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THE PRICING OF BANK LOANS WITH CONTINGENT ASSETS AND LIABILITIES

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The Pricing of Bank Loans with Contingent Assets and Liabilities

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ABSTRACT

In recent years there has occurred a transformation of bank services. While customers traditionally controlled quantities for many deposits, loan quantities were a decision variable for the bank. Increasingly, however, customers are controlling loan quantities, e.g., under credit lines and commitments. Therefore, the bank must price loan contracts bearing in mind the stochastic characteristics of credit utilization and deposit supply. In particular the bank must bear the risk that losses could occur if utilization is high at the same time that deposit supply is low, forcing the bank to rely on more expensive sources of funds. Pricing decisions are affected by reserve requirements, the elasticities of credit line demand and deposit supply, and the stochastic characteristics of utilization and deposits.
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I. Introduction

A major source of uncertainty for banks and other financial institutions is that associated with deposits and withdrawals. When deposit supply is low (or withdrawals are large) the bank may be forced to pay considerably higher interest rates to service customer creditor utilization, thus reducing solvency. A good example is the "end of year effect," when credit line customers tend to draw down funds in unusually large amounts for accounting and liquidity considerations, while short term interest rates (such as Fed Funds) rise to unusually high levels. In addition, when interest ceilings are in effect shifts in deposits can cause large reductions in the amount of credit extended by the institutions affected. These factors will be reflected in credit pricing.

Since the work of Orr and Mellon (1961) researchers have noted the impact of deposit withdrawals on optimal bank reserves. More recently Stigum (1976), Sealy (1980) and others have investigated the effect of stochastic changes in deposits on the loan market. However, relatively little research has been devoted to the impact of stochastic loan changes on loan pricing decisions.

In recent years a growing body of research has addressed the questions of bank behavior, particularly in lending and portfolio decisions, when the bank itself is unable to exercise control over asset quantities. This problem could arise due to contingent assets, where asset quantities are subject to customer discretion. Until now the role
of contingent assets and of contingent liabilities have generally been modeled separately. This is unfortunate because the bank clearly deals with both problems simultaneously. Moreover the importance of both contingent assets and liabilities has grown in recent years. Today the bulk of commercial and industrial loans is made under commitments, and so credit utilization is subject to customer discretion. The share of highly liquid deposits among bank deposits has grown, partly due to removal of interest regulation. Today it is the customer who often controls quantity decisions regarding both bank assets and bank liabilities.

In this paper we develop a model of the bank loan market and portfolio behavior where both assets and liabilities are contingent, but interrelated. Banks must take into account the stochastic behavior of both assets and liabilities when pricing bank loans and determining their commitment "exposure." Because of the nature of loan contracts, the timing of pricing decisions is different for assets and liabilities. Banks can alter deposit interest at any time to attract deposits, but loan pricing is determined in advance when credit facilities are established.

In the next section we present a simple model of a bank whose assets and liabilities are "contingent," subject to customer discretion. Following that, we analyze the bank's pricing decisions and portfolio design. We show how various parameters affect loan pricing, including the stochastic characteristics of loans and deposits. We end the paper with a section of conclusions.
II. The Setup

We focus our analysis on a bank whose managers are risk averse. The bank is assumed to maximize the expected utility of wealth. The environment within which it operates is characterized by the following assumptions:

1. All bank assets and liabilities are "contingent" in the sense that their quantities are at customer discretion. All liabilities are demand deposits paying the same interest rate. No other bank liabilities exist. All loans are made under loan commitment contracts or credit lines where loan utilization is left to customer discretion. All borrowers are homogeneous. All credit lines are for one period. The bank holds no other assets.

2. The bank is required by law to hold reserve requirements in zero-interest accounts at the central bank in the proportion $q$ of deposits. Banks never hold excess reserves nor insufficient reserves. They hold exactly an amount $A = qD$ where $D$ is the deposit level. There are no minimum capital requirements. The bank is restricted so that its total loan volume plus reserves must not exceed its equity plus its deposits. The equity is assumed to remain constant throughout the model.

3. All banks are imperfectly competitive in the sense that they face a "traditional" downward sloping demand curve for credit facilities. In addition, an upward sloping supply curve for deposits is assumed. This is in line with the approach initiated by Klein (1970), and developed by Sealey (1980), Flannery (1982) and Landskroner and Ruthenberg (1985), among others. The representative bank's portfolio is shown in Figure 1. The bank is contractually obligated to supply total credit under credit line
facilities with total amount \( L \). The actual loan size is equal to the loan commitment exposure, \( L \), multiplied by a random variable \( \psi \), that varies between 0 and 1. \( \psi \) reflects the uncertain level of utilization of the commitments by bank customers. While \( \psi \) is not known, its density function is known by all at time 0.

As noted, the level of reserves is exactly \( A = qD \), and this must be maintained at all times. The bank accepts deposits at time 1, where the supply of deposits by bank customers is

\[
D = D_o \theta^\eta .
\]

(1)

Here \( \eta \) is the deposit supply elasticity and is assumed constant and positive. \( \theta \), the exogeneous deposit change parameter, is a random variable with values ranging between 0 and 1, whose density function is known. Since \( \eta > 0 \), the bank may always raise more funds from depositors by raising the interest rate it pays.

The representative bank has initial equity of \( K \). At time 0 the bank signs contracts with its customers granting them access to credit lines whose total aggregate amount is \( L(R) \). \( R \) is the interest charged on credit utilized under commitment contracts and is the same for all customers. \( L \) is the demand for credit facilities by customers. The interest elasticity of demand is \( \delta = -\frac{\partial L}{\partial R} \frac{R}{L} = -L \frac{R}{L} \), which is assumed to be constant and greater than 1. It follows that \( \frac{\partial^2 L}{\partial R^2} > 0 \). (This is needed below to establish second-order conditions for optimality.)
The bank is restricted so that

\[(2) \quad K + D \geq \psi L + A = \psi L + qD \]

or \[K + (1 - q)D \geq \psi L.\]

Equation (2) says that the bank may not make loans whose quantity exceeds equity plus the fraction of deposits not held as reserves. The bank makes decisions concerning \(L\) and \(R\) in advance, at time 0, but may alter \(r\) at any time (see Figure 2). Since \(r\) and \(D\) increase together, the bank will wait until \(\psi\) and \(\theta\) are revealed, and then will set \(r\) so that either (2) holds as a strict equality or \(r\) is zero, that is

\[(2') \quad rK + (1 - q)Dr = rK + (1 - q)D_0 \theta r^{(\eta + 1)} = \psi Lr.\]

From (2'), therefore, either

\[(3) \quad r = \left[ \frac{\psi L - K}{D_0(1 - q)\theta} \right]^{\frac{1}{\eta}} \quad \text{or} \quad r = 0,\]

whichever is greater.

The bank's decision process may be illustrated in Figure 3. The demand for commitments or credit facilities is a downward function of \(R\). Actual utilization depends on \(\psi\), revealed after time 0. Once utilization is determined by the customers, the bank must
see that deposits are sufficient to maintain the portfolio relation (2). Once \( \theta \) is revealed, deposit supply is an upward function of the deposit interest rate \( r \). Should \( \psi \) be large and \( \theta \) small, \( r \) could even exceed \( R \), meaning the bank would be losing money on its credit operations.

The bank's wealth includes the initial capital plus the present value of profits from lending. That is

\[
W = K + \beta (R \psi L - rD),
\]

where \( \beta \) is the subjective discount factor used by the bank. The objective is to maximize \( E(U(W)) \), where \( U \) is a continuous, differentiable function and \( U' = \frac{\partial U}{\partial W} > 0 \) and \( U'' = \frac{\partial^2 U}{\partial W^2} < 0 \). At time 0, the bank chooses \( L \) (and hence \( R \)) in order to maximize the expected utility of \( W \), subject to (2').

Before examining optimality conditions, we note that (2') may be differentiated to yield

\[
\psi dL = \eta D_0 (1 - q) \theta r^{\eta} dr
\]

or

\[
\frac{dr}{dL} = \frac{\psi}{D_0 (1 - q) \theta \eta} r^{1-\eta} \geq 0 .
\]

That is, an increase in loan commitments is associated with a future increase of the deposit rate, other things equal.
It similarly follows that

\[
\frac{d(Dr)}{dL} = \frac{\eta + 1}{\eta(1 - q)D_o} \psi_r \geq 0.
\]

This says that larger loan commitments will result in larger expenditures on deposits in the future.

III. The Pricing of Bank Loans

The first-order condition for an optimum may now be derived. The bank chooses \( L \) to maximize expected utility,

\[
\max_L E(U(W)),
\]

subject to (2'), (4) and non-negatively constraints on \( L \) and \( R \).

The first partial of \( E(U) \) with respect to \( L \) is

\[
\beta E \left[ U' \left( \psi R(1 - \frac{1}{\delta}) - \frac{\eta + 1}{\eta(1 - q)D_o} \psi_r \right) \right] = 0.
\]

Second-order conditions for a maximum may be shown to hold and the optimum is unique.
Equation (8) may be solved to yield the bank's optimal $L^*$ or the optimal $R^*$. The latter is equal to

$$
R^* = \frac{(\eta + 1)\delta}{\eta(1-q)(\delta - 1)D_0} \frac{E(\psi U')}{E(\psi U')'.}
$$

$R^*$ is the interest rate charged bank customers for credit utilized under credit facilities. $L^* = L(R^*)$ is the size of total credit line "exposure" of the bank. It is noted that $r^*$ is finite and $L^*$ positive only as long as $\delta > 1$.

Equation (9) describes the optimal level of interest charged by the banks on credit lines, given the parameters of the model and stochastic characteristics of contingent assets and liabilities. Using (9) we obtain the following comparative statics results for the direction of changes of the loan rate variable:

1. $\frac{\partial R^*}{\partial q} > 0$ from (9). Any increase in the required reserve ratio will make it more expensive to raise funds from deposits for extending loans. Extending credit facilities is thus more expensive for the bank and it will provide lower amounts of loan commitments at higher interest rates.

2. $\frac{\partial R^*}{\partial \delta} < 0$. Any increase in the elasticity of demand (by bank customers) for credit facilities leads to a higher interest charge and lower commitment exposure. This is similar to the price-output response of a monopolist to an increase in demand elasticity.

3. $\frac{\partial R^*}{\partial \eta} < 0$. Any increase in the elasticity of the deposit supply function implies that $r$ will be lower (from equation (3)) in every state of nature (any $\psi$ and $\theta$). Since
lower $r$ implies that providing credit to customers is cheaper for the bank, it contracts for a larger credit exposure and charges a lower interest rate $R$. This is similar to the price-output response of a monopsonist to increases in supply elasticity.

4. $\frac{\partial R^*}{\partial \beta} = 0$. Loan commitment exposure and interest rates are not affected by the bank's discount factor.

5. $\frac{\partial R^*}{\partial K} < 0$. This follows from the fact that $\frac{\partial r}{\partial K} \leq 0$ in all states of nature in period 1. In effect a higher level of equity enables the bank to fund any given level of commitments more cheaply, with less deposits and so with a lower deposit rate $r$. It follows that the bank contracts for a larger volume of commitments and does so at a lower $R$ when $K$ increases.\(^4\)

6. $\frac{\partial R^*}{\partial D_o} < 0$. An increase in $D_o$ raises the supply of deposits for any $r$ in every state of nature. Therefore the cost of raising funds for supplying credit falls when $D_o$ rises. Banks extend a greater volume of credit lines, charging a lower $R$.

IV. The Effect of Stochastic Characteristics on Loan Pricing

The bank, as we have seen, prices loan commitments while bearing in mind the effects of future takedowns and deposit supply shifts on the future costs of funding the commitments. It is therefore of interest to see how shifts in the distributions of takedowns and deposits will affect loan pricing.

First let us note that from (9), $R^*$ is proportional to
\[ E(r) + \frac{\text{cov}(\psi U', r)}{E(\psi U')} = \frac{E(\psi U')}{E(\psi U')} \]

where \( \text{cov} \) indicates covariance. If \( \psi \) and \( \theta \) are negatively correlated, then when \( \theta \) rises (falls), \( \psi \) will tend to fall (rise), and hence both \( r \) and \( U' \) will tend to fall (rise) at the same time. When \( r \) and \( \psi \) are positively correlated, deposit rates will be high in the same states of nature when credit line takedown is high. This would mean that credit line exposure would be a riskier undertaking for the bank. The more (less) \( r \) and \( \psi \) are correlated, the higher (lower) will be \( \text{cov}(\psi U', r) \), and so the higher (lower) would be \( R \).

We now turn to other aspects of the stochastic behavior of credit takedown and deposit supply. Other distributional changes in stochastic variables may be explored formally by substituting transformations of the random variables into the model. We begin with the stochastic characteristics of deposits.

1. Let us replace the deposit withdrawal parameter \( \theta \) with a new random variable, \( \tilde{\theta} = \theta + aE(\theta) \), where \( \theta \) is the same as previously defined. It is noted that \( E(\tilde{\theta}) = (1 + a)E(\theta) \) and \( \text{VAR}(\tilde{\theta}) = \text{VAR}(\theta) \). By substituting \( \tilde{\theta} \) for \( \theta \) in (9) and differentiating with respect to \( a \), we may examine the effects of variance-preserving increases in the mean of \( \theta \).

It can be shown that the expression \( \frac{E(\psi U')}{E(\psi U')} \) in (9) falls when \( a \) rises. Therefore a variance preserving increase (decrease) in \( E(\theta) \) would lower (raise) \( R \) and raise (lower) \( L \). The intuition here is that an increase in \( E(\theta) \), like an increase in \( D_o \), would mean that the "average" supply of deposit funds would be higher for any \( r \), and so it would
be "cheaper on average" to extend any volume of credit commitments. Therefore this volume would increase and $R$ would fall.

2. Let us now change the definition of $\widetilde{\theta}$ to $\widetilde{\theta} = a\theta + (1 - a)E(\theta)$. It is noted that now $E(\tilde{\theta}) = E(\theta)$ and $\text{VAR}(\tilde{\theta}) = a^2\text{VAR}(\theta)$. By substituting $\tilde{\theta}$ for $\theta$ in (9) and differentiating with respect to $a$, we may examine the effects of mean-preserving increases in the variance of $\theta$.

Now from (9), the partial of $E(\psi U'r)$ with respect to $a$ is equal to

$$
(10) \quad -\frac{1}{n}E\left[\psi U'r\frac{1}{n}\left[\frac{\theta - E(\theta)}{\tilde{\theta}}\right]\right] = -\frac{1}{n}\text{cov}\left[\frac{\psi U'r}{\tilde{\theta}}, \theta\right].
$$

The sign of (10) is ambiguous. This is because $\psi$ and $\theta$ could have high positive or high negative correlation, affecting the covariance in (10). However, it can be shown that a sufficient (but not necessary) condition for (10) to be positive is for $\text{cov}(\psi, \theta)$ to be small or negative.\textsuperscript{5} If that were the case, then any increase (decrease) in the mean-preserving spread of $\theta$ would lead to a rise (fall) in $R^*$ and a decrease (increase) in $L^*$. Any increase in $\text{VAR}(\theta)$ would then make the bank's portfolio more risky. The bank would respond by contracting credit line exposure and would raise $R$.

3. We now turn to consider the effect of changes in the stochastic behavior of takedowns. Let us define $\widetilde{\psi} = \psi + aE(\psi)$, where $\psi$ is the same as it was previously defined. It is noted that $E(\widetilde{\psi}) = (1 + a)E(\psi)$ and $\text{VAR}(\widetilde{\psi}) = \text{VAR}(\psi)$. By substituting
\( \hat{\psi} \) for \( \psi \) in (9) and differentiating with respect to \( a \), we may examine the effects of variance-preserving increases in the mean of \( \psi \).

Now the partial of \( R^* \) with respect to \( a \) is proportional to

\[
E(U' \hat{\psi}) \left[ E(U'r)E(\psi) + E(U'\psi) \frac{dr}{da} \right] - E(U'\psi r)E(\psi)E(U') .
\]

The sign of (11) is ambiguous. The first term is positive and the second is negative. Increasing \( E(\psi) \) raises the revenue from loan extension as well as the deposit interest costs \( rD \). The bank could respond by either raising or lowering \( R \) (or \( L \)). Noting that \( E(U' \hat{\psi}) > E(\psi)E(U') \) and that \( \frac{dr}{da} > r \) if \( \eta < 1 \), a sufficient condition from (11) for \( R^* \) to rise and \( L^* \) to fall when \( E(\psi) \) increases is for \( \eta < 1 \).

4. Finally we will change the definition of \( \hat{\psi} \) to \( \hat{\psi} = a\psi + (1 - a)E(\psi) \). We note that \( E(\hat{\psi}) = E(\psi) \) and \( \text{VAR}(\hat{\psi}) = a^2\text{VAR}(\psi) \). If we substitute \( \hat{\psi} \) for \( \psi \) in equation (9) and differentiate with respect to \( a \), evaluated at \( a = 1 \), we may examine the effects of a mean-preserving increase in the spread of \( \psi \). The sign of \( \frac{\partial R}{\partial a} \) is ambiguous. However, as in the previous case, \( \eta < 1 \) is a sufficient (but not necessary) condition to insure that \( \frac{\partial R}{\partial a} > 0 \). In that case, a mean-preserving increase in the variance of \( \psi \) makes the bank's portfolio more risky. The bank responds by contracting its commitment exposure, and raises \( R \).
Conclusions

When bank assets and liabilities are "contingent," that is, when customers retain control over the quantities of both deposits and credit utilization, the supply of bank loans and bank portfolio design is quite different from what it would be in traditional portfolio selection models. The bank must price credit commitments recognizing that actual utilization and therefore also bank revenues will depend on customer discretion. When utilization increases, banks must mobilize an increased amount of deposits, and so deposit rates would rise. This would be particularly costly if deposit supply fell (θ fell) at the same time that utilization rose, which could occur as a result of an overall increase in market interest rates.

Recognizing this, banks will limit their exposure to reduce deposit costs. At the margin the bank equates ex ante the "average" expected cost of raising additional funds from depositors to the "average" expected revenue from extending credit commitments. Ex post the bank must manipulate the deposit rate to insure sufficient liquidity. Deposit rates could rise above the lending rate if utilization were high and deposit supply were low.

We modeled contingent assets and liabilities and derived "rules" for the pricing and the supply of bank commitments under such conditions. It was shown that the "exposure" or volume of credit commitments was a negative function of the reserve ratio and a positive function of the elasticity of demand for credit lines, the elasticity of supply for deposits, bank equity, and expected deposit supply. Changes in the variance of deposit supply and of loan utilization, and in the average loan utilization
rate were analyzed, and factors determining their impact on portfolio design were explored.

A major problem for the bank would be if -- as seems likely -- loan utilization (ψ) and deposit supply (θ) were negatively correlated. Risk would then be particularly high since whenever overall market rates rose (fell), utilization would increase (decrease) while deposit supply decreased (increased). Bank profits would be quite volatile.

The traditional methods of immunization through duration matching may not apply when assets and liabilities are contingent. The bank would unavoidably bear considerable interest risk. This risk may be controlled somewhat through the bank’s pricing decisions. In a more sophisticated model with several contingent assets and liabilities, risk would be somewhat more manageable through judicious selection of the mix of these.

More generally, the importance of contingent assets and liabilities may provide one explanation for the existence and importance of intermediary institutions. Banks may exist alongside other financial institutions and markets precisely because they offer customers contingent assets and liabilities, where customers freely alter quantities at a minimum of cost. It may be in exchange for quantity flexibility and low transaction costs that the public agrees to bear the costs of intermediation in the first place. By providing contingent assets and liabilities, the bank "manufactures" a technology of low-cost transactions, from which customers meet current credit needs and purchase payments mechanisms.
FOOTNOTES

1. A large literature has arisen analyzing bank lending and portfolio decisions. Much of this literature has focused on price uncertainty regarding the bank’s assets and liabilities. In many papers the bank faces a portfolio problem that differs from standard optimization models due to institutional characteristics, but involves some form of optimization through portfolio choice under price (or interest) uncertainty. For example, see Klein (1971) and Tobin (1982).

2. The most noteworthy example of contingent assets would be loan commitment contracts or revolving credit facilities. Under these, banks commit themselves to providing credit under previously agreed pricing formulas, up to some maximum ceiling. Customers may utilize any amount of funds up to that limit for the duration of the contract. Papers that have analyzed such contracts would include Campbell (1978), James (1981), Thakor, Greenbaum and Hong (1981), Thakor (1982), and Melnik and Plaut (1986).

3. The innovative work of Flannery (1982) provides both empirical evidence and theoretical justification for the view that the retail deposit market is not perfectly competitive. A recent paper by Prisman, Slovin and Sushka (1986) argues that in local loan markets the bank faces a downward sloping demand curve.

4. In our model $R$ is not a decision variable, but an exogenous parameter. The relative cost of capital is not a relevant factor. An interesting extension would be to include capital decisions as an endogenous variable.
5. It is likely to be negative since when overall market interest rates rise, $\psi$ will probably rise and $\theta$ fall, *ceteris paribus*.
Figure 1

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<tr>
<th>Assets</th>
<th>Liabilities</th>
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<td>Loans (Contingent)</td>
<td>$\psi L$</td>
</tr>
</tbody>
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Figure 2
Figure 3
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