A MULTI-ATTRIBUTE COMPARATIVE EVALUATION OF RELATIVE RISK FOR A SAMPLE OF BANKS

BY

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A Multi-Attribute Comparative Evaluation of Relative Risk for a Sample of Banks

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Abstract

This paper proposes a comparative analysis of five different attributes of commercial banks' liabilities that may be thought of as either directly measuring the default risk, or are perceived by the market as impacting upon that default risk. Two of these attributes are credit ratings assigned by Moody's and Standard and Poor's to debt issues. The third attribute that we consider is yield to maturity, which is typically assumed to be positively related to default risk. The fourth attribute that is proposed is the "fair value," risk-adjusted deposit insurance premium, derived from option-theoretic modeling and estimated from the daily data on equity prices. Finally, we propose a term structure-adjusted difference in yield between the risky debt and default-free Government debt as a more precise and theoretically sound measure of the default risk premium, and develop a methodology for eliciting it on a composite basis for a given bank from the market prices of its various bond issues. This term structure-adjusted risk premium provides an analytical approximation to the valuation of corporate bonds' call features and sinking fund provisions. In view of the ordinality of the credit ratings given by the investment agencies, Spearman's rank correlation coefficient is employed as an analytical tool. The analysis is empirically applied to a sample of 22 major banks.
A Multi-Attribute Comparative Evaluation of Relative Risks for a Sample of Banks

Introduction

Default risk of the debt issued by commercial banks is usually assessed by several different measures. On one hand, there are credit ratings assigned by investment agencies -- principal among them, Moody's Investors Service and the Standard & Poor's Corporation -- and it has been suggested that these ratings are the most commonly used measures of default risk by investors. On the other hand, one could conceive of default risk reflected in the market's pooled assessment of bond prices. Default risk premium would then be thought of as the difference in yield between the risky and default-free debt. Particularly for commercial banks, another important measure of default risk is the "fair-value" risk-adjusted deposit insurance premium, which -- as shown, among others, by Ronn and Verma (1986) -- can be estimated from a time series of market prices of *equity*. The yield to maturity implied by the market prices of debt, though certainly an imperfect measure of default risk, is taken into consideration by most private and institutional investors.

This study proposes a comparative analysis of the above five attributes for a sample of banks, and of how each of them is pairwise related to the others. As the credit ratings by

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1 The authors gratefully acknowledge the helpful suggestions of Cheng F. Lee and George Pennachi, while remaining solely responsible for any remaining errors. The original version of this paper was presented at the Asset Securitization and Off-Balance Sheet Risks of Depository Financial Institutions, Northwestern University, February 1987.

2 For example, the Banking Act of 1936 constrained Federally chartered banks to investing in "investment-grade bonds," defined as bonds rated Baa3 or better.
investment agencies are ordinal, Spearman's rank correlation will be utilized as one of the tools of analysis.\textsuperscript{3}

The rest of this paper is organized as follows. Section II describes the data and the sample. Section III contains an analytical description of the five attributes of risk: (1) risk-adjusted, "fair-value" deposit insurance premium; (2) term structure-adjusted risk premium; (3) yield-to-maturity; (4) and (5) Moody's and Standard & Poor's bond ratings. Section IV combines the five attributes in an overall empirical assessment of risk for the sample banks, and Section V concludes with a brief summary of our empirical findings.

II. Data and Sample

The study was restricted to a sample of 22 banks on the basis of availability of data. As described in Ronn and Verma (1986), deposit insurance premium was estimated as at the end of 1983 using a sample of 43 banks for which both a daily time series of equity prices, and total deposit liabilities were available on CRSP and COMPUSTAT tapes respectively. The sample was further narrowed down to 22 by the availability of November 30, 1983 market prices of bonds issued by the banks.\textsuperscript{4}

\textsuperscript{3} We also compute a measure of concordance indicating the strength of agreement among the rankings provided by these five attributes. See footnote 19.

\textsuperscript{4} The choice of November 30, 1983 was determined by the availability of an estimate of term structure of spot interest rates on that date as estimated by Ronn (1986). A description of the estimation procedure and the data set is provided in Appendix A. Moreover, in the empirical analysis, we also excluded all bonds with sinking fund provisions, though the methodology proposed herein can readily be adapted to accommodate them.
III. Specification of Risk Attributes

III.1 Risk-Adjusted “Fair-Value” Deposit Insurance Premium

Merton (1977) pioneered the application of option pricing to deposit insurance. Marcus and Shaked (1984) used his model to estimate deposit insurance premiums, and found that such risk-adjusted premiums would be far below the FDIC’s current flat premium of a twelfth of one percent. Ronn and Verma (1986) modified Merton’s model to take into explicit consideration market perceptions of implementation of closure rule, and also improved Marcus and Shaked methodology in such other details as, inter alia, inferring and using market values of assets rather than book values.\(^5\) Chiefly with a view to ensuring that this article is self-contained and complete, we recapitulate the model used in Ronn and Verma (1986). Let

\[ V \] the unobserved post-insurance value of the bank’s assets.

\[ B \] face value of total debt liabilities.\(^6\)

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\(^5\) Other differences between Marcus and Shaked (1984) and Ronn and Verma (1986) include: (1) the use of market riskless interest rates (rather than Regulation Q rates) to proxy for the riskless interest rate; (2) the use of post-insurance values to assess the magnitude of the deposit premium; and (3) recognition that equity, being the recipient of dividends, is fully dividend-protected.

\(^6\) The use of \( B \), total debt liabilities, rather than only the insured debt liabilities may be justified in either of two ways: (1) all debt is of equal seniority at time of liquidation; or (2) existing FDIC Purchase & Assumption policies extend, at least for larger banks, de facto insurance to all liabilities of an insured bank. Ronn and Verma (1986) demonstrate that either assumption gives rise to the deposit premium being a function of all debt liabilities rather than solely dependent on insured deposits.
\( \sigma_V \) - the instantaneous standard deviation of the rate of return on the value of the bank's assets.

\( T \) - time until next audit of the bank's assets.

\( \delta \) - dividend per dollar of value of the assets, paid \( n \) times per period.

The isomorphism between a put and the insurance (arising from identical return patterns), together with standard assumptions of the Black-Scholes option pricing model [see, for example, Merton (1973)], enables us to represent the per dollar deposit insurance premium, \( d \), as

\[
d = N(y + \sigma_V \sqrt{T}) - (1 - \delta)^n (V/B) N(y)
\]  

(1)

where

\[
y = \frac{\log \left[ \frac{B}{V(1 - \delta)^n} \right] - \frac{\sigma_V^2 T}{2}}{\sigma_V \sqrt{T}}
\]

and \( N(\cdot) \) is the cumulative density of a standard normal random variable.

While full details of modeling and methodology are discussed in Ronn and Verma (1986), it is sufficient to note here that the values of the two unknowns in equation (1), i.e. \( V \) and \( \sigma_V \), can be elicited from the market data by simultaneously solving the following two equations.
\[ E = VN(x) - \rho BN(x - \sigma_V \sqrt{T}) \] (2)

where

\[ x \equiv \frac{\log(V/\rho B) + \sigma_V^2 T/2}{\sigma_V \sqrt{T}} \]

and

\[ \sigma_V = \frac{\sigma_E E}{VN(x)} \] (3)

where \( E \) stands for the market value of the equity of the bank, and \( \sigma_E \) for the instantaneous standard deviation of its return. \( \rho \) is a policy parameter taking values between 0 and 1 at the discretion of the FDIC, and is assumed to be so perceived by the market.\(^7\) In empirical estimation it was set at 0.97 so that the aggregate average deposit insurance premium for the industry turned out to approximately equal the 1/12 of 1 percent gross deposit premium charged to banks.

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\(^7\) As discussed in Ronn and Verma (1986), the isomorphic relationship between equity as a call, modified to build into it the implementation of the bank closure rule, can be used to invert market prices of equity for \( V \) and \( \sigma_V \). The FDIC is justly concerned about containing the disruptive effect of an individual bank failure to ensure that it never reaches the magnitude of a bank run. These concerns, which are reflected in their preference for purchase and assumption or direct assistance to a depositor payoff, not only have the effect of allowing a bank to operate up to a certain point beyond complete erosion of net worth, but, more importantly, are perceived by the market to have such an effect. In order to be able to use market data on equity prices, we therefore need to define this point, and we do so in units of total debt. The closure rule is therefore modeled as follows: the FDIC liquidates a bank if \( V_T < \rho B \) where \( V_T \) is the terminal value of assets at time \( T \) and \( \rho \leq 1 \) is the policy parameter.
III.2 Composite Measures of Default: Risk Premium and Yield to Maturity

III.2.A Derivation of a Term Structure-Adjusted Risk Premium

While default risk premium, denoted $\varphi$, is normally defined as the yield differential between a risky and a default-free security, we present below a more precise attribute for each bank that properly accounted for a non-flat term structure. For those banks for which we had the prices of more than one bond, $\varphi$ was calculated as the solution to

$$
\sum_j N_j B_j = \sum_j \sum_i \frac{N_j CF_j t}{(1 + K_i + \varphi)^i}
$$

(4)

where $B_j$ - market price of the given bank’s $j^{th}$ bond

$N_j$ - amount outstanding for the $j^{th}$ bond

$K_i$ - the term structure of default-free spot interest rates

$CF_j t$ denotes the cashflow of the $j^{th}$ bond at time $t$. For simple bonds, this would be the coupon and the principal on maturity. For callable bonds, an analytical approximation to the valuation of callability was employed. This methodology, described in the Appendix below, has the intuitive interpretation of minimizing the “cost of debt” on the particular bond; this optimizing behavior on the part of firms is fully anticipated by bondholders and reflected in the bond’s market value.9

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8 Completeness of notation would require subscripting all variables (excepting $K_i$) by a bank-specific subscript. We have parsimoniously avoided such notation whenever the formulation is unambiguous.
A similar composite figure for the yield to maturity for each bank can be obtained from equation (4) in a straightforward manner by setting $K_t = 0$, all $t$.

A number of observations concerning $\psi$ are appropriate here. First, estimates of $\psi$ are of obvious interest for the purpose of investment, since, as observed earlier, $\psi$ is the risk premium as perceived from bond investors' point of view. The larger is the value of $\psi$, the greater is the rate of return on the bond in excess of the default-free term structure: it is important to note that since it is the promised rather than the expected cashflows that are being discounted in equation (4), $\psi$ is the promised and not the expected premium over the default-free return. Moreover, under the same set of assumptions that lead to equations (1), (2), and (3) of the previous section, enabling us to estimate $\omega$, we can express $\psi$ in terms of the same primary determinants: the volatility of the assets and the market value leverage ratio. Thus, as shown in Merton (1974),

$$\psi = -\frac{1}{T} \ln \left[ N(-z) + \frac{1}{\omega} N(z - \sigma \sqrt{T}) \right]$$  \hspace{1cm} (5)

where

$$\omega = \frac{Fe^{-K_T T}}{V(1 - \delta)^2},$$

$$z = \frac{\ln \omega + \sigma^2 T/2}{\sigma \sqrt{T}}.$$

9 Essentially, the procedure estimates, for each bond, eq. (4) to maturity and to the first call date and -- taking cognizance of the bond's maturity and callability dates -- selects the minimal resulting $\psi$ as indicative of call/no-call. The minimal $\psi$ would be analogous to minimizing the "cost of debt" on the particular bond.
and $F$ is the promised value of the debt. Thus, $\phi$ represents the risk premium reflecting the investment opportunity set that the managers face at the time of estimation. This implied volatility extends over the horizon encompassing the longest maturity bond in the sample of the concerned bank.\(^\text{10}\)

### III.2.B Empirical Validation

The study afforded us an opportunity to conduct a more detailed examination of the relationship between $\phi$ and a temporal measure of maturity. Figures 1a-b contain plots of $\phi$ against Macaulay’s duration for the four banks in our sample for which the largest number of publicly-traded bond issues was available.

The two patterns of relationship which are discernible from Figures 1a-b are reminiscent of Silvers’ (1973) discussion of the yield spreads between risky and riskless cash flows. Whereas his analysis considers broad categories of bond ratings,\(^\text{11}\) our analysis focuses on each particular firm; moreover, we do not require the bond rating as an input to our procedure. Nevertheless, we find evidence of similar patterns of risk. Figure 1a contains the plots for the Manufacturers’ Hanover Corp. and Security Pacific Corp., both of which evidence Silvers’ “normal risk adjustment” pattern of increasing risk. Conversely, BankAmerica Corp., in Figure 1b, displays the “crisis-at-maturity” pattern enunciated by Johnson (1967): this view suggests that the

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\(^{10}\) Obviously, bond prices cannot reflect any unanticipated changes in the investment opportunity set nor any attendant future change in the volatility of the banks’ assets that may change the risk premium.

\(^{11}\) Consequently, his analysis may be contaminated by bond ratings errors as well as an averaging bias across different firms with the same bond ratings. Also, the use of maturity rather than duration can seriously bias the analysis, as maturity is an imperfect measure of the bond’s cash flow timing.
Risk Premium, $\Phi$, by Bond Duration

Figure 1b

Risk Premium, $\Phi$ (in %)

Macaulay Duration (in Years)

[Graph showing a trend with markers for BofACORP: 0.452% and FIRST INTST: 0.873%]
difficulty of refinancing and of meeting the balloon payments at maturity would cause short-term maturities to be riskier than long-term ones.

Merton (1974) has provided an explicit theoretical analysis of the response in \( \phi \) to changes in time to maturity.\(^\text{12}\) We reproduce his graphical representation, as corrected by Lee (1981), in Figure 2.\(^\text{13}\) The correspondence in shape between our empirical graphs and his theoretical one for the solvent firm is readily discernible.

Finally, the overall \( \phi \) calculated for each bank may still be considered an "average," cardinal measure of perceived riskiness. The overall \( \phi \) of the Manufacturers' Hanover Corp., .852 percent, is greater than that of the Security Pacific Corp., .707 percent, and is accurately reflective of the higher \( \phi \)'s for the former throughout the duration spectrum. In cases where such strict dominance does not occur, the bank's \( \phi \) is the best bond prices-based estimate of the term structure-adjusted risk premium for the bank as long as inter-bank duration differences are relatively "small."\(^\text{14}\)

Figure 3 presents the plot of the overall composite \( \phi \)'s for each bank against their aggregate respective Macaulay durations. As Merton (1974, p. 460) has noted, "while com-

\(^\text{12}\) Recall that in his idealized setting there is only a simple homogeneous issue of debt, and therefore time until maturity is the same as the duration.

\(^\text{13}\) Merton's variable \( d \) is not to be confused with our risk-adjusted deposit insurance premium. His \( d \) denotes the "quasi" debt-to-firm value ratio where debt is valued at the riskless rate; in terms of the notation of eq. (5), this is given by the variable \( w = F \exp(-K_r T)/[V(1 - \delta)] \).

\(^\text{14}\) Additional reasons why \( \phi \) might vary in intra-bank analysis include: (1) the relationship between the risk premiums and business cycles which has been commented upon in the literature [see, e.g., Van Horne (1984)]; (2) the fact that more frequently traded, and therefore more marketable, issues will in addition have a liquidity premium; (3) finally, possible errors in the estimation of the term structure which may also be a source of variability of \( \phi \) across bonds of the same bank.
FIGURE 2

TIME UNTIL MATURITY

\[ d \geq 1 \]

\[ d < 1 \]
paring the term premiums on bonds of the same maturity does provide a valid comparison of the riskiness of such bonds, one cannot conclude that a higher term premium on bonds of different maturities implies a higher standard deviation [of debt returns].” Thus, Figure 3 demonstrates that the 22 banks in the data sample may be assigned to six broad duration categories. Within each category, $\phi$ may efficiently serve as a measure of the banks’ default risk.

Further, Figures 1a-b reveal that the bell-shaped graph of $\phi$ as a function of duration peaks at a duration of approximately 3.7 years, which in turn implies that $\phi$ is a monotonically declining function of duration for banks with aggregate duration in excess of 3.7 years. To the extent this phenomenon applies to all 22 banks, an inter-category comparison of $\phi$ as a relative measure of default risk may also be appropriate. For example, this criterion implies that Citicorp is riskier than the banks in Categories I-III. Also, the banks found in the northeasterly quadrant relative to banks in Categories III-VI are all pairwise riskier than their southwesterly counterparts.15

III.3 Credit Ratings

In order to arrive at a composite rating for each bank, we had to convert the credit ratings into cardinal numbers, weigh them with amounts outstanding and then calculate ranks on the

15 For example,

1. First City Bancorporation, Chase Manhattan, First Pennsylvania and Marine Midland Banks are all pairwise riskier than Wells Fargo;
2. Chase is riskier than all banks in Category IV; and
3. First Pennsylvania and Marine Midland are riskier than all banks in Category IV and most banks (excepting Chase) in Category V.
basis of the weighted average. The scheme of cardinalization presented in Table 1 was used. Appendix C demonstrates that the rankings induced by this cardinalization scheme are invariant to any positive linear transformation of the cardinalization.
<table>
<thead>
<tr>
<th>Moody's</th>
<th>Standard &amp; Poor's</th>
<th>Cardinalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>AAA</td>
<td>1</td>
</tr>
<tr>
<td>Aa1</td>
<td>AA +</td>
<td>1.66</td>
</tr>
<tr>
<td>Aa2</td>
<td>AA</td>
<td>2</td>
</tr>
<tr>
<td>Aa3</td>
<td>AA -</td>
<td>2.33</td>
</tr>
<tr>
<td>A1</td>
<td>A +</td>
<td>2.66</td>
</tr>
<tr>
<td>A2</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>A -</td>
<td>3.33</td>
</tr>
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<td>Baa1</td>
<td>BBB +</td>
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<td>Baa2</td>
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<td>Baa3</td>
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<td>BB +</td>
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<td>B1</td>
<td>B +</td>
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<tr>
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<td>B</td>
<td>6</td>
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<tr>
<td>B3</td>
<td>B -</td>
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<tr>
<td>Caa</td>
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<td>C</td>
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<td>C</td>
<td>C</td>
<td>9</td>
</tr>
</tbody>
</table>
IV. Empirical Results

Table 2 presents the yields, composite credit ratings, and estimates of the default risk and deposit insurance premium for the 22 banks sorted into the six aggregate duration categories discussed in Section III.2.b and ranked in ascending order of $\phi$.

While $\phi$ and yield-to-maturity are highly correlated, that correlation is not perfect. As at any given point in time, yield is a weighted average of $\phi$ and the spot interest rates, the correlation between the yield and $\phi$ would be higher the lower the variability in interest rates over the term structure. Figure A1 in Appendix A presents a graph of the term structure, and while spot rates rise in the near to medium term,\(^{16}\) the flat long term structure reduces the overall variability, and therefore the presence of long-term bonds in the sample would seem to account for the high correlation between yield and $\phi$.

To the extent that credit ratings and $\phi$ are good measures of default risk, a positive intra-category association attests to an increasing relationship between yields and default risk. Broadly speaking, since investment agencies are hired for a fee at the time of issue, it is conceivable that their ratings are based on information not accessible to the market, and it would be reasonable to think of these two measures as belonging to the same broad category. While all of the remaining three measures are based on the market price data, it should be noted that $d$ is computed from, among other things, daily series of data on equity prices as opposed to the bond price data at a given point in time for the yield and $\phi$. Relatively low association between $d$ and the other attributes may therefore be interpreted as indicative of differences in perception and information in the two markets. Also, while $d$ is computed from

\(^{16}\) A term structure implied by Federal Reserve Board data is provided for comparative and robustness purposes.
Table 2 -- Measures of Default Risk for a Sample of Banks

Date: IVth Quarter, 1983

<table>
<thead>
<tr>
<th>Duration Category</th>
<th>Bank Name</th>
<th>Duration (Years)</th>
<th>Yield-to-Maturity</th>
<th>Bond Ratings S&amp;P</th>
<th>Bond Ratings Moody's (b.p.)</th>
<th>Deposit Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>TEXAS COMM BANCSHARES CITICORP</td>
<td>1.55</td>
<td>0.80%</td>
<td>11.1%</td>
<td>1.00</td>
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<td></td>
<td></td>
<td>1.44</td>
<td>1.77%</td>
<td>12.1%</td>
<td>2.00</td>
<td>1.68</td>
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<td>II</td>
<td>FIRST CHICAGO CORP</td>
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<td>11.2%</td>
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<td></td>
<td>MORGAN J P &amp; CO INC</td>
<td>2.10</td>
<td>0.22%</td>
<td>11.0%</td>
<td>1.00</td>
<td>1.00</td>
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<td></td>
<td>BANK OF BOSTON CORP</td>
<td>3.07</td>
<td>0.44%</td>
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<td>2.00</td>
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<td></td>
<td>SECURITY PAC CORP</td>
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<td>11.9%</td>
<td>1.70</td>
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<td>MANUFACTURERS HANOVER</td>
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<td></td>
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<td>III</td>
<td>WELLS FARGO &amp; CO</td>
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<td>1.47%</td>
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<td>IV</td>
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<td>SOUTHWEST BANCSHARES</td>
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<td>FIRST ATLANTA CORP</td>
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<td>12.7%</td>
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<td>1.49%</td>
<td>13.3%</td>
<td>3.00</td>
<td>2.66</td>
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<tr>
<td></td>
<td>CHASE MANHATTAN CORP</td>
<td>5.80</td>
<td>2.06%</td>
<td>13.9%</td>
<td>2.00</td>
<td>2.00</td>
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<td>VI</td>
<td>MARINE MIDLAND BKS IN</td>
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<td>1.73%</td>
<td>13.5%</td>
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<td>NR</td>
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<tr>
<td></td>
<td>FIRST PA CORP</td>
<td>6.85</td>
<td>1.77%</td>
<td>13.6%</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

NR -- Not Rated
b.p. -- basis points
the perspective of the insurer, \( \varphi \) is the premium as a bond investor would view it, and bond investors may already be taking into consideration effective FDIC protection to all liabilities implied by the bailing-out policies.

Under Merton’s (1974) idealized conditions,

\[
d = 1 - e^{-\varphi \tau T}
\]

and as \( \frac{\partial d}{\partial \varphi \tau} = T e^{-\varphi \tau T} > 0 \), the intra-category rank correlation between \( \varphi \) and \( d \) should be perfect, since a monotonically increasing transformation preserves ranks.\(^{17}\) Quite apart from the two reasons adduced above, the rank-order correlations may be low owing to different empirical approximations implicit in the two estimation procedures. In estimating \( d \), we treated all debt as a single homogeneous issue with a maturity equal to one year, the approximate periodicity of audit by regulators, and Ronn and Verma (1986) discuss the justification for this treatment in detail. In sharp contrast, estimation of \( \varphi \), on one hand, takes into explicit consideration different bond issues with different maturities, and, on the other, is procedurally restricted to only that part of debt for which the market price is available. In addition, whereas \( d \) evaluates the volatility of equity using historical data, \( \varphi \) presumably incorporates the market’s assessment of ex ante volatilities.\(^{18}\)

\(^{17}\) Technically, the relationship should be formulated as \( d_T = 1 - \exp (-\varphi \tau T) \), where \( d_T \) denotes the (non-annualized) \( T \)-period deposit insurance premium. Thus, \( \frac{\partial d_T}{\partial \varphi \tau} = -T \exp(-\varphi \tau T) \) implying a theoretical unitary rank-order correlation between \( d_T \) and \( \varphi \tau \). The discussion in the text analyzes the intra-category association between \( d(\equiv d_1; \text{that is, } T = 1) \) and \( \varphi \tau \). We justify the calculation of a single, non-duration dependent \( d \) by appeal to the results of Ronn and Verma (1986), who demonstrate the empirical robustness of rank-order correlations of \( d_T \) across alternative values of \( T \) ranging from three months to five years.

\(^{18}\) An analysis of Table 2 would point to Fleet Financial Group as a clear outlier. Its deposit insurance premium is among the lowest, while by all other attributes it is ranked much higher. Its low insurance premium is partly explained by a relatively low standard deviation of equity returns (an annualized 10% for the last quarter of 1983).
Finally, it is instructive, if unsurprising, to note that the two unrated issues -- those of Marine Midland Banks and the First Pennsylvania Corp. -- are those of the two institutions displaying high degrees of risk by the four other risk attributes.

Table 3 contains the rank orderings of banks by the three measures appropriate for inter-category comparisons: bond ratings and fair deposit premium \(d\). Table 4 contains the Spearman's rank correlation matrix based on these three attributes.\(^{19}\)

The high rank order correlation between the two bond ratings agencies is expected. More surprising is the substantially greater agreement of \(d\) with Standard & Poor's ratings vis-a-vis those of Moody's.\(^{20}\) This result should be treated with caution, as the analysis of Table 2 reveals substantially similar associations between each of the two bond ratings and the other four risk attributes.

---

19 Kendall's coefficient is an alternative measure of the correlation in rank orderings. Spearman's correlation coefficient is more appropriate for our purposes because its computations takes into consideration the magnitude of differences in ranks, giving proportionately greater weight to inversions of rank which are farther apart. In contrast, Kendall's coefficient gives equal weight to all changes in ranks from one ranking to the other. See, e.g., Kendall (1970) for further details.

20 While the pairwise comparisons afford insight into differences in perception implicit in these risk attributes, an investigation of the general relationship among rankings may also be of interest. Such an investigation would call for a measure of concordance of the attributes taken as a group. Kendall (1970) proposes exactly such a measure as \(W = \frac{12S}{m^2(n^3 - n)}\) where \(m = 3\) is the number of rankings of \(n = 22\) different banks, and \(S\) is the sum of squares of deviations of the sum of the five ranks of each bank from its mean \(mn(n + 1)/2\). Since if all the rankings were identical \(S = m^2(n^3 - n)/12\), \(W\) reaches its highest value at one and is bounded below by zero. In the empirical analysis its value came out to be 0.659, which indicates a broad agreement between private institutions and equity market participants on relative assessment of risk; however, this agreement is considerably less than perfect.
Table 3 -- Rank Ordering of Banks by Bond Ratings and "Fair" Deposit Premium

Date: IVth Quarter, 1983

<table>
<thead>
<tr>
<th>Bank Name</th>
<th>Bond Ratings</th>
<th>Deposit Premium</th>
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<tbody>
<tr>
<td>BANK OF BOSTON CORP</td>
<td>7</td>
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<td>BANKAMERICA CORP</td>
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</tr>
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<tr>
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<td>6</td>
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</tr>
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</tr>
<tr>
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</tr>
<tr>
<td>WELLS FARGO &amp; CO</td>
<td>12</td>
<td>14</td>
</tr>
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</table>

NR -- Not Rated
<table>
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<td>Standard &amp; Poor's Rating</td>
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<td>.407</td>
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<tr>
<td>Moody's Rating</td>
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<td>.195</td>
</tr>
<tr>
<td>d</td>
<td>.407</td>
<td>.195</td>
<td>.1</td>
</tr>
</tbody>
</table>

[^1] The only bonds in the sample pertaining to First Pennsylvania and Marine Midland Banks happened not to be rated by either investment agency. The rank correlation coefficients in these columns are therefore computed after excluding these two banks from the sample.
V. Summary

In conclusion, we performed an analysis of how rankings of a sample of commercial banks calculated on the basis of five different attributes of risk are related to each other. While these attributes were all positively correlated, the correlation was not perfect. Thus, to the extent these indicators are not perfectly correlated and/or contain measurement errors, each indicator is complementary to the others, implying that the overall picture contains more information than any single component.
Appendix A

Estimation of Riskless Term-Structure
of Spot Interest Rates

A.1 The Primal Problem

The underlying motivation for the LP model used in the current analysis is the relative valuation of coupon bonds.\textsuperscript{21}

The following notation is assumed:

\[ p^a_j \quad \text{asked price}\textsuperscript{22} \text{ of bond } j \]
\[ p^b_j \quad \text{bid price of bond } j \]
\[ x^a_j \quad \text{amount bought of bond } j \]
\[ x^b_j \quad \text{amount of bond } j \text{ sold short} \]

\[ J \equiv \{1, \ldots, j, \ldots, N\} \quad \text{- set of riskless bonds} \]

\textsuperscript{21} Note than in arbitrage-free and complete markets, the prices of riskless securities would reflect the present values of their future promised cash flows. Thus, all bonds would be “fairly priced” by a unique term structure of spot interest rates.

\textsuperscript{22} All prices are inclusive of the flat price and the accrued interest.
As noted in Litzenberger and Rolfo (1984a), "the long side of an arbitrage position must be established at ask prices while the short side of the position must be established at bid prices." Thus, the objective of the program is

\[
\text{Max } Z = \sum_{j=1}^{N} X^b_j p^b_j - \sum_{j=1}^{N} X^a_j p^a_j \quad (A1)
\]

Now consider the future cash-flow implications of the portfolio \((X^b_j, X^a_j, j \in J)\). In one period (month), the net cash flow, \(C_1\), will be

\[
C_1 = \sum_{j=1}^{N} X^a_j a_j^1 - \sum_{j=1}^{N} X^b_j a_j^1 \quad (A2)
\]

where

\[
a_j^t \quad \text{coupon and/or principal payment on bond } j \text{ at time } t = 1.
\]

For the portfolio to be riskless,

\[
C_1 \geq 0 \quad (A3)
\]

For \(t \geq 2\),

\[
\]

\[23\] For completeness of notation, all variables \(P^b, P^a, X^b, X^a\) and \(J\) should be indexed by a time subscript so as to specify the current date at which the problem is formulated. The static analysis of the problem permits the suppression of the time subscript without loss of generality. Thus, the current date is taken to be \(t = 0\).
\[ C_t = (1 + \rho)C_{t-1} + \sum_{j=1}^{N} X_j^q a_j^t - \sum_{j=1}^{N} X_j^p a_j^t \]  

(A4)

where

\( \rho \) - minimal (exogenous) riskless reinvestment rate at all future dates \( t, t = 1, ..., T \) \(^{24}\)

and

\( a_j^t \) - coupon or principal payment on bond \( j \) at time \( t \)

In analogy to (A3), the non-negative cash flow constraints require

\[ C_t \geq 0 \quad t = 2, ..., T \]  

(A5)

A number of comments are appropriate here. To begin, consider the intuitive economic interpretation of program (A1) - (A5). Two such explanations may be offered. The first is arbitrage-based and takes a literal view of the problem. Supposing one could take both long (at the ask price) or short (at the bid price) positions in bonds, then the program defines the arbitrage opportunities offered by the marketplace. Viewed alternatively, if the agent's bond portfolio contains all available coupon bonds, then program (A1) - (A5) risklessly maximizes the market value of the bond portfolio without altering the bond portfolio's cash flows.

---

\(^{24}\) The notion of a minimal (possibly time-dependent) reinvestment rate is offered by Hodges and Schaefer (1979), who also allow for (higher) borrowing rates. The current paper excludes borrowing to emphasize the conservative nature of the estimation procedure; the model uses an (annualized) value of \( \rho = 5.5 \) percent.
The second interpretation of (A1) - (A5) may be encapsulated in the term "quasi-arbitrage." Note that the LP program exploits price differentials of an optimally selected stream of cash flows. The program purchases "underpriced" bonds while attempting to sell short "overpriced" bonds. This formulation thus elicits possible tax-clientele effects. Moreover, in the presence of such effects, the program is able to select those bonds which should "rationally be held" by a given tax bracket. Consequently, the program is able to construct an optimally chosen bond portfolio, composed of bonds which, for a given tax bracket, are "underpriced" relative to other bonds.25

Since the bid-ask spread has been explicitly modeled, it is clear that $X_j^a \geq 0$ and $X_j^b \geq 0$ for $j \in J$ are required. Now, (A1) - (A5) admits of two possible solutions. Either all bonds are priced to within the bid-ask spread by the fitted term structure26 (i.e., $Z^* \equiv 0$), or infinite arbitrage profits may be attained ($Z^* \rightarrow \infty$). Clearly, any attempt to exploit price differentials (assuming one could take both long and short positions in bonds) by taking extremely large positions in these bonds would cause price movements: the bonds being bought would appreciate in price; the bonds being sold short would decline in value. Thus, in order to provide a finite solution, the constraints $X_j^a \leq 1$ and $X_j^b \leq 1$ are imposed. These constraints implicitly model the relative liquidity/availability of bonds. They state that the bid-ask spread places all bonds on an equal footing: less frequently traded bonds will have larger spreads; however, once these spreads have been explicitly recognized, the bonds are equally responsive/

---

25 Consider the RHS of the inequality constraints (A3) - (A5): the non-negativity of the $C_i$'s is well in keeping with this objective. The program optimally selects "underpriced" bonds to meet the cash flows of "overpriced" ones.

26 Note that for $Z^*$ to be strictly positive, there must exist at least two bonds whose prices deviate sufficiently from the fitted term structure, such that it is profitable to purchase one bond at the ask price, sell one at the bid, and nevertheless realize a profit.
irrespective to demand price pressures. Consequently, the constraints $X_j^a \leq 1$ and $X_j^b \leq 1$ have the effect of "standardizing" the position taken in each bond to a maximum of one unit.

Thus, with

$$0 \leq X_j^a \leq 1$$

$$j \in J$$

$$0 \leq X_j^b \leq 1$$

the complete problem is now specified as (A1) - (A6).

\[\text{A.2 The Dual Problem}\]

The Dual Problem to (A1) - (A6) is given by

---

27 Note that the bid-ask spread plays a dual role in this model. In the "arbitrage-based" interpretation, its role is obvious: the long position is established at ask prices, the short at bid prices. However, as explained in the Appendix's text, the spread also serves to equilibrate bonds with respect to their liquidity.

28 No real loss of generality is thus incurred. If the constraints $X_j^b \leq b$ and $X_j^b \leq b$, $j \in I$, for some $b > 0$ were inserted instead, the optimal value function would be scaled up by $b$; however, the linearity of the problem would ensure that no other changes would occur. Non-scalar permutations of the constraint are permissible so long as they do not violate the LP program's sensitivity analysis' values.
Min \frac{\sum_{j=1}^{2N} y_j}{(d_t, y_j)}  \tag{A7}

\sum_{i=1}^{T} c_{i}^j d_i + y_j \geq P_{j}^b \quad j = 1, \ldots, N

- \sum_{i=1}^{T} c_{i}^j d_i - y_{N+j} \geq -P_{j}^a \quad j = 1, \ldots, N

- d_t + (1 + \rho)d_{t+1} \geq 0 \quad t = 1, \ldots, T - 1

\ y_j \geq 0 \quad j = 1, \ldots, 2N

The dual vector \((d_1, \ldots, d_T)\) represents the value, to the objective function \(Z^*\), of an additional dollar at time \(t\). Thus, it is the present value of $1 at time \(t\) and may be solved for the term structure of spot interest rates, \(R_t\), given by the relation \(R_t \equiv (1 / d_t)^{1/t} - 1\).

A.3 Empirical Implementation

The above procedure was utilized to generate the term structure of U.S. Government spot interest rates for November 30, 1983 using U.S. Government bond prices obtained from the CRSP Government Bond Tape. Due to the large number of U.S. Government bonds and

\[29 \text{ To be mathematically precise, } d_t = \frac{\delta Z^*}{\partial C_t} \text{ and thus } R_t \equiv (1 / |d_t|)^{1/t} - 1, \text{ where } |\cdot| \text{ denotes the absolute value operator. For parsimony of notation, the equation in the text will be utilized.}\]
computer capacity limitations, that procedure yielded spot interest rates up to a maturity of 12 years. The Federal Reserve's term structure estimate is the theoretical spot rate curve corresponding to the U.S. Treasury yield curve out to a maturity of 30 years. The LP-based estimate of the term structure is then extended beyond year 12 based on a comparison with the Federal Reserve estimate: since the U.S. Treasury (par) yield curve declines two basis points between years 12 and 30, the implied par yield curve for the LP-based estimates was also constrained to decline an identical magnitude over that period. The two term structures of spot interest rates are presented in Figure A1 below.
Appendix B

An Analytical Approximation to the
Valuation of Callable Bonds

B.1 Analysis

In the modern-day financial markets characterized by interest rate uncertainty, callable bonds are more the norm than the exception, particularly in long-term markets. Yet, theoretical literature on the valuation of the callable debt is remarkably inadequate. Pye (1966) demonstrated that given interest rate certainty, callable bonds will always sell below the call price, and developed a model for the value of the option under interest rate uncertainty. Bodie and Taggart (1978) argue that differences in expectations about interest rates and in risk-aversion may not altogether account for increasing popularity of callable debt. In addition, callable debt may also serve to ensure that the benefits of any stochastic shifts in the investment opportunity set of the firm accrue entirely to shareholders.

An analytical valuation of callable bonds can be obtained by writing the fundamental equation describing stochastic dynamics for these securities as a function of the underlying value of the firm’s assets and interest rates. [See, e.g., the valuation of mortgages in Dunn and Spatt (1986), or Bodie and Friedman (1978) for a model based on interest rate uncertainty.] However, empirical estimation of the underlying parameters is a nontrivial, if not an intractable, proposition.
For our limited purposes of estimating risk premiums and yields, we assumed a simplified setting of constant $K$ and $\varphi$.\footnote{The constancy of $K$ and $\varphi$ is maintained here for simplicity of exposition. Empirical implementation will explicitly account for the term structure of riskless interest $K_t$ (i.e., $K$). As for the theoretical dependence of $\varphi$ on $T$ postulated by Merton’s (1974) model, that property is considered in conjunction with the empirical findings.} In this setting, bond investors would be willing to pay only the lower of the two values arrived at by assuming either no call or call at the earliest opportunity.

For example, if callable bond with a coupon of $C$, face value of $F$, and a maturity of $T_1$ can be called at $T_2 < T_1$ for $F + P$ where $P \geq 0$, then the two alternative values are given by

$$V_1 = \frac{C}{K + \varphi} \left[ 1 - \frac{1}{(1 + K + \varphi)^{T_1}} \right] + \frac{F}{(1 + K + \varphi)^{T_1}}$$

(B1)

$$V_2 = \frac{C}{K + \varphi} \left[ 1 - \frac{1}{(1 + K + \varphi)^{T_2}} \right] + \frac{F + P}{(1 + K + \varphi)^{T_2}}$$

(B2)

The argument below demonstrates that $V_2 > V_1$ cannot be an equilibrium price.

To take a numerical example, suppose $V_2 = 103$, $V_1 = 100$, and suppose, by counterexample, that $\$103$ was the market price, so that the bond is priced at early exercise. Then the company should:

- issue zero coupon bonds for $t = 1, \ldots, T_1 - 1$ with a face value of $C$; and zero coupon bonds with a face value of $C + F$ for $T$;
issue the callable bond at $103;

repurchase its zero coupon bonds to equal cash flow (at $T_1$) for $100.

The obvious conclusion therefore is that for a given $\phi$, the bond will always be selling at the lower price.

In the foregoing analysis, we have been considering only a dichotomy of two out of a continuum of possible outcomes -- premature exercise at the earliest possible opportunity, and no exercise until maturity, while in principle the exercise can occur at any time after the bond becomes callable, yielding a continuum. In other words, we have modeled the probability of early exercise as a Bernoulli variable taking a value of one if the current market price exceeds the call price and zero otherwise, while it may well be a continuous random variable over the interval $[0, 1]$ taking values depending on the interest rate, its volatility and other possible determinants discussed earlier. In options terminology, we have valued the call at its premature exercise price. Rational bondholders will deduct the value of the call option from the price based on option-theoretic valuation of early exercise. Our analysis treats the given market price data as if it pertained either to a bond that will not be called or to one that will certainly be called at the earliest possible opportunity; since a truly noncallable bond will always sell for more, our estimates of $\phi$ may be upward biased.

Geometrically, if the value of a callable bond is plotted against the coupon payment, we should theoretically get a smooth concave curve as shown in Figure B1 plotted here for a 15-year 8 percent coupon bond that is immediately callable at a premium of 8.5 percent above par and becomes callable after two years at 4 percent above par.\(^{31}\) The environment is one with $K = 6$ percent and $\phi = 2$ percent. Our empirical procedure approximates the smooth
Callability: Price by Bond Coupon Rate

Figure B1

Immediate Call at $108.5

Priced to Call at $104 at T = 2

Priced to Maturity at T = 15

Environment: K = 6%, phi = 2%

Price (Per $100 Face Value)

6.5% 7.0% 7.5% 8.0% 8.5% 9.0% 9.5% 10.0% 10.5% 11.0% 11.5% 12.0% 12.5%

Model Value

Theoretical Value
theoretical curve by kinked lines, where the kinks occur at the points when the coupon payments are so high that it becomes optimal to recall. Figure B2 contains a similar plot of value against the interest rate with the coupon fixed at 8 percent, and all other parameters as in Figure B1. In either graph the straight line after the kink defines the asymptotical bound, and the bound in the other direction is defined by a parallel straight line because of the curvature of the value function. The error implicit in our empirical approximation at a given point therefore is given by the vertical distance between the kinked line and the curve, and its magnitude depends on the parameters that give the call option its value -- the volatility, and $T_1$ and $T_2$ or times until maturity and callability respectively.

Now, the well-known relationship between the semi-elasticity of price and duration can be adapted as below for our purposes:\textsuperscript{32}

$$\frac{\partial B}{\partial \phi} \cdot \frac{1}{B} = - \frac{DUR}{1 + Y}$$

(B3)

where $DUR$ is the duration and $Y$ is the yield to maturity. Therefore, for a 1 percent error in price on account of failure to take into consideration the value of the call option ($dB/B = .01$) of a bond with a yield to maturity of 10 percent and duration of 11 years,

\textsuperscript{31} These call premia are assumed to be inclusive of whatever transaction costs may be involved in the call process.

\textsuperscript{32} The equation is derived from

$$\frac{\partial B}{\partial (K + \phi)} \cdot \frac{1}{B} = - \frac{DUR}{1 + Y},$$

using

$$\frac{\partial B}{\partial (K + \phi)} = \frac{\partial B}{\partial \phi} \frac{\partial \phi}{\partial (K + \phi)}$$

and

$$\frac{\partial \phi}{\partial (K + \phi)} = 1.$$
Callability: Price by Interest Rate

Figure B2

Notes: 1) Bond Coupon Rate = 8%
2) Risky Discount Rate = Riskless Rate + 2.0%
our $\phi$ will be upward biased by .001. This may at first appear as an error of alarming magnitude. On a deeper level of analysis, we find that the pricing error, $dB/B$, will be largest for long maturity bonds, where changes in interest rate over a longer horizon make exercise between $T_2$ and $T_1$ more probable. For these same bonds, however, the duration will also be high, thus mitigating a high $dB/B$ in eq. (B3)'s expression for $d\phi$, the error in the estimation of $\phi$.

### B.2 Empirical Implementation

In the empirical counterpart of this callability analysis, we take price (or value) as given and attempt to infer the value of $\phi$ from the bond's market price. For example, using the notation of Section B1, if $V=100$, then

$$100 = PV(C, F, T_1, \varphi_1) = PV(C, F + P, T_2, \varphi_2),$$

where $PV(\cdot)$ is the present value operator defined in eqs. (B1) or (B2) and $T_2$ is the first maturity date. The choice between $\varphi_1$ and $\varphi_2$ cannot, however, simply be predicated on $\min(\varphi_1, \varphi_2)$.

As demonstrated in Merton's analysis (pictured in Figure 2 in the text), $\varphi(T)$ has a bell-shaped or declining curve as a function of $T$. Thus, the bond may be priced to $T_1(T_2)$ even if $\varphi_1 > ( <) \varphi_2$. Rather, the analysis requires empirical calculation of $\varphi_1$ and $\varphi_2$ and a judicious selection between these values.
Consequently, for the callable bonds in our sample, the computed values of \( \varphi_1 \) and \( \varphi_2 \) are presented in Table B1. Discount bonds which may be immediately callable are clearly priced to maturity, as "\( \varphi \) to call" is arbitrarily large. Currently callable premium bonds are also priced to maturity, as the issuing institution is not calling in these bonds presumably due to implicit transaction costs or other non-price related effects.\(^{33}\) For the remaining bonds, either the difference in \( \varphi \)'s is sufficiently large, or the difference in duration (between first call and maturity) sufficiently small, such that the utilization of the \( \min(\varphi_1, \varphi_2) \) criterion is appropriate.

\(^{33}\) In this case, however, the value of the option to call is presumably different from its premature exercise value.
<table>
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<th>Bank Name</th>
<th>Maturity Date</th>
<th>S &amp; P Rating</th>
<th>Moody's Rating</th>
<th>Amount Outstanding (in '000s)</th>
<th>First Call Rate (in %)</th>
<th>Price (c)</th>
<th>Yield to First Call Date (in %)</th>
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Notes: (a) "Price" includes accrued coupon
(b) Callable at $101.38
(c) Callable at $104.27
(d) Immedately Callable => phi "infinite"
(e) Maturity Call Price
NR -- Not Rated
Appendix C

Cardinalization of Bond Ratings

Though the cardinalization scheme presented in the text was chosen arbitrarily, it is shown below that ranks across the banks will be invariant to any linear and monotonically increasing transformation of the cardinalization.

Let \( x_i \in \mathbb{R}^m \) and \( x_j \in \mathbb{R}^n \) be vectors of cardinalized ratings of \( m \) bonds of the \( i^{th} \) bank and \( n \) bonds of the \( j^{th} \) bank, and \( a_i \) and \( a_j \) are corresponding vectors of weights, each summing to unity, and let

\[
a_i \cdot x_i \leq a_j \cdot x_j.
\]

If we now choose a new scheme of cardinalization, \( y \), such that \( x_i = ay_i + \beta I^m \), \( x_j = ay_j + \beta I^n \) where \( a, \beta \in \mathbb{R} \), \( a > 0 \), and \( I^m \) and \( I^n \) are vectors of ones, then

\[
a_i \cdot (ay_i + \beta I^m) \leq a_j \cdot (ay_j + \beta I^n)
\]

Since \( \beta a_i \cdot I^m = \beta \cdot a_j I^n = \beta \), we have

\[
a_i \cdot y_i \leq a_j \cdot y_j
\]

Since \( a > 0 \), this implies

\[
a_i \cdot y_i \leq a_j \cdot y_j,
\]
\[ a_i \cdot y_i \leq a_j \cdot y_j , \]

or that the ranks will be preserved.
Bibliography


