Competitive Pricing of Demand Deposits

by

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Abstract

The existing literature on the pricing of demand deposits demonstrates that regulation or other departures from perfect markets are *sufficient* to account for observed bank pricing behavior, in particular the failure of banks to pay interest rates on demand deposits equal to the market rate of interest. We show that neither of these assumptions are *necessary* to explain that behavior. Using a model that is similar to one introduced by Fischer (1983), but in which consumer labor input to checking transactions is explicitly modelled, we find that the pricing separation between deposits and the transactions aspect of banking that was the centerpiece of Fischer's results does not prevail. This separation in pricing fails to appear as the only optimal solution and, in fact, turns out to be suboptimal for consumers.
I. Introduction

Commercial banking activities have been modeled from a variety of perspectives (see Baltensperger (1980) or Santomero (1984) for critical surveys). One objective of the theoretical models has been to explain various aspects of observed institutional behavior. We are particularly interested in rational explanations of demand deposit pricing by banks. It has been suggested (e.g. Black (1975), Mitchell (1979)) that, in an environment characterized by perfect competition and absence of interest rate ceilings, it would be optimal for banks to pay market interest rates on demand deposits and to recover the full cost of demand deposit services through service charges. The implication is that observed pricing practices that are at odds with this optimum can only be explained by appeals to market imperfections (e.g. monopoly power) or to distortions introduced by regulation. In this vein, Startz (1983) makes a case against assuming that deposit markets are competitive and develops a model of monopolistic competition to interpret the frequently observed practice of providing services below cost. In contrast, Flannery (1982) uses account set-up costs to explain how bank deposit rates can be in excess of negotiable CD rates.

The literature demonstrates that regulation or other departures from perfect markets are sufficient to account for observed bank pricing behavior. We propose to show that neither of these assumptions may in fact be necessary to explain that behavior. We choose a perfectly competitive setting for our model and attempt to show that, contrary to what has been conjectured in this setting, deregulation of demand deposit interest rate ceilings will not necessarily lead to payment of interest on demand deposits at the market rate available on non-checkable assets. Our chief insight into this problem and the source of the differential results that we obtain is that consumers of banking services also make a labor contribution to the production of these services. The household production aspects of consumption have been recognized since Becker's (1965) pioneering work on the subject. Chapter 2 of Baxter, Cootner and Scott (1977) contains a graphical model of deposit pricing that triggered our interest in this problem, but to our knowledge their insight has not been fully exploited in a formal model of demand deposit pricing.

The work most closely related to ours is contained in an article by Fischer (1983) in which he developed a dynamic model of banking in a general equilibrium framework. His model, though different in focus, defines our point of departure. In view of its relevance, we shall briefly recapitulate the main features of his model and comment on what we consider to be its limitations.
A typical household in Fischer's model chooses the size of its currency holdings and the time to be spent on cash purchases to maximize the discounted sum of the utility of consumption \( U(C_t) \) over an infinite horizon. This choice is subject to a labor constraint whereby a given amount of labor, \( L \), is allocated between work, \( L_{M} \), and making purchases for consumption, \( L_{C} \). Banks, which hold all non-currency assets and provide transactions services, choose the size of operation to maximize profits. The first order condition for an optimum calls for the payment of the market interest rate on deposits. In addition, a zero profit condition equates the cost of providing transaction services to its price. Both results are typical of the previously noted consensus of the literature on the subject.

However, the treatment of the household production aspects in Fischer's model is somewhat one-sided. While he allows the dollar volume of cash purchases to depend on household labor, no household labor is assumed necessary for purchase transactions effected through banks. Both symmetry and empirical observation would suggest that transactions through banks should likewise depend on a household labor input.

Moreover, Fischer chooses the bank's capital as the size variable that serves as control in profit maximization. We will show that this choice together with the specification of a per dollar charge for services (as opposed to a more general service charge formulation) leads to the separation between the financial and transactional aspects of the pricing decision. Payment of market interest rates on deposits and full-cost pricing of services follows as a consequence.

Further, as Fischer explicitly states, the separation between the deposits and the transactions aspect of banking follows from the assumption that there is no necessary link between these two aspects. This absence of linkage is reflected in the manner in which the size of deposits with the bank is kept out of either part of his model: on banks' side, it is assumed not to enter the production function, and on consumers' side it is neither a control variable, nor an argument of the function that captures the volume of checking transactions. The fact that asset management and transactions services have traditionally been provided by the same institution may have originated, as he discusses, in information advantages (on depositors' ability to pay) or cost advantages (one or both parties to a given transaction may be a depositor with the bank). Whatever the manner in which the practice emerged, the empirical evidence is incontestable that deposits are in fact an input to the transactions volume effected through checking. If we further modify the modeling to make room for the other important input to the production of checking transactions, the household labor, then, as we subsequently demonstrate, not only does the separation in pricing of the two aspects fail to appear as the only optimal solution, it in fact turns out to be suboptimal for consumers.
Finally, Fischer contrasts "high-powered money" economies and "inside money" economies and focuses on technical change on the production side of his model to generate this contrast. We propose an alternative way of considering the same question by admitting corner solutions in the household optimization problem. This allows for "high-powered money" and "inside money" sectors within an economy and may be relevant to the question of providing so-called "lifeline banking services."

In the next section, we present our alternative model. The objective is to analyze demand for demand deposits taking into consideration the fact that consumption of goods and services is necessarily attended by time costs of making purchase transactions. The magnitude of these costs is assumed to differ depending on whether the transactions are carried out via checks or via cash.

II. The Model

A. Depositors

Depositors are assumed to be able to specify exogenously the total number of transactions that they engage in over a given period to achieve their consumption goals for that period. This total number of transactions, \( Q \), can be effected either by cash or by check. The number of cash transactions, \( T \), is given by a nicely-behaved function, \( T(M,L_C) \) where \( L_C \) is the total amount of time expended on cash transactions, and \( M \) is the amount of cash. It is reasonable to assume that \( T_1 \geq 0, T_2 \geq 0 \), and that \( T_{11} \leq 0, T_{12} \leq 0 \). On the other hand, transactions via checks presuppose a strictly positive level of deposits in the bank, and their number, denoted \( S \), is given by function \( S(D,L_B) \), where \( D \) is the amount of deposits, and \( L_B \) is the total amount of time to be allotted to check transactions. Again, it is reasonable to assume that \( S_1 \geq 0, S_2 \geq 0, S_{11} \leq 0, S_{12} \geq 0, S_{22} \leq 0 \) and \( S_{11} \leq 0 \), where the sign of the last derivative captures the substitutability of deposits for some of the checking transaction time; for example, the fact that large balances will lower monitoring time. The total cost of producing the predetermined number of transactions, \( Q \), has four components:

\[
\begin{align*}
\Delta(D) & = \text{the opportunity cost per unit of deposits, which may depend on the level of deposits,} \\
f & = \text{a fixed, periodic service charge for a deposit account,} \\
c & = \text{a service charge per check transaction,} \\
w & = \text{unit cost of time allotted to transactions.}
\end{align*}
\]
For the analysis here, it is assumed that $\Delta(D) = r_M - r_B$, a constant, which represents the difference between the market riskfree rate of interest, $r_M$, and the rate of interest paid on demand deposits, $r_B$. Then the depositor's problem is to choose $L_C$, $M$, $L_B$, and $D$ so as to

$$\text{MIN} \quad \{(r_M - r_B) \cdot D + w \cdot (L_C + L_B) + r_M \cdot M + c \cdot S(D, L_B) + f \cdot I(D)\}$$

subject to: $S(D, L_B) + T(M, L_C) = Q$

where $I(D) = \begin{cases} 
0 & D = 0 \\
1 & \text{otherwise}.
\end{cases}$

The choice between the two competing transactions technologies is parameterized by the transactions volume, $Q$, and the unit cost of time, $w$, which characterize depositors; by $r_B$, the rate paid on deposits, $c$, the per check charge, and $f$, the per account charge per period, which constitute the pricing policy of the bank; and by $r_M$, the rate paid in the market on those deposits that do not produce transactions.

Before we go on to state and interpret the Kuhn-Tucker conditions, a comparison with the Fischer's setting may be in order. We seek to derive the optimal checking balance and cash mix by means of cost minimization rather than by utility maximization, as in Fischer. Cost minimization in our opinion, brings into a sharper focus the household production aspects of consumption, which are captured in the functions $S(D, L_B)$ and $T(M, L_C)$. It may also be noted that in his model the Kuhn-Tucker conditions are entirely in terms of the derivatives of transactions producing functions, and utility derivatives enter only to characterize the dynamic condition on consumption. This dynamic condition does not bear upon the issue of concern here, the demand for demand deposits. Further, it is the labor constraint in Fischer's model which brings the time cost of transactions to bear on the optimal solutions through the wage rate. A cost minimization specification allows us to take time cost directly into consideration.

(i) Interior Solution

For an interior optimum, it is necessary that

$$\frac{r_M}{T_1} = \frac{w}{T_2} \quad (M > 0, L_C > 0) \quad (1)$$

$$\frac{(r_M - r_B) \cdot S_1}{S_2} = w \quad (D > 0, L_B > 0) \quad (2)$$

$$\frac{(r_M - r_B) \cdot S_1 + c}{S_2} = \frac{r_M}{T_1} \quad (D > 0, M > 0) \quad (3)$$

$$\frac{w}{S_2} + c = \frac{w}{T_2} \quad (L_B > 0, L_C > 0) \quad (4)$$
In general, these first-order conditions lend themselves to standard marginal interpretations in a straight-forward manner. However, one of the standard conditions of efficiency in production is that the marginal rate of substitution between inputs should be the same across technologies. In the context of our model, this would require

\[ \frac{T_1}{T_2} = \frac{S_1}{S_2} \]

which does not hold for \( r_B > 0 \) because by (1) and (2)

\[ \frac{S_1}{S_2} = \frac{(r_M - r_B)}{w} < \frac{r_M}{w} = \frac{T_1}{T_2} \]

The usual equality of the marginal rates of input substitution results from the fact that each factor usually commands a single price common across differing technologies. In our context cash and deposit balances have an opportunity cost that differs across the two technologies for \( r_B > 0 \). It is interesting to note, however, that since

\[ \frac{S_2}{T_2} > \frac{S_1}{T_1} \]

the ratio of the marginal product of labor in checking transactions to the marginal product of labor in cash transactions is higher than the same ratio for checking and cash balances. At the optimal allocation, therefore, labor has a comparative advantage over deposit balances in producing checking transactions. This is because with \( r_B > 0 \) more balances are shifted to deposits than would have been optimal if the cost of balances were the same across the two technologies. What also follows from \( \frac{S_1}{S_2} = \frac{(r_M - r_B)}{w} \) is that if \( r_B \) were to equal \( r_M \), labor would have to become infinitely more efficient at producing checking transactions than deposits.6

(ii) "No-Cash" Solution

The Kuhn-Tucker conditions necessary for a "no-cash" solution are

\[ M = 0, \quad r_M > \lambda T_1 \]
\[ L_c = 0, \quad w > \lambda T_2 \]

where \( \lambda \), the Lagrange multiplier for the transactions volume constraint, is given by the interior solution with strictly positive checking activity (\( D > 0, \ L > 0 \)) as

\[ \lambda = \left( \frac{w}{S_2} \right) + c \]
\[ \lambda = \frac{(r_M - r_B)}{S_1} + c + f \cdot \delta(D) \]

and \( \delta() \) is Dirac's delta function defined as

\[ \delta(x) = \begin{cases} \infty & x = 0 \\ 0 & \text{otherwise.} \end{cases} \]
Therefore, taken together with interior checking, the necessary conditions for "no-cash" solution are

\[
(r_M - r_B)/S_1 + c < r_M/T_1
\]

and

\[
c < w \cdot (1/T_2 - 1/S_2)
\]

Utilization of the cash technology by all consumers can be strongly argued in the context of condition (6). \((1/T_2)\) and \((1/S_2)\) can be thought of as the time spent on the marginal cash and checking transactions, and condition (6) can be interpreted to state that, for a no-cash solution, it is necessary that the per check charge is less than the dollar value of time saved by opting for checking rather than cash technologies. As there will always be transactions that can be more expeditiously carried out by cash (e.g. bus rides), \((1/T_2) < (1/S_2)\) will always be true of some transactions so that even if \(c\) were set at zero, (6) will be violated, contrapositively implying occurrence of some cash transactions.

(iii) "No-Checks" Solution

The Kuhn-Tucker conditions necessary for a "no-checks" solution, together with those for strictly positive cash technology variables \((M > 0, L_C > 0)\) are

\[
D = 0, \quad (r_M - r_B)/S_1 + c + \delta(D)/S_1 > r_M/T_1
\]  

\[
L_B = 0, \quad w/S_2 + c > w/T_2
\]

where \(\delta(\cdot)\) is as previously defined.

The no-checks solution will prevail for individuals for whom the value of time devoted to transactions is low and the marginal productivity of time devoted to checking transactions at \(S = 0\) is less than or equal to the marginal productivity of time devoted to cash transactions when all transactions are in cash.

The need for "lifeline banking services" is one of the issues that the deregulation of the banking industry, with the resulting increase in service charges, has brought to the fore. There has been concern that as explicit pricing of checking services replaced the implicit cost due to deposit interest rate shortfalls, certain sections of the population may have been priced out of the banking sector. The previous analysis suggests that transacting outside the banking system may be the norm for some low income consumers. The strongest case for lifeline checking services for low income individuals (or for some substitute for those services) can be made if there are transactions for which the marginal productivity of time devoted to checking is substantially greater than the marginal productivity of time via cash transactions, i.e. where inequality (8) fails to hold at \(S = 0\) even though inequality (7) does hold.
B. Banks

(i) *Price-Taking Behavior*

Banks in the economy are assumed to be identical, and they choose the size of their operations to maximize net profit. Thus, if we denote by $N$ the number of accounts served, then the total costs of producing banking services, $G$, can be thought to depend upon both the number of accounts served, and the number of checks processed, i.e., $G = G(N,N \cdot S)$. The profit maximization problem, therefore is to choose $N$ so as to

$$\text{MAX} \ \{N \cdot \left[(r_m - r_B) \cdot D + c \cdot S + f\right] - G(N,N \cdot S)\}$$

The first-order condition necessary for a maximum is

$$(r_m - r_B) \cdot D + c \cdot S + f = G_1 + G_2 \cdot S$$

or

$$r_m - r_B = \left(1/D\right) \left[G_1 - f + S \cdot (G_2 - c)\right]$$

and zero-profit competitive condition is

$$N \cdot \left[(r_m - r_B) \cdot D + c \cdot S + f\right] = G(N,N \cdot S).$$

If the per account service charge, $f$, is equated to the marginal cost of serving an additional account, $G_1$, and the per check service charge, $c$, is equated to the marginal cost of an additional check processed, $G_2$, then the first-order condition reduces to

$$r_m = r_B$$

which is identical to Fischer's condition. Moreover, in that case, the zero-profit condition reduces to

$$N \cdot \left[c \cdot S + f\right] = G(N,N \cdot S)$$

or

$$N \cdot G_1 + N \cdot S \cdot G_2 = G(N,N \cdot S)$$

which is equivalent to the requirement that $G$ be a function homogenous of degree one, in which case, $N$ drops out of either side of the equation, and

$$c \cdot S + f = g(1,S)$$

where $g$ is the *per-account* cost function. Again analogous to Fischer, the scale is determined exogenously by demand, and the service charges $c$ and $f$ depend only on the per account cost function.

However, this marginal cost pricing of services, which leads to payment of the market interest rate on deposits, was exogenously imposed and not dictated by optimality. Thus, in our more general specification of the
problem, profit maximization by itself does not lead to price separation for deposits and services, even in a perfectly competitive and unregulated setting.

Since $r_M = r_B$ need not obtain in a competitive setting, we turn to the question whether it can in fact obtain as an optimum. Whatever the extent of competition in the banking market, banks can be assumed to be aware of, and therefore to take into consideration, the household labor input in the production of checking transactions, as reflected in the first order conditions of the depositors' problem. Given substitutability of labor for deposits in transactions production, as Baxter, Cootner, and Scott (1977) observed, "[the optimal behavior] is to increase balances up to the point where the opportunity cost of a further balance addition is just equal to the value of time saved by that addition." Mathematically, the depositors increase $D$ up to the point when

$$r_M - r_B = w \cdot (-dL/dD)|_S$$

(11)

where $(-dL/dD)|_S$ denotes the saving in labor attained by a marginal increase in deposits for a fixed number of checking transactions.

However, as

$$dS = S_1 \cdot dD + S_2 \cdot dL$$

we have for $dS = 0$

$$(-dL/dD)|_S = S_1/S_2$$

so that (11) amounts to

$$r_M - r_B = w \cdot S_1/S_2$$

which is a first-order condition in the previous section [equation (2)].

Clearly, if $S_1 > 0$ and $S_2 < \infty$ for all $D$, then $r_M > r_B$. However, given our starting assumptions, we cannot rule out the possibility that the function $S_1$ reaches zero for some finite $D$, and stays there for all higher values of $D$. Thus, there may very well be a finite limit on deposits beyond which they fail to substitute for labor, though this seems unlikely to be the case for all consumers. Furthermore, even if $S_1$ were to reach zero at finite $D$ for all consumers, it is unlikely that this finite level of deposits would be within the economic means of all depositors to maintain. With no cost to checking due to interest foregone, depositors transfer all such wealth as is not needed for optimal cash transactions to the checking account. However, if even at this maximum feasible $D$, $S_1$ happens to be greater than zero, then, ceteris paribus, the total cost will be higher than the total cost at some $r_B$ (less than $r_M$) that does not push depositors to seek to maintain deposits at the limit of their means. This argument is analytically demonstrated in a straightforward manner by explicitly introducing a wealth constraint of the form

$$B \geq M + D$$
where $B$ is the total wealth available over a given period to be allocated between cash balances, $M$, or demand deposits, $D$. Denoting by $\lambda_1$ the Lagrange multiplier to the original transactions volume constraint and $\lambda_2$ that of the new constraint, the first order conditions with respect to the checking technology variables are

$$D > 0, \quad r_M - r_B - c \cdot S_1 = \lambda_1 \cdot S_1 + \lambda_2$$

$$L_B > 0, \quad w + c \cdot S_2 = \lambda_1 \cdot S_2$$

which together imply that

$$r_M - r_B = \lambda_2 + w \cdot S_1 / S_2.$$ 

Since $w$, $S_1$, and $S_2$ are all clearly nonnegative, and $\lambda_2$, the shadow price of the budget constraint, is strictly positive for those depositors for whom the budget constraint is binding, it is clear that when $r_B \geq r_M$ any allocation will be suboptimal from these depositors' viewpoint. In which case, the market rate pricing of deposits can be undone by a competitor who lures away depositors by lowering their cost by means of a pricing policy which involves $r_B < r_M$, but which leaves his total profit unaffected through suitable adjustments in the other instruments of the price schedule. Therefore, in equilibrium, the rational pricing that takes depositor cost minimization into account will dictate a deposit interest rate less than the market interest rate.

**(ii) Socially Optimal Allocation**

Before going on to characterize price-setting for banks, it may be interesting to look at the allocation of labor that would be optimal if there were a social planner who brought the costs that are incurred by depositors as well as those by banks under the purview of a single optimization exercise. If the input of household labor in the production of transactions introduces distortions in optimal allocation of labor, then such a minimization of the costs in the aggregate will yield a solution that is free of such distortions. The bank rate, $r_B$, is not relevant to this problem because its impact cancels out of the social cost problem. If $i$ is an index which runs over different kinds of depositors, there being $N_i$ of each kind, then the aggregate problem is to choose $L_{Bi}$ and $L_{Ci}$ to

\[
\text{MIN} \{ G(\Sigma_i N_i, \Sigma_i S_i, S(D_i, L_{Bi})) + r_M \cdot \Sigma_i N_i \cdot M_i + \Sigma_i N_i \cdot w_i \cdot (L_{Bi} + L_{Ci}), \Sigma_i \lambda_i \cdot N_i \cdot (Q_i - S(D_i, L_{Bi}) - T(M_i, L_{Ci})) \}
\]

Then for an interior solution it is necessary that

$$L_{Bi} > 0, \quad G_2 \cdot S_2 + w_i = \lambda_i \cdot S_2$$

$$L_{Ci} > 0, \quad w_i = \lambda_i \cdot T_2$$
However, it may be recalled from section A above that for each type \(i\), the first order conditions for a solution to the consumers' minimization problem are satisfied when

\[
c \cdot S_2 + w_i = \lambda_i \cdot S_2
\]

\[
w_i = \lambda_i \cdot T_2
\]

Clearly, for the individual allocation to be socially optimal, we must have

\[
c = G_2.
\]

Conversely, when \(c \neq G_2\), there must be inefficiency either on the banks' side or on the depositors' side of the problem. It is not surprising that the social optimum is characterized entirely in terms of \(c\). This is because the per account charge \(f\) leaves marginal behavior unaffected and, as noted earlier, \(r_B\) cancels itself out of aggregate consideration.\(^{12}\)

The social optimum may also be characterized by the relation between consumer utilization of labor and the efficient use of resources by the bank. An interior solution to the social optimization problem implies that

\[
G_2 = w_i (1/T_2 - 1/S_2) \text{ for all } i
\]

According to equation (13), each consumer will adjust his utilization of labor in producing transactions so that the increase his marginal labor cost from shifting from checking to cash transactions equals the bank's marginal cost of producing an additional checking transaction. Suppose that the bank's per transaction marginal cost, \(G_2\), was to increase. Consumers' checking transaction cost, \(c\), would increase inducing a shift from checking transactions to cash transactions, thereby reducing \(T_2\) and increasing \(S_2\) until the equilibrium condition given by (13) was restored.

(iii) **Price-Setting Behavior**

We now allow the bank the flexibility of choosing a price schedule, \([r_B, c, f]\) so as to maximize its profits by taking into account the inverted demand functions of the depositors as given by their cost minimization problem. Now, in general, both the number of accounts, and the balance deposited in each account may respond to changes in the price schedule. However, if each bank in its given geographical area can offer different pricing schedules (kinds of accounts) to suit the needs of depositors with different wages and transactions volumes (different "products" for different "segments"), then depositors will not have an incentive to go outside their area to seek a more suitable pricing schedule. Therefore, we can think of each bank working as a perfectly discriminating monopolist within the area of its geographical draw.
However, quite apart from the issues of legality, implementation of explicit perfect first-degree price discrimination (climbing down the demand curve by means of a different price for each customer) is likely to be fraught with informational issues having to do with, say, revelation of the depositor type. Therefore, while we do assume that the total number of depositors that a bank has is exogenously given, we postpone until later a more detailed discussion of implicit price discrimination. For now, the bank chooses a single price schedule uniform across all depositors within its area.

Let the index $i$, as before, distinguish the depositor type, let $N_i$ be the number of depositors of type $i$, and let

$$D_i = D(r_B, c, f, w, Q_i, r_M)$$

$$L_i = L(r_B, c, f, w, Q_i, r_M)$$

and

$$S_i = S(D_i, L_i).$$

Then the bank chooses $r_B$, $c$, and $f$ to

$$\text{MAX} \{ \Sigma_i N_i [(r_M - r_B) \cdot D_i + c \cdot S_i + f] - G(\Sigma_i N_i, \Sigma_i N_i \cdot S_i) \}$$

To keep the notation simple without loss of generality, we consider the bank’s pricing behavior where there are only two depositor types. In this circumstance the first order conditions for profit maximization are

$$r_B > 0, (r_M - r_B) \cdot [N_1 \cdot (\partial D_1 / \partial r_B) + N_2 \cdot (\partial D_2 / \partial r_B)] - (N_1 \cdot D_1 + N_2 \cdot D_2)$$

$$+ (c - G_2) \cdot [N_1 \cdot (\partial S_1 / \partial r_B) + N_2 \cdot (\partial S_2 / \partial r_B)] = 0$$

(14)

$$c > 0, (r_M - r_B) \cdot [N_1 \cdot (\partial D_1 / \partial c) + N_2 \cdot (\partial D_2 / \partial c)] + (N_1 \cdot S_1 + N_2 \cdot S_2)$$

$$+ (c - G_2) \cdot [N_1 \cdot (\partial S_1 / \partial c) + N_2 \cdot (\partial S_2 / \partial c)] = 0$$

(15)

The first two terms in (14) can be interpreted as the marginal change in profit due to a change in the deposit interest rate arising from the depositary aspect of operations alone. However, a change in $r_B$ also leads to changes in optimal $D$ and $L$, and eventually to a change in the number of checking transactions. The last term in (14) captures the marginal change in profit due to a change in number of checking transactions provided. Similarly, the first term in (15) accounts for the interest that could have been earned on the marginal decrease in deposits that occurred on account of an increase in $c$, while the last two terms represent marginal changes in service charges recovered due to the same increase in $c$.

From (14) it follows that setting the bank rate at or beyond the market rate will be necessarily accompanied by a per check charge strictly higher than the marginal cost $G_2$. With this price schedule, it may be recalled that the allocation of labor across technologies is suboptimal for all types of depositors. Equation (14) also shows that, at the socially optimal $c = G_2$, it is necessary that $r_B < r_M$. In fact, we can solve for the exact difference as
\[ r_M - r_B = \frac{N_1 \cdot D_1 + N_2 \cdot D_2}{N_1 \cdot (\partial D_1 / \partial r_B) + N_2 \cdot (\partial D_2 / \partial r_B)} = \frac{N_1 \cdot S_1 + N_2 \cdot S_2}{N_1 \cdot (-\partial D_1 / \partial c) + N_2 \cdot (-\partial D_2 / \partial c)} \]

It is therefore necessary for efficiency that the extent of underpricing of deposits should be equal both to the ratio of average balances to the average sensitivity of balances to interest rate changes, and to the ratio of average number of checking transactions to the average sensitivity of deposits to changes in the per transaction charge.

While \( c = G_2 \) is socially optimal, all that the analysis so far has shown is that, given other instruments of pricing, \( c = G_2 \) is not inconsistent with profit maximization as characterized by (14) and (15); there is nothing in (14) and (15) that prevents the bank from setting \( c \neq G_2 \) and maintaining maximal profit. But, if \( c \neq G_2 \) does not lower bank’s profits, then the inefficiency that it leads to must occur by way of a rise in depositors’ cost. If depositors’ costs are higher, then there will be some depositors who will seek out a competitor beyond the geographical domain with a pricing schedule such that \( c = G_2 \). Thus, \( c \neq G_2 \) will not endure as a profit-maximizing solution.

The fact that the efficient and socially optimal allocation should be attainable in what is effectively a monopolistic setting without any kind of price discrimination at the same time appears to be counterintuitive at first. However, as depositors differ in the value they put on their household labor, the total of explicit and implicit costs of accessing banking services does differ across depositors, and an implicit Pigovian discrimination, as in Baxter, Cootner, and Scott’s (1977) geometrical model, does in fact occur in terms of what they call the “delivered price” of banking services.

However, if an explicit discrimination were both legal and feasible, then as the profit function is additively separable in depositor types, the first order conditions reduce to

\[ (r_M - r_B) \cdot \partial D_i / \partial r_B - D_i + (c_i - G_2) \cdot \partial S_i / \partial r_B = 0, \text{ all } i \]  
\[ (r_M - r_B) \cdot \partial D_i / \partial c_i - S_i + (c_i - G_2) \cdot \partial S_i / \partial c_i = 0, \text{ all } i \]

Now, for reasons similar to those discussed earlier

\( c_i = G_2, \text{ all } i \)

and the discriminatory bank rate will be given by

\[ r_M - r_B = D_i / (\partial D_i / \partial r_B) = S_i / (-\partial D_i / \partial c_i). \]

Although in the earlier setting the uniform rate differential was determined by population averages, explicit discrimination leads to setting a specific rate differential for each type of depositor. An alternative way of looking at this problem is to think of the per check charge \( c \) as that component of a two-
part tariff which varies with the production level, while $r_m - r_B$ and $f$ are lump-sum charges extracted in return for access to the checking technology. Then, as Oi (1971) has demonstrated, the optimal solution for a discriminating monopolist, and one which is equivalent to Pigovian first-degree discrimination and, as such, is socially efficient, is to set the variable component of the two-part tariff equal to the marginal cost of production. This would of course make it uniform across all consumers. The monopolist, however, charges each consumer a different lump-sum fee totally exhausting his or her consumer surplus. The social efficiency of $c = G_2$ is thus in agreement with Oi's proposition, and whether or not $r_B$ and $f$ are allowed to differ across depositors, the delivered price of a transaction, depending as it does on the input of depositors' labor, can be thought of as perfectly discriminatory.

### III. Conclusion

In concluding, we recapitulate briefly our main results. The characteristics of depositors' problem make disappearance of cash technology relatively unlikely. Household production aspects invalidate the conjectured separation between depository and transactional aspects of banking, and cost is minimized at least for some depositors when the deposit interest rate is lower than the market interest rate. If this underpricing is taken into consideration on the bank's side, then together with the socially efficient marginal pricing of checking transactions $c = G_2$, a zero profit condition would force the per account charge below its marginal cost ($f < G_1$). Thus, market rates on balances and marginal cost pricing of services are not only not necessarily implied by perfect competition, they in fact do not occur as an optimal. Finally, if banks can be thought of as operating as a discriminating monopolist within their geographical area, then either explicit or implicit Pigovian perfect discrimination makes the socially efficient allocation feasible, and the underpricing of deposits can be exactly determined.
Endnotes

1. This amounts to assuming that the choice of the consumption schedule for utility maximization determines $Q$ without being affected by the cost of effecting the transactions.

2. For analytical tractability, we assume that $Q$ (and its component functions $S$ and $T$) and $N$ are infinitely divisible.

3. This will be true, for example, if increasing $L_C$ involves going to increasingly distant shops, so that while $T$ increases with $L_C$ it does so at a decreasing rate.

4. This assumes that the replenishment costs incurred between securities and bank deposits will be equal to those due to similar transactions between securities and cash, so that they can be left out.

5. If $S$ and $T$ are concave, i.e., in addition to the signs of partials already noted, if we also assume that

$$S_{11} S_{22} - (S_{12})^2 > 0$$
$$T_{11} T_{22} - (T_{12})^2 > 0$$

then depositors have a well-defined problem in the sense that the bordered Hessian matrix of second order partial derivatives is positive semi-definite so that the second order sufficiency conditions are satisfied. The concavity of the technological functions agrees with intuition to the extent that it is reasonable to assume that increasing either input would increase the number of transactions, but would do so at a decreasing rate. Moreover, since $S_{12} \geq 0$ captures the substitutability between deposits and labor (e.g., higher balances lead to lower monitoring time), it is reasonable that for low values of $D$ the inequality will be strict. As there is no reason to suppose why such substitutability should also obtain in cash technology, only $T_{12}$ may very well be zero over the entire range of $M$ or $L_C$.

6. We reserve further discussion on the possibility and optimality of $r_M = r_B$ until the next section.

7. We pause here to discuss how our modeling differs from Fischer. As noted above, the only service charges in Fischer’s model were those levied on a per dollar basis. The total dollar amount of banking services in his model depends on size of operations through labor and capital inputs. The notion of size of operation is captured in the productive assets, or the loans made by the bank, which is one of the variables over which the profit maximization is carried out. The critical difference in our modeling is that we directly bring into the picture the number of accounts served. By
letting this number be the only variable that has to do with the size of the operations, we again emphasize the service aspect of banking operations.

8. p. 27.

9. The alternative to $S_1 = 0$ is $S_2 = \infty$ for some $D$ and $L$, and as $S_{22} \leq 0$, $S_2$ decreases in $L$ so that we must have $S_2 = \infty$ at $L = 0$, which lacks intuitive appeal.

10. The socially optimal allocation is also relevant as a standard against which other possibly suboptimal solutions, such as those arising in a price-setting situation, are measured.

11. For notational simplicity, we are assuming that $S$ and $T$ are functions that are exogenously given for all depositors, their shape being determined by factors that equally affect all consumers. However, even if their shape differed for each kind of depositor, our results would not be effected. Moreover, $D_i$ and $M_i$, which may be subject to individual budget constraints, are not variables of concern in the social problem. The goal of the social optimization is efficient allocation of household labor.

12. However, it remains true that the consumer equilibrium will still, in general, require $r_M > r_B$.

13. This follows since the partials with respect to $r_B$ in equation (14) are all positive.

14. The partials with respect to $c$ in equation (15) are all negative.
References


