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DEBT AND MARKET INCOMPLETENESS

by

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Debt and Market Incompleteness

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Abstract

This paper provides a capital structure equilibrium analysis in an environment characterized by market incompleteness and risky debt. We demonstrate a Miller-type equilibrium at an industry level wherein each firm faces within an industry an indeterminate debt level. However, corporations as a whole act as financial intermediaries to generate cross-sectional capital structure patterns among industries, despite the fact that rents associated with financial intermediation services dissipate in equilibrium for any particular firm. The main result of the paper is a direct consequence of extending the Senbet-Taggart (1984) analysis into risky debt with multiple industries. Moreover, we provide an economic rationale for financial synergy and complex financial attributes, such as option characteristics and prioritization of debt, when we extend the analysis into a richer setting. Also, we find that in this setting firms have an incentive to use their productive decisions to augment financial decisions in enhancing market completeness.
I. Introduction

The key element in the Modigliani-Miller capital structure theorem is an ability on the part of investors to costlessly duplicate any return patterns that might be created by corporate financing decisions. The subsequent literature has focused on market imperfections that may render imperfect that substitution between corporate leverage and personal leverage, most notably the impact of financial distress costs and taxation. More recently, however, Miller (1977) has shown that a limited Modigliani-Miller theorem obtains even under taxation so long as corporations costlessly adjust their supply of debt. In equilibrium, the tax deductibility advantage of debt on corporate account is offset by the taxability disadvantage on personal account. Senbet and Taggart (1984) show that Miller's irrelevance result can be generalized to any kind of capital market imperfection or incompleteness. As long as corporations possess a comparative advantage in dealing with these imperfections, they will have an incentive to act as financial intermediaries. While there is a positive theory of corporate finance at the aggregate level, capital structure is indeterminate at the individual firm level.

Following several previous authors -- e.g., Williams (1938), Durand (1959), Litzenberger and Sosin (1978), Litzenberger (1980) and Senbet and Taggart (1984) -- we emphasize the role of debt to complete the market. Thus, the firm acts as a financial intermediary on behalf of investors. In particular, the current paper extends Senbet and Taggart (ST, 1984) by explicitly introducing risky debt. In further contrast to that earlier work, we employ the more realistic assumption that individuals can lend risklessly by virtue of government securities but are precluded from borrowing on their personal account. ST motivate market incompleteness or imperfections as a result of transaction costs. However, there is no differentiation between
borrowing and lending activities in terms of the level of transaction costs, although these costs are assumed lower for corporations. We depart from ST not only in terms of introduction of risky debt but in the manner in which market incompleteness is introduced. We assume for the most part that riskless lending is available to all agents (corporations as well as individual investors) at a relatively low cost by virtue of the availability of guaranteed government securities. While the assumption of riskless lending is realistic at least in nominal terms, it is unrealistic to assume that individuals can borrow at the riskless rate. However, we will maintain ST's view that firms possess comparative advantage in borrowing relative to investors. Indeed, to dramatize this view and for the sake of simplicity, we shall assume that personal borrowing is prohibitively expensive. Investors are also prohibited from undoing this restriction through short sales. Short-sales restrictions are also realistic in light of the existing financial system which disallows the use of short-sale proceeds, requires margin, and disallows interest on the proceeds residing with brokers. Thus, firms use their capital structure decisions to enhance the scope of the investment opportunity set by providing investors with the type of high risk-high return securities that individuals could not otherwise obtain. In other words, the short-sale restrictions on personal debt imply that levered equity is the avenue by which firms can span the capital market line beyond the risk-return tradeoff provided by unlevered equity.

In the current analysis, we demonstrate a Miller-type equilibrium at the industry level wherein each firm faces an indeterminate debt level. Thus, despite the fact that capital structure has no impact on the value of a particular firm, we can observe capital structure regularities among industries. This is somewhat surprising, because the extant literature on capital structure tends to associate industry effect with significant valuation effect of capital structure. Our industry-based capital structure equilibrium is a direct consequence of extending the ST
analysis into risky debt with multiple industries. Firms perform a financial intermediary function by providing investors with an expanded set of opportunities for trading off risk and return. This particular corporate function could potentially give rise to rents, but firms' attempts to take advantage of these rents will drive these profits out. The absence of financial monopoly power of a particular firm within each industry leads to indeterminate capital structure at the individual firm level but an optimal capital structure at the industry level. We also provide the precise rules for industry-level capital structure.

When we expand the degrees of freedom for corporations in their use of financial decisions in completing the market, we provide a strong economic rationale for financial synergy and complex attributes, such as option characteristics and classification or prioritization of debt. A by-product of this is a clear outcome from our analysis for ST's initial observation about basic identity between corporate finance and financial intermediation so that the existence issue in the area of financial intermediation can be considered in the broader framework of the theory of corporate finance. We also find that firms can use productive decisions to augment financial decisions in enhancing market completeness. Indeed, we find it rather surprising that the corporate role in providing financial intermediation services gives rise to a rich menu of issues in finance, such as industry patterns of capital structure, evolution of complex financial securities, mergers or spin-offs, financial intermediation, and linkage between production and finance.

The paper is organized as follows. Section II presents the basic model and capital structure implications. Section III extends the analysis into a richer setting giving rise to financial synergy and complex financial securities. Section IV concludes the paper along with some empirical implications.
II. Capital Structure with Market Imperfections and Incompleteness

II.1 A Single-Firm Economy

We shall begin with a single-firm economy in order to dramatize the role of capital structure in undoing market imperfections or incompleteness. Subsequent sections will progressively move into a more complex setting with multiple firms. The capital structure implications of a single-firm economy are derived under the following set of assumptions:

(A1) Markets are frictionless and trading takes place continuously, but personal borrowing and short-selling are disallowed.

(A2) The prices of options are such that these contingent claim contracts are neither a dominant nor a dominating security.¹

(A3) There exists a single firm in the entire economy, currently financed with unlevered equity, whose instantaneous return on the market value of assets, \( V \), follows the diffusion process

\[
\frac{dV}{V} = \mu_r dt + \sigma dz ,
\]

¹ As Merton (1973) notes in his alternative derivation of the Black-Scholes option pricing model, "The assumptions of unrestricted borrowing and short-selling can be weakened and still have the results obtained by splitting the created portfolio [containing the common stock, the option and riskless bonds] ... into two portfolios: one containing the common stock and the other containing the warrant plus a long position in bonds. Then [given this paper's Assumption (A2)] the formulas of the current section follow immediately" (p. 162, footnote 41).
where $\mu_t$ may be allowed to vary across time.

Assumptions (A1) - (A3) are sufficient to imply the valuation of European call and put options according to the Black-Scholes (1973)/Merton (1973) option pricing model. Assumption (A1) is critical to our analysis and needs further explanation. Under this assumption no transaction costs are incurred in taking long positions (in equity or debt), but that these costs are prohibitively costly for individuals desiring to take short positions. By the same token and to maintain consistency, personal borrowing is also assumed prohibitively costly. However, the frictionless aspect of the market is satisfied for a subset of agents -- the corporate sector -- which can take long and short positions. Thus, we are modeling the differential comparative advantage between investors and corporations in an extreme form. This is in contrast to Senbet and Taggart (ST, 1984) who differentiate corporations and individual investors on the basis of divergent borrowing and lending rates, where the divergence is assumed lower for the corporate sector. We take a view here that both sets of agents can risklessly lend by virtue of holding government securities, but investors are unable to engage in riskless personal borrowing. Moreover, as an important departure from ST, our framework is capable of handling risky debt as a means of completing the market. As we shall see in later sections,

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2 The literature has uncovered a number of alternative derivations of the Black-Scholes option pricing model:

- Black-Scholes (1973): continuous trading opportunities with riskless borrowing and lending, or risk neutrality.
- Samuelson and Merton (1969): constant relative risk averse (CRRA) preferences, existence of three securities (riskless bond, option and associated stock), zero net supply of options and bonds.

Of these alternative set of postulates, only the set of assumptions in the text provide for the joint consistency of the Black-Scholes option pricing model with the multi-period applicability of Black's (single-period) zero-beta version of the CAPM.
the introduction of risky debt along with multiple industry classes generates interesting capital structure implications of empirical content.

Now, turning to a single-firm economy case, we wish to append the following assumptions so as to provide the model with an equilibrium asset pricing framework.

(A4) \( \mu_t = \mu \) for all \( t \); or

(A4') Investors' preferences for consumption are described by the logarithmic utility function, \( \log (C_t) \).

Assumption (A4) guarantees a stationary investment opportunity or, by (A4'), an indifference/unwillingness to hedge a stochastically changing investment opportunity set. This, together with assumption (A1) provides for the applicability of Black's (1972) "zero-beta" version of the Capital Asset Pricing Model.\(^3\)

Assumptions (A1) - (A4) have established the concurrent validity of the Black-Scholes option pricing framework and a multi-period applicability of the (single-period, mean-variance driven) Black version of the CAPM. Moreover, these assumptions are sufficient to derive initial capital structure implications for the case under consideration.

\(^3\) Merton's (1973) Theorem 1, p. 878, demonstrates the appropriateness of the single-period CAPM under the assumption of a constant investment opportunity set.
It is important to note that the results derived below constitute the conditions for an "instantaneous" optimal capital structure; the optimal capital structure is an instantaneous one as the stochastic process governing asset values is continuously moving through time. Consequently, parsimoniously dropping the time subscript, assume the following notation:

\[ E_m - \text{the expected rate of return on the unlevered equity}^4 \]

\[ V - \text{the market value of the unlevered firm's equity; this value is assumed fixed and unaffected by the capital structure decision}^5 \]

\[ \sigma_m^2 - \text{the variance of the rate of return of unlevered equity} \]

\[ F - \text{the face value of zero-coupon risky debt; initially, } F \equiv 0 \]

\[ E_2 - \text{expected return on the asset whose return is uncorrelated with the market portfolio}^6 \]

---

4 The single firm represents the "market portfolio of all risky assets."

5 Actually, fixing \( V \) is imprecise, given that \( V \) may be affected in the process of the firm's intermediation activity (or in the equilibrating process). In that case, the optimality of the debt decision \( \hat{F} \) can be motivated as a myopic optimization along the expansion path of \( V \). We argue in Section II.2 below that any attempt by firms to enhance value by providing an investor-desired pattern of risk and return would be competed away by other firms' entry into the industry and concomitant supply adjustments.

6 In a single-asset case, \( E_2 \) is not explicitly identifiable. Thus, it is interpreted as the expected rate of return on a portfolio uncorrelated with the market portfolio in a multiple-firm economy whose market portfolio and individual-investor behavior are identical to the (otherwise-equivalent, single-firm) economy examined in the text. Moreover, the single-firm construct used in the text was chosen to focus attention on the capital structure of the corporate sector. At the current stage of analyses, a multiple-firm representation would increase the technical complexity without yielding additional insights; consequently, we defer the consideration of the multi-firm case to Section II.2.
$R_F$ - the continuously-compounded riskless interest rate

$S$ - the market value of levered equity

$B$ - the market value of risky debt, with promised end-of-period payment of $F$

Theorem 1 now presents the initial results of this section:

**Theorem 1:**

Given the risk-return tradeoff implied by the expected return on the lognormally distributed levered asset, let $E^*$ ($\geq E_m$) denote the optimally **maximal** expected rate of return desired by the most risk-tolerant investor. In that event, the optimally **minimal**, or lower-bound, face value of the one-period pure discount risky debt $F^*$ exists, and it can be given as the implicit solution to the non-linear equation (1):

$$\frac{F^*}{V} = \frac{N(d)}{N(d - \sigma_m)} \left( \frac{E^* - E_m}{E^* - E_z} \right)$$  \hfill (1)

where

$$d = \frac{\ln(V/F^*) + (R_F + \frac{1}{2} \sigma_m^2)}{\sigma_m}$$
and \( N(*) \) is the cumulative distribution function of a standard normal random variable.

**Proof:**

For a given value of \( F \), Ito's lemma yields

\[
dS = \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial t} dt + \frac{1}{2} \frac{\partial^2 S}{\partial V^2} (dV)^2 ,
\]

where the latter two terms on the RHS are deterministic.

Thus,

\[
\text{Cov} \left( \frac{dS}{S} , \frac{dV}{V} \right) = \sigma_S \sigma_V dt = \frac{\partial S}{\partial V} \frac{V}{S} \sigma^2_m dt \equiv N(d) \frac{V}{S} \sigma^2_m dt ,
\]

where \( \partial S / \partial V \equiv N(d) \) is implied by the isomorphism between levered equity and a European call option.

Now, by Assumption (A4), Black's (1972) framework implies that

\[
E_S = E_z + (E_m - E_z) \frac{\text{Cov} (\tilde{R}_S, \tilde{R}_m)}{\text{Var} (\tilde{R}_m)}
\]
where $\tilde{R}_S$ is the random return on levered equity$^7$ and $E_S = E(\tilde{R}_S)$.

Now,

$$
\frac{\text{Cov}(\tilde{R}_S, \tilde{R}_m)}{\text{Var}(\tilde{R}_m)} = N(d) \frac{V}{S}
$$

where $S$ is obtained by the Black-Scholes (1973) valuation:

$$
S =VN(d) - F \exp(-R_F)N(d-\sigma_m)
$$

The most risk-tolerant investor requires $E_S = E^*$; to solve, then, set $F^*$ such that

$$
E_z + (E_m - E_z)N(d) \frac{V}{S} = E^*
$$

Substituting (2) into (3) and rearranging yields eq. (1), and $F^*$ as given by eq. (1) is in fact the minimal value to yield the required $E^*$.$^8$ To see this, note that $\partial[N(d)V/S]/\partial F > 0$

---

$^7$ An arbitrage-based valuation of the call option implies that

$$
E_S - E_c = \beta_S (E_m - E_z),
$$

where $\beta_S \equiv \text{Cov}(\tilde{R}_S, \tilde{R}_m)/\text{Var}(\tilde{R}_m)$.

See Cox and Rubinstein (1985). The use of the zero-beta version of the CAPM has been utilized here due to the model's constrained-borrowing assumption [Assumption (A4)].

$^8$ Note that if $F > F^*$, the most risk-tolerant investor will lend a portion of his portfolio to restore the desired risk-return tradeoff of his position.
and \( \lim_{F \rightarrow \infty} N(d) \frac{V}{S} \rightarrow \infty \), so that \( \frac{\partial E^*}{\partial F} > 0 \) and \( \lim_{F \rightarrow \infty} E^* \rightarrow \infty \). The intuition for this result is that, for a low-priced call infinitely deep-out-of-the-money, the elasticity of the call with respect to the stock price is infinite; analogously, when \( F \) increases, equity becomes an out-of-the-money call, and \( V N(d) / S \) grows without bound. See Merton (1974, pp. 466-467) for a rigorous proof of this particular result. Thus, \( E^* \) can be made arbitrarily large by increasing the firm's leverage ratio.9

This, however, does not complete the proof. It remains to be shown that the infra-marginal investors are indifferent to the selection of \( F^* \); if such is the case, \( F^* \) can indeed be determined by \( E^* \).10 The proof of this result is relegated to Appendix A.

Q.E.D.

Graphically, the position of various investors is depicted in Figure 1. The most risk-tolerant individual(s) holds levered equity, attaining point \( [E(F^*), \sigma(F^*)] \). Infra-marginally risk-tolerant individuals, positioned between \( R_F \) and \( [E(F^*), \sigma(F^*)] \), attain their optimal portfolio position by combining holdings of levered equity and either risky debt or riskless lending to the government. The minimal optimal level of corporate debt \( (F^*) \) determines 100 percent investment in levered corporate equity by the most risk-tolerant group of investors. Capital structure matters only in a minimum sense for the aggregate economy. If the firm were to issue debt beyond \( F^* \), the most risk-tolerant investors will find it a matter of indifference

9 Moreover, as shown immediately below, an increase in the leverage ratio in no way detracts from the investment opportunities afforded to the less-leveraged positions held by other individual investors.

10 As noted in the previous footnote, this is equivalent to stating that the most risk-tolerant investor is indifferent to \( F > F^* \).
Risk-Return Tradeoff

Figure 1

Market Portfolio

( Corporate Debt + Equity )

$E(R_p)$

$R_f$

$\sigma(R_p)$

Equity

Risky Debt

0%
because they can undo this through personal lending to the government sector. Although we shall investigate a multiple firm (industry) case later, we note here a special case implied by the single-firm economy case. If the firm were to split up into multiple firms of identical return characteristics, capital structure, like the ST case, is again indeterminate at the level of the individual firm. However, the determinacy at the aggregate level is unlike the earlier extremum, in the sense that what is being determined here is something *minimally* optimal.

### II.2 Multiple-Firm Economy

We have already touched upon earlier as to what might happen if we consider multiple firms. If a single firm economy were to split into multiple firms of identical characteristics, our analysis indicated indeterminate capital structure at a particular firm's level but a minimally optimal capital structure at the aggregate level. In this subsection we shall consider a more interesting case where the economy is differentiated by industry classes, although each industry would be composed of firms of identical risk classes. We will first examine capital structure decision rules of a particular firm or industry facing the economy's capital structure equilibrium. We will then conclude with some capital structure implications drawn from the introduction of risky debt with multiple industry classes.

To simplify consideration of a multiple-firm economy, we consider the two-firm case whereby assumption A3 of Section II.1 is replaced by
(A3') There exist two firms in the economy, both financed by levered equity and debt, whose asset returns are jointly lognormally distributed with correlation coefficient of ρ (where |ρ| < 1).

Assumption (A3') can also imply two distinct industry classes including many firms of identical (intra-industry) risk. Under this interpretation, an industry is defined here as an investment in a stochastic process with a particular volatility; the rates of return on firms' assets within a given industry are perfectly correlated.

Letting firm 1 represent the "market portfolio of all risky assets excluding firm 2," assume the following notation:

For \( i = 1, 2 \), let

\[
V_i \quad \text{the market value of firm } i
\]

\[
S_i \quad \text{the market value of firm } i's \text{ equity}
\]

\[
F_i \quad \text{the face value of firm } i's \text{ zero coupon bond}
\]

\[
d_i = \frac{\ln(V_i/F_i) + \left( R_F + \frac{1}{2} \sigma_{V_i}^2 \right)}{\sigma_{V_i}}
\]

\[
N(*) \quad \text{cumulative distribution function of standard normal random variable}
\]
$w_{Si} \quad \text{portfolio weight of firm } i\text{'s equity}$

$w_{Bi} \quad \text{portfolio weight of firm } i\text{'s risky debt}$

and

$\alpha \equiv \frac{V_1}{V_1 + V_2}$

With this notation, the following lemma initiates the analysis of the two-firm case:

**Lemma 1**

Consider the solution to the mean-variance optimization problem

$$\min_{\{F_1, F_2, w_{Si}, w_{S2}, w_{Bi}, w_{B2}\}} \text{Var} (R_p)$$

$$\text{s.t. } \sum_{i=1}^{2} (w_{Si} + w_{Bi}) = 1; \quad w_{Si}, w_{Bi} \geq 0 \quad i = 1, 2$$

and

$$E(R_p) = \bar{E}$$
where

\[ \tilde{R}_p = \sum_{i=1}^{2} (w_{Si} \tilde{R}_{Si} + w_{Bi} \tilde{R}_{Bi}) \]  

(4d)

The solution to problem (4) results in a degeneracy -- that is, the resulting problem is under-identified -- hence precluding the possibility of solving for a unique optimal vector \((F_1^*, F_2^*, w_{S1}, w_{B1}, w_{B2})\).

Prior to presenting the lemma's proof, a brief discussion is appropriate. The lemma's result should not be surprising: in the single-asset case, \(F^*\) was a lower-bound to the set of optimal \(F\) values.\(^{11}\) Given the redundancy evidenced in the single-asset case (Section II.1), it is reasonable to anticipate an analogous result for the two-asset case.

**Proof of Lemma 1: Appendix B**

Having proved this degeneracy, it is now possible to reformulate the optimal capital structure problem with the added constraint that \(w_{B1} = w_{B2} = 0\) and with no attendant loss of generality. The import of the assumption that \(w_{B1} = w_{B2} = 0\) is that the most risk-tolerant investor will hold only the levered equity of the two firms, and none of these firms' debt.

\(^{11}\) That is, the set of optimal \(F\) values is given by the semi-open interval \(\{F^*, \infty\}\).
**Theorem 2**

Assume the postulates underlying Lemma 1. In addition, suppose that the firms completing the market (by issuing risky debt) desire that investors be able to obtain their optimal risk-return tradeoffs by holding *market-value* proportions of risky assets.\(^{12}\) Then the instantaneously-optimal capital structure for firms 1 and 2 is to select \( F_1 \) and \( F_2 \) such that

\[
N(d_2) = N(d_1) \tag{5}
\]

Some brief comments are in order. It is instructive to note what the optimal capital structure (5) does not depend on: it is not influenced by \( \alpha \) or \( \rho \equiv \text{corr}(\tilde{R}_{V_1}, \tilde{R}_{V_2}) \) but hinges solely on the operating volatility \( \sigma_{V_2} \) and the riskless interest rate \( R_F \).\(^{13}\)

**Proof of Theorem 2**

Lemma 1 has yielded the result that the optimal capital structure can be solved with \( w_{B_1} = w_{B_2} = 0 \). Thus, the problem is formulated as:

\[
\min_{(w_i, F_i)} \sum_{i=1}^{3} w_i^2 \left[ \frac{N(d_i) V_i}{S_i} \right]^2 + 2w_1 w_2 N(d_1) N(d_2) \frac{V_1 V_2}{S_1 S_2} \rho \sigma_1 \sigma_2
\]

\(^{12}\) The rationale for this constraint is simple: a capital structure decision must permit an arbitrary number of investors to hold any mix of assets. As this can be done at no cost to other investors, firms must choose their leverage so that investors may attain their desired positions by holding market-value weights of assets.

\(^{13}\) We have assumed that the two firms are "distinct," in that \(|\rho| < 1\).
\[
\text{s.t. } w_1 \frac{N(d_1) V_1}{S_1} [\alpha \sigma_1 + (1 - \alpha) \rho \sigma_2 ] \sigma_1 + w_2 N(d_2) \frac{V_2}{S_2} [(1 - \alpha) \sigma_2 + \alpha \rho \sigma_1] \sigma_2 = \bar{y}
\]

and

\[
w_1 + w_2 = 1, \quad w_i \geq 0
\]

where \( \alpha \) = proportion of value of asset 1 in market portfolio of risky assets.

Make the following substitutions

\[
a_i = \frac{N(d_i) V_i}{S_i}
\]

\[
x = w a_1 \quad [w \equiv w_1]
\]

\[
y = (1 - w) a_2
\]

\[
A = \sigma_1 [\alpha \sigma_1 + (1 - \alpha) \rho \sigma_2]
\]

\[
B = \sigma_2 [(1 - \alpha) \sigma_2 + \alpha \rho \sigma_1]
\]

With these substitutions, the problem reduces to
\[
\begin{align*}
\min_{\{x,y\}} \quad & x^2\sigma_1^2 + y^2\sigma_2^2 + 2xy\rho \sigma_1\sigma_2 \\
\text{s.t.} \quad & Ax + By = \bar{\gamma}
\end{align*}
\]

Using standard Lagrangian techniques yields the solution \((x^*, y^*)\) as given by:

\[
x^* = \frac{\bar{\gamma}(A\sigma_2 - \rho B\sigma_1)\sigma_2}{B^2\sigma_1^2 + A^2\sigma_2^2 - 2AB\rho\sigma_1\sigma_2} \tag{6a}
\]

\[
y^* = \frac{\bar{\gamma} - Ax^*}{B} \tag{6b}
\]

Note that the denominator of \((x^*, y^*)\) is \(\text{Var}(B\tilde{R}_1 - A\tilde{R}_2)\). It is trivial to show that \(\text{Var}(B\tilde{R}_1 - A\tilde{R}_2) = 0 \iff |\rho| = 1\).\(^{14}\) This result is as expected, since under perfect correlatedness, the single-asset redundancy explained in Section 11.1 results.

Dividing (9b) by (9a) yields

\[\text{Var}(B\tilde{R}_1 - A\tilde{R}_2) = B_1^2\sigma_1^2 - 2AB\rho\sigma_1\sigma_2 + A^2\sigma_2^2\]

\(^{14}\) The variance of the difference of any two random variables is zero if and only if the two assets are perfectly correlated. Thus, \(\text{Var}(B\tilde{R}_1 - A\tilde{R}_2) > 0\), since the determinant of the quadratic equation satisfies

\[(AB\rho\sigma_2^2 - A^2B^2\sigma_2^2) = A^2B^2\sigma_2^2(\rho^2 - 1) < 0,
\]

unless \(|\rho| = 1\).
\[
\frac{x^*}{y^*} = \frac{A\sigma_2^2 - \rho B\sigma_1 \sigma_2}{B\sigma_1^2 - \rho A\sigma_1 \sigma_2}
\]

which, upon substitution of \(A\) and \(B\) yields

\[
\frac{x^*}{y^*} = \frac{a}{1 - \alpha} \quad \rightarrow \quad \frac{\omega a_1}{(1 - \omega)a_2} = \frac{a}{1 - \alpha}
\]

(7)

Now, to solve for \(a_1/a_2\), we need to determine \(\omega\) and \((1 - \omega)\). The most risk-tolerant investors in the economy will hold the "riskiest" portfolio. Consider now the composition of that "riskiest" portfolio. If that portfolio were, say, comprised of one security, and a significant number of investors were desirous of investing in it, its price would be bid up to accommodate that demand. But that would not be an equilibrium state since other financial intermediaries would be enticed to replicate that asset's risk-return tradeoff. To guarantee equilibrium conditions, we impose the requirement that \(\omega\) and \(1 - \omega\) reflect market-value proportions. That is,

\[
\omega = \frac{a \frac{S_1}{V_1}}{\left[ a \frac{S_1}{V_1} + (1 - a) \frac{S_2}{V_2} \right]}
\]

(8)

Thus,15 substituting this and \(a_i = V_i N(d_i)/S_i\) back into the optimal solution yields

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15 Note that \(\omega\) is indeed equal to the market-value proportion \(S_1/(S_1 + S_2)\). This can be verified by multiplying the numerator and denominator by \((V_1 + V_2)\) and noting that \(a = V_1/(V_1 + V_2)\).
\[
\frac{w a_1}{(1 - w)a_2} = \frac{\alpha N(d_1)}{(1 - \alpha)N(d_2)} = \frac{\alpha}{1 - \alpha}
\]

Consequently, the instantaneously-optimal capital structure is given by

\[
N(d_1) = N(d_2) \iff d_1 = d_2 \iff
\]

\[
\frac{\ln \left( \frac{V_1}{F_1} \right) + R_F + \frac{1}{2} \sigma_1^2}{\sigma_1} = \frac{\ln \left( \frac{V_2}{F_2} \right) + R_F + \frac{1}{2} \sigma_2^2}{\sigma_2}
\]

(9)

Q.E.D.

If asset 1 is the "market portfolio" excluding asset 2, then one can obtain estimates of the LHS by looking at aggregate market values; this then determines the leverage ratio for the "individual firm" on the RHS. As stated earlier, this leverage ratio could also be viewed as the optimal solution for a particular industry consisting of many firms from an identical risk class.

We can now examine the effects of interest rate changes on the optimality condition (9):

\[
\frac{\partial \ln \left( \frac{V_2}{F_2} \right)}{\partial R_F} = \frac{\sigma_2}{\sigma_1} - 1 \geq 0 \quad \text{as} \quad \sigma_2 \geq \sigma_1
\]

The optimality condition implies that, for "high-risk" firms \((\sigma_2 > \sigma_1)\), higher interest rates imply a decline in the leverage ratio. The result should be interpreted with caution, as.
the *ceteris paribus* rule is clearly invoked under these comparative statics conditions. With this caveat in mind, it appears that a higher interest rate increases expected returns without a concomitant increase in risk. Thus, risk-tolerant individuals can achieve their desired risk-return tradeoff with a lower leverage for the "riskier" firm. Conversely, for "low-risk" firms \((\sigma_2 < \sigma_1)\), an increase in interest rates triggers an increase in the optimal leverage ratio, perhaps resulting from the need to align the expected rate of return with the new slope of the capital market line.\(^{16}\)

The implication of the preceding analysis follows from the introduction of risky debt, which was suggested by the concluding paragraphs in Senbet and Taggart (ST, 1984). As anticipated in that article, we have shown that "risky debt can still play a role in completing the market." Now, however, we find a Miller-type equilibrium at the *industry* level. The precise rule for that industry-level capital structure is as follows: each industry examines eq. (9) and consequently establishes its optimal capital structure as a function of the economy-wide structure, the industry's operating volatility and the riskless rate of interest. Corporations perform a financial intermediary function by providing investors with an expanded set of opportunities for trading off risk and return. This might give rise to rents within a particular industry class with unique technological characteristics. However, without barriers to entry and exit, the attempts of firms to take advantage of these rents would be competed away by other firms through supply adjustments or proper financial intermediation services. Consequently, capital structure equilibrium obtains in which capital structure is a matter of indif-

\(^{16}\) The effect of operating volatility on leverage is indeterminate, since

\[
\frac{\partial \ln \left( \frac{V_2}{F_2} \right)}{\partial \sigma_2} = \frac{1}{\sigma_1} \left[ \ln \left( \frac{V_1}{F_1} \right) + R_p + \frac{1}{2} \sigma_1^2 \right] - \sigma_2 = d_1 - \sigma_2 \geq 0.
\]
ference to an individual firm in a particular industry; yet it is determinate at that industry’s level.

The determinacy of capital structure both at the industry and economy-wide levels occurs without adding value to any particular firm in the corporate sector. However, the resulting equilibrium characterized by risky debt and multiple industry classes established industry-based regularities in observed capital structure behavior.\(^1\)\(^7\) This is in contrast to the prevailing view that detection of industry effect in capital structure implies significant value creation by corporate debt decisions. One can only infer without ambiguity a positive role of corporate finance in an aggregate even when one observes industry-based patterns in corporate financial behavior. This role, which presumably results in “consumer-investor surplus,” arises from its enhancement of market completeness.

### II.3 Alternative Stochastic Production Processes

As previously noted in Section II.1, the production technology employed in the lognormal distribution gives rise to a required rate of return on levered equity, \( E_S = E(\tilde{R}_S) \), given by

\[
E_S = E_x + (E_m - E_x)N(d) \frac{V}{S},
\]

where \( E_S \) increased without bound as \( F \) increases (equivalently, as the debt-equity ratio

\(^1\)\(^7\) For analyses of such industry effects on capital structure, see, e.g., Bradley, Jarrell and Kim (1984), Dammon and Senbet (1987) and Raymar (1986).
increases). The required return on risky debt, $E_B$, is then given by

$$E_B = E_z + (E_m - E_z)[1 - N(d)] \frac{V}{B} . \tag{11}$$

These results are displayed in Figure 2 below.\textsuperscript{18} As observable from Figure 2, the required return on risky debt rises as the debt-equity ratio increases. This is quite true in general and is independent of the underlying stochastic process: naturally, as the equity cushion vanishes, debt becomes increasingly risky until, in the extreme, it assumes the properties of unlevered equity. What does not necessarily follow is that, for \textit{arbitrary} distributions of asset returns, $E_S$ must necessarily diverge as the debt-equity ratio increases without bound.

To begin, the value-additivity principle $E_m = [S/(S+B)]E_S + [B/(S+B)]E_B$ implies

$$E_S = E_m + \left[ E_m - E_B \left( \frac{B}{S} \right) \right] \frac{B}{S} , \tag{12}$$

where $E_m$ is the required return on unlevered equity and $E_B(B/S)$ is the required return on risky debt as a function of the debt-equity ratio. Eq. (12) is the well-known Modigliani and Miller (1958) Proposition II. It is derived here under constrained borrowing by appeal to the value additivity principle.\textsuperscript{19} In contrast to $E_B(B/S)$ under the lognormal distributions [eq. (11)], assume now that $E_B(B/S)$ is given by the functional form.

\textsuperscript{18} The convexity of the $E_S$ curve (as a function of the debt-value ratio) may surprise readers of Merton's (1974) derivation and graphical representation, wherein $E_S$ is a concave function of the "quasi" debt-value ratio. The distinction arises from the current paper's use of the \textit{market} debt-value ratio as the independent variable, whereas Merton uses $F \exp \left(-Rd/V \right)$ (where $F$ is the \textit{face value of debt}) as the independent variable in his analysis.
Leverage Effects on Required Returns

Figure 2

Excess Return / Risk Premium on Firm

Equity

Debt

Market Value of Debt / Firm Value

\( V \frac{N(d_1)}{S} \)

\( \frac{[1 - N(d_1)] V}{B} \)
\[ E_B \left( \frac{B}{S} \right) = R_F + (E_m - R_F)(1 - e^{-\lambda B/S}). \] (13)

An examination of the required returns given by eqs. (9) - (10) can be instructive.\(^{20}\) First, note that \( E_B(B/S = 0) = R_F, \)

\[
\frac{\partial E_B}{\partial (B/S)} = (E_m - R_F)(-e^{-\lambda B/S})(-\lambda) > 0
\]

and

\[
\lim_{B/S \to \infty} E_B \left( \frac{B}{S} \right) = E_m + (E_m - R_F) = E_m.
\]

Thus, eqs. (12) - (13) conform to the intuitive requirements for return on risky debt. Now, substituting (13) into (12) yields

\[ E_S = E_m + (E_m - R_F)e^{-\lambda B/S} \frac{B}{S}, \] (14)

which gives an explicit functional form for \( E_S(B/S) \). Differentiating (18) w.r.t. \( B/S \) and setting the result equal to zero yields

\[ \text{Note that if a weighted average of } E_S \text{ and } E_B \text{ exceeded } E_m, \text{ this disequilibrium would induce new corporate entrants until the equilibrium relationship was restored.} \]

\[ \text{In contrast to eqs. (10) - (11), eqs. (12) and (13) have not been motivated by an explicit underlying stochastic process. There is, however, nothing to rule out the possibility of such a structure for required returns.} \]
\[
\frac{\partial E_S}{\partial (B / S)} = (E_m - R_F)e^{-\lambda B / S} \left(1 - \lambda \frac{B}{S}\right) = 0
\]

which implies \((B / S)^* = 1 / \lambda\). The required return on equity, \(E_S\), reaches an \textit{internal maximum} at the point where the debt-equity ratio equals the reciprocal of the parameter determining the rate by which \(E_B(B / S)\) approaches \(E_m\). At the point of maximal required return for levered equity,

\[
E_S \left(\frac{B}{S} = \frac{1}{\lambda}\right) = E_m + \frac{1}{\lambda} \left(E_m - R_F\right)
\]

whereas, by L’Hospital’s rule, \(\lim_{B / S \to \infty} E_S(B / S) = E_m\). Consequently, after attaining a maximum, \(E_S\) declines back to return on unlevered equity. This result is graphically demonstrated in Figure 3.

Under the alternative stochastic process postulated in Section II.3, the condition for optimality prescribes a capital structure which maximizes the required return to levered equity.

---

21 The second-order conditions for a maximum are easily verified:

\[
\frac{\partial^2 E_S}{\partial (B / S)^2} = - (E_m - R_F)\lambda e^{-\lambda B / S} (2 - \lambda B / S)
\]

and

\[
\left.\frac{\partial^2 E_S}{\partial (B / S)^2}\right|_{B / S = 1 / \lambda} = - (E_m - R_F) \frac{S}{Be} < 0.
\]

22 An economy wherein a maximal \(E_S\) is achieved (at value \(B / S = 1 / \lambda\)) may suffer in a welfare comparison with an economy (as in Section II.2) wherein arbitrarily large \(E_S\) are attainable.
This contrasts with the earlier result where the required return diverges as the face value of debt increases without bound. There, the economy-wide capital structure prescription required only minimally optimal debt ratios. However, the results in the multiple firm case with multiple industry risk classes remain qualitatively unaltered and, indeed, they are reinforced. The precise rule for capital structure has, nonetheless, changed as can be inferred from comparing eq. (9) with the new rule calling for the debt ratio to be equal to $1/\lambda$.

III. Some Extensions

III.1 Corporate Bankruptcy and Financial Synergy

By virtue of their taxation and seignorage functions, governments are able to provide investors with nominally riskless lending opportunities. In an uncertain inflationary environment, however, it is less clear that governments have a comparative advantage in the issuance of securities with real, inflation-adjusted payoffs. For example, corporations may have such an advantage with respect to securities whose payoffs are highly correlated with the corporation’s product prices: dividends and interest on corporate debt may be scaled to changing product prices. In this fashion, corporate securities are better suited for the purposes of hedging relative price changes.\textsuperscript{24} We can no longer assume that individuals rely on the government

\textsuperscript{23} It would, naturally, be desirable to extend the result of Appendix A -- that infra-marginal investors are not hampered by this choice of capital structure -- to the current case. Unfortunately, that is not possible as the underlying stochastic distribution has not been explicitly stated. Rather, the analysis of Section II.3 has relied on the “reduced-form” equations for required returns, eqs. (15) and (16).
sector for real riskless lending; corporations may play a role in enhancing the investment opportunity set through the issuance of near-riskless real debt securities. We now turn our attention to this possibility.

Thus far, corporate capital structure adjustments are shown to augment the set of portfolio opportunities that investors could construct on their own account. The capacity of any given firm to issue debt with the desired return attributes, however, depends on the characteristics of its operating cash flow stream. It can be seen now that financial synergy of firms enhances the capacity of risky debt in completing the market. In a subsequent subsection, we shall argue that altering the structure of the debt claim itself into alternative complex attributes, such as call provisions and conversion privileges, and using the production sector itself, can further enhance the role of corporate financial intermediation.

We can illustrate the role of financial synergy by considering a firm engaged in splitting up its original equity securities into debt and levered equity securities. This is accomplished by the expedient of using the proceeds from the newly issued debt to retire part of the outstanding equity. Accordingly, the total income, \( Y \), is partitioned into return streams available to the two classes of security holders as depicted in Figure 4. The terminal cash flow, \( Y \), is measured along the horizontal axis, and the partitioned return stream is measured along the vertical axis. If the promised payment, \( F \), on the firm's debt is relatively small, then the debt will be virtually riskless (that is, the actual payment will equal the promised payment in almost every state of nature). But as the firm alters its capital structure from the

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24 Indeed, future research might profitably extend the current analysis to economies which lack the opportunity of riskless lending in real terms.
FIGURE 4

Partitioning of Firm's Operating Income Stream
Through the Use of Debt and Equity Securities

Partitioned Income Stream

\[ Y^U = Y \]
\[ Y^D = \text{Max}(0, Y - F) \]
\[ Y^D_0 = \text{Min}(F^*, Y) \]

Total Operating Stream

45°

45°

45°

45°
promised payment \( F \) to \( F' = F + dF \), the corresponding return streams to debt-holders and equityholders are altered to \( Y_D \) and \( Y_S^L \), respectively. Indeed, if this process continues, the firm’s debt will no longer have the near-riskless property potentially desired by net lending clienteles, because its return stream becomes increasingly indistinguishable from that available to unlevered equity. High probabilities of bankruptcy can thus play a negative role in completing the market, because they reduce the available supply of near-riskless debt. From the standpoint of investor welfare, this is much like a bankruptcy cost, but it is fundamentally different from traditional notions of costs associated with formal bankruptcy proceedings either in liquidation or reorganization. This cost arises naturally in the market place through the process of altering the debt security into an “equity-like” security.

The cost of market incompleteness due to bankruptcy or conglomerate merger could be mitigated, or even eliminated, through financial synergy. A conglomerate merger is a case in point.\(^{25}\) Consider two firms, A and B, having operating cash flow streams that are imperfectly correlated, with equal standard deviations, \( \sigma_A = \sigma_B \). The pre-merger risk-return vector composition \( (\sigma_A, Y_A) \) and \( (\sigma_B, Y_B) \) in all possible proportions) is now altered to \( (\sigma_{AB}, Y_{AB}) \) where \( Y_{AB} = Y_A + Y_B \) and \( \sigma_{AB} < \sigma_A = \sigma_B \). The return stream available to debt holders in the merged firms is less “equity-like” than the combination of streams of the unmerged firms. With merger it is possible to support the issuance of additional debt without additional cost of bankruptcy in the sense of market incompleteness. Thus, merger increases the capacity of debt to complete the market. If the process of merging were costless, this activity alone

\(^{25}\) This is not the whole story, however. Mergers also restrict investors’ portfolio flexibility by cementing together the equity streams of the two firms. If good market substitutes exist elsewhere in the capital market for the two pre-merger equity streams, this creates no problem. Otherwise, there might be a tradeoff between the enhanced ability to issue near-riskless debt and reduced flexibility for equity investors. An additional problem is that equityholders in the two separate firms lose some of the benefits of limited liability through merger.
would not affect equilibrium market valuation, but would simply arise as part of the competition among firms to meet investors’ desired return patterns. Nonetheless, there would be an optimal economy-wide configuration of firms with various sizes. Whenever departures from this configuration arose, an incentive would exist for certain firms to merge. In this sense, depending upon the underlying investment opportunity sets available to firms, not only mergers but spin-offs would be endogenous in financial equilibrium.

III.2 Complex Financial Structures

Thus far, we have focused only on debt and equity securities, but a truly complete market may call for the existence of other forms of securities to be supplied by firms, such as hybrid securities with option characteristics (e.g., warrants, callable debt, convertible debt, etc.). Moreover, in a multiperiod environment, a complete market calls for securities that span the available time-state space. When investors’ desires include return patterns across time, then securities with multiple periods become relevant. Thus, corporate debt maturity structure, at least in an economy-wide sense, evolves naturally as a means of completing the market.

The role of complex financial securities can be seen readily if we restate the key aspects of Senbet-Taggart (1984) somewhat qualitatively. If corporations can costlessly transform their return streams into any securities package, as in the discussion thus far, the certainty-equivalent yield differentials (or, equivalently, cost of capital differentials to firms) among various classes of securities must be identical in equilibrium. Therefore, the advantages that firms possess in completing the market get “priced out,” as long as firms act as price-taking
competitors. This is depicted graphically in Figure 5. The construction of the graph can be explained as follows. There exists a (certainty-equivalent) yield differential between debt and equity, \(-\theta_f\), at which no lender demands corporate debt. At this point, the benefit that corporate debt offers to investors in completing the market is outweighed by the yield differential. Stated another way, the negative yield differential on corporate debt is at least as high as the cost associated with financial transformation at the investor level. As long as this personal transformation remains costly, however, there will be some yield differential at which there is a demand for corporate debt. Thus, the demand curve for corporate debt is a set of tradeoffs between its benefit in completing the market and its yield disadvantage relative to equity. This graphic analysis does not capture the whole story, however. An expanded version must consider supply and demand adjustments for riskless debt, risky debt, levered equity, and unlevered equity.

Pursuant to our discussion in the previous subsection regarding corporate bankruptcy, as the level of corporate debt increases, its marginal benefit in completing the market decreases. Therefore, the demand curve must be upward sloping as investors require a lower yield differential to tradeoff against lower marginal benefits. Indeed, a zero yield differential obtains when the marginal benefits of both debt and equity are driven to zero. This occurs at the point of intersection \((Z)\) between the flat supply curve and the upward sloping demand curves, when firms adjust their supply of debt to meet the desired combinations of debt and equity demanded in the aggregate. At this point, financial transformation has effectively eliminated the initial effects of an incomplete market at the investor level.

Although this equilibrium is posited in the context of two classes of securities, it can be expanded to multiple classes with a whole range of payoff characteristics. One can envision
FIGURE 5

An Equilibrium Characterization of Financial Transformation
the development of complex financial securities, such as callable debt and convertible debt, and thus the process of completing the market via corporate financial behavior has the potential for rationalizing observed complex finance even in the absence of agency costs and taxes. Nonetheless, financial transformation at a particular firm level is of no consequence to its market value unless the transformation itself is costly. Again, this underscores the fact that the limited MM theorem obtains so long as corporate financial policy is costless, irrespective of the degree to which markets are incomplete at the investor level. In particular, the familiar homemade leverage assumption is not necessary to this result. Thus, corporate financial policy can complete the market not only by splitting equity securities into levered equity and debt securities, but also by creating more complex attributes with option characteristics and differential maturity structure.

III.3 Financial Transformation by Nonfinancial Firms and Financial Institutions

Up to this point, the discussion focused on costless financial transformation at the corporate level. However, the immediate consequence of costly corporate supply adjustment is that yield differentials occur among securities. This is depicted in the downward sloping supply curve in Figure 5. The curve is generated by the process of firm supply adjustments up to the point where the marginal benefit (cost) of substituting debt for equity is equal to the positive (negative) yield differential between debt and equity. A yield differential can now occur. The differential is positive or negative depending upon the elasticities of the supply and demand curves and their intersection point. If these transformation costs were firm-specific, there would exist optimal mixes of financial claims not only in an economy-wide sense but
also at the level of individual firms. This optimality depends upon relative efficiencies in packaging financial securities across firms and industry groups. More generally, we might expect specialized financial transformers (financial intermediaries) to emerge in such an environment. Thus, financial institutions can compete with non-financial firms.

Thus far, we have considered firms and individual investors at the two groups competing with one another to perform the desired financial transformation activities. Once financial institutions are brought into the picture, this transformation process might best be thought of as occurring in two stages. First, those with expertise in managing real assets organize corporations and partnerships for the purpose of holding and operating these assets. They in turn issue financial claims against these assets in the form of bonds, bank loans, stock, and partnership shares. These claims can split up the return stream from the real assets in a variety of ways desired by households. This first element in the economy's financial structure thus consists of the nonfinancial sector's capital structure, consisting of debt, equity and complex securities.

The second step in the process of transforming asset characteristics is performed by financial institutions. Although households find it convenient to hold directly many of the financial claims issued by nonfinancial firms, the degree of transformation inherent in these claims may still be insufficient in some respects to match their desired consumption plans. Commercial banks, for example, buy debt claims of nonfinancial firms (as well as of households themselves) and enhance their liquidity and divisibility by transforming them into demand and savings deposits; or mutual funds buy stock and bonds and overcome the barriers to diversification implied by security indivisibilities by in turn issuing small-denomination claims on
their overall portfolios; or insurance companies hold claims on nonfinancial firms and rearrange their return streams so as to pay off households in the event of death or illness.

As previously noted by Senbet and Taggart (1984), it is important to recognize that the second stage in financial transformation can be performed by nonfinancial firms as well. There is nothing inherent in the institutional arrangement of a financial institution that makes it superior to a nonfinancial firm in performing financial transformation. The theory of financial intermediation is usually thought of as distinct from the theory of corporate finance, but to the extent that both types of institutions help to complete the capital market, they are competing with one another to perform the same function. Apart from distinguishing institutional features such as government regulation, then, we would argue that there is a basic identity between the phenomena that the two theories are attempting to explain. The theory of corporate finance is really a theory of financial intermediation.

III.4 The Role of Real and Financial Production Functions

Our focus has been on the role of corporate financial decisions to complete the market. We have seen how regularities in the debt decisions among industries, even firms, emerge and how debt claims themselves take complex attributes in their characteristics. In this section we recognize the possibility that productive decisions can also enhance the ability of corporations to provide their intermediation services. It is, indeed, somewhat surprising that the productive decisions enhance the efficiency of financial markets.\textsuperscript{26}

\textsuperscript{26} A genesis to this basic point is in the economics literature that considers the trade-off between scale economies and consumer markets.
It is important to note that just as the productive process of transforming raw materials into finished goods requires capital and labor resources, so too does the process of financial transformation. This has been widely recognized for the case of financial institutions which have long been thought of as having production functions. It has not been explicitly recognized for nonfinancial corporations, however, where the dominance of perfect market models has confined discussion to the case where financial transformation is costless.

The nonfinancial firm can be thought of as operating with two production functions: one that uses capital and labor to produce real goods and services and one that uses capital and labor to transform the characteristics of its real assets into alternative financial asset characteristic vectors. From this perspective, the importance of nonfinancial firms’ capital structure depends on the characteristics of their financial production functions relative to those of households, financial institutions and one another. If nonfinancial firms are identical to one another in this regard but possess relative advantages over financial institutions and households, a limited MM theorem will again result. Capital structure will not affect the equilibrium valuations of firms in a given risk class, but the aggregate corporate capital structure will nevertheless be determinant. When viewed in the context of financial production functions, in fact, this limited MM result is analogous to the constant returns to scale case in ordinary microeconomic theory, where industry output alone is determinate, but the size and the output of any one firm are not.

However, the production decisions themselves can also augment corporate financial decisions in completing the market. We can illustrate the basic point without the benefit of a detailed analysis by beginning with a single firm economy which is initially unlevered. Can such a firm meet the needs of diverse investor interests? Suppose that there are three distinct
clienteles in the parlance of the traditional capital market line, namely the net borrowers, net lenders, and unlevered investors. Clearly, the firm can split its unlevered equity into debt and levered equity, but it can only satisfy the net borrowing investors (and perhaps to a limited degree unlevered investors). However, the net lending clientele would wish that the firm lend rather than borrow. This illustrates the limitation of corporate finance alone in completing the market.

The firm has another option, namely to split into divisions on the production side rather than split financial securities. Three separate divisions can separately cater to the three investment clienteles. Indeed, this suggests an optimal configuration of firms even in a single commodity world, emerging for the sole purpose of providing financial services. Thus, there is a natural linkage between production and finance. The problem is even more interesting if the productive role of market completeness leads to suboptimal scale. The optimal level of output can be thought of as being determined by minimizing the average cost (\( AC \)); alternatively, this level obtains at the point of equality between \( AC \) and marginal cost (\( MC \)).

Clearly, when the firm splits itself into divisions, it may depart from the efficient level of production. This characterizes costly supply adjustment in using the production sector to complete the market. However, the departure from the point \( \min AC = MC \) is compensated by the financial cost saving which results from the demand for financial intermediary services and which consequently lowers the required rate of return. At any rate, the firm faces productive means as well as financial means to complete the market. The final equilibrium would be reached by trading off the costs of production and financial adjustments.\(^{28}\)

\(^{27}\) The full development of this idea requires a more rigorous modeling to link the production and financial decisions of the firm. An example of such an analysis focusing on taxation is Dammon and Senbet (1987).
On the financial side, the firm faces something more than just splitting unlevered equity into levered equity and debt. It can begin to classify the debt itself into a priority structure (e.g., subordinated debt). However, if the firm is not split, the market observes only the aggregate earnings and not disaggregated earnings or payoffs catering to diverse clienteles. Consequently, the firm faces, as Townsend (1979) puts it, "costly verification." It might be difficult for some firms to make their project returns on divisional units known to outside investors. Indeed, without costly verification, the firm could again have conducted all of its intermediation services through a design of complex and classified securities.

IV. Conclusions and Implications

This paper has developed a theory of capital structure in an environment in which corporations act as financial intermediaries in a restricted personal borrowing market. The role of corporate financial intermediation has been examined both in a single firm economy and an economy with multiple industry risk classes. A minimally optimal level of debt obtains for the single-firm economy case which allows a maximally optimal levered return for most risk-tolerant groups of investors. However, the introduction of multiple firms and risky debt leads to a Miller-type equilibrium at an industry level. Although capital structure is a matter of indifference to a particular firm and has no impact on the value of the corporate sector, the resulting capital structure equilibrium establishes observable industry regularity. Thus, an industry effect in capital structure obtains even when capital structure is of no consequence.

28 This trade-off between production and financial decisions may imply systematic differences in debt-equity ratios between service and manufacturing industries.
to both firm and industry valuations. An alternative stochastic return process may generate a concave function for the levered equity return in the parlance of the well-known Modigliani-Miller Proposition II. While this is somewhat surprising, it only reinforces our results which establish industry regularity, although the precise capital structure decision rules are altered in comparison with the lognormal return distribution assumed in the earlier part of the analysis.

Further extensions of the model into a richer framework provide a rationale for important issues that characterize corporate finance, namely financial synergy, complex characteristics of financial securities, classification or prioritization of corporate debt claims, and a linkage between production and financial decisions. The immediate empirical implication is the emergence of an industry effect resulting from corporate financial intermediation. However, unlike the traditional explanation, an industry regularity emerges despite the fact that capital structure is inconsequential to both firm valuation and valuation of the corporate sector as a whole.

The significance of capital structure in completing the market has implications for international differences in capital structure. There is a long-standing view that certain countries, such as Japan, are more highly levered than the United States. Differential market incompleteness across national boundaries can give rise to differential debt-equity ratios on a country basis. For example, if the domestic capital market is more complex and relatively more complete, foreign leverage ratios may be higher than domestic levels to afford greater market completing power. By the same token, multinational firms can generally be viewed as more complex and more differentiated, and hence more highly levered, than firms domiciled entirely in the domestic economy. Moreover, our analysis has implications for time series behavior of capital structure. As capital markets became more efficient and complete by virtue of introduction of new and derivative securities, such as options and futures, capital structure
plays a less significant role in enhancing market completeness, and hence corporate debt ratios decline commensurately. It has been observed that debt ratios in the U.S. have not changed significantly in response to changes in the U.S. Internal Revenue Code. This may be explained in part by the fact that changes in the tax code, while they encourage corporate debt, are accompanied by changes in the financial system (i.e., financial innovation) that diminish the role of corporate financial intermediation through debt financing.

The latter part of the paper illustrated the role of productive means to augment financial mechanisms to complete the market. Costly verification associated with more complex and prioritized financial securities encourages firms to utilize the production sector, such as splitting up into divisions, so as to better cater to individual investment opportunities. This has an interesting implication suggesting that industries that face lower costs of production diseconomies or face close to constant return technology would be characterized by relatively simple forms of debt and equity securities. Conversely, those which face relatively high costs of production diseconomies will exhibit more complex forms of finance. The issue relating to an index of financial complexity and a more precise measure of production scale is left for future research.
Appendix A: Proof that the Selection of $F^*$ Does Not

Diminish Infra-Marginal Investors' Risk-Return Opportunities

To proceed with the proof, consider the generation of the frontier of efficient portfolios using all available risky assets: levered equity and risky debt.

The efficient frontier of risky assets is obtained by solving the following constrained minimization problem

$$\min_{\{w,F\}} \text{Var}(R_p) \equiv \text{Var}\left[wR_S + (1 - w)R_B\right]$$

(A1)

s.t.

$$E_p \equiv wE_S + (1 - w)E_B = \bar{E} ; \quad 0 \leq w \leq 1$$

(A2)

where

$$E_S = E_z + (E_m - E_z)N(d) \frac{V}{S}$$

(A3)

$$E_B = E_z + (E_m - E_z)[1 - N(d)] \frac{V}{B}$$

(A4)

$S$ is as given in eq. (2) and
\[ B = V - S \]

Now, using Itô's lemma it follows that

\[ \text{Var}(R_B) = \left\{ \frac{V[1 - N(d)]}{B} \right\}^2 \sigma_V^2 \]

and

\[ \text{Cov}(R_B, R_S) = N(d)[1 - N(d)] \frac{V^2}{SB} \sigma_V^2 \]

Thus,

\[ \text{Var}(R_p) = w^2 \left[ N(d) \frac{V}{S} \right]^2 \sigma_V^2 + 2w(1 - w)N(d)[1 - N(d)] \frac{V^2}{SB} \sigma_V^2 + (1 - w)^2 \left\{ [1 - N(d)] \frac{V}{B} \right\}^2 \sigma_V^2 \]  

(A5)

Further, the constraint (A2) can be rewritten, using (A3) and (A4),

\[ w \frac{V}{S} N(d) + (1 - w) \frac{V}{B} [1 - N(d)] = \bar{\gamma} \]  

(A6)
for an appropriate selection of \( \gamma \). Consequently, the minimization problem (A1) - (A2) can be stated in terms of equations (A5) - (A6). Close inspection of eqs. (A5) and (A6) reveals that the “reduced-form” problem bears the form

\[
\min \quad x^2 \\
\text{s.t.} \quad x = c
\]

In words, once the expected return has been fixed, the variance is a trivial monotonic transformation of expected return. In simple language, “there is nothing left to optimize.”

This conclusion has important implications for infra-marginal investors. Say the aggregate capital structure has been determined by eq. (1), so that \( S/V, \ B/V \) and \( N(d) \) have been determined. The infra-marginal investors can choose \( w \) given by (A6) to reduce their risk exposure (variance) while reducing the expected return of their portfolio concomitantly. The infra-marginal investors will be indifferent to the selection of \( F^* \), as they can “costlessly” (i.e., at no loss in terms of the risk-return trade-off) establish their desired mean-variance efficient allocation by diversifying their holdings in equity and debt.

Q.E.D.
Appendix B: Proof of Lemma 1

Under the twin assumption of Black-Scholes option pricing and the Black zero-beta version of the CAPM, the expected return constraint $E(R_p) = \bar{E}$ [eq. (7c)] can be restated.

Consider the security market lines

$$E(R_{Si}) = E_z + (E_m - E_z) \frac{V_iN(d)}{S_i} \beta_i$$

and

$$E(R_{Bi}) = E_z + (E_m - E_z) \frac{V_i[1 - N(d)]}{B_i} \beta_i$$

where

$$\beta_i = \frac{\text{Cov} [R_i, \ aR_{Y1} + (1 - a)R_{Y2}]}{\text{Var} [aR_{Y1} + (1 - a)R_{Y2}]}$$

The above value of $\beta_i = \text{Cov} (R_i, R_m)/\text{Var} (R_m)$ follows from the recognition that this is a two asset world, composed of assets 1 and 2, weighted by $a$ and $(1 - a)$, respectively. Naturally, the $\beta_i$'s satisfy

$$a\beta_1 + (1 - a)\beta_2 = 1$$
Now,

\[ E_p = \sum_{i=1}^{2} (w_{Si}E_{Si} + w_{Bi}E_{Bi}) = \]

\[ = E_z + (E_m - E_z) \left[ \sum_{i=1}^{2} w_{Si}N(d_i) \frac{V_i}{S_i} \beta_i + \sum_{i=1}^{2} w_{Bi}[1 - N(d_i)] \frac{V_i}{B_i} \beta_i \right] \]

\[ E_p = \bar{E} \rightarrow \]

\[ \sum_{i=1}^{2} w_{Si}N(d_i) \frac{V_i}{S_i} \beta_i + \sum_{i=1}^{2} w_{Bi}[1 - N(d_i)] \frac{V_i}{B_i} \beta_i = \bar{Y} \]

In a two-asset world,

\[ \alpha \beta_1 + (1 - \alpha) \beta_2 = 1 \rightarrow \]

\[ \beta_2 = \frac{1 - \alpha \beta_1}{1 - \alpha} \]

Furthermore,

\[ \beta_1 = \frac{\text{Cov}[R_{\psi_1}, \alpha R_{\psi_1} + (1 - \alpha)R_{\psi_2}]}{\text{Var}[\alpha R_{\psi_1} + (1 - \alpha)R_{\psi_2}]} = \]
Thus, \( E_p = \bar{E} \leftrightarrow \)

\[
\frac{w_{S1}N(d_1)}{S_1} \frac{V_1}{\beta_1} + w_{S2}N(d_2) \frac{V_2}{S_2} \frac{1 - \alpha \beta_1}{1 - \alpha} +
\]

\[
+ \frac{w_{B1}[1 - N(d_1)]}{B_1} \frac{V_1}{\beta_1} + w_{B2}[1 - N(d_2)] \frac{V_2}{B_2} \frac{1 - \alpha \beta_1}{1 - \alpha} = \bar{v}
\]

\[
\frac{w_{S1}N(d_1)}{S_1} \frac{V_1}{S_1} [\alpha \sigma_{V1}^2 + (1 - \alpha) \rho \sigma_{V1} \sigma_{V2}] +
\]

\[
\frac{w_{S2}N(d_2)}{S_2} \frac{V_2}{S_2} [(1 - \alpha) \sigma_{V2}^2 + \alpha \rho \sigma_{V1} \sigma_{V2}] +
\]

\[
\frac{w_{B1}[1 - N(d_1)]}{B_1} \frac{V_1}{\beta_1} [\alpha \sigma_{V1}^2 + (1 - \alpha) \rho \sigma_{V1} \sigma_{V2}] +
\]

\[
\frac{w_{B2}[1 - N(d_2)]}{B_2} [\alpha \sigma_{V2}^2 + \alpha \rho \sigma_{V1} \sigma_{V2}] = \bar{v}
\]
With appropriate manipulation,

\[ w_{S_1}N(d_1) \frac{V_1}{S_1} \sigma_{\nu_1}[(1-\alpha)\rho \sigma_{\nu_2} + (1-\alpha)\rho \sigma_{\nu_1}] \]

\[ + w_{S_2}N(d_2) \frac{V_2}{S_2} \sigma_{\nu_2}[(1-\alpha)\sigma_{\nu_2} + a \rho \sigma_{\nu_1}] \]

\[ + w_{B_1}[1-N(d_1)] \frac{V_1}{B_1} \sigma_{\nu_1}[(1-\alpha)\rho \sigma_{\nu_1} + (1-\alpha)\rho \sigma_{\nu_2}] \]

\[ + w_{B_2}[1-N(d_2)] \frac{V_2}{B_2} \sigma_{\nu_2}[(1-\alpha)\sigma_{\nu_2} + a \rho \sigma_{\nu_1}] = \overline{\gamma} \quad (B1) \]

for an appropriate selection of \( \overline{\gamma} \). Thus, problem (4) can be restated with (B1) replacing (4c).

For compactness of notation, define

\[ a_i \equiv \frac{N(d_i)V_i}{S_i}; \quad 1 \leq a_i < \infty \]

\[ b_i \equiv \frac{[1-N(d_i)]V_i}{B_i}; \quad 0 \leq b_i < 1 \]

With this notation, eq. (4a) can be expressed as
\[
\text{Var}(R_p) = \sum_{i=1}^{2} w_{S_i} a_i^2 \sigma_{\nu_i}^2 \\
+ \sum_{i=1}^{2} w_{B_i} b_i^2 \sigma_{\nu_i}^2 + 2 \sum_{i=1}^{2} w_{S_i} w_{B_i} a_i b_i \sigma_{\nu_i}^2 \\
+ 2w_{S_1} w_{B_1} a_1 b_2 \rho \sigma_{\nu_1} \sigma_{\nu_2} + 2w_{S_2} w_{B_1} a_2 b_1 \rho \sigma_{\nu_1} \sigma_{\nu_2} \\
+ 2w_{S_1} w_{S_2} a_1 a_2 \rho \sigma_{\nu_1} \sigma_{\nu_2} + 2w_{B_1} w_{B_2} b_1 b_2 \rho \sigma_{\nu_1} \sigma_{\nu_2} \\
\text{s.t. } \sum_{i=1}^{2} (w_{S_i} + w_{B_i}) = 1 ; \quad w_{S_i}, w_{B_i} \geq 0 \quad i = 1, 2
\]

\[
w_{S_1} \sigma_{\nu_1} a_1 [\alpha \sigma_{\nu_1} + (1 - \alpha) \rho \sigma_{\nu_2}] + w_{S_1} \sigma_{\nu_1} a_1 [\alpha \sigma_{\nu_1} + (1 - \alpha) \rho \sigma_{\nu_2}] + \\
+ w_{S_2} \sigma_{\nu_2} [1 - \alpha] \sigma_{\nu_2} + a \rho \sigma_{\nu_1} + w_{B_1} b_1 \sigma_{\nu_1} [\alpha \sigma_{\nu_1} + (1 - \alpha) \rho \sigma_{\nu_2}] \\
+ w_{B_2} b_2 \sigma_{\nu_2} [1 - \alpha] \sigma_{\nu_2} + a \rho \sigma_{\nu_1} = \bar{Y}
\]

This can be transformed into

\[
\frac{\partial \bar{t}}{\partial w_{S_1}} = 2w_{S_1} a_1^2 \sigma_{\nu_1}^2 + 2w_{B_1} a_1 b_1 \sigma_{\nu_1}^2 + 2w_{B_2} a_1 a_2 \rho \sigma_{\nu_1} \sigma_{\nu_2} + 2w_{S_2} a_1 a_2 \rho \sigma_{\nu_1} \sigma_{\nu_2} \\
- \lambda_1 - \lambda_2 a_1 \sigma_{\nu_1} [\alpha \sigma_{\nu_1} + (1 - \alpha) \rho \sigma_{\nu_2}] = 0
\]
\[
\frac{\partial \ell}{\partial w_{B1}} = 2b_1^2\sigma_{v1} \sigma_{v1} w_{B1} + 2w_{S1} a_1 b_1 \sigma_{v1}^2 + 2w_{S2} a_1 b_1 \rho \sigma_{v1} \sigma_{v2} + 2w_{B2} b_1 b_2 \rho \sigma_{v1} \sigma_{v2} \]

\[-\lambda_1 - \lambda_2 b_1 \sigma_{v1}[a_1 \sigma_{v1} + (1 - a_1)\rho \sigma_{v2}] = 0\]

\[
\frac{\partial \ell}{\partial w_{S2}} = 2w_{S2} a_2 \sigma_{v2}^2 + 2w_{B2} a_2 b_2 \sigma_{v2}^2 + 2w_{B1} a_2 b_1 \rho \sigma_{v1} \sigma_{v2} \]

\[+ 2w_{S1} a_1 a_2 \rho \sigma_{v1} \sigma_{v2} - \lambda_1 - \lambda_2 a_2 \sigma_{v2}[(1 - a_1)\sigma_{v2} + a_1 \rho \sigma_{v1}] = 0\]

\[
\frac{\partial \ell}{\partial w_{S2}} = 2w_{S2} a_2 \sigma_{v2}^2 + 2w_{B2} a_2 b_2 \sigma_{v2}^2 + 2w_{B1} a_2 b_1 \rho \sigma_{v1} \sigma_{v2} + 2w_{S1} a_1 a_2 \rho \sigma_{v1} \sigma_{v2} \]

\[-\lambda_1 - \lambda_2 a_2 \sigma_{v2}[(1 - a_1)\sigma_{v2} + a_1 \rho \sigma_{v1}] = 0\]

\[
\frac{\partial \ell}{\partial w_{B2}} = 2b_2^2 \sigma_{v2} \sigma_{v2} w_{B2} + 2a_2 b_2 \sigma_{v2}^2 w + 2w_{S1} a_1 b_2 \rho \sigma_{v1} \sigma_{v2} \]

\[+ 2w_{B1} b_1 b_2 \rho \sigma_{v1} \sigma_{v2} - \lambda_1 - \lambda_2 \sigma_{v2} b_2[(1 - a_1)\sigma_{v2} + a_1 \rho \sigma_{v1}] = 0\]

\[\sum_{i=1}^{2}(w_{Si} + w_{Bi}) = 1 \]

and the non-negativity constraint. In matrix form,
\[
\begin{pmatrix}
X_1 \\
Y_1 \\
Y_1' \\
\theta
\end{pmatrix} = c
\]

where

\[
X_1 = \begin{pmatrix}
2a_1^2\sigma_{V1}^2 \\
2a_1 b_1 \sigma_{V1}^2 \\
2a_1 b_1 \sigma_{V1}^2 \\
2a_1 b_1 \sigma_{V2}^2 \\
2a_1 b_2 \sigma_{V1}^2 \\
2a_2 b_1 \sigma_{V1}^2 \\
2a_2 \sigma_{V1}^2 \\
2a_2 \sigma_{V1}^2 \\
2a_2 \sigma_{V2}^2 \\
2b_2 \sigma_{V2}^2
\end{pmatrix}
\]

\[
Y_1' = \begin{pmatrix}
1 & a_1 \sigma_{V1} [a_0 \sigma_{V1} + (1 - a_0) \rho \sigma_{V2}] \\
1 & b_1 \sigma_{V1} [a_0 \sigma_{V1} + (1 - a_0) \rho \sigma_{V2}] \\
1 & a_2 \sigma_{V2} [(1 - a_0) \sigma_{V2} + \alpha \rho \sigma_{V1}] \\
1 & b_2 \sigma_{V2} [(1 - a_0) \sigma_{V2} + \alpha \rho \sigma_{V1}]
\end{pmatrix}
\]

and

\[
c' = (0, 0, 0, 1, \bar{\gamma}).
\]
Finally, consider the $X_1$ matrix; its determinant is given by

$$|X_1| = (2^2 a_1 b_1 a_2 b_2 \sigma_{\nu_1}^2 \sigma_{\nu_2}^2)^2 \begin{vmatrix} 11 & \rho \rho \\ 11 & \rho \rho \\ \rho \rho & 11 \\ \rho \rho & 11 \end{vmatrix} = 0$$

Thus, as posited in the lemma, the matrix \(\begin{pmatrix} X_1 & Y_1 \\ Y_1 & 0 \end{pmatrix}\) has singular matrices along the block-diagonal, indicating a redundancy in the control variables.

Q.E.D.
Bibliography


