Research Program in Finance
WORKING PAPER SERIES

WORKING PAPER NO. 167

Stock Splits,
Volatility Increases
And Implied Volatilities

by

Aamir Sheikh

Research Program in Finance Working Papers are preliminary in nature; their purpose is to stimulate discussion and comment. Therefore, they should not be cited or quoted in any publication without the permission of the author. Single copies of a paper may be requested from the Institute of Business and Economic Research.
RESEARCH PROGRAM IN FINANCE AT THE
WALTER A. HAAS SCHOOL OF BUSINESS,
UNIVERSITY OF CALIFORNIA, BERKELEY

The Research Program in Finance in the Walter A. Haas School of Business at the University of California has as its purpose the conduct and encouragement of research in finance, investments, banking, securities markets, and financial institutions. The present reprint and working paper series were established in 1971 in conjunction with a grant from the Dean Witter Foundation.

INSTITUTE OF BUSINESS AND ECONOMIC RESEARCH
Carl Shapiro, Director

The Institute of Business and Economic Research is an organized research unit at the University of California, Berkeley, whose mission is to promote research by faculty and graduate students in the fields of business and economics. The Institute carries out its mission by organizing programs and activities that enrich the research environment, administering extramural research awards, publishing working papers, and making direct grants for research.
Stock Splits, Volatility Increases and Implied Volatilities

Aamir Sheikh

Doctoral Student
Graduate School of Business Administration
350 Barrows Hall
University of California, Berkeley, CA 94720

June, 1986
Revised July, 1986
Revised September, 1986

ABSTRACT

A test of the efficiency of the Chicago Board Options Exchange, relative to post-split increases in the volatility of common stocks, is presented. The Black-Scholes and Roll option pricing formulas are used to examine the behaviour of implied standard deviations (ISD's) around split announcement and ex-dates. Comparisons with a control group of stocks find no relative increase in ISD's of stocks announcing splits. However, a relative increase is detected at the ex-date. Therefore, the joint hypothesis that 1) the Black-Scholes and Roll formulas are true, and 2) the CBOE is efficient, can be rejected.

I am grateful to Greg Connor, Roy Henriksson, James Hoag, Tee Lim, Stephen Penman, Ehud Ronn and Mark Rubinstein for helpful comments and advice. I alone am to blame for any errors and omissions.
1. Introduction

Recent studies by Ohlson and Penman (1985), Dravid (1984) and Dubofsky and French (1985) have documented significant increases in the observed variance of common stock returns subsequent to splits larger than 25%. This is surprising because, in theory, no real event is associated with the actual day of the split. Information effects of the split (which may signal good earnings prospects and is usually accompanied by a dividend increase) should be imputed into the distribution of stock returns at the announcement of the split.¹ It is possible, but unlikely that firms align other information releases pertinent to their future earnings distribution with the split date.

In the absence of any real changes at the ex-split date, one must turn to irrational behaviour on the part of investors, institutional factors or the effect of stock price measurement rules to explain the post-split variance increase. The first explanation is understandably distasteful to economists. Dravid (1984) and Gottlieb and Kalay (1985) provide some evidence that the observed increase in volatilities may be due to measurement effects. Simulations reveal that the observed variance of returns on stocks with low prices is biased upward when the stock price is rounded to the nearest $ 0.125.² Also, an ex-

¹ Abnormal variance increases are not the only anomaly associated with stock splits. As noted below, there are also significant positive abnormal stock returns at the ex-date.
date increase in the percent bid-ask spread may account for the post-split increase in common stock return variances.\(^3\)

While the causes of the volatility increase are not apparent, one effect of an increase in the real (population) variance of a common stock’s returns is an increase in the value of call options on the stock.\(^4\) *If it is anticipated*, the post-split variance increase should result in an increase in the prices of calls (on the splitting stock) that expire after the ex-date relative to those that expire before the ex-date. In an efficient options market, these relative price changes should occur at the announcement of the split.

An examination of the relative price movements of post-split expiration calls and pre-split expiration options around the split announcement should reveal if the post-split variance increase is properly anticipated.\(^5\) In the same spirit, one may study the relative price movements, around the split announcement, of calls on stocks that split vs. those that did not. Reilly and Gustavson (1985) find that prices of calls on splitting stocks do increase relative to prices of calls on a control group of stocks on the day of, and the three days following the announcement of the split.

There are two problems with these tests. Call values are increasing in the price of the underlying stock and in the time to maturity. Charest (1978a), Grinblatt, Masulis and Titman (1984) and Dravid (1984) find that splitting stocks earn abnormal positive returns both at the announcement and ex-split dates.\(^6\)

\(^3\) The percent bid-ask spread may increase if the dollar spread is not simply proportional to the stock price.

\(^4\) An American Call option gives the holder the right to buy the underlying stock at a specified price (the "Exercise" price) before a specified date (the "Expiration" or "Maturity" date). European options may be exercised only at maturity. Merton (1973) shows that the value of a call option is increasing in the variance of the underlying stock.

\(^5\) This is suggested by Ohlson and Penman.

\(^6\) The usual explanation for these price movements is that splits signal good earnings prospects. See Grinblatt, Masulis and Titman for a discussion of other possible explanations. There is no good explanation for the ex-date returns.
These increases in the price of the underlying stock result in call price increases around the split announcement and ex-dates. Since the call is levered in the stock, the relative increase in the call value is considerably larger than the movement in the stock price. Moreover, call values are increasing in the time to maturity and any study of option prices that covers a reasonable period of time (one month in the present study) around the relevant dates will capture a downward move in the values of calls as time progresses. Thus, no clear inferences about anticipated stock volatility can be drawn from a simple examination of call price movements around the split announcement and ex-dates.

In this paper, I use the Black-Scholes (1973) and Roll (1977) call option pricing formulas to control for stock price and maturity changes. These formulas allow a solution for the standard deviation of stock price returns implicit in call prices and thus permit us to study the behaviour of the options market's beliefs about future stock return volatility. An increase, on average, over the month surrounding the split announcement, in the implied standard deviations (ISD's) of call options maturing after the ex-split date would indicate that the options market properly anticipates the post-split increase in stock return volatility. However, the existence of maturity biases (discussed below) may also account for such increases. The relevant question, therefore, is whether ISD's of stocks that announce a split increase relative to ISD's of other stocks.

Non-parametric tests reveal no such relative increases at the split announcement. There is also no relative increase in ISD's during the period between the announcement and the split. However, the ex-date is found to have a significant positive impact on the ISD's of calls on splitting stocks relative to a

---

7 Note that no increase in ISD's does not imply that the options market does not anticipate the variance increase. This is because the models used to obtain the ISD's may be wrong.
control group of stocks. The relative increase in ISD's is smaller than the increase in standard deviations estimated from daily returns data, indicating that part of the post-split increase in stock return variances is due to stock price measurement rules. At the same time, however, the increase in ISD's is found to continue beyond the seventh day after the split, but in a manner which appears to be unrelated to actual changes in stock volatilities. The Chicago Board Options Exchange (CBOE) seems to have been slow in learning of volatility increases associated with stock splits and may have over-reacted when these increases actually occurred.

The next section discusses the Black-Scholes and Roll option pricing models and the properties of ISD's obtained from these models. In section three, I describe the methodology of my study. The results are discussed in section four and then the paper concludes.

2. The Black-Scholes and Roll Option Pricing Models, and Implicit Volatilities:

The Black-Scholes formula provides a simple expression for the value of a call option in terms of the current stock price, the exercise price, the time to maturity, the interest rate and the variance of the stock's returns. Roll has extended the Black-Scholes model by explicitly considering the early exercise of calls due to dividend payouts by the underlying stock. The empirical evidence suggests that other pricing models do not yield significant improvements over the above formulas. However, ISD's obtained from these models differ across options on the same stock. It is argued that ISD's from close-to-the-money calls provide the most accurate measures of the options market's expectations of future stock return volatilities.

The Black-Scholes options pricing formula describes the call option value, \( C \), as a function of the stock price, \( S \), the exercise price, \( E \), the expiration date,
$T$, current time, $t$, the riskless interest rate, $r$, and the instantaneous standard deviation of stock returns, $\sigma(t)$:

$$C = SN(d_1) - e^{-rt}N(d_2) \quad (1)$$

$$d_1 = \frac{\ln(S/E) + (r + \sigma^2/2)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t};$$

$$\tau = T-t;$$

$$\sigma^2 T = \int_t^\tau \sigma^2(s)ds$$

$N(.)$ is the cumulative standard normal density of $(.)$.

Assumptions underlying the derivation of (1) are

1. the stock, option and riskless bond markets operate continuously;
2. the riskless interest rate, $r$, is constant;
3. the stock price follows the diffusion

$$dS(t) = \mu S(t) + \sigma(t)S(t)dz(t)$$

where $\sigma(t)$ is known and non-stochastic and $dz(t)$ is a standard Wiener process;

4. there are no penalties for short sales;
5. there are no transactions costs or taxes;
6. the stock makes no payouts;
7. the option is European and may only be exercised at maturity.\(^8\)

\(^8\) The above formulation is provided by Merton (1973), who extended the Black-Scholes analysis to the case of stochastic interest rates and variance changing as a non-stochastic function of time. This assumes that the ex-date variance increase is known and not stochastic. Clearly, this a strong assumption. However, the problem is intractable when the variance changes stochastically in a realistic manner, e.g., the variance is not a simple function of the stock price.
Early exercise of American calls is never optimal if the underlying stock does not make any payouts. Thus, an American call on a stock that does not pay dividends is equivalent to a European call and may be valued by the Black-Scholes formula. However, most stocks, with options traded on the CBOE, do pay dividends regularly. CBOE options are American in nature and are not payout protected. This complicates the analysis in two ways. The stock price has to be adjusted for dividends and the possibility of early exercise has to be considered.

Under the assumption that the stock pays a finite number of known dividends during the life of the option, there is a sufficient condition under which early exercise is never optimal. In the case of a single dividend, $D$, this reduces to:

$$E\{1 - e^{-(T-t_i)}\} > D \quad (2)$$

where $t_i$ is the date of the dividend. When (2) (or a multiple dividend analogue) holds, the stock price may be adjusted by subtracting the present value of all dividends paid during the life of the call.

Merton (1973) has shown that early exercise of American calls may be optimal only at times just prior to the ex-dividend dates. Black (1975) suggests that early exercise may be handled by computing the call value contingent on exercise just before the last ex-dividend date before maturity and on holding the call to maturity. The larger of the values thus obtained approximates the correct value of the call. However, this solution forces the probability of early exercise to be zero or one. In fact, this probability lies between zero and one and the right to exercise early results in call values larger than those obtained

---

9 See Merton (1973)
10 For multiple dividends, $D_i$, paid at times $t_i$, the sufficient condition is $E\{1 - e^{-(T-t_i)}\} > D_i$, all $i$. The dividends may be uncertain. Then $D_i$ is replaced by $\sup\{D_i\}$, the largest dividend possible.
from the Black method. Roll (1977) has solved the problem of valuing an American call by explicitly taking into account the probability of early exercise.\textsuperscript{11} He assumes that the stock price net of the present value of all (known) dividends follows a lognormal diffusion. Denote by $P(t)$ the current stock price and for simplicity assume that the stock pays a single dividend, $D$, at time $t_1$. Then the Roll assumption is that

$$S(t) = P(t) - De^{-(t_1 - t)}$$

follows a lognormal diffusion.\textsuperscript{12} For the single dividend case, the Roll formula reduces to:

$$C = S(t)[N(a_1) + M(b_1, -a_1; -\sqrt{\tau_1/\tau})]$$

$$-e^{-\tau_1 M(a_2, -b_2; -\sqrt{\tau_1/\tau})}$$

$$-(E-D)e^{-\tau_1}N(b_2)$$

(3)

$$a_1 = \frac{\ln(S/E) + (\tau + \sigma^2/2)\tau}{\sigma\sqrt{\tau}};$$

$$a_2 = a_1 - \sigma\sqrt{\tau};$$

$$b_1 = \frac{\ln(S/S^*) + (\tau + \sigma^2/2)\tau_1}{\sigma\sqrt{\tau_1}};$$

$$b_2 = b_1 - \sigma\sqrt{\tau_1};$$

$$\tau_1 = t_1 - t;$$

$$\tau = T - t;$$

$S^*$ solves $C(S^*, t_1) = S^* + D - E;$

\textsuperscript{11} Cox, Ross and Rubinstein (1979) develop a discrete time algorithm for dividend adjusted call values. The dividends are assumed to be known and the stock price follows a binomial process.

\textsuperscript{12} This says that the dividend(s) will be paid under all circumstances. One would think, however, if the stock price fell enough the dividend would be reduced or not paid at all. However, the problem is intractable when discrete, stochastic dividends are considered.
where \( C(\ldots) \) is the Black-Scholes value and \( M(a, b; \rho) \) is the standard bivariate cumulative normal density with upper limits of integration \( a \) and \( b \) and correlation coefficient \( \rho \).\(^{13}\) Roll notes that the probability of early exercise increases with the size of the dividend and declines with the time between the dividend and maturity dates.

Tests of the Black-Scholes model by Black and Scholes (1972), Macbeth and Merville (1979, 1980) and Rubinstein (1985) reveal that model prices are systematically biased relative to observed market prices. These biases are related to the extent to which the call is in- or out-of-the-money, the time to maturity of the call and the estimated volatility of the underlying stock.\(^{14}\) However, the in- and out-of-the-money and maturity biases shift over time in a manner that none of the alternative models, e.g. Cox-Ross (1976) Constant Elasticity of Variance formula, Merton (1976) Jump Diffusion formula, Geske (1983) Compound Option formula, is able to explain.

The Roll model has been tested for the single dividend case by Sterk (1982, 1983) and Whaley (1982). They find that the Roll formula is effective in reducing the in- or out-of-the-money and maturity biases, but the estimated variance bias remains at statistically significant levels.\(^{15}\)

An attractive feature of the models for our purpose is that \( C, P, E, r \) and \( \tau \) are observable. \( S \) may be easily obtained from \( P \) if future dividends are known.\(^{16}\) An analytic solution for \( \sigma \) is not possible because of the functional

\(^{13}\) \( S^* \) is the critical stock price above which the call will be exercised just prior to the ex-dividend date.

\(^{14}\) A call is said to be in-the-money (out-of-the-money) if \( S > B e^{-r\tau} \) (\( S < B e^{-r\tau} \)).

\(^{15}\) Sterk (1983) finds that the reduction in bias is not significant unless the dividend is larger than \( $1 \).

\(^{16}\) There is some question as to how synchronized the readings of \( C \) and \( S \) are and as to the appropriate interest rate. Non-synchronous readings on \( C \) and \( S \) may be treated as measurement errors in one or the other and this leads to biases in ISD’s as measures of the stock return’s standard deviation. The call pricing function is relatively insensitive to interest rate specifications and alternative interest rates have not been found to affect results in past studies, e.g. Galai (1977).
forms of (1) and (3). Numerical search techniques are used to obtain a solution. This implicit standard deviation is an estimate of the options market's expectation of the average daily \( \sigma \) during the life of the option.

Previous studies of the properties of ISD's by Latané and Rendleman (1976), Chiras and Manaster (1978) and Beckers (1978c) find them to be better predictors of the future volatility of stock returns than estimates obtained from the past time series of returns.\textsuperscript{17} Patell and Wolfson (1979) find that ISD's increase before annual earnings announcements and take this as indicative of the stock (and options) market anticipating the earnings release. Thus, ISD's do seem to contain information about the future volatility of the underlying stock.

However, studies by Brenner and Galai (1981) and Rubinstein (1985) find significant differences across ISD's obtained from different calls on the same stock. Again, the differences are related to the extent to which a call is in- or out-of-the-money and the maturity of the call and shift in a manner that is not captured by alternative pricing models. Rubinstein finds one bias to persist: for out-of-the-money or deep out-of-the-money calls, the shorter the time to maturity, the higher the ISD.\textsuperscript{18}

Note that the partial derivative of the Black-Scholes formula with respect to \( \sigma \) is relatively large for options that are close-to- the-money (it is largest for calls that are slightly out- of-the-money).\textsuperscript{19} Thus, good measures of the options

\[ C_s = S \sqrt{T} N(d_1), \]

which is maximized when \( d_1 \) is zero, i.e., \( \ln(S/Ke^{-rT}) = -\sigma^2T/2 \).

\textsuperscript{17} Predictive power is measured by the \( R^2 \) obtained in regressions of realized standard deviations (variances) on ISD's (\( ISD^2 \)) vs. historic standard deviation (variance).

\textsuperscript{18} Butler and Schachter (1964) have noted that ISD's are biased estimates of the true standard deviation of stock returns due to the non-linearity of the Black-Scholes formula in \( \sigma \). However, simulations in Cox and Rubinstein (1985, p. 218) reveal that, except for small values of \( \sigma \), \( C \) is virtually linear in \( \sigma \) and thus the biases in ISD's are negligible. This is supported by simulations in Butler and Schachter that show that computed biases are small in both absolute and relative terms. Also, the biases change sign from positive for at-the-money calls to negative elsewhere. Averaging over several close-to-the-money calls thus tends to cancel out the biases in individual ISD's (as noted by Butler and Schachter, p. 11).

\textsuperscript{19}
market's forecast of a stock's future volatility may be obtained from options that are relatively close-to-the-money. This is supported by Beekers' (1978c) finding that the ISD obtained from a single, slightly out-of-the-money option out-performs a weighted ISD in predicting the future standard deviation of stock returns.\(^{20}\)

Given the simplicity of the Black-Scholes and Roll formulas and the absence of a clearly superior alternative, this study uses the former to compute ISD's when the sufficient condition for no early exercise is satisfied and the latter when this condition is violated.\(^{21}\) Attention is confined to options that were relatively close-to-the-money during the period under consideration. Also, a control group of stocks that did not split is employed to check for biases that may have been induced due to maturity changes.

3. Methodology:

3.1. Data:

The study includes all CBOE optionable stocks that split between December 1, 1976 and December 31, 1983, had split factors larger than 0.25 and did not undergo major structural changes, such as mergers, in the nine months

\(^{20}\) The weighted ISD is obtained by a weighted non-linear regression. The weights employed are the partial derivatives of the Black-Scholes formula with respect to \(\sigma\) for the different calls on a stock.

\(^{21}\) It is unlikely that the results are sensitive to changes in the maintained hypothesis of the validity of the Black-Scholes and Roll formulas. For example, Geske's (1979) compound option formula yields call values that are, for realistic debt-equity ratios, very close to the Black-Scholes values, as noted in Rubinstein (1985, p. 458). Simulations by Beekers (1978a), Macbeth and Mervelle (1980) and Rubinstein (1985) show that the Cox-Ross (1978) constant elasticity of variance formula gives call values that are generally higher than Black-Scholes values for in-the-money options and lower for out-of-the-money options. Thus, our computed ISD's will be biased upward for in-the-money calls and downward for out-of-the-money calls. Averaging over different classes will mitigate the individual biases. For close-to-the-money calls, Merton's (1978) diffusion-jump formula, with a zero jump drift, yields values that are close to the Black-Scholes values for the same variance of expiration stock price. Thus, our choice of close-to-the-money calls should give estimates of stock volatilities that are insensitive to alternative call pricing formulas.
following the split. This leads to a sample of 83 stock splits, of which 30 had split factors smaller than one. The smallest length of time between announcement and ex-dates was 34 days, the largest 185 days. The splits were identified from the CRSP Monthly Master file and checked against a list of stocks with CBOE options. Daily return data was obtained for these stocks for 30 trading days preceding the split till 31 trading days after the split. The daily returns were read from the CRSP Daily Returns file.

The Ohlson and Penman test was repeated on this sample: squared daily returns in the month after the split date were compared, by day of the week, to squared daily returns in the month before the split. Out of 1553 comparisons, the value after the split exceeded the one before the split in 885 cases. Based on a paired-sample sign test of the null hypothesis of no increase in volatilities, the associated z-statistic is 4.47, which is significantly positive at the 1% level. Considering each split as one data point, the post-split estimated standard deviation was found to be larger than the pre-split estimated standard deviation in 55 out of 83 cases, with an associated z-statistic of 2.85, which is significantly positive at the 1% level. The median change in estimated standard deviations was 20.6%. Thus, the split sample does exhibit a significant ex-date increase in return variances.

It is important to identify the earliest date at which the CBOE may have known about the split. The Wall Street Journal Index was scanned for six months prior to the split announcement date on the CRSP files for articles that discussed a possible split before the CRSP date. 18 splits were found to have been proposed before the date given by CRSP. For these, the trading day before the date of the Wall Street Journal article was taken as the announcement date.

Each split was matched with a stock that did not split or announce a split in the period 15 days before the announcement of the split to 15 days after the
ex-date. The split and match had to have comparable betas and monthly return variances in the year preceding the split. Betas and variances were estimated by an OLS regression of (when available) 60 monthly stock returns on the returns to a value weighted market index. When possible, the stocks were also matched on the first two digits of their SIC codes.\textsuperscript{22} The return data was retrieved from the CRSP Monthly Returns file, while the SIC codes were obtained from the CRSP Monthly Master file.

Transactions data on call options was collected for one month surrounding the split announcement and ex-dates. The data was retrieved from the Consolidated Data Tapes of the Berkeley Options Data base.\textsuperscript{23} An option record had to satisfy the following six criteria to be included in the study:

1. the option matured after the ex-split date;
2. $0.85E \leq P \leq 1.15E$;
3. the record did not occur in the first 1000 seconds after 9:00 A.M. or in the last 1000 seconds before 3:00 P.M.;
4. the high and low option prices did not differ by more than \$0.25;
5. at least three contracts were traded during the constant stock price interval;
6. at least three raw records were consolidated.

The first condition is necessary for the purpose of the study and needs no discussion. The second criterion aims at eliminating options that are far in- or

\textsuperscript{22} The match had to have a standard deviation of monthly returns within 20\% of the split's standard deviation of monthly returns. Within this class, the match with the same first two digit SIC code and closest beta was selected for the control group. However, if the resulting beta of the match was not within 20\% of the beta of the split, the matching on the basis of SIC codes was dropped, and the match with the closest beta, within a 20\% standard deviation range, was selected for the control group.

\textsuperscript{23} The consolidated format compresses all records on an option during a constant stock price interval into a single record and makes the analysis simpler. Some information is necessarily lost in the process. For example, only the high and low option price are stored.
far out-of-the-money. The remaining four criteria are to ensure that we use records that reflect market prices. The third condition is to eliminate trades that may have been held over from the previous day and market maker quotes at the end of the day to influence their margin requirements. The tight option price interval is desirable so as to obtain a relatively accurate option price. The last two criteria ensure that the option is traded in some depth. The call price used in the computations was a weighted average of the high and low option prices, with contract volumes acting as weights. If both these volumes were zero, a simple arithmetic average of the high and low prices was used.

The interest rate used was the yield to maturity of a T-bill with maturity closest to the maturity of the option. T-bill yields were obtained from the CRSP Fama Term Structure files. Since the T-bill data is recorded on a monthly basis, a simple convex combination was used to interpolate the interest rates between the record dates.

Dividend data was collected from the CRSP Monthly Master file. Past researchers have used actual dividends in the computation of Black-Scholes or Roll values or to obtain ISD's. This study tries to use the market's expectations of future dividends. Since most stocks that pay dividends seldom change their dividends and tend to pay dividends at the same time (usually the same day of the week) every year, it is reasonable that the market uses the last quarterly (monthly, semi-annual, annual) dividend when a normal quarterly (monthly, etc.) dividend is expected. Therefore, if a dividend is expected but not announced, the last similar announced dividend is used. When a stock follows a predictable pattern with extra or special dividends, these are treated as

---

24 I am grateful to Mark Rubinstein for suggesting these.
25 Any errors this may induce should not affect the results significantly due to the insensitivity of the pricing formulas to the interest rate specification.
26 Rubinstein (1985) uses both actual and last year's dividends.
regular dividends. Other extra dividends are considered only as they are announced. It is possible that since the stocks under consideration had done well in the past (most splitting stocks are like this), a dividend increase is expected. My dividend adjustment would not capture this. To check the effect of the dividend adjustment, part of the analysis was repeated using actual dividends. The results did not change significantly.

Early exercise was considered only at the last ex-dividend date before maturity. The Roll formula with more than one dividend involves the cumulative multivariate normal distribution with more than two variables and becomes difficult and expensive to implement. Restricting the sample to options with a single dividend during their lives would have resulted in a very small sample. The probability of early exercise is declining in the time between the ex-dividend date and the maturity date of the option. This period is usually three months or more for dividends before the last one before the maturity of the option. Thus, it is unlikely that an option will be exercised prior to the last ex-dividend date before maturity. This is supported by Beckers’ (1978c) finding that only eight out of 6,179 options may have been optimally exercised even at the last ex-dividend date before maturity. Therefore, the computation of ISD’s used the Black-Scholes formula when the no early exercise condition was satisfied at the last ex-dividend date. In this case, the stock price was reduced by the present value of all dividends expected during the life of the option. Otherwise, the Roll formula was used with early exercise only at the last ex-dividend date before maturity. Note that this may result in ISD’s that are biased upward because the right to exercise at other dividend dates has

---

27 This is based on the large number of quarterly dividends paid by the stocks in the sample. Extra and monthly dividends weaken this assertion.
28 However, Beckers uses the modified Black approach in considering early exercise. We have already noted the weakness in this method.
positive value. Thus, part of the value we are ascribing to the stock volatility is actually due to the possibility (albeit small) of exercising at an earlier dividend date. However, this occurs for ISD's obtained both before and after the relevant events and should not bias the results in any one direction.

Implied standard deviations of daily returns were obtained using numerical search procedures until the resulting call value was within $0.01 of the market price. The search was restricted to the interval [0.001, 0.1] because these were felt to be reasonable limits for daily stock return standard deviations. The sampling criteria resulted in 20206 option record observations in the month around the announcement date. 20104 of these records provided convergent solutions. There were 28329 observations, with 28149 convergent solutions, during the month surrounding the ex-split date.

3.2. Tests:

Four measures are used to analyse the impact of the split announcement and ex-dates on anticipated volatility. The first categorizes the options on a given stock into 15 classes, three based on $P/E_t^{t-t_t}$ times five based on the time to maturity, and replicates the day of the week comparisons used by Ohlson and Penman. The remaining three measures regard each split as a single observation and use different methods to aggregate the ISD's for a given stock.

The option categories used in the first test are the following:

1. $\frac{P}{E_t^{t-t_t}}<0.95$;

---

29 This restriction may explain some of the solutions that did not meet the convergence criterion. A Newton-Raphson search was used for the Black-Scholes formula. The Black formula was solved using the method of bisection. An IMSL routine MDNBOR was used to compute the cumulative bivariate normal density.

30 Violation of parity, i.e., $C>S-E_t^{t-t_t}$, explains most of the non-convergent solutions. Some of the solutions may not have converged because of the shape of the option pricing functions. A number were found to tax the computational bounds of the computer.
2. \( 0.95 \leq \frac{P}{E_k^{1+\tau}} \leq 1.05; \)

3. \( 1.05 < \frac{P}{E_k^{1+\tau}}; \)

4. \( \tau \leq 70; \)

5. \( 71 \leq \tau \leq 120; \)

6. \( 121 \leq \tau \leq 170; \)

7. \( 171 \leq \tau \leq 220; \)

8. \( 220 < \tau. \)

For a given stock, ISD's in a given category are averaged for each day to obtain a single implied volatility for that stock, for the given option category, for the day. The ISD's thus obtained before and after the relevant event are compared by day of the week. For example, all the ISD's obtained from IBM options with \( P/E_k^{1+\tau} < 0.95 \) and \( 121 \leq \tau \leq 170 \) on the first Monday after the split are averaged and compared to an average for the same class on the first Monday in the two weeks prior to the split. The number of positive changes in all such comparisons are then added across all option classes and stock splits for a single paired-sample sign test of the hypothesis that ISD's do not change after stock splits (or split announcements) versus the one-sided alternative of an increase in ISD's. This is referred to as “Test 1” in what follows.

Cross-sectional and serial dependence in ISD's for the same stock invalidates the above test. This problem is alleviated if we regard each split as a single observation and obtain single expected volatility measures before and after the relevant event. Since the tests may be sensitive to the method in which ISD's are aggregated, three methods are used. The first simply uses the sample mean for each stock, e.g., all ISD's for IBM before a split announcement are averaged

\[^{31}\text{These are derived from those used by Rubinstein (1985).}\]
into an aggregate, pre-announcement ISD for IBM. The same measure is obtained after the announcement. Changes in this measure across all splits are then used to see if the event caused a general increase in the ISD's of splitting stocks.

The second measure uses only those options with \( \tau > 70 \) and \( 0.95 \leq \frac{P}{E^{\tau}} \leq 1.05 \) and employs the mean from the resulting smaller sample. Following Macbeth and Merville (1979, 1980) and Sterk (1982, 1983), the third method regresses all ISD's on a given stock on the ratio of the stock price (net of escrowed dividends) to the present value of the exercise price. The resulting estimates are used to obtain the ISD for an at the money option \((S/E^{\tau} = 1)\). These three measures are denoted by \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \) respectively.\(^{32} \)

The paired-sample sign test and the Wilcoxon signed-rank tests tests are used to check for increases in these measures at the announcement and ex-split dates. The null hypothesis of no change in ISD's is tested against the one-sided alternative of an increase in ISD's. Further, I test if the ratio of post-event ISD's to pre-event ISD's for splits exceeds the same ratio for the control group. Only exact matches are used in the latter test. The null hypothesis of no difference in ISD ratios is tested against the one-sided alternative of a larger ratio for the split sample. The announcement date is excluded from the entire analysis because it is not known when during the day the split was announced. However, the ex-date is considered as the first day after the split and used as such in the comparisons.

\(^{32}\) Tests based on sample medians, instead of means, for \( \sigma_1 \) and \( \sigma_2 \) yield very similar results.
4. Results:

4.1. Using Expected Dividends:

The results of the binomial and signed-rank tests are given in tables 1 through 6. The number of observations reported in the tables are usually less than 83 because the option data screens resulted in no observations for some of the splits and matches.

Both the announcement and the actual split are seen to increase the ISD's of stocks that split. However, the announcement date effects disappear when compared with the control group. In contrast, the ex-date increase in ISD's of splitting stocks is found to exist relative to the control group.

Table 1 provides the announcement date effects separately for the split and match samples. The day of the week, by option class, comparisons yield a statistically insignificant decline in ISD's for the split group. The aggregate measures indicate a significant increase in the ISD's of the split group. The control group shows significant increases in implied variances, except for tests based on $\sigma_2$. The results of the comparisons between the two groups are given in table 2. For the set of exact matches, none of the six tests shows a significant increase in the ISD's of the split group relative to the control group. Further, as shown in table 5, there is very low and statistically insignificant Spearman rank correlation between the percent change in ISD's and the ex-date percent change in estimated standard deviations.33

33 A low correlation between changes in ISD's and changes in estimated standard deviations does not, of itself, imply that the post-split variance increase is not anticipated. Since the ex-date increase is stochastic, it is perfectly rational to increase the implied variance rate of all splitting stocks by the expected increase in variance associated with stock splits. This would also result in zero correlation between actual changes in standard deviations of returns and changes in ISD's at the split announcement. The fact that we find no increase in split ISD's relative to the control group, however does show that the post-split volatility increase is not anticipated. Note that the ex-date changes in ISD's should be correlated with changes in actual standard deviations.
The announcement date increase in ISD's thus seems to be due to reasons other than anticipation of a post-split variance increase. One possible explanation lies in a negative relationship between maturity and implicit variances, as noted for out-of-the-money options by Rubinstein (1985).

The ex-date results show that the CBOE does pick up the increase in variance as it occurs. Table 3 contains the results for each group separately. Test 1 provides the most significant z-statistic for the split sample. All of the test statistics are positive for the split group and, with the exception of the paired-sample test for $\sigma_9$, are significantly positive at the 10% level. In contrast, the control group exhibits, for the most part, a statistically insignificant decrease in ISD's during the split period.

The between group comparisons confirm the increase in ISD's of splitting stocks relative to the control group. All of the statistics are positive and significantly so for the Wilcoxon signed-rank test for $\sigma_1$ and $\sigma_2$. Table 5 gives the Spearman rank correlations between the percent changes in ISD's and the percent changes in estimated standard deviations. 34 All three measures show a significantly positive correlation with actual changes, with the $\sigma_1$ providing the highest correlation.

However, the median difference between splits and matches in percent increases in $\sigma_1$ is 7.59% (those for $\sigma_2$ and $\sigma_9$ are smaller). This difference is less than one half the 20.6% median increase in estimated standard deviations. 35 It is closer to the Ohlson and Penman finding of an 11% increase in estimated

---

34 Actual standard deviations are estimated using the daily returns data described above. The number of observations used depends on the particular test. When we center around the ex-date, thirty observations per-split are used for the pre-split standard deviations, while 31 are used for the post-split standard deviation. Centering around calendar day seven after the split results in, at most, five observations per split for both the post- and pre-seventh day estimated standard deviations.

35 It even smaller than the 35% median increase in estimated standard deviations found by Ohlson and Penman.
standard deviations of monthly returns. This indicates that a large proportion of the ex-date increase in estimated stock return standard deviations may be due to measurement effects, such as the bid-ask spread and rounding of stock prices to the nearest $0.125.

Interesting details were uncovered by a closer examination of ISD changes over the entire period. I looked at changes after the announcement and before the split (comparing the 15 days after the announcement with the 15 days before the ex-date), 15 days (seven days before the ex vs. eight days after) around the split date, and in the sixteen days after the split (days zero to seven after the split vs. days eight to 15). Tests were conducted using $\sigma_1$ and the results of the between group comparisons are given in table 6. No relative increase is found in ISD's between the announcement and the ex-dates. A significant increase is found to occur in the fifteen days surrounding the ex-date and ISD's of the split sample continue to increase relative to the control group beyond the seventh day after the split.

However, no increase in actual volatilities is found beyond the seventh day after the split. Out of 323 squared return comparisons, only 160 register an increase. The standard deviation of returns in days eight to fifteen exceeded that of days zero to seven in only 36 out of 83 cases. It is possible that the increase in ISD's following day seven may have been due to a slow reaction to the ex-date increase in actual variances. However, this last increase in ISD's has a low and statistically insignificant Spearman rank correlation with the ex-date increase in estimated standard deviations.

In sum, the CBOE does not appear to have anticipated post-split increases in common stock return variances. These increases are imputed in option prices only as they occur. The increase in ISD's is considerably smaller than recorded increases in estimated standard deviations of stock returns,
indicating that a large part of the increase in measured variances is probably due to changes in the percent bid-ask spread or the rounding of stock prices to $0.125. Moreover, there seems to have been an over-reaction to the sudden variance increases, with ISD’s increasing beyond the seventh day after the split in a manner unrelated to actual post-split standard deviation increases.

4.2. Economic Significance:

The next natural question is: given that the CBOE did not react to the post-split variance increase in a timely manner, were there economic profits to be made? To abstract from the noted increase in the prices of splitting stocks, I study percent changes in the value of the following index:

\[ BS = N\left(\frac{\sigma\sqrt{180}}{2}\right) - N\left(-\frac{\sigma\sqrt{180}}{2}\right). \]

This is simply the ratio of the Black-Scholes value of a six month, at-the-money option, to the price of the underlying stock. The value of this index is seen to depend only on the variance of stock returns and percent changes in \( BS \) are indicative of the returns accruing to call options aside from stock price and maturity changes.

Table 7 records the median returns from this index around ex-split dates for all three aggregate implied volatility measures. The median return to the split sample exceeds that of the control group for all three cases, with median excess returns (over at most one month) ranging from 4.17% for \( \sigma_3 \) to 7.54% for \( \sigma_1 \). The table also contains the results of statistical tests on the differences between the index returns for the split and control groups. These results mirror the implied variance results and need no discussion.

However, transaction costs allow only market makers to capture these excess returns.\(^{36}\) For each split, returns inclusive of transaction costs are

\(^{36}\) I am grateful to Tee Lim for pointing out the possible impact of transaction costs and for helpful discussions on the topic.
obtained for an at-the-money, six month call. The value of this call is based on
the pre-split median stock price and pre- and post-split values of $\sigma_1$. Transac-
tion costs are then added to the value of this call when it is purchased before
the split and subtracted when it is sold after the split. Public traders and arbi-
trageurs are assumed to pay a $0.25 bid-ask spread, while market makers are
assumed to earn the same spread. Commission costs for public traders are
based on a schedule for multiple trading unit orders in Cox and Rubinstein
(1985). Clearing costs are assumed to be $1.70 per contract for arbitrageurs
and $1.00 per contract for market makers.\footnote{Public traders and arbitrageurs buy the pre-split call at its value plus$0.125 and sell
it after the split for its value minus$0.125/(1 + the split factor). Market makers buy before
the split at the call value minus$0.125 and sell after the split at value plus$0.125/(1 + the
split factor). The $0.25 spread is the largest we need consider because this study only uses
transaction records that had a maximum spread of $0.25 between the high and low call
prices for a given stock price interval. Multiplication of the post-split spread by $(1 + the
split factor)$ is necessary because a $0.25 spread after the split is equivalent to a $0.25/(1 + the
split factor)$ spread before the split, as noted by French and Dubofsky (1988). The
commission schedule is given on p. 111 of Cox and Rubinstein and is modified here by as-
suming a uniform fee of 0.5% for order values larger than $30,000. The commission and
clearing costs are at the higher end of those given in Phillips and Smith (1980).

The above costs represent the higher end of the transaction costs spec-
trum for calls. Additional tests were conducted with a $0.125 bid-ask spread,
commission costs for the public based on a schedule from Charles Schwab and
Co., Inc., and clearing costs of $1.50 and $0.50 for arbitrageurs and market
makers respectively.\footnote{The commission schedule was obtained from Mark Rubinstein.}
The results of these tests are given in table 8.b and the
only noticeable change is that arbitrageurs’ returns now become slightly posi-
tive. However, the returns are not statistically significant and little can be said
of whether these returns are commensurate with the risks that may be borne by following such a trading strategy. The returns to market makers remain abnormally high, with 42 out of 51 splits providing positive returns. Since these returns abstract from stock price movements, the positive variance and stock price effects together should make calls on splitting stocks an unusually attractive investment for market makers.

4.3. Alternative Dividend Adjustment:

It is possible that the results may be driven by my dividend adjustment method. To check for this, test 1 and tests based on $\sigma_1$ were repeated, for the announcement and ex-dates, using actual dividends. For brevity, the tests are reported in table 9 only for the between group comparisons and are found to be virtually identical to the previous results.

5. Conclusion:

It appears that the CBOE did not anticipate post-split volatility increases in common stock returns. Although there does appear to be an increase in ISD's of stocks announcing splits, this increase seems to be due to factors that affect the expected volatilities of all stocks and is not attributable to proper anticipation of post-split variance increases. This study also does not find any increase in implied volatilities in the period between the announcement of the split and the split date.

On the other hand, there is an increase in ISD's of splitting stocks at the ex-split date and this increase is related to the actual variance changes and probably not due to other economy wide factors. The fact that the proportionate increase in ISD's is smaller than the increase in estimated standard deviations provides some evidence that the post-split increase in stock return
variances is partly due to measurement effects. Also, the CBOE may have over-
compensated for the post-split jump in variances by continuing to increase its
expectation of split stock variances beyond the seventh day after the split in a
manner that does not reflect the actual ex-date variance increases. These
increases in implied variances translate into excess returns to market makers
from calls on splitting stocks, independent of any excess returns due to stock
price movements.

One may question the use of the Black-Scholes and Roll formulas to study
changes in anticipated volatilities when the actual variance changes may be
stochastic. All stock splits are not accompanied by an increase in return vari-
ance and the the magnitude of the increase is stochastic. Thus, our tests only
allow us to reject the joint hypothesis that (1) the CBOE rationally anticipated
post-split volatility increases in common stock returns and (2) the Black-
Scholes and Roll option pricing formulas are valid. In defense of the models,
however, it is not clear why they would capture an increase in expected volatili-
ties at the ex-date but not at the announcement.
BIBLIOGRAPHY


Geske, Robert, "A Note on an Analytical Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends," in Graduate School of Management, University of California, Los Angeles, 1979.


### Table 1

<table>
<thead>
<tr>
<th>Method of Comparison</th>
<th>Number of Observations</th>
<th>Positive Changes¹</th>
<th>Median Percent Change</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Splits:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>341</td>
<td>163</td>
<td>-0.95</td>
<td>-0.76</td>
<td>-</td>
</tr>
<tr>
<td>σ₁</td>
<td>62</td>
<td>39</td>
<td>2.76</td>
<td>1.9₁ᵇ</td>
<td>2.4ᶜ</td>
</tr>
<tr>
<td>σ₂</td>
<td>52</td>
<td>30</td>
<td>2.76</td>
<td>0.97</td>
<td>1.8₅ᵇ</td>
</tr>
<tr>
<td>σ₃</td>
<td>59</td>
<td>34</td>
<td>2.18</td>
<td>1.04</td>
<td>1.2₈</td>
</tr>
<tr>
<td>Matches:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>291</td>
<td>161</td>
<td>0.57</td>
<td>1.7₆ᵇ</td>
<td>-</td>
</tr>
<tr>
<td>σ₁</td>
<td>58</td>
<td>37</td>
<td>2.3₆</td>
<td>1.9₇ᵇ</td>
<td>1.9₁ᵇ</td>
</tr>
<tr>
<td>σ₂</td>
<td>46</td>
<td>25</td>
<td>0.9₆</td>
<td>0.4₄</td>
<td>0.2₅</td>
</tr>
<tr>
<td>σ₃</td>
<td>53</td>
<td>28</td>
<td>1.4₈</td>
<td>0.2₇</td>
<td>1.4₈ᵃ</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Method of Comparison</th>
<th>Number of Observations</th>
<th>Positive Differences²</th>
<th>Median Difference in % Changes³</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-Rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ₁</td>
<td>49</td>
<td>24</td>
<td>-0.05</td>
<td>-0.1₄</td>
<td>0.4₂</td>
</tr>
<tr>
<td>σ₂</td>
<td>29</td>
<td>14</td>
<td>-1.7₀</td>
<td>0</td>
<td>0.4₆</td>
</tr>
<tr>
<td>σ₃</td>
<td>43</td>
<td>21</td>
<td>-5.5₇</td>
<td>0</td>
<td>-0.1₂</td>
</tr>
</tbody>
</table>

¹ This is number of cases in which the post-event ISD exceeded the pre-event ISD.

² This is number of cases in which the percent change in the split ISD exceeded the percent change in the match ISD.

³ This is the percent change in the split ISD minus the percent change in the match ISD.

* Significant at α=0.1.

*b Significant at α=0.05.

*c Significant at α=0.01.
Table 3

<table>
<thead>
<tr>
<th>Method of Comparison</th>
<th>Number of Observations</th>
<th>Positive Changes</th>
<th>Median Percent Change</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Splits:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>180</td>
<td>119</td>
<td>5.29</td>
<td>4.25(^c)</td>
<td></td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>51</td>
<td>35</td>
<td>5.73</td>
<td>2.52(^c)</td>
<td>3.12(^c)</td>
</tr>
<tr>
<td>(\sigma_2)</td>
<td>40</td>
<td>28</td>
<td>8.33</td>
<td>2.37(^c)</td>
<td>3.19(^c)</td>
</tr>
<tr>
<td>(\sigma_3)</td>
<td>46</td>
<td>27</td>
<td>3.97</td>
<td>0.72</td>
<td>1.82(^b)</td>
</tr>
<tr>
<td><strong>Matches:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>528</td>
<td>261</td>
<td>-0.19</td>
<td>-0.22</td>
<td></td>
</tr>
<tr>
<td>(\sigma_1)</td>
<td>63</td>
<td>22</td>
<td>-3.57</td>
<td>-2.27(^b)</td>
<td>-1.9(^a)</td>
</tr>
<tr>
<td>(\sigma_2)</td>
<td>43</td>
<td>22</td>
<td>0.014</td>
<td>0</td>
<td>-0.6</td>
</tr>
<tr>
<td>(\sigma_3)</td>
<td>58</td>
<td>25</td>
<td>-2.18</td>
<td>-0.92</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Table 4

<table>
<thead>
<tr>
<th>Method of Comparison</th>
<th>Number of Observations</th>
<th>Positive Differences</th>
<th>Median Difference in % Changes</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-Rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_1)</td>
<td>45</td>
<td>32</td>
<td>7.59</td>
<td>2.5(^c)</td>
<td>2.94(^c)</td>
</tr>
<tr>
<td>(\sigma_2)</td>
<td>24</td>
<td>15</td>
<td>6.50</td>
<td>1.02</td>
<td>1.91(^b)</td>
</tr>
<tr>
<td>(\sigma_3)</td>
<td>41</td>
<td>23</td>
<td>4.17</td>
<td>0.82</td>
<td>1.08</td>
</tr>
</tbody>
</table>

\(^1\) This is number of cases in which the post-event ISD exceeded the pre-event ISD.

\(^2\) This is number of cases in which the percent change in the split ISD exceeded the percent change in the match ISD.

\(^3\) This is the percent change in the split ISD minus the percent change in the match ISD.

\(^a\) Significant at \(\alpha=0.1\).

\(^b\) Significant at \(\alpha=0.05\).

\(^c\) Significant at \(\alpha=0.01\).
Table 5
Spearman Rank Correlations of ISD Changes with Actual Standard Deviation Changes\(^1\)

<table>
<thead>
<tr>
<th>Announcement:</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td>$\sigma_1$</td>
<td>1</td>
<td>0.85(^b)</td>
<td>0.76(^b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(52)</td>
<td>(59)</td>
<td>(62)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>1</td>
<td>0.72(^b)</td>
<td>(52)</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(52)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>1</td>
<td>0.72(^b)</td>
<td>(52)</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(59)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ex-Date:</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td>$\sigma_1$</td>
<td>1</td>
<td>0.83(^b)</td>
<td>0.88(^b)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(40)</td>
<td>(49)</td>
<td>(51)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>1</td>
<td>0.87(^b)</td>
<td>(40)</td>
<td>0.34(^a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(40)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>1</td>
<td>0.41(^b)</td>
<td>(48)</td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses are the number of observations used in the computation.

\(^{1}\) Actual standard deviations are estimated from daily returns, using 30 trading days prior to the split and 31 trading days following the split. Correlations are between percent changes.

\(^{a}\) Significant at $\alpha=0.05$.

\(^{b}\) Significant at $\alpha=0.01$. 
Table 8

Analysis of ISD's by Period:

<table>
<thead>
<tr>
<th>Period</th>
<th>Number of Observations</th>
<th>Positive Differences(^1)</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-rank Test</th>
<th>Correlation with Actual Changes(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Announcement to Split</td>
<td>52</td>
<td>27</td>
<td>0.13</td>
<td>0.67</td>
<td>-0.101</td>
</tr>
<tr>
<td>15 Days Around Split</td>
<td>37</td>
<td>26</td>
<td>2.3(^b)</td>
<td>2.3(^b)</td>
<td>0.34(^b)</td>
</tr>
<tr>
<td>After the Split</td>
<td>39</td>
<td>26</td>
<td>1.92(^a)</td>
<td>2.27(^b)</td>
<td>0.018</td>
</tr>
</tbody>
</table>

\(^1\) These are the number of cases for which the percent change in the split's ISD exceeded the percent change in the corresponding match's ISD.

\(^2\) The Spearman rank correlation between percent changes is given.

\(^a\) Significant at \(\alpha=0.05\).

\(^b\) Significant at \(\alpha=0.01\).
Table 7
Comparison of Index Returns in the Month Around the Split Ex-Date

<table>
<thead>
<tr>
<th>Tests</th>
<th>Number of Observations</th>
<th>Positive Differences(^1)</th>
<th>Median Difference in % Returns</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-Rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_1)</td>
<td>48</td>
<td>32</td>
<td>7.54</td>
<td>2.5(^b)</td>
<td>2.94(^b)</td>
</tr>
<tr>
<td>(\sigma_2)</td>
<td>24</td>
<td>15</td>
<td>6.47</td>
<td>1.32</td>
<td>1.91(^a)</td>
</tr>
<tr>
<td>(\sigma_3)</td>
<td>41</td>
<td>23</td>
<td>4.17</td>
<td>0.62</td>
<td>1.07</td>
</tr>
</tbody>
</table>

\(^1\) These are the number of cases in which the return to the split exceeded the return to the corresponding match.

\(^a\) Significant at \(\alpha=0.05\).

\(^b\) Significant at \(\alpha=0.01\).
### Table 8.a

**Index Returns Around Ex-dates for Calls on Splitting Stocks**

<table>
<thead>
<tr>
<th></th>
<th>Number of Observations</th>
<th>Positive Returns</th>
<th>Median % Return</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public</td>
<td>51</td>
<td>14</td>
<td>-10.7</td>
<td>-3.09&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-3.07&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Arbitrageurs</td>
<td>51</td>
<td>21</td>
<td>-3.0</td>
<td>-1.12</td>
<td>-0.45</td>
</tr>
<tr>
<td>Market Makers</td>
<td>51</td>
<td>43</td>
<td>11.5</td>
<td>4.76&lt;sup&gt;b&lt;/sup&gt;</td>
<td>5.05&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

### Table 8.b

**Index Returns Around Ex-dates for Calls on Splitting Stocks**

<table>
<thead>
<tr>
<th></th>
<th>Number of Observations</th>
<th>Positive Returns</th>
<th>Median % Return</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-rank Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public</td>
<td>51</td>
<td>12</td>
<td>-10.6</td>
<td>-3.64&lt;sup&gt;b&lt;/sup&gt;</td>
<td>-3.10&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Arbitrageurs</td>
<td>51</td>
<td>29</td>
<td>2.0</td>
<td>0.84</td>
<td>1.37</td>
</tr>
<tr>
<td>Market Makers</td>
<td>51</td>
<td>42</td>
<td>8.7</td>
<td>4.48&lt;sup&gt;b&lt;/sup&gt;</td>
<td>4.26&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> Significantly different from zero at α=0.05.

<sup>b</sup> Significantly different from zero at α=0.01.
<table>
<thead>
<tr>
<th></th>
<th>Number of Observations</th>
<th>Positive Differences¹</th>
<th>Paired-Sample Sign Test</th>
<th>Wilcoxon Signed-rank Test</th>
<th>Correlation with Actual Changes²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Announcement</td>
<td>49</td>
<td>22</td>
<td>-0.57</td>
<td>-0.22</td>
<td>-0.055</td>
</tr>
<tr>
<td>Ex-Date</td>
<td>46</td>
<td>32</td>
<td>2.5⁰</td>
<td>3.02⁰</td>
<td>0.49⁰</td>
</tr>
</tbody>
</table>

¹ These are the number of cases for which the percent change in the split's ISD exceeded the percent change in the corresponding match's ISD.
² The Spearman rank correlation between percent changes is given.
⁰ Significant at α=0.05.
⁰⁰ Significant at α=0.01.