FINANCE WORKING PAPER NO. 170

NON-ADDITIVE PREFERENCES
AND THE MARGINAL PROPENSITY
TO CONSUME

BY

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WALTER A. HAAS SCHOOL OF BUSINESS,
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Non-Additive Preferences and the Marginal Propensity to Consume

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December 1985
Revised: April 1987

Finance Working Paper 170
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I. Introduction

Many authors have made the simplifying assumption of a time-additive utility function. Examples include Phelps (1962), Mirrlees (1965), Hakansson (1970) and Merton (1971). The advantage of time additivity follows from its analytical tractability. Time additivity, however, incorporates the implicit assumption of zero complementarity across time. It is thus of interest to see what effects the relaxation of their assumption yields.¹

Some of the earliest work concerning the extension of consumption-investment decisions from certainty to uncertainty was carried out by Phelps (1962) and Mirrlees (1965). Both authors considered isoelastic multi-period time-additive utility functions with stationary investment opportunity sets. Their results were extended by Levhari and Srinivasan (1969) and Mirman (1971); the former incorporated a stationary investment opportunity set to derive the optimal consumption-investment decision while the latter considered a two-period model.

Samuelson (1969) analyzed discrete-time time-additive logarithmic and isoelastic utility functions. His results were extended and generalized in discrete-time by Hakansson (1970)

¹ The author gratefully acknowledges the helpful suggestions of Nils Hakansson, Hayne Leland, Scott Richard, Mark Rubinstein and an anonymous referee, while remaining solely responsible for any errors contained herein.

² Thus, for the purposes of this paper, time-complementarity is definitionally equivalent to non-additivity in intertemporal consumption choices.
and in continuous-time by Merton (1971). Further, Kraus and Litzenberger (1973) and Rubinstein (1976) derived market equilibrium implications of identical economy-wide logarithmic preferences. Their analyses, however, fail to account for any time-complementarity (in that their posited utility functions are time-additive), and that is one of the objectives of this paper.

Leland (1968) considered the effect on saving of the uncertainty of future income, the "precautionary" demand for saving, in the context of a two-period model with arbitrary preferences. In contrast, the current paper investigates the effect of larger current certain wealth on the marginal propensity to consume. Leland found that risk aversion per se was not a sufficient condition for greater savings in the presence of increased uncertainty. Rather, additional assumptions were required, such as time-additive preferences displaying decreasing absolute risk aversion. The current paper demonstrates that, under non-additive preferences, risk aversion is not a sufficient condition for a positive marginal propensity to consume.

Richard (1975) defined multivariate risk aversion over two or more attributes:

Let \( u(x, y) \) be a decision maker's utility function for two attributes [where \( u_x > 0, u_y > 0 \)]. If for any \( x_0 < x_1 \) and any \( y_0 < y_1 \), the decision maker prefers lottery two which gives an even chance for \( (x_0, y_1) \) or \( (x_1, y_0) \) to lottery one which gives an even chance for \( (x_0, y_0) \) or \( (x_1, y_1) \), then the decision maker is considered multivariate risk averse [p. 12].

Richard then demonstrated that a necessary and sufficient condition for (strict) multivariate risk aversion is the negativity of the second cross-partial derivative of the utility function with respect to any two arguments of the utility function. Richard did not, however, analyze the consumptive implications of this form of risk aversion, and that is the objective of this paper.
The main result of this paper is as follows. Consider the marginal propensity to consume of an individual at his/her consumptive-investment optimum. If this individual is sufficiently multivariate risk averse (in the Richard sense), and if the investment opportunity set is sufficiently disadvantageous, then the individual's marginal propensity to consumer (MPC) will be negative. If the individual is multivariate risk preferrer \( \frac{\partial^2 U}{\partial C_t \partial C_s} > 0 \) for \( t \neq s \) or risk neutral (displays time-additive preferences), then the MPC is positive and its positivity is independent of investment opportunities. Thus, a startling result obtains: the intuitively appealing feature of multivariate risk aversion can, in the presence of "low" (real) interest rates, give rise to negative MPC's.

Section II below presents the intuition underlying multivariate risk aversion while Section III derives the result cited above. Concluding thoughts are contained in Section III.

II. Multivariate Risk Aversion and Non-Additive Preferences

This section analyzes the relationship of non-additive preferences to the marginal propensity to consume in the context of complete (Arrow-Debreu) capital markets. One of the appealing features of the current result is that it provides a natural definition and role for a multivariate extension of the well-known coefficients of absolute and relative risk aversion. Thus, define the coefficient of multivariate risk aversion, MVRA, as

\[
\text{MVRA}(s) \equiv - \frac{\frac{\partial^2 U}{\partial C_0 \partial C_s}}{\frac{\partial U}{\partial C_s}} 
\]  

(1)
for a given state of the world \( s \). This definition of MVRA(s) as given in eq. (1) may be justified by noting that \( \text{sign}(\text{MVRA}) = -\text{sign}(\partial^2 U / \partial C_0 \partial C_x) \) since non-satiety assures that \( \partial U / \partial C_x > 0 \). Thus, MVRA > 0 implies multivariate aversion to risk and MVRA = 0 is synonymous with time-additive utilities. Further, the well-known coefficient of absolute risk aversion, \(- (\partial^2 U / \partial C_x^2) / (\partial U / \partial C_x)\), similarly features the (own) second derivative divided by marginal utility, and both coefficients are invariant to increasing linear transformations of the utility function.

Moreover, in analogy to the coefficient of absolute risk aversion, the MVRA as defined in eq. (1) can be shown to prescribe the magnitude of the risk premium an individual would pay to avoid (an infinitesimally small) gamble. Thus, proceeding in a multivariate extension of Pratt’s (1964) seminal analysis, define

\[
U(C_0^*, C_1^*, \ldots, C_n^*) = \max_{(C_0^*, C_1^*, \ldots, C_n^*)} \mathbb{E}[U(C_0, C_1)]
\]  

(2)

subject to a budget constraint, where \( C_0 \) is current consumption and \( C_i \) is state-contingent consumption in state \( i \) (of period 1). Now fix \((C_2^*, \ldots, C_n^*)\), so that \( \nu(C_0^*, C_1^*) = U(C_0^*, \ldots, C_1^*, C_n^*) \), and define the state 1 risk premium \( p \) as the implicit solution to

\[
\nu(C_0^*, C_1^* - p) = \mathbb{E}[\nu(C_0^* + \tilde{z}, C_1^* + \tilde{z})],
\]

(3)

where \( \mathbb{E}(\tilde{z}) = 0 < \text{Var}(\tilde{z}) = \sigma_x^2 \). Relation (3) states that an individual would rather sacrifice \( p \) units of state 1 consumption than entertain an actuarially fair gamble impacting current consumption.
and state 1 consumption. Proceeding as in Pratt (1964) and letting subscripts denote partial derivatives (i.e., $v_0 \equiv \partial v / \partial C_0$, $v_1 \equiv \partial v / \partial C_1$, etc.),

$$v(C_0^*, C_1^* - p) = v(C_0^*, C_1^*) - v_1 p + O(p^2)$$

(4a)

and

$$E[\nu(C_0^* + \bar{Z}, C_1^* + \bar{Z})] =$$

$$= E\left[v(C_0^*, C_1^*) + v_0 \bar{Z} + v_1 \bar{Z} + \frac{1}{2} v_{00} \bar{Z}^2 + \frac{1}{2} v_{11} \bar{Z}^2 + v_{01} \bar{Z}^2 + O(\bar{Z}^3)\right] =$$

$$= v(C_0^*, C_1^*) + \left(\frac{1}{2} v_{00} + \frac{1}{2} v_{11} + v_{01}\right) \sigma_z^2 + o(\sigma_z^2).$$

(4b)

Equating (4a) to (4b) and noting the definition (2) yields

$$p = \left[\frac{1}{2} \left(- \frac{U_{00}}{U_1}\right) + \frac{1}{2} \left(- \frac{U_{11}}{U_1}\right) + \left(- \frac{U_{01}}{U_1}\right)\right] \sigma_z^2$$

(5)

Thus, while the risk premium $p$ depends on the coefficient of absolute risk aversion, $-U_{11}/U_1$, and the term $-U_{00}/U_1$, it clearly follows from eq. (5) that $\partial p / \partial (-U_{01}/U_1) > 0$. Consequently, the MVRA as given in eq. (1) determines the multivariate risk aversion "con-
tribution” to the magnitude of the risk premium $p$. And finally, as indicated in the Introduction, the MVRA as given in eq. (1) dictates the impact of the behavioral property of multivariate risk aversion on the marginal propensity to consume.

The relationship between the behavioral attributes of multivariate risk aversion and complementarities across time induced by non-additive utilities is of intuitive interest. As previously noted, multivariate risk aversion is uniquely identified by the sign of the second cross-partial derivative. The definition of complementarity in consumption preferences lacks a similar uniqueness.

In the current context, the most useful definition of complementarity is provided by Katzner (1970), who notes that one possible definition of complementarity is the “well-known notion of Auspitz and Leiben that goods $i$ and $j$ are complements, independent, or substitutes according as the derivative $u_y(x)$ ” is greater than, equal to, or less than zero (p. 147). This definition is intuitively appealing as the decreasing marginal utility of the good is extended to its substitute via the property $\frac{\partial^2 U}{\partial C_0 \partial C_x} < 0$.

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4 Richard’s (1975) Theorem 3 considers two utility functions $u(x, y)$ and $v(x, y)$ satisfying 1) $u_{xy} \equiv \frac{\partial^2 u}{\partial x \partial y} < 0$, 2) $v_{xy} \equiv \frac{\partial^2 v}{\partial x \partial y} < 0$, 3) $-v_{xx}/v_y \geq -u_{xx}/u_y$, 4) $-v_{xy}/v_y \geq -u_{xy}/u_y$, 5) $-v_{yy}/v_y \geq -u_{yy}/u_y$, and 6) $-v_{xy}/v_y \geq -u_{xy}/u_y$. These conditions imply that $v$ is more risk averse than $u$ in both the univariate and multivariate dimensions. Thus, for $x_0 < x_1$ and $y_0 < y_1$,

$$u(x_1 - p_x, y_0) + u(x_1, y_1 - p_y) = u(x_0, y_0) + u(x_1, y_1)$$

will result in

$$v(x_1 - p_x, y_0) + v(x_0, y_1 - p_y) \geq v(x_0, y_0) + v(x_1, y_1).$$

That is, $v$ would be willing to pay a greater risk premium than $u$ to avoid the all-or-nothing gamble. Thus, while Richard similarly identifies a role for MVRA $\equiv -u_0/i_1$, his different methodology yielded a “global” condition encompassing univariate and multivariate risk aversion, whereas the current derivation yields the multivariate-specific comparative result $\frac{\partial p}{\partial (-U_0/U_1)} > 0$.  

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6
The analytical focus of the above definitions imply the equivalence of multivariate risk aversion and substitutability. An intuitive interpretation may also be offered. As noted by Richard, a multivariate risk averter prefers a lottery between (High, Low) and (Low, High) to a lottery between (High, High) and (Low, Low). Thus, a multivariate risk averter will attempt to smooth his/her intertemporal consumption flows. An individual exhibiting the attribute of substitutability will also prefer the lottery (High, Low), (Low, High) to (High, High), (Low, Low). In other words, the substitutability of consumption in periods 0 and 1 implies that the (High, Low), (Low, High) lottery assures that individual a higher level of expected utility. Thus, substitutability through time induces the individual to maximize his/her expected utility by minimizing the intertemporal variation of consumption flows; conversely, utility maximization under complementarity causes the individual to maximize the variability of intertemporal consumption choices.

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5 Note that the above definition of complementarity is not invariant to non-linear positive transformations of the utility function. Nor is this definition unique. See Katzner (1970), pp. 145-150, for a critical discussion of this and alternate definitions of complementarity.

6 Of course, the proof of the consumption choices exhibited under substitutability -- under the mathematically equivalent property of multivariate risk aversion -- is provided by Richard's (1975) Theorem 1, pp. 14-15.
III. MPC Under Arbitrary Preferences

With these preliminaries, the results are stated in Theorem 1:

Theorem 1:

Let $U$ be an arbitrary strictly concave utility function defined over present consumption $C_0$ and state-contingent consumption $C_r$, and exhibiting non-satiation w.r.t. both arguments. Assume that $U$ obeys the von Neumann-Morgenstern axioms of rational choice under uncertainty, including the strong independence axiom.

Now, consider the state-specific ratio of the coefficient of multivariate risk aversion to the coefficient of absolute risk aversion [i.e., $\frac{-U_{0i}/U_{i0}}{-U_{0i}/U_{i0}}$]. Then the marginal propensity to consume (MPC) for an individual exhibiting multivariate risk aversion will be negative if the weighted sum of those state-specific ratios, weighted by the (real) Arrow-Debreu state-contingent prices, exceeds unity. Further, the positivity of MPC for time-additive preferences and for multivariate risk preferrers is independent of the investment opportunity set.

A technical explanation of the theorem’s import is warranted here. We consider an individual at his/her consumptive-investment optimum: i.e., the first-order conditions for an optimal allocation are satisfied. Now, we wish to consider the $\text{MPC} = \frac{\partial C_0}{\partial W_0}$ of a given individual under alternative behavioral assumptions. These behavioral postulates pertain to

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7 Strictly speaking, the theorem is stated in a two-period (now and then) model. However, note that $C_r$ may be redefined as state-contingent wealth. In permitting the investor to choose state-contingent wealth, the model adopts multi-period characteristics.
the second cross-partial derivatives $U_{0i} = \frac{\partial^2 U}{\partial C_0 \partial C_i}$ for a given state of the world $i$. If the reference individual is multivariate risk neutral, $U_{0i} = 0$ for all $i$. If the individual is multivariate risk averse (preferrer), then $U_{0i} < (>) 0$ for all $i$.  

**Proof:**

We first establish the formal framework within which the proof is worked out.

For the given utility function $U$, the consumer's constrained utility maximization problem is

$$
\max_{C_0, \{C_i\}} E[U(C_0, \widetilde{C}_i)] = \sum_{i=1}^{n} \pi_i U(C_0, C_i) \tag{6}
$$

$$
s.t. \quad P_0 C_0 + \sum_{i=1}^{n} \phi_i P_i C_i = W_0
$$

for $n$ possible states of nature, and where

$\pi_i$ - probability of state $i$ materializing at time $1$

$P_0$ - current price of one unit of the consumption good in terms of the numeraire ("money")

---

8 Thus, throughout this paper it is assumed that sign $(U_{0i}) = \text{sign} (U_{0j})$ for all $i, j > 0$. 

9
\( \varphi_i \) - current price of an Arrow-Debreu security paying $1 if state \( i \) materializes and 0 otherwise

\( P_i \) - price of the consumption good in state \( i \)

Forming a Lagrangean in the usual fashion, the resulting first-order conditions for optimum are obtained:

\[
L = \sum_{i=1}^{n} \pi_i U(C_0, C_i) + \lambda(W_0 - P_0 C_0 - \sum_{i=1}^{n} \varphi_i P_i C_i) \rightarrow \\

\sum_{i=1}^{n} \pi_i \frac{\partial U(C_0, C_i)}{\partial C_0} - \lambda P_0 = 0 \\

\pi_i \frac{\partial U(C_0, C_i)}{\partial C_i} - \lambda \varphi_i P_i = 0 \\

W_0 - P_0 C_0 - \sum_{i=1}^{n} \varphi_i P_i C_i = 0
\]

where \( \lambda \) is a Lagrangean multiplier.

Now, define

---

9 \( P_0 \) and \( P_i \), \( i = 1, \ldots, N \), may be set identically equal to unity without loss of generality. However, the existence of such price indices in a multi-commodity world is by no means a trivial problem. Samuelson and Swamy (1974) have shown that a wealth-invariant unique price index obtains only under the restrictive assumption of a homothetic utility function. Namely, if \( q_j^i \) is the quantity of commodity \( j \) consumed in state \( i \), we must have \( C_i = f(\prod_j q_j^i) \) where \( a_j \), \( f(\cdot) > 0 \) and \( \sum_j a_j = 1 \).
\[ V(C_0, C_1, \ldots, C_n) = \sum_i \pi_i U(C_0, C_i) \]

\[ p_0 = P_0 \]

\[ p_i = \varphi_i P_i \quad \text{for } i = 1, \ldots, n \]

Under this formulation, the problem reduces to the consumer comparative statics analyzed in graduate microeconomics textbooks, e.g., Katzner (1970), Henderson and Quandt (1971) and Samuelson (1976). Thus, use the following notation:

\[ \frac{\partial^2 V}{\partial C_0^2} = \sum_i \pi_i \frac{\partial^2 U}{\partial C_0^2} = V_{00} \]

\[ \frac{\partial^2 V}{\partial C_i^2} = \pi_i \frac{\partial^2 U}{\partial C_i^2} = \pi_i U_{ii} \quad \text{for all } i \]

\[ \frac{\partial^2 V}{\partial C_0 \partial C_i} = \pi_i \frac{\partial^2 U}{\partial C_0 \partial C_i} = \pi_i U_{0i} \quad \text{for all } i \]

Now, from standard micro analysis,

\[ \frac{\partial C_0}{\partial W_0} = \frac{|M_{12}|}{|M|} \quad \text{(7)} \]

---

10 Note that the strong independence axiom has implied \( \frac{\partial^2 V}{\partial C_i \partial C_j} = 0 \) for all \( i \neq j, i, j > 0 \). \( \frac{\partial V}{\partial C_i} = \pi_i (\frac{\partial U}{\partial C_i}) \) implies \( \frac{\partial^2 V}{\partial C_i \partial C_j} = 0 \) for \( i \neq j, i, j > 0 \).
where

\[ |M| = \begin{vmatrix} 0 & P_0 & \phi_1 P_1 & \phi_2 P_2 & \cdots & \phi_n P_n \\ P_0 & V_{00} & \pi_1 U_{01} & \pi_2 U_{02} & \cdots & \pi_n U_{0n} \\ \phi_1 P_1 & \pi_1 U_{01} & \pi_1 U_{11} & 0 & \cdots & 0 \\ \phi_2 P_2 & \pi_2 U_{02} & 0 & \pi_2 U_{22} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\ \phi_n P_n & \pi_n U_{0n} & 0 & 0 & \cdots & \pi_n U_{nn} \end{vmatrix} \]  

(8)

and \(|M_{12}|\) is the cofactor of the (1,2) element of \(M\).

A solution to the RHS of (7) may be obtained from Bellman (1970). We may solve for \(|M|\) and \(|M_{12}|\) using the following linear algebra theorem:

\[ \begin{vmatrix} A & B \\ C & D \end{vmatrix} = |D| \begin{vmatrix} A - BD^{-1} C \end{vmatrix} \]  

(9)

where

\[ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \]

\(A\) and \(D\) are square matrices and \(|D| \neq 0\).
Set

\[ D = \begin{pmatrix} \pi_1 U_{11} & 0 \\ \pi_2 U_{22} \\ 0 \end{pmatrix} \begin{pmatrix} \pi_n U_{nn} \end{pmatrix} \]

\[ A = \begin{pmatrix} 0 & P_0 \\ P_0 & V_{00} \end{pmatrix} \]

\[ B = \begin{pmatrix} \phi_1 P_1 & \hdots & \phi_n P_n \\ \pi_1 U_{01} & \hdots & \pi_n U_{0n} \end{pmatrix} \]

\[ C = B' \]

Then

\[ |M| = \begin{vmatrix} \pi_1 U_{11} & 0 \\ \pi_2 U_{22} \\ 0 \end{vmatrix} \begin{vmatrix} \pi_n U_{nn} \end{vmatrix} \begin{vmatrix} 0 & P_0 \\ P_0 & V_{00} \end{vmatrix} - \]

\[ \begin{pmatrix} \phi_1 P_1 & \hdots & \phi_n P_n \\ \pi_1 U_{01} & \hdots & \pi_n U_{0n} \end{pmatrix} \begin{pmatrix} 1 \\ \pi_1 U_{11} \\ 0 \\ \pi_n U_{nn} \end{pmatrix} \begin{pmatrix} \phi_1 P_1 & \pi_1 U_{01} \\ \pi_1 U_{01} & \hdots & \pi_n U_{0n} \end{pmatrix} \]
We have $|D| = \prod_{i=1}^{n} \pi_i U_{ii}$. Performing the multiplication and addition $A = BD^{-1}C$, we obtain

$$|M| = - \left( \prod_{i=1}^{n} \pi_i U_{ii} \right) \left| - \sum_i \frac{\phi_i^2 p_i^2}{\pi_i U_{ii}} \begin{array}{c} P_0 - \sum_i \frac{\phi_i p_i}{U_{ii}} \frac{U_{0i}}{U_{ii}} \end{array} \right| =$$

$$- \left( \prod_{i=1}^{n} \pi_i U_{ii} \right) \left[ \left( \sum_i \frac{\phi_i^2 p_i^2}{\pi_i U_{ii}} \right) \left( V_{00} - \sum_i \frac{\pi_i U_{0i}^2}{U_{ii}} \right) + \left( P_0 - \sum_i \frac{\phi_i p_i U_{0i}}{U_{ii}} \right)^2 \right]$$

Note here that by the strict concavity of $U$, \(^{11}\) $V_{00} - \sum_i \pi_i U_{0i}^2 / U_{ii} < 0$ and $U_{ii} < 0$. Thus, sign $|M| = \text{sign} (-1)^{n+1}$ as expected.

The calculation of $M_{12}$ proceeds along similar lines. We have

---

\(^{11}\) The strict concavity of $U(\cdot, \cdot)$ implies

$$\frac{\partial^2 U}{\partial C_0 \partial C_i} (C_0, C_i) - \frac{\partial^2 U}{\partial C_0^2} (C_0, C_i) \frac{\partial^2 U}{\partial C_i^2} (C_0, C_i) < 0 \quad \text{for all } i.$$

Thus, $U_{0i} - U_{00} U_{ii} < 0 \rightarrow U_{0i} / U_{ii} - U_{00} > 0 \rightarrow U_{00} - U_{0i} / U_{ii} < 0$. Multiplying by $\pi_i$ and summing across all $i$ implies

$$\sum_i \pi_i U_{00} - \sum_i \pi_i U_{0i} / U_{ii} < 0$$

or

$$V_{00} - \sum_i \pi_i U_{0i} / U_{ii} < 0.$$
$$|M_{12}| = - \begin{pmatrix}
P_0 & \pi_1 U_{01} & \pi_2 U_{02} & \ldots & \pi_n U_{0n} \\
\varphi_1 P_1 & \pi_1 U_{11} & 0 & \ldots & 0 \\
. & 0 & \pi_2 U_{22} & \ldots & . \\
. & . & . & \ldots & . \\
\varphi_n P_n & 0 & 0 & \ldots & \pi_n U_{nn}
\end{pmatrix}$$

Thus, set

$$D_1 = D$$

$$A_1 = P_0$$

$$B_1 = (\pi_1 U_{01}, \ldots, \pi_n U_{0n})$$

$$C_1 = \begin{pmatrix}
\varphi_1 P_1 \\
. \\
\varphi_n P_n
\end{pmatrix}$$

Now, proceeding precisely as in the calculation of $|M|$, we have

$$|M_{12}| = - \left( \prod_{i=1}^{n} \pi_i U_{ii} \right) \left( P_0 - \sum_{i=1}^{n} \frac{\varphi_i P_i U_{0i}}{U_{ii}} \right)$$

Finally, eq. (10) considers the marginal propensity to consumer from (nominal) wealth, $\partial C_0 / \partial W_0$. \cite{12}
\[
\frac{\partial C_0}{\partial W_0} = \frac{P_0 - \sum_i \frac{\varphi_i P_i U_{0i}}{U_{ii}}}{\left(\sum_i \frac{\varphi_i^2 P_i^2}{\pi_i U_{ii}}\right) \left(V_{00} - \sum_i \pi_i \frac{U_{0i}^2}{U_{ii}}\right) + \left(P_0 - \sum_i \frac{\varphi_i P_i U_{0i}}{U_{ii}}\right)^2}
\]

As the denominator of the RHS of (6) is positive,\(^{13}\)

\[
\text{sign} \left(\frac{\partial C_0}{\partial W_0}\right) = \text{sign} \left(1 - \sum_i \frac{\varphi_i P_i}{P_0} \frac{U_{0i}}{U_{ii}}\right)
\]

\[
= \text{sign} \left[1 - \sum_i \frac{\varphi_i P_i}{P_0} \left(-\frac{U_{0i}}{U_i}\right) / \left(-\frac{U_{ii}}{U_i}\right)\right]
\]

\[
= \text{sign} \left(1 - \sum_i \frac{\varphi_i P_i}{P_0} \frac{\text{MVRA}_i}{\text{ARA}_i}\right)
\]

Q.E.D.

\(^{12}\) The MPC from real wealth is given by \(\delta C_0 / \delta (W_0 / P_0)\). For constant \(P_0\),

\[
\frac{\partial C_0}{\partial (W_0 / P_0)} = P_0 \frac{\partial C_0}{\partial W_0} = \frac{\partial (P_0 C_0)}{\partial W_0},
\]

where \(\partial (P_0 C_0) / \partial W_0\) is the marginal consumption expenditure of nominal wealth. Naturally, since \(P_0 > 0\), sign
\[\left[\frac{\partial C_0}{\partial (W_0 / P_0)}\right] = \text{sign} \left(\frac{\partial C_0}{\partial W_0}\right).
\]

\(^{13}\) Recall from footnote 11 that

\[
V_{00} - \sum_i \pi_i \frac{U_{0i}^2}{U_{ii}} = \sum_i \pi_i \frac{\partial^2 U}{\partial C_0^2} (C_0, C_i) - \sum_i \pi_i \frac{U_{0i}^2}{U_{ii}} = \sum_i \pi_i \left[\frac{\partial^2 U}{\partial C_0^2} (C_0, C_i) - \frac{U_{0i}^2}{U_{ii}}\right] < 0,
\]
Thus, for a given investment opportunity set -- i.e., prices of (real) state-contingent claims \( \{\varphi_t, P_t\} \) -- the marginal propensity to consume will be negative only if the individual is "sufficiently" multivariate risk averse.\(^{14}\)

IV. Summary

Thus, differential attitudes towards multivariate risk aversion yield different results with regard to the marginal propensity to consume. Further analysis is required to investigate the results with regard to the optimal portfolio policy.\(^{15}\) These additional analyses would be directed at eliciting, for both additive and non-additive preferences, the impact of behavioral attributes on the (marginal) investment in the real/nominal riskless assets. A judicious use of first-order conditions, coupled with the well-known comparative statics techniques utilized in this paper, might yield interesting insights on these portfolio decision rules.

The general results obtained in this paper dealt explicitly with changes in consumption induces by marginal changes in nominal wealth. The transition from marginal analysis to infra-marginal analysis is no simple matter. For one, we know from Pye's (1972) analysis of the time-multiplicative power utility function that Theorem 1 does not simply generalize; i.e., it is not possible to suppress the word "marginal" in that theorem. For another, any

\[\text{where the negativity of } [\ast] \text{ above follows from the concavity of } U(\ast), \text{ which implies } U_{\theta 0} - U_{\theta 1} / U_{\theta 2} < 0 \text{ for all } t > 0.\]

\(^{14}\) Note that in a world of certainty, \( \pi_1 = 1, \pi_2 = 0 \) for all \( q \neq x \), and sign \( \partial C_0 / \partial W_0 \) reduces to sign \( [1 - (\text{MVR} / \text{ARA}) / (1 + R)] \), where \( 1 / (1 + R) = \varphi_2 P_2 / P_0 \) and \( R \) is the interest rate on a real riskless bond.

\(^{15}\) For example, this might be achieved by analyzing \( \partial C_t / \partial W_0 \) or \( (\partial C_t / \partial W_0) / [1 - P_0 (\partial C_0 / \partial W_0)]. \)
infra-marginal analysis based on an aggregation (taking integrals)\textsuperscript{16} of the marginal results expounded above would necessarily have to account for the fact that, under differential attitudes toward time-complimentarity, $\delta C_0 / \delta W_0$ is evaluated at different levels of $C_0$ and $(C_r)$. It would thus require further assumptions concerning the third-order derivatives of the utility function. Such assumptions follow from Arrow's conjectures regarding decreasing absolute risk aversion and increasing relative risk aversion and these postulates may yet yield additional results.

\textsuperscript{16} That is, $C_0 = \int_0^W (C(q)/W(q))dq$. 
Bibliography


