Research Program in Finance
WORKING PAPER SERIES

WORKING PAPER NO. 164

INFORMATIONAL EFFICIENCY
AND THE PRIVATE VALUE OF INFORMATION

BY
MARK LATHAM

Research Program in Finance Working Papers are preliminary in nature; their purpose is to stimulate discussion and comment. Therefore, they should not be cited or quoted in any publication without the permission of the author. Single copies of a paper may be requested from the Institute of Business and Economic Research.
RESEARCH PROGRAM IN FINANCE AT THE
WALTER A. HAAS SCHOOL OF BUSINESS,
UNIVERSITY OF CALIFORNIA, BERKELEY

The Research Program in Finance in the Walter A. Haas School of Business at the University of California has as its purpose the conduct and encouragement of research in finance, investments, banking, securities markets, and financial institutions. The present reprint and working paper series were established in 1971 in conjunction with a grant from the Dean Witter Foundation.

INSTITUTE OF BUSINESS AND ECONOMIC RESEARCH
Carl Shapiro, Director

The Institute of Business and Economic Research is an organized research unit at the University of California, Berkeley, whose mission is to promote research by faculty and graduate students in the fields of business and economics. The Institute carries out its mission by organizing programs and activities that enrich the research environment, administering extramural research awards, publishing working papers, and making direct grants for research.
INFORMATIONAL EFFICIENCY AND THE PRIVATE VALUE OF INFORMATION

Mark Latham *

August 1986
revised December 1986

Finance Working Paper No. 164

* School of Business Administration, University of California, Berkeley, CA 94720. Telephone (415)-642-6452. Financial support from the Berkeley Finance Foundation, and from the University of California, Berkeley, School of Business Administration is gratefully acknowledged. For valuable discussions, I thank participants in finance seminars at Berkeley, Stanford, London Business School, C.E.S.A.-H.E.C. (France), Universitat Konstanz (Germany), and University of Southern California. Suggestions from Gregory Connor and Gérard Gennai were particularly helpful.
The proposition that the private value of information in an informationally efficient security market is zero, is widely believed in the finance field. This paper shows that the proposition is in general false (except for one narrow unconventional interpretation of it). An investor might rationally pay to learn something that everyone else already knows. Therefore informational efficiency is possible even when information is costly, in spite of the well known argument to the contrary. An explicit formula for the value of information is derived in a specific model in which the market is informationally efficient. These findings have implications for the question of what is the most appropriate definition of informational efficiency.
INFORMATIONAL EFFICIENCY AND THE PRIVATE VALUE OF INFORMATION

Informational efficiency ("market efficiency" in the finance literature) is often associated with questions of the value of information. As Baja [4, page 7] mentions,

"... it is generally accepted that if markets are efficient with respect to some information investors need not use ... this information in the determination of their optimal market position."

In fact, Jensen's [11] definition can be interpreted as having this property built in:

"A market is efficient with respect to information set $\Theta_t$ if it is impossible to make economic profits by trading on the basis of information set $\Theta_t$."

This is because one might expect the following four conditions to be equivalent to each other: 1

1. It is impossible to make economic profits using information set $\Theta$.
2. Investors need not use $\Theta$ in the determination of their optimal market position.
3. No investor would (rationally) pay to acquire $\Theta$.
4. The private value of $\Theta$ is zero for all investors.

The focus of this paper is the widely believed proposition that if the securities market is efficient with respect to information $\Theta$ then the private

---

1Of course there are other, non-equivalent, ways to interpret these conditions.
value of θ to each investor is zero. Let us call this Proposition Z. Further examples of the widespread acceptance of Proposition Z are found in the empirical literature. An accumulation of evidence in favor of semi-strong-form efficiency and evidence against strong-form efficiency would typically be taken to imply that, while inside information is valuable, once a piece of information has been published, it is of no incremental value to investors.

Grossman and Stiglitz [8] point out an important consequence of this: If informational efficiency removes the incentive to pay for information, then in a rational expectations equilibrium, the market can never become efficient with respect to information that is costly. However, their premise (Proposition Z) is false: Informational efficiency does not, in general, remove the incentive to acquire costly information (except in a narrow sense described in Section I.A below). The main goal of this paper is to prove that claim. The paper also mentions conditions under which efficiency does remove this incentive.

Grossman and Stiglitz use a nonstandard definition of informational efficiency: The market is "Grossman-Stiglitz" efficient with respect to θ if and only if equilibrium prices reveal a sufficient statistic for θ, meaning that prices reveal everything about θ that is relevant for investors' portfolio decisions. But consider a situation where all investors have acquired θ from other sources, such as the newspaper. This market would be

---

2 This was also discovered independently by Startz [16].
3 "Revealing a sufficient statistic for θ" will often be abbreviated as "revealing θ" where this would not cause confusion.
considered efficient with respect to \( \theta \) in the standard "Efficient Markets" literature. For example, Fama [7, page 387] states that information set \( \theta \) being known to all investors is generally accepted to be a sufficient condition for efficiency with respect to \( \theta \) in frictionless security markets — prices would then "fully reflect" \( \theta \). But Grossman-Stiglitz efficiency may not hold — prices may not reveal \( \theta \) even when all investors know \( \theta \). The benchmark equilibrium in Section I.B of the present paper is a specific example. Furthermore, Jordan [12] argues that revelation of \( \theta \) by prices is unlikely in general, although he expresses his results using the Grossman-Stiglitz efficiency terminology.

It is true that Grossman-Stiglitz efficiency removes the incentive to acquire costly information. This is because, in a rational expectations model, no one would pay for information that is going to be revealed by prices. But it does not hold in general for more standard definitions in the finance literature. Unfortunately, there is no single standard definition. Latham [14] gives two definitions which conform reasonably well to the empirical and theoretical finance literature: "S-B efficiency", which essentially means that security prices would not change if everyone knew \( \theta \); and "E-efficiency", which means that neither prices nor portfolios would change if everyone knew \( \theta \). This paper shows that Proposition Z is false.

\[ \text{\footnotesize{\textsuperscript{4}}"S-B" is for Sharpe-Beaver. Beaver [3] wrote the formal definition following a suggestion by William Sharpe to amend a definition proposed by Rubinstein [15]. "E" is for equilibrium, since the whole equilibrium (prices and portfolios) must be invariant. Both definitions are ex ante: for the market to be considered efficient, invariance must hold for all states of nature. The interpretation of Proposition Z in this paper is correspondingly ex ante.} \]
for S-B efficiency, for E-efficiency, and for any definition that satisfies Fama's sufficiency condition mentioned above. A microscopic (price-taking) investor would sometimes pay to acquire information that everyone else already knows.

Like Grossman and Stiglitz (and others\(^5\)), this paper uses exponential utility, joint normality, and asset-supply uncertainty to solve for a partially revealing rational-expectations equilibrium, in which uninformed traders would be willing to pay for information. But the distinctive feature here is to show that this can occur in an informationally efficient market. Other related papers include Hakansson, Kunkel and Ohlson [9], who analyze the social value of information (rather than the private value as in Proposition Z), starting from an equilibrium which is typically not informationally efficient. Jaffe and Rubinstein [10] assert that disseminating existing private information can have positive value in an informationally efficient market, although they use a rather weak "signal"-type definition of efficiency and do not model a rational-expectations equilibrium.

There are two distinct ways to interpret Proposition Z. This is seen most clearly when the E-efficiency definition is used. Notice that in a market that is E-efficient with respect to \( \theta \), each investor has a sufficient statistic for \( \theta \). In such a context, there are two ways to ask about the private value of information \( \theta \) for a given investor. The first is, would she

---

\(^5\) These include Diamond and Verrecchia [6], Bray [5], and Admati and Pfleiderer [2].
now pay to acquire \( \theta \)? (Obviously not, since she already has a sufficient statistic.) The second is, is she better off now than if she did not have the sufficient statistic for \( \theta \) in the first place? So in the former interpretation, informational efficiency is taken as the starting point for finding the value of acquiring information, while in the latter interpretation, efficiency is the ending point for finding the value of acquired information.

Section I below takes the former interpretation ("acquiring information"), and shows in Theorem 1 that Proposition Z is trivially true for \( E \)-efficiency, but the converse is more interesting: Assuming microscopic investors, if \( \theta \) has zero private value to each investor then the market is \( E \)-efficient; thus \( E \)-efficiency is the only definition for which Proposition Z holds in this context. Part I.B gives a counterexample to Proposition Z under \( S-B \) efficiency. Section II takes the other interpretation of Proposition Z ("acquired information"), and presents a detailed example showing that it is false for both \( E \)-efficiency and \( S-B \) efficiency. In this example, an infinitesimal investor would pay to get information that everyone else already knows; it is fully reflected in prices and all other portfolios, but prices do not fully reveal it. It is argued in Section III that these results tend to make \( E \)-efficiency the preferred definition of informational efficiency.

Section IV concludes the paper.
I. The Value of Acquiring Information

We use a simple one-period rational-expectations model, with trading at
time $t=0$ and consumption only at $t=1$. The endowment matrix $\bar{Z}$ consists of $N$
column vectors $\bar{Z}_i$ for investors $i=1,\ldots,N$, with element $\bar{z}_{ik}$ being the number
of shares of security $k$ originally held by investor $i$. These are traded in a
frictionless market at price vector $P$ to reach equilibrium allocations $z_{ik}$
forming matrix $Z$. Investor $i$ observes information-signal $S_i$ privately before
trading, as well as using any information that can be deduced from the
equilibrium price vector $P$. (Typically all these quantities are random
variables. $S_i$ may be a scalar, a vector, or most generally a subset of the
space of possible $t=1$ states of nature.) Investor $i$ chooses $\tilde{Z}_i$ to maximize
her expectation of $U_i(c_i)$, where consumption $c_i$ is the $t=1$ value of her
portfolio. $U_i$ is strictly increasing and strictly concave.

Definition. A rational-expectations equilibrium is a pair $(\tilde{P},\tilde{Z})$ such that
(a) portfolios are optimal: $\tilde{Z}_i$ maximizes $E(U_i(\tilde{c}_i)|\tilde{S}_i,\tilde{P})$ subject to the budget
$\tilde{p}'\tilde{Z}_i \leq \tilde{p}'\bar{Z}_i$ and subject to informational feasibility — $\tilde{Z}_i$ measurable
with respect to $\{\tilde{S}_i,\tilde{P}\}$;
(b) markets clear: $\Sigma_i\tilde{Z}_i = \Sigma_i \bar{Z}_i$; and
(c) prices only reveal information known to someone: $\tilde{P}$ measurable with
respect to $\{\tilde{S}_i\}_{i=1}^{N}$.

It is well known that these standard equilibrium conditions are
incomplete in the sense that they are not sufficient to determine a unique
equilibrium in all cases. (Nonexistence can also occur.) In the theorems proved below, no attempt is made to complete the conditions by adding a rule for choosing among multiple equilibria. Rather, the theorems hold in the sense that there exists an equilibrium with the property being claimed.

To examine the private value of some piece of information $\theta$ in a market that is informationally efficient with respect to $\theta$, it is convenient to define three configurations of endowed information $\{\tilde{S}_i\}_{i=1,...,N}$:

Economy 1 — $\tilde{S}_i^1$ = any arbitrary $\tilde{S}_i$.
Economy 2 — $\tilde{S}_i^2 = (\tilde{S}_i^1, \tilde{\theta})$ for all $i$.
Economy 3j — $\tilde{S}_i^3j = \tilde{S}_i^1$ for $i \neq j$; $\tilde{S}_j^3j = (\tilde{S}_j^1, \tilde{\theta})$.

Thus Economy 1 is the actual economy we are studying, Economy 2 is the benchmark for assessing informational efficiency of Economy 1 with respect to $\theta$, and Economy 3j is the benchmark for measuring the private value to investor $j$ of acquiring information $\theta$.

A. $E$-efficiency

An equilibrium in Economy 1 is $E$-efficient with respect to $\theta$ if there is an equilibrium in Economy 2 with the same prices and portfolios. It is easy to see that under this interpretation, Proposition Z holds. Each investor already has a sufficient statistic for $\theta$ in Economy 1, and so would not pay to acquire $\theta$.
Theorem 1. E-efficiency with respect to θ ⇒ the value of acquiring θ
privately is zero for each investor.

Proof: Let \((\hat{P}^*, \hat{Z}^*)\) be the equilibrium that Economies 1 and 2 have in
common. We will show that it is also an equilibrium for Economy 3j for any j.

For investor j, equilibrium condition (a) in Economy 3j is identical to
equilibrium condition (a) in Economy 2 (with \((\hat{P}^*, \hat{Z}^*)\) substituted in as the
possible equilibrium) — prices, endowments, and investor j’s information are
the same, so \(\hat{Z}_j^*\) is still optimal.

For investors \(i \neq j\), equilibrium condition (a) in Economy 3j is identical
to that in Economy 1 (with \((\hat{P}^*, \hat{Z}^*)\) substituted in) — prices, endowments, and
investor i’s information are the same, so \(\hat{Z}_i^*\) is still optimal.

Equilibrium condition (b) (market clearing) is satisfied by \(\hat{Z}^*\) in Economy
3j because \(\hat{Z}^*\) clears the market in Economies 1 and 2, and endowments are the
same in all three economies (only the information is different).

Equilibrium condition (c) is satisfied by \(\hat{P}^*\) in Economy 3j because the
set of all information known in Economy 3j is the same as that in Economy 2,
so \(\hat{P}^*\) must be measurable with respect to it.

Therefore \((\hat{P}^*, \hat{Z}^*)\) satisfies all equilibrium conditions of Economy 3j for
any j — investor j would not make use of information θ even if he had it. By
rational expectations he knows this, and knows that his expected utility is
the same with or without θ. So the value of acquiring θ privately is zero for
each investor. Q.E.D.
To further explore the link between $E$-efficiency and zero value of information, consider the converse of Theorem 1. If private knowledge of $\theta$ has zero value to each investor, then is the market $E$-efficient with respect to $\theta$? Not necessarily: The attempt to trade on $\theta$ may reveal $\theta$, which could cause an adverse price change that prevents any utility gain, rendering $\theta$ valueless. (For example, this would happen if all investors were identical in endowments, preferences, and prior beliefs.) But if public knowledge of $\theta$ causes prices to change, then the original equilibrium was not $E$-efficient with respect to $\theta$.

However, in an economy where all investors are microscopic, the converse of Theorem 1 does hold. Such an economy can be constructed directly by assuming a continuum of investors as in Admati [1], or indirectly by taking limits in a sequence of economies where the number of investors grows infinitely, as in Section II of this paper.<sup>6</sup>

---

<sup>6</sup> In the case of a sequence of finite-investor economies, there must be some "background noise" in equilibrium prices to ensure that the amount of information revealed by an investor's trades becomes small as the trades become small. Let us include this feature in the meaning of "microscopic investor". (In fact, with this noise, the converse of Theorem 1 does not need to assume infinitesimal investors, provided that a more sophisticated definition of equilibrium is used. This would involve non-price-taking behavior. $E$-inefficiency means that an investor would benefit from trading on $\theta$ at existing prices. Noise permits a "small" amount of trading at existing prices. Sophisticated non-price-taking behavior lets an informed investor restrain her trading before the negative impact of price changes wipes out the initial gains. Going one step further, one can even argue that noise is not necessary, but that leads to theoretical planes beyond the scope of this paper. Here we will stay with the standard price-taking equilibrium definition.)
Theorem 2. In an economy of microscopic investors, if the value of acquiring \( \theta \) privately is zero for all investors, then \( E \)-efficiency holds with respect to \( \theta \).

Proof: Let \( \tilde{p}^1, \tilde{z}^1 \) be the equilibrium of reference in Economy 1 (the actual economy). We will show that it is also an equilibrium for Economy 2.

For any investor \( j \), zero private value of \( \theta \) implies that, comparing Economies 1 and 3\( j \), no portfolio \( \tilde{z}_j \) gives a higher value of \( E[u_i(S_{3j}, \tilde{p}^1)] \) than \( \tilde{z}_j^1 \) (subject to the budget and informational feasibility in Economy 3\( j \)). (The "microscopic" assumption justifies the use of \( \tilde{p}^1 \) here.) Therefore \( \{\tilde{p}^1, \tilde{z}^1\} \) satisfies equilibrium condition (a) for investor \( j \) in Economy 3\( j \). But this is identical to condition (a) for investor \( j \) in Economy 2, so that too is satisfied by \( \{\tilde{p}^1, \tilde{z}^1\} \).

Condition (b) is obviously satisfied. Condition (c) holds by transitivity: \( \tilde{p}^1 \) is measurable with respect to \( \{\tilde{s}_{i1}^1\}_{i=1..N} \) which is measurable with respect to \( \{\tilde{s}_{i2}^2\}_{i=1..N} \).

Therefore \( \{\tilde{p}^1, \tilde{z}^1\} \) satisfies all equilibrium conditions for Economy 2, so by definition is \( E \)-efficient in Economy 1. Q.E.D.

Taking Theorems 1 and 2 together, \( E \)-efficiency is the only informational efficiency definition that is a necessary and sufficient condition for acquiring \( \theta \) to have zero value to all investors, in an economy of microscopic investors. Any definition with this property must be equivalent to \( E \)-efficiency.
B. S-B efficiency

An equilibrium in Economy 1 is S-B efficient with respect to \( \theta \) if there is an equilibrium in Economy 2 (all investors knowing \( \theta \)) with the same prices. Proposition 2 is false for this definition of informational efficiency. Information \( \theta \) may be valuable to an investor even if the market is S-B efficient with respect to \( \theta \). Proof is given by the following example, which is a simplified version of one in Latham [14].

There are two investors, maximizing expected utility with

\[
U_1(c) = -(8-c)^2, \quad \text{for } c < 7;
\]
\[
U_2(c) = -(12-c)^2, \quad \text{for } c < 7.
\]

Each investor is endowed with half a share of security 1 and half a share of security 2. Both securities are risky (there is no riskless asset), with payoffs in the four possible states of nature as shown in Table I. Prior beliefs of both investors give equal probability (.25) to all states. The information \( \theta \) has two equiprobable outcomes, \( a \) and \( b \), inducing posterior beliefs as shown in Table I.
Table I
Payoffs and Probabilities

<table>
<thead>
<tr>
<th></th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Security-1 payoff</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Security-2 payoff</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>No-information prob.</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Probabilities given signal a</td>
<td>0.4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td>Probabilities given signal b</td>
<td>0.1</td>
<td>0.4</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

In the actual equilibrium that we are interested in (Economy 1), neither investor is informed of \( \theta \). Equilibrium conditions are easily solved to give a price ratio of 1.0 and no trading. Expected utilities are -20.0 for investor 1 and -68.0 for investor 2. To check whether this is S-B efficient with respect to \( \theta \), we compare it with a benchmark equilibrium in which both investors know \( \theta \) (Economy 2): If \( \theta \) gives signal \( a \), then security 1 is riskier than security 2, so the less risk-averse investor 2 will buy more of it (0.12 shares more, as it turns out). But security 1 also has a higher expected payoff given \( a \), just enough so that the equilibrium price ratio is again 1.0. If \( \theta \) gives signal \( b \), then security 2 is riskier than security 1, and a symmetrical equilibrium results, in which investor 2 holds 0.62 shares of security 2, and the price ratio is again 1.0. So prices in Economies 1 and 2 are equal, and the equilibrium in Economy 1 is S-B efficient with respect to \( \theta \).

But the value of information \( \theta \) is positive for both investors. In the fully informed (benchmark) equilibrium, expected utilities are -19.356 for
investor 1 and -67.856 for investor 2 — both are better off than in the uninformed (actual) equilibrium. \(^7\) So Proposition Z is false for S-B efficiency.

Therefore it is false for a class of price-oriented informational efficiency concepts implied by S-B efficiency (or satisfied by the above example), including those intended by Fama, Jensen, and Beaver. One way to see this is by reference to the conditions stated in Fama [7, p. 387], which are generally accepted to be sufficient for informational efficiency. The conditions are that there be no transaction costs, that all information be costlessly available to all investors, and that all agree on how to interpret the information. In the above example, the benchmark economy has all these features, although the actual economy does not (since no one knows \(\theta\)). But prices (and expected returns) are the same in both economies, so any definition that requires only that prices and/or expected returns be "right" or "fully reflect the information" in the sense of Fama’s conditions, must deem the actual economy informationally efficient. And yet both investors would rationally pay to learn \(\theta\).

---

\(^7\) If only one investor knows \(\theta\), then technically no rational-expectations equilibrium exists. This is because, if you look for a non-revealing equilibrium, there is no price ratio under which the informed investor would make the same trade regardless of whether the signal were \(a\) or \(b\); and if you look for a fully revealing one, the only equilibrium where both investors know \(\theta\) is non-revealing, as described in the text. But that is an unrealistic knife-edge which can be resolved several ways, such as by allowing the informed investor to give the information away. A similar example is given by Kreps [13].
Also notice that both the actual and the benchmark economies illustrate the difference between conventional definitions of informational efficiency and that of Grossman and Stiglitz [8]: prices fully reflect \( \theta \) but do not reveal \( \theta \).

II. The Value of Acquired Information

As mentioned in the introduction to this paper, there is widespread belief that Proposition Z is true. Theorem 1 proves that it is true, but only for a recent definition of informational efficiency (\( E \)-efficiency), and then only because each investor already knows \( \theta \) (or a sufficient statistic). It is all too obvious that you would not pay to learn something you already know, but that is probably not what most people mean by "the value of information is zero" in Proposition Z. For price-oriented definitions like S-B efficiency this is not a problem — Proposition Z is false anyway, since not everyone has a sufficient statistic for \( \theta \), as in the example in Section I.B above. But even for \( E \)-efficiency, when everyone in effect knows \( \theta \), a more meaningful and conventional interpretation of "the value of information \( \theta \)" is by the impact, on some investor's welfare, of not knowing \( \theta \). The "value of acquired information" is the payment that would neutralize that impact. So in this interpretation, Proposition Z would be false if we could find a case where an investor would pay for information that everyone else already knows.
Of course, such an equilibrium where one investor is ignorant of \( \theta \), would not be \( E \)-efficient. But this analysis is appropriate for the question of whether a market can ever become efficient when information is costly, as addressed by Grossman and Stiglitz [5]. Furthermore, the model below will consider an economy so large that the one ignorant investor has negligible impact on prices, and owns a negligible fraction of the economy’s assets. So even this equilibrium has “virtually” efficient prices and portfolios, in a sense that will be made precise below.

The easiest way to create an example where you would pay to learn something known to everyone else, is with differential prior information. If you alone have the key (piece of prior information) that makes \( \theta \) meaningful, then it comes as no surprise that you might pay. For example, if you alone knew of a huge vein of gold where a certain company owned the mineral rights, you would pay to find out when and if the company was drilling exploratory samples there, even if everyone else already knew the drilling schedule.

But differential priors are not necessary for the desired counterexample, and more insight can be gained from working with homogeneous priors. Furthermore, allowing differential priors seems to stretch the assertion that the market is “virtually” efficient by exclusion of just one microscopic investor, now that that investor possesses a key piece of information. Therefore the examples below assume homogeneous priors.
Even with homogeneous priors, you might rationally pay for information that everyone else already has, that is, information that is fully reflected in prices. For this to happen, it must be that a component of the information is not revealed by prices, and is relevant to your portfolio decision.

A. CAPM Discussion

Suppose the following is common knowledge: All the usual assumptions of the Sharpe-Lintner CAPM hold, except that there is one piece of information, $\theta$, that everyone knows but you. You are a microscopic price-taker. You know that your optimal portfolio is a combination of the riskless asset and the market portfolio of risky assets. Your only problem is to calculate the best weights for that combination. To do this, you need to estimate the mean and variance of the return on the market portfolio.

If $\theta$ is firm-specific information, and has insignificant relevance to the parameters of the market portfolio, then it is of no use to you — you would not pay for it. Because everyone else knows $\theta$, the price of that security (to which $\theta$ is relevant) has already moved to the level that makes the CAPM return-versus-$\beta$ relationship hold. So the market portfolio is mean-variance efficient both unconditionally and conditionally on $\theta$. Therefore, for the special case of firm-specific information in a CAPM world, Proposition Z is true: informational efficiency removes the incentive to acquire costly information. Notice that almost all empirical work on informational efficiency examines firm-specific information, and prices risk using the CAPM
or a similar two-mutual-fund framework. That is probably why so many people believe that Proposition Z holds in general.

But Proposition Z will typically not hold if \( \theta \) gives information about the return on the market.\(^8\) For it not to hold, all that is needed is for prices not to reveal \( \theta \), and for your preferences to be sufficiently different from those of other investors, so that the effect of \( \theta \) on prices does not exactly neutralize your incentive to trade on \( \theta \).

The following is a specific example of such a situation. It is designed to somewhat resemble that of Grossman and Stiglitz \([8]\). The example in Section I.B above could be reused here, and expanded by adding pairs of informed investors, but it would be less satisfactory for several reasons. It is harder to compare with Grossman and Stiglitz; the state-preference framework (especially without a riskless asset) is less familiar and intuitive than multivariate normality; and the non-revealing feature is not robust to slight perturbations of parameters.

---

\(^8\) In a more complex model than the CAPM, such as when the equilibrium portfolios can be formed using \( m > 2 \) mutual funds, the class of information for which Proposition Z is false becomes larger. You would pay for any information that helps you choose the right mix of the mutual funds, even if all other investors know it, as long as you can't get it for free some other way (such as from prices).
B. The Model

The general structure is the same as in Section I.A above, but here we consider a more specific case. Notation is summarized in Table II. There is a large number, \( N \), of identical investors, plus one investor, you, who are different. Your utility function is \( U_y(c_y) = -\exp(-bc_y) \); theirs is \( U_i(c_i) = -\exp(-ac_i) \). There are two securities: a riskless bond paying $1 at \( t=1 \), and a risky stock paying \( \tilde{u} \) at \( t=1 \). \( \tilde{u} \) has two components:

\[
\tilde{u} = \tilde{\theta} + \tilde{\varepsilon}
\]

(1)

Everyone knows \( \theta \) except you, and no one knows \( \varepsilon \). Also, you do not know the other investors' endowment quantity of \( x \) shares of stock, although you do know \( N \) and their utility parameter \( a \). You perceive that \( \tilde{\theta}, \tilde{\varepsilon}, \) and \( \tilde{x} \) have a multivariate normal distribution, with

\[
\begin{align*}
E\tilde{\theta} &= \tilde{\theta} \\
E\tilde{\varepsilon} &= 0 \\
E\tilde{x} &= \tilde{x} \\
\text{Var}(\tilde{\theta}) &= \sigma_\theta^2 \\
\text{Var}(\tilde{\varepsilon}) &= \sigma_\varepsilon^2 \\
\text{Var}(\tilde{x}) &= \sigma_x^2
\end{align*}
\]

(2) \((\tilde{\theta}, \tilde{\varepsilon}, \tilde{x} \text{ mutually independent})\).

The other investors know \( \theta \) and \( x \), and perceive the same distribution as you for \( \varepsilon \). Your ignorance of \( x \) provides the noise necessary to prevent you from deducing \( \theta \) from equilibrium prices.

You are endowed with one share of stock and no bonds. Each of the other investors is endowed with \( x \) shares of stock and no bonds. For any investor to buy bonds, another investor would have to short bonds (borrow).
Table II  
Notation for Section II

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>the information with respect to which efficiency is being assessed; here, it is also a component of ( u ), the stock's payoff</td>
</tr>
<tr>
<td>( N )</td>
<td>number of informed investors</td>
</tr>
<tr>
<td>( U )</td>
<td>utility function</td>
</tr>
<tr>
<td>( c )</td>
<td>consumption at time ( t=1 )</td>
</tr>
<tr>
<td>( y )</td>
<td>subscript for &quot;you&quot;, the uninformed investor</td>
</tr>
<tr>
<td>( i )</td>
<td>subscript for the informed investors</td>
</tr>
<tr>
<td>( b )</td>
<td>your absolute risk aversion parameter</td>
</tr>
<tr>
<td>( a )</td>
<td>the informed investors' risk aversion parameter</td>
</tr>
<tr>
<td>( \sim )</td>
<td>indicates a random variable</td>
</tr>
<tr>
<td>( u )</td>
<td>payoff (value) of the stock at ( t=1 )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>noise component of ( u ) (( u = \theta + \epsilon ))</td>
</tr>
<tr>
<td>( x )</td>
<td>endowment of shares for each informed investor</td>
</tr>
<tr>
<td>( p )</td>
<td>price per share of stock, at ( t=0 ), using the bond as numéraire</td>
</tr>
<tr>
<td>( n )</td>
<td>number of shares of stock you hold at equilibrium</td>
</tr>
<tr>
<td>( n_i )</td>
<td>number of shares of stock held by each informed investor at equilibrium</td>
</tr>
<tr>
<td>( \sim )</td>
<td>indicates a corresponding value in the fully informed equilibrium</td>
</tr>
<tr>
<td>( \lambda \equiv a \sigma^2 x )</td>
<td>(to minimize algebra)</td>
</tr>
<tr>
<td>( p^* \equiv \theta - \lambda )</td>
<td></td>
</tr>
<tr>
<td>( \phi \equiv \sigma^2 \sigma^2 + \sigma^2 \sigma^2 + \sigma^2 \sigma^2 )</td>
<td>(to minimize algebra)</td>
</tr>
<tr>
<td>( X )</td>
<td>the value to you of information ( \theta )</td>
</tr>
</tbody>
</table>
C. Equilibrium

We assume that you use the information about $\theta$ that is revealed by the equilibrium stock price $p$. (The bond is the numéraire.) Your optimization problem at $t=0$ is to choose $n$, the number of shares of stock to hold. Your endowment is one share of stock and no bonds, so your budget constraint will make you hold $p-np$ bonds:

$$\max_n E_y \{ U_y (\tilde{c}_y) \} = E_y \{ -\exp[-b(p-np+n\tilde{u})] \}$$

$$= -\exp \left[ -b(p-np+nE_y(\tilde{u})) + b^2 n^2 \text{Var}_y(\tilde{u})/2 \right]. \quad (3)$$

The first-order condition then determines that

$$n = \frac{E_y(\tilde{u}) - p}{b \cdot \text{Var}_y(\tilde{u})}, \quad (4)$$

so your expected utility simplifies to

$$E_y \{ U_y (\tilde{c}_y) \} = -\exp \left[ -bp - \frac{(E_y(\tilde{u})-p)^2}{2\text{Var}_y(\tilde{u})} \right]. \quad (5)$$

(The $y$ subscript on the expectations and variances in equations (3), (4), and (5) indicates that they are conditioned on your information set, which is [the realization of] $p$.)
We next calculate the price $p$, as a function of $n$, that will induce the other investors to hold the remaining supply of securities. These $N$ investors are identical, so each must end up holding \( \frac{n-1}{N} \) bonds and \( x + \frac{1-n}{N} \) shares of stock. By an analysis similar to equations (3) and (4) above, they will each want to hold

\[
\frac{E_i(\tilde{u}) - p}{a \cdot \text{Var}_i(\tilde{u})} = \frac{\theta - p}{a \sigma^2_{\tilde{\epsilon}}}
\]

shares of stock, so for markets to clear we must have

\[
\frac{\theta - p}{a \sigma^2_{\tilde{\epsilon}}} = x + \frac{1-n}{N} \tag{6}
\]

\[\Rightarrow\]

\[
p = \theta - a \sigma^2_{\tilde{\epsilon}} \left[ x + \frac{1-n}{N} \right] \tag{7}
\]

An equilibrium pricing function \( p(\tilde{\theta}, \tilde{x}) \) is one such that there is a demand function \( n(p) \) that satisfies equations (4) and (7) simultaneously. To simplify the algebra, let \( \tilde{\lambda} = a \sigma^2_{\tilde{\epsilon}} \tilde{x} \). We can restate (4) and (7) as

\[
n(p) = \frac{E_y(\tilde{u} | p) - p}{b \cdot \text{Var}_y(\tilde{u} | p)} \tag{4'}
\]

and

\[
p(\tilde{\theta}, \tilde{\lambda}) = \tilde{\theta} - \tilde{\lambda} + a \sigma^2_{\tilde{\epsilon}} \left[ \frac{n(p) - 1}{N} \right] \tag{7'}
\]

This emphasizes the fact that the function \( p(\tilde{\theta}, \tilde{\lambda}) \) affects \( n \) both as a price and as information about \( \tilde{u} \). In general, such a rational-expectations equilibrium may not exist or, if it does, it may not be unique. But in this model we have both existence and uniqueness, as the following analysis shows:
Let \( \hat{p}^* (\tilde{\theta}, \tilde{\lambda}) = \tilde{\theta} - \tilde{\lambda} \). Then \( E_y (\tilde{u} | p) = E_y (\tilde{u} | p^*) \)

and \( \text{Var}_y (\tilde{u} | p) = \text{Var}_y (\tilde{u} | p^*) \)

because you can calculate \( p - p^* = \frac{a \sigma^2}{\sigma^2 + \sigma^2_\tilde{\theta}} \) from information known to you.

(Also notice that \( \frac{p^*}{N} = \lim_{N \to \infty} p \).

Based on your prior distribution for \( \{\tilde{\theta}, \tilde{\epsilon}, \tilde{\lambda}\} \), you deduce that

\[
E_y (\tilde{u} | p^*) = E_y (\tilde{\theta} | p^*) = \bar{\theta} + \left( \frac{\sigma^2_{\tilde{\theta}}}{\sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\lambda}}} \right) \left( p^* - \bar{p}^* \right) \tag{8}
\]

and

\[
\text{Var}_y (\tilde{u} | p^*) = \text{Var}_y (\tilde{\theta} | p^*) + \sigma^2_{\tilde{\epsilon}} = \frac{\sigma^2_{\tilde{\theta}} \sigma^2_{\tilde{\lambda}}}{\sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\lambda}}} + \sigma^2_{\tilde{\epsilon}} = \frac{\phi}{\sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\lambda}}} \tag{9}
\]

(Where

\[
\phi = \sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\theta}} \sigma^2_{\tilde{\lambda}} + \sigma^2_{\tilde{\lambda}} \sigma^2_{\tilde{\epsilon}}
\]

\[
\bar{p}^* = \bar{\theta} - \bar{\lambda}
\]

\[
\bar{\lambda} = a \sigma^2_{\tilde{\epsilon}} \bar{\lambda}
\]

and

\[
\sigma^2_{\tilde{\lambda}} = a^2 \sigma^2_{\tilde{\epsilon}} \sigma^2_{\tilde{\lambda}} \).\]

Substituting into (4') gives

\[
n = \frac{(\sigma^2 + \sigma^2_{\tilde{\lambda}})(\bar{\theta} - p) + \sigma^2_{\tilde{\theta}} (p^* - \bar{p}^*)}{\phi \sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\lambda}}}
\]

\[
= \frac{\sigma^2_{\tilde{\theta}} \bar{\lambda} + \sigma^2_{\tilde{\epsilon}} \bar{\theta} - \sigma^2_{\tilde{\theta}} p + a \sigma^2_{\tilde{\epsilon}} \sigma^2_{\tilde{\lambda}} \left( 1 - r/N \right)}{\phi \sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\lambda}} \sigma^2_{\tilde{\epsilon}} \sigma^2_{\tilde{\theta}} + \sigma^2_{\tilde{\lambda}} \sigma^2_{\tilde{\epsilon}} \sigma^2_{\tilde{\theta}} \sigma^2_{\tilde{\lambda}} \sigma^2_{\tilde{\lambda}}}. \tag{10}
\]
Solving out for \( n \) then gives
\[
- n = \frac{\sigma^2_{\theta} \hat{\lambda} + \sigma^2_{\lambda} - \sigma^2_{\lambda} p + \left( a \sigma^2_{\lambda} \sigma^2_{\theta}/N \right)}{b \phi + \left( a \sigma^2_\theta \sigma^2_{\lambda}/N \right)}.
\] (11)

Finally, the equilibrium pricing function \( p(\tilde{\hat{\theta}}, \tilde{\lambda}) \) is found by substituting (11) into (7') and solving out for \( p \):
\[
p(\tilde{\hat{\theta}}, \tilde{\lambda}) = \tilde{\hat{\theta}} - \tilde{\lambda} + \frac{1}{N} \left( a \sigma^2_{\lambda} \left( \sigma^2_{\theta} + \sigma^2_{\lambda} - \sigma^2_{\lambda} (\tilde{\hat{\theta}} - \tilde{\lambda}) - b \phi \right) \right) \left( b \phi + \left( a \sigma^2_\theta (\sigma^2_{\lambda} + \sigma^2_{\lambda})/N \right) \right)^{-1}
\] (12)

Because (11) and (12) are logical consequences of the assumptions of the model, this equilibrium is unique.

D. Efficiency for large \( N \)

If you knew \( \theta \), then the resulting equilibrium would be \( E \)-efficient and \( S-B \) efficient with respect to \( \theta \). But you don't. Nonetheless, this section examines in what sense the equilibrium computed in II.C above is "virtually" efficient when \( N \), the number of informed investors, becomes very large. Both \( E \)-efficiency and \( S-B \) efficiency are assessed using the fully informed equilibrium as a benchmark, so first we compute that.

If everyone knew \( \theta \), then the analysis in Section II.C above is unchanged until equation (8), where it becomes
\[
E_y(\tilde{u}) = \theta
\] (13)
\[
\text{Var}_y(\tilde{u}) = \sigma^2_\epsilon
\] (14)
Using a caret (^) to distinguish the fully informed equilibrium, equations (4) and (7) now give

\[
\hat{n} = \frac{\theta - \bar{p}}{b \sigma_\epsilon^2} = \frac{a}{b} \left( x + \frac{1-n}{N} \right) \quad (15)
\]

\[\Rightarrow \]

\[
\hat{n} = \frac{ax + \frac{a}{N}}{b + \frac{a}{N}} \quad (16)
\]

\[\Rightarrow \]

\[
\hat{p}(\bar{\theta}, \bar{\lambda}) = \bar{\theta} - \bar{\lambda} + \frac{1}{N} \left( \frac{a \lambda - ab \sigma_\epsilon^2}{b + (a/N)} \right) \quad (17)
\]

Therefore equations (12) and (17) imply

\[
\lim_{N \to \infty} p = \theta - \lambda = \lim_{N \to \infty} \hat{p} \quad . \quad (18)
\]

So S-B efficiency (\(p = \hat{p}\)) holds in the limiting case of large \(N\). You become a microscopic price-taker in an economy that is fully informed (except for you).

\[E\text{-efficiency requires that portfolios also be the same as if everyone knew } \theta. \text{ Recall that } n \text{ is the number of shares in your (equilibrium) portfolio. Equations (11) and (12) imply}
\]

\[
\lim_{N \to \infty} n = \frac{\sigma_\lambda^2 \lambda + \sigma_\bar{\theta}^2 \bar{\theta} - \sigma_\lambda^2 (\theta - \lambda)}{b \phi} \quad (19)
\]

but (16) implies

\[
\lim_{N \to \infty} \hat{n} = \frac{ax}{b} \quad (20)
\]
so \[ \lim_{N \to \infty} n \neq \lim_{N \to \infty} \hat{n} \] in general. \hspace{1cm} (21)

Let \( n_i \) be the number of shares in each of the other investors' portfolios, so \( \hat{n}_i \) is the corresponding number in the hypothetical equilibrium where you know \( \theta \). Then

\[
n_i = x + \frac{1 - \theta}{N}
\]

and

\[
\hat{n}_i = x + \frac{1 - \hat{\theta}}{N}
\]

so

\[
\lim_{N \to \infty} n_i = \lim_{N \to \infty} \hat{n}_i = x.
\] \hspace{1cm} (22)

Regarding \( E \)-efficiency therefore, at least we can say that for large \( N \) the proportion of portfolios that approach the fully-informed-equilibrium portfolios, approaches 1. The limit of that proportion is given by

\[
\lim_{N \to \infty} \left( \frac{\text{value of other investors' portfolios}}{\text{value of all portfolios}} \right) = \lim_{N \to \infty} \left( \frac{Nxp}{Nxp + p} \right) = 1.
\] \hspace{1cm} (23)

Here again the idea is that the equilibrium is "virtually" \( E \)-efficient for large \( N \), because virtually all of the economy's assets are allocated efficiently.
E. Positive Value of Information

This section adopts the context of a large-\( N \) economy, because of the near-efficiency features noted above and for computational simplicity. From (18) and (21), knowledge of \( \theta \) affects your portfolio decision but does not affect prices. This will imply an increase in your expected utility and a positive value of information.

The question of how much you would pay to learn \( \theta \) can be asked conditionally on \( p \) or unconditionally. If you could observe \( p \) before buying \( \theta \), the stock price you observe might affect the amount you would be willing to pay for \( \theta \). But in this simple example, the conditional (on \( p \)) value of knowing \( \theta \) is the same for all \( p \), and thus equals the unconditional value. The conditional value is calculated below by first comparing your expected utilities with and without knowledge of \( \theta \), conditional on observing stock price \( p \).

Your expected utility without knowing \( \theta \), conditional on \( p \), is denoted \( E_y\{U_y | p\} \), and can be expressed, using equations (5), (8), and (9), as:

\[
E_y\{U_y | p\} = -\exp\left[-bp - \frac{(E_y\{\hat{\theta} | p\} - p)^2}{2(\text{Var}_y\{\hat{\theta} | p\} + \sigma_e^2)}\right]. \tag{24}
\]
If you knew \( \theta \) as well as \( p \) then you would calculate

\[
E_y(\hat{u}| \theta, p) = \theta \quad \text{and} \quad \text{Var}_y(\hat{u}| \theta, p) = \sigma^2_\epsilon.
\]

This would affect your portfolio decision and the probability distribution of your utility. Equation (5) can be used again, to compute your expected utility conditional on \( \theta \) and \( p \), in the fully informed equilibrium:

\[
E_y(\hat{U}_y| \theta, p) = -\exp\left[-bp - \frac{(\theta - p)^2}{2\sigma^2_\epsilon}\right].
\]

But when you are deciding whether to buy \( \theta \), you don't know \( \theta \) yet, so you must average this across the distribution of \( \theta \) given \( p \). By reasoning similar to that for equations (8) and (9), this distribution is Normal with

\[
E_y(\hat{\theta}|p) = \theta + \frac{\sigma^2_{\hat{\theta}}}{\sigma^2_{\hat{\theta}} + \sigma^2_{\lambda}}(p - \bar{p}) \quad (25)
\]

and

\[
\text{Var}_y(\hat{\theta}|p) = \frac{\sigma^2_{\hat{\theta}}\sigma^2_{\lambda}}{\sigma^2_{\hat{\theta}} + \sigma^2_{\lambda}}. \quad (26)
\]

Therefore, knowing \( p \) but not \( \theta \), your expectation of utility-after-learning-\( \theta \) is

\[
E_y(E_y(\hat{U}_y| \theta, p)| p) = E_y\left\{-\exp\left[-bp - \frac{(\hat{\theta} - p)^2}{2\sigma^2_\epsilon}\right] | p\right\}
\]

\[
= \left[\frac{-e^{-bp}}{1 + 2\left(\frac{1}{2\sigma^2_\epsilon}\right)\text{Var}_y(\hat{\theta}|p)}\right] \cdot \exp\left[\frac{-1}{2\sigma^2_\epsilon}\left(E_y(\hat{\theta}|p) - p\right)^2\right] \quad (27)
\]

- 27 -
It turns out that your expected utilities with and without knowledge of $\theta$ are related by a simple proportion, independent of $p$. Dividing equation (24) by equation (27) gives

$$
\frac{E_y(U_y|p)}{E_y(E_y(\hat{U}_y|\theta,p)|p)} = \sqrt{1 + \frac{\sigma^2 \sigma^2_{\lambda}}{(\sigma^2_{\theta} + \sigma^2_{\lambda}) \sigma^2_\varepsilon}} > 1. \tag{28}
$$

Your utility function is always negative, so this shows that you are better off knowing $\theta$. Because you have constant absolute risk aversion, any fee you pay for $\theta$ will not affect the amount of stock you then buy. Therefore the maximum fee $\$X$ that you would be willing to pay, i.e. the value to you of the information $\theta$, is found by solving

$$
e^{bX} = \sqrt{1 + \frac{\sigma^2 \sigma^2_{\lambda}}{(\sigma^2_{\theta} + \sigma^2_{\lambda}) \sigma^2_\varepsilon}}
$$

so

$$X = \frac{1}{2b} \ln \left[1 + \frac{\sigma^2 \sigma^2_{\lambda}}{(\sigma^2_{\theta} + \sigma^2_{\lambda}) \sigma^2_\varepsilon}\right] > 0. \tag{29}
$$

Since the numéraire is one bond, $X$ can be thought of as a number of bonds paid at $t=0$, or a number of dollars paid at $t=1$. Although $X$ is negligible compared to the value of all the assets in the (large-$N$) economy, $X$ is not negligible to you, compared to the value of your portfolio.

---

9 Admati and Pfleiderer [2] find the value of information in the more general case of $n$ risky assets. They also show that $X$ can be written as

$$X = \frac{1}{2b} \ln \left[\frac{\text{Var}(\hat{U}|\hat{P})}{\text{Var}(\hat{U}|\hat{P},\theta)}\right].$$
III. Implications for Defining Informational Efficiency

In addition to clearing up a widespread misconception about Proposition Z, the above results cast some light on the issue of how to define informational efficiency, in the sense used in the finance literature. It will be argued here that they support $E$-efficiency over price-oriented definitions like those of Fama [7], Jensen [11], and Beaver [3].

The strange innovation in the $E$-efficiency definition is the introduction of a portfolio benchmark. It is strange because the empirical literature (event studies, mainly) tests efficiency by checking prices, not portfolios. Latham [14] gives several reasons for checking portfolios, but does not answer the following objection: Informational efficiency is traditionally associated with a zero value of information, or a lack of profit opportunities based on the information. For an individual investor trying to profit on some information, what matters is whether prices are out of line, not whether some other investors' portfolios are out of line. But the present paper shows that this connection does not generally hold. Beaver [3] and Latham [14] argue that the "no abnormal returns" concept boils down to the question "Would prices change if $\theta$ were revealed?" This paper shows that such a definition does not have the operational economic significance of zero value of information.
IV. Conclusions

The widespread belief that the value of information for investors is zero if the market is informationally efficient with respect to that information, is shown to be false except in one unusual interpretation. It is true only for $E$-efficiency, and then only in the sense that if you already have a particular piece of information (or a sufficient statistic for it), then there is zero value to learning it again, though there may well have been positive value to learning it the first time.

Because information can be valuable in an efficient market, the Grossman-Stiglitz [8] impossibility argument does not in general go through: the market can become efficient with respect to costly information.

These results lend support to the idea of including a portfolio benchmark in the definition of informational efficiency, as in $E$-efficiency. An economically significant property that had been attributed to conventional price-oriented efficiency definitions, has been shown to hold only for $E$-efficiency. This raises the more general question of what is the economic significance of prices being "right" when portfolios are not.
REFERENCES


   paper, Department of Economics, University of Washington, April 1985.