from the author.

13 Stephen Smith has pointed out that the one subperiod where Model 1 exhibited worse predictive power was characterized by a brief period of price controls imposed in the United States. If the term structure reflected the temporary controls in the short rates, but continued to respond to inflationary pressures in the long rates, the linkages between the short and long rates observed during other periods could have undergone some alterations or erosion during the latter part of the fourth subperiod, resulting in the inferior predictive power. Although the price controls continued during the first part of the fifth subperiod as well, Smith has hypothesized that the market was already preparing for their imminent removal.
Table 3

FORECASTING ERRORS FOR PREDICTIONS WITH DATA OF UNITED STATES

<table>
<thead>
<tr>
<th>Period</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Random Walk</th>
<th>Sub- mart.</th>
<th>Forward Rate</th>
<th>Best* Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/52 - 1/82</td>
<td>0.1094</td>
<td>0.0949</td>
<td>0.1158</td>
<td>0.1158</td>
<td>0.1160</td>
<td>M1</td>
</tr>
<tr>
<td>4/52 - 3/57</td>
<td>0.1225</td>
<td>0.1196</td>
<td>0.1292</td>
<td>0.1269</td>
<td>0.1316</td>
<td>M1</td>
</tr>
<tr>
<td>4/57 - 3/62</td>
<td>0.1746</td>
<td>0.1480</td>
<td>0.1921</td>
<td>0.1916</td>
<td>0.1899</td>
<td>M1</td>
</tr>
<tr>
<td>4/62 - 3/67</td>
<td>0.0693</td>
<td>0.0728</td>
<td>0.0735</td>
<td>0.0735</td>
<td>0.0752</td>
<td>M1†</td>
</tr>
<tr>
<td>4/67 - 3/72</td>
<td>0.0872</td>
<td>0.0728</td>
<td>0.0849</td>
<td>0.0854</td>
<td>0.0858</td>
<td>RW</td>
</tr>
<tr>
<td>4/72 - 3/77</td>
<td>0.0831</td>
<td>0.0678</td>
<td>0.0849</td>
<td>0.0832</td>
<td>0.0855</td>
<td>M1</td>
</tr>
<tr>
<td>4/77 - 1/82</td>
<td>0.0825</td>
<td>0.0529</td>
<td>0.0843</td>
<td>0.0877</td>
<td>0.0830</td>
<td>M1</td>
</tr>
</tbody>
</table>

Root Mean Square Errors Measured in Percentage Terms

<table>
<thead>
<tr>
<th>Period</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Random Walk</th>
<th>Sub- mart.</th>
<th>Forward Rate</th>
<th>Best* Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/52 - 1/82</td>
<td>0.5467</td>
<td>0.3919</td>
<td>0.5492</td>
<td>0.5638</td>
<td>0.5543</td>
<td>M1</td>
</tr>
<tr>
<td>4/52 - 3/57</td>
<td>0.2081</td>
<td>0.1929</td>
<td>0.2135</td>
<td>0.2168</td>
<td>0.2189</td>
<td>M1</td>
</tr>
<tr>
<td>4/57 - 3/62</td>
<td>0.3776</td>
<td>0.3059</td>
<td>0.3789</td>
<td>0.3799</td>
<td>0.3921</td>
<td>M1</td>
</tr>
<tr>
<td>4/62 - 3/67</td>
<td>0.2508</td>
<td>0.2718</td>
<td>0.2621</td>
<td>0.2683</td>
<td>0.2700</td>
<td>M1†</td>
</tr>
<tr>
<td>4/67 - 3/72</td>
<td>0.4882</td>
<td>0.1249</td>
<td>0.4729</td>
<td>0.4793</td>
<td>0.4843</td>
<td>RW</td>
</tr>
<tr>
<td>4/72 - 3/77</td>
<td>0.5667</td>
<td>0.4373</td>
<td>0.5741</td>
<td>0.5681</td>
<td>0.5856</td>
<td>M1</td>
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<tr>
<td>4/77 - 1/82</td>
<td>1.0044</td>
<td>0.6132</td>
<td>1.0113</td>
<td>1.0575</td>
<td>1.0073</td>
<td>M1</td>
</tr>
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</table>

Mean Errors Measured in Percentage Terms

<table>
<thead>
<tr>
<th>Period</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Random Walk</th>
<th>Sub- mart.</th>
<th>Forward Rate</th>
<th>Best* Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/52 - 1/82</td>
<td>0.0031</td>
<td>-0.0069</td>
<td>0.0113</td>
<td>0.0103</td>
<td>-0.0044</td>
<td>M1†</td>
</tr>
<tr>
<td>4/52 - 3/57</td>
<td>0.0108</td>
<td>-0.0141</td>
<td>0.0187</td>
<td>0.0132</td>
<td>-0.0051</td>
<td>FR†</td>
</tr>
<tr>
<td>4/57 - 3/62</td>
<td>0.0153</td>
<td>-0.0043</td>
<td>0.0116</td>
<td>0.0089</td>
<td>-0.0171</td>
<td>SM</td>
</tr>
<tr>
<td>4/62 - 3/67</td>
<td>0.0016</td>
<td>-0.0228</td>
<td>0.0093</td>
<td>0.0157</td>
<td>0.0006</td>
<td>FR†</td>
</tr>
<tr>
<td>4/67 - 3/72</td>
<td>0.0081</td>
<td>-0.0071</td>
<td>0.0017</td>
<td>0.0001</td>
<td>-0.0107</td>
<td>SM†</td>
</tr>
<tr>
<td>4/72 - 3/77</td>
<td>-0.0054</td>
<td>-0.0016</td>
<td>0.0082</td>
<td>0.0060</td>
<td>-0.0042</td>
<td>FR</td>
</tr>
<tr>
<td>4/77 - 1/82</td>
<td>-0.0125</td>
<td>-0.0060</td>
<td>0.0183</td>
<td>0.0184</td>
<td>-0.0104</td>
<td>FR</td>
</tr>
</tbody>
</table>

Mean Errors Measured in Basis Points (1.00 = 100 basis points)

<table>
<thead>
<tr>
<th>Period</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Random Walk</th>
<th>Sub- mart.</th>
<th>Forward Rate</th>
<th>Best* Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/52 - 1/82</td>
<td>0.0074</td>
<td>-0.0158</td>
<td>0.0314</td>
<td>0.0345</td>
<td>-0.0334</td>
<td>M1†</td>
</tr>
<tr>
<td>4/52 - 3/57</td>
<td>0.0185</td>
<td>-0.0290</td>
<td>0.0261</td>
<td>0.0211</td>
<td>-0.0209</td>
<td>M1†</td>
</tr>
<tr>
<td>4/57 - 3/62</td>
<td>0.0749</td>
<td>-0.0053</td>
<td>-0.0047</td>
<td>-0.0088</td>
<td>-0.0970</td>
<td>RW</td>
</tr>
<tr>
<td>4/62 - 3/67</td>
<td>0.0056</td>
<td>-0.0882</td>
<td>0.0262</td>
<td>0.0537</td>
<td>-0.0095</td>
<td>M1†</td>
</tr>
<tr>
<td>4/67 - 3/72</td>
<td>0.0577</td>
<td>0.0549</td>
<td>0.0081</td>
<td>-0.0128</td>
<td>0.0806</td>
<td>RW†</td>
</tr>
<tr>
<td>4/72 - 3/77</td>
<td>-0.0228</td>
<td>0.0003</td>
<td>0.0227</td>
<td>0.0225</td>
<td>-0.0542</td>
<td>SM</td>
</tr>
<tr>
<td>4/77 - 1/82</td>
<td>-0.0926</td>
<td>-0.0301</td>
<td>0.1297</td>
<td>0.1349</td>
<td>0.0648</td>
<td>FR</td>
</tr>
</tbody>
</table>

* Model 2 not included
† Also better than Model 2

Note: The errors associated with forecasts implied by futures prices during the final subperiod for which data exist were as follows:
Rt.M.S.E. (measured in percentage terms) = 0.1246; Rt. M.S.E. (measured in basis points) = 1.5471; Mean Error (in percentage terms) = -0.0734; Mean Error (in basis points) = -0.8148. While these errors compare unfavorably with those of any of the other models, some of the error is obviously derived from the necessity of making interpolations due to noncoinciding time periods between futures data and term structures.
competitor. Indeed, Model 1 even had a lower average error than Model 2, where in-sample information about the contemporaneous percent change in the long rate was utilized. While the mean error clearly is of less importance to the forecaster, it is noteworthy that during several of the subperiods the extended model did not produce the lowest mean errors.

Several other results are worthy of mention. In the Meese and Rogoff (1983) studies which employed out-of-sample rolling regression techniques similar to ours, they showed that even perfect foresight of the values of the contemporaneous independent variables did not provide better forecasts than a simple random walk, an indication of misspecifications in the structural models they tested. Thus, it would be important to the confirmation of our model's validity that it produces lower forecasting (root mean squared) errors than the naïve benchmarks and imperfect foresight (Model 1) alternatives. The final column summarizes the fact that Model 2 did, in fact, produce the lowest forecast errors in all but the 4/62-3/67 period, during which its imperfect information counterpart produced the lowest errors. The marked instability in the coefficient on the contemporaneous change in the long rate was the cause of this finding, and during those years it would have been better to ignore this aspect altogether.

It would appear from the data presented above that a model has been put forth which is (with high probability) capable of producing better forecasts of short-term interest rates over time than use of the naïve alternatives. The question of how much better has yet to be addressed. An answer to this question requires a closer look at the data.

We first note that the average level of the short-term rate of interest throughout the overall forecasting period was approximately 4.81 percent, ranging from .80 to 15.98 percent. This rate changed, on average and in absolute terms, by about 34 basis points each month. The highest change during any single month was 396 basis points and occurred during the sixth subperiod, when the one-year interest rate was 14.87 percent. (This translates into a relative change in the interest rate of -26.6 percent.) The highest relative change during any single month was 125.9 percent and occurred during the second subperiod, when the interest level was 1.27 percent. (This translates into a 160 basis points rise.) The smallest change during any single month was zero. Thus, in terms of basis points, absolute changes from month to month ranged from 0 to 395, averaging 34. These figures may be interpreted directly as errors associated with the random-walk model, which predicts zero change.

With these parameters in mind, we are prepared to examine the additional accuracy achieved by the extended forecasting model. First it is observed that in terms of absolute changes (not reported in Table 3), the forecasting model also produces average errors of 34 basis points per month over the entire period. Indeed, it was only able to correctly predict the direction of change 46 percent of the time.\(^\text{14}^\) When coupled with the information given in Table 3 that the forecasting model generally achieved lower root mean squared errors, we are led to believe that the extended and naïve models differential in success is due to the errors incurred when there are sizable changes in the interest rate. In these cases, further checking revealed that the extended model usually came closer to the new interest rate.\(^\text{15}^\) Because the use of a forecasting standard like the root mean squared error places higher weights upon the larger errors, the extended model generally scored better than the naïve models. The magnitude of

\(^{14}\) As an experiment, we turned our forecasting model around backwards, after removing the term relating to the lagged percentage change in the long rate, so that the forecasting model focused upon changes in the long rate. In out-of-sample tests similar to those done here, our backwards model was able to correctly predict the direction of change in the long rate about 60 percent of the time, while never achieving a success rate during any subperiod of less than 53.3 percent. Nonetheless, the backwards model was never able to beat the Random Walk model in terms of forecasting accuracy, when accuracy was judged on the basis of minimum root mean squared errors.

\(^{15}\) For example, the model averaged seven basis points closer to the actual change in the interest rate during the maximum absolute change of each subperiod. Another item of note is that whenever the model predicted a relative change in the interest level of nine percent or greater, it had 100 percent accuracy in terms of predicting direction of change. When the signals were below this threshold, the predictive accuracy in terms of direction of change was worse than the toss of a coin.
this improvement is disappointingly slight, however. In terms of predicted relative changes in the interest rate, the forecasting model averaged about 6 percent closer to the actual percentage changes than the random walk. In terms of the actual interest rate forecast, the model came only about one half of one basis point closer, on average. When these averages are stripped of their implied weights (i.e., those weights implied by using the root mean squared errors standardization) and are simply viewed in terms of absolute percentage errors and absolute errors, the figures are even less provocative—2 percent and zero basis points, respectively.

Thus, even though our interest forecasting model was able to achieve a feat—beat the naive random walk, submartingale, and forward rate alternatives—that none of the premier structural exchange rate models could accomplish when judged over out-of-sample fits, its success when viewed in this overall context is marginal at best. Judging the extended models forecasting success by the standard of its ability to help an investor speculate successfully, the model did not fare well. To test the model in this context, we restricted our attention to the period since January of 1976 when interest futures began trading. The model was used to provide a signal whether our hypothetical investor should go long or short on an interest futures contract. Two strategies were considered when it came to cashing out of the position taken: cash out at the end of the month, or cash out after fifteen days. The first strategy was consistent with the periodicity of the model estimations, whereas the latter was in recognition of the fact that even though the lagged terms of (5) exerted an influence in the model, we cannot say that the influence culminated at the end of one month. We checked to find if greater lags were important and found nothing beyond one month of any significance; however, it could well be that the influence was primarily felt in the first week or two and that the regression would still detect the effect over the monthly sampling horizon. Thus, we also used the fifteen day holding period as an alternative. Neither strategy was able to generate positive profits after transaction costs were taken into account.

This is not to say that the model is without potential use. For example, the pure expectations hypothesis can also be shown to offer little predictive value—indeed, less than the present model when it comes to predicting short-term interest rates—yet it has served as a useful standard for assessing market expectations. That model was used by Babbel [1983] in developing immunized bond portfolio strategies. The duration measures used required a knowledge of the sensitivity of movements (shocks) in the long rates relative to movements (shocks) in the short rate. By defining “shocks” as something unanticipated, and using the pure expectations hypothesis rather than the random walk hypothesis to model expectations of future interest rates, the resulting duration measures became more valuable for immunizing properly against these shocks. In out-of-sample testing over a period of 271 months, he was able to reduce the downside risk (i.e., getting a rate of return below the target level of return) of his immunized portfolios by 83 percent below that associated with the immunized portfolios based on strategies using the traditional duration measure, which does not take into account the expected drift in interest rates. Additionally, while reducing downside risk, the strategy also produced higher average returns.

It would appear that the forecasting model presented in this paper could be used in an application similar to the one mentioned above, where it would provide an alternative to the expected drift in interest rates specified by the pure expectations hypothesis. Such an exercise is left for a subsequent paper. What appears to be certain, however, is that the model has only marginal predictive value. This is particularly striking in light of the favorable in-sample fits. It should serve as a reminder to empiricists to carefully examine their models using out-of-sample techniques before pronouncing their beneficitions upon them.

Summary

In the present paper we have reexamined the Brennan and Schwartz two-factor model of the term structure of interest. We used spot rates of interest rather than the yields to maturity used by the other authors to proxy for their short and long-term rates of interest. This kind of data was used in an attempt to measure better the relationships which the authors put forth.
While we found more stability in the revised estimates, and generally better explanatory power, the level of explanatory power remained so low that the model was reformulated to focus strictly upon movements in the short rate.

The reformulated model was extended to include the lagged effects of changes in the long rate on movements in the short rate. This model was then estimated for both the United States and the United Kingdom. While the model appeared to have some explanatory power for both countries, it was much better fitted to the pattern of interest rates in the United States.

Next, the extended model was tested out of sample. For the United Kingdom, the model generally performed worse than the random walk and submartingale models, as expected, but for the United States, it consistently seemed to rank higher in terms of forecasting accuracy than the alternative naïve models. Upon closer examination, however, the extended model’s improvement over the random walk and submartingale alternatives was only marginal.

BIBLIOGRAPHY


