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Capital Controls and Bank Risk

Gerard Gennotte and David Pyle

Haas School of Business
University of California at Berkeley

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Abstract

We analyze the effects of more stringent capital controls on the behavior of a bank that maximizes shareholder value where there are deposit guarantees and imperfect regulatory control of the risk of the bank’s assets. In contrast to earlier work, we take the view that loan evaluation costs and loan monitoring costs make bank loans intrinsically different from zero NPV investments (e.g., market securities). We show that deposit guarantees lead banks to engage in inefficient investment, and that there are plausible circumstances in which an increase in capital requirements will result in a decrease in the level of investment, but an offsetting increase in asset risk. Furthermore, there are situations in which the asset risk increase leads to an increase in the probability of default.

In some cases, the increase in capital reduces the probability of default and improves the economic efficiency of bank lending, but not by as much as if the risk offset did not occur. In other cases, the probability of bankruptcy increases with a tightening of the capital constraint, leading to higher expected costs associated with the transfer of assets to regulatory agencies and with financial instability.

Increases in bank capital are not a substitute for the monitoring and control of asset risk by the bank regulator and may imply an increased need for such surveillance.
Introduction*

The key role played by commercial banks in developed economies and the memory of the bank runs of the Great Depression have led to extensive regulation of these financial institutions by most governments. The social costs of bank failure have induced governments to provide financial support and other forms of protection from failure. For example, in the U.S., the FDIC explicitly guarantees the principal of bank deposits (up to a limit that has varied over time) and, through their closure policies, U.S. regulators have implicitly guaranteed all deposits in large banks; in other cases such as postwar France, the government nationalized major banks. It is indeed commonly perceived that bank deposits should be riskless in nominal terms and that government guarantees, explicit or implicit, are directed toward this end.\(^1\) If such deposits were risky, the aggregate costs of gathering information related to their risk by a large number of small investors would be large, creating the need for a more efficient monitoring procedure. By guaranteeing deposits, governments have redistributed bankruptcy risks but have also been led to play the role of principal in order to prevent agents --commercial banks-- from generating excess profits at the social cost of increasing the risk of and losses due to failure. The recent financial distress experienced by financial

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\(^1\) See Diamond and Dybvig (1986) for a synthesis of banking theory that supports this perception.
institutions in the U.S. and elsewhere has intensified the debate over the role of regulatory agencies.

Corporate finance scholars and agency theorists have long recognized the inefficiency of simple debt contracts in cases where the likelihood of bankruptcy is large. Since bondholders ultimately bear the costs of bankruptcy, it is in the interest of shareholders -- and hence of managers who represent shareholders -- to increase investment risk in the hope of receiving a high return if the economy evolves favorably while limiting their losses if it does not. These "financial distress" games by stockholders being anticipated, the required interest rate on corporate debt includes a risk premium and the debt contract includes a variety of covenants which restrict the actions of managers. By guaranteeing deposits, the government allows banks to offer a riskless or nearly riskless rate on their deposits. In the absence of covenants so strict that they essentially rule out bankruptcy, this constitutes a windfall profit, which is larger if the bank faces larger risks on its investments.

A value maximization framework has been frequently used in previous research on the effects of deposit guarantees. Proponents of this analytic framework argue that the managers of a widely-held financial institution will choose investments so as to maximize the value of the deposit guarantee. Equivalently, the costs (net of monitoring costs) borne by regulatory institutions (and the taxpayer) are maximized in equilibrium. Deposit guarantees without effective restrictions on the recipient's investment policy constitute a transfer from the government to bank shareholders or bank customers or both. Furthermore, the deadweight costs incurred in bankruptcy make such a contract economically inefficient. Regulation of bank capital and bank portfolio restrictions have been introduced to reduce the incentive for banks to undertake highly risky positions.
Capital controls limit the banks' ability to lever their investment portfolios. But, banks may then have an incentive to shift their investments to riskier assets, thereby increasing the risk of bankruptcy and at least partially defeating the purpose of capital controls. The question of a risk offset to tighter capital regulation has been addressed in the academic and professional literature and conflicting answers have been provided. Kahane (1977) and Koehn and Santomero (1980) demonstrate that higher capital requirements may induce a bank to increase its asset portfolio risk. The validity of these results were challenged by Furlong and Keeley (1987 and 1989) and Keeley and Furlong (1990) who make the important point that the mean-variance framework employed in the earlier studies is inappropriate because, in the case of insolvency, asset return distributions are truncated. Insolvency is a central element in the bank's decision problem. After taking account of the truncation in the return distribution of banks that hold market assets, Furlong and Keeley go on to show that imposing stricter leverage limits unambiguously results in a decrease in total bank risk with no increase in asset risk. Our objective is to reopen that debate by considering the asset risk-leverage trade-off when bank assets are not restricted to be zero net present value investments.

Our results in the subsequent analysis, may be contrasted with those of Furlong and Keeley (1987). We show that the inclusion of non-zero net present value investments in the bank's opportunity set has a profound effect on the efficiency of regulation. In particular, banks may not choose the highest available risk level, increases in capital requirements can lead to an increase in asset risk and in the probability of bank failure.
II Value maximization in a static model

In this section, we develop a model of the bank's investment decision. We assume that bank managers seek to maximize the market value of its equity. Deposit insurance allows the bank to borrow at a subsidized rate and this affects equity value and the investment decision. We show that it induces risk-taking and, more importantly, inefficient investment.

Our analysis departs significantly from previous research by relaxing the key assumption that banks invest in zero net present value projects (i.e. traded assets). In our view, banks differ from other agents in their ability to identify and efficiently manage real project financing. The net present value of investments available to banks thus depends on competitive conditions in the loan market. We assume that the bank's investment opportunity set is a set of loan portfolios. The present value of the payoffs of loan portfolios and their probability distribution affect the bank's decision. To keep the analysis tractable, we assume that the distribution of loan portfolio payoffs are characterized by their present value \( v \) and by their risk index \( \sigma \). We denote \( J(\sigma, v) \) the net present value to the bank of the asset whose cash flows have a present value \( v \) and a risk index \( \sigma \).

In the absence of deposit insurance, banks are indifferent between financing investments through equity or by raising deposits. Banks thus seek to maximize the net present value of the asset portfolio, \( J(\sigma, v) \).

Governmental intervention in the form of deposit guarantees modifies the bank's optimization problem. Assume now that the bank raises an amount \( D \) of deposits, with \( D' \) being the amount due to depositors at a future date \( T \). The amount available for investment \( I \) is now equal to the sum of the bank's shareholders
capital and the amount borrowed D. At maturity, if assets exceed the amount owed, D', bank's shareholders receive the balance. In the other case, the government takes over, repays depositors and the shareholders receive nothing. The government's guarantee represents a subsidy because it allows the bank to borrow at the riskless rate r: $D' = D \exp(rT)^2$. On behalf of shareholders, bank managers choose the scale $v$ and the risk index $\sigma$ which maximize the net present value of their claim on the bank, inclusive of the value of the subsidy:

$$V(\sigma,v,D) = (C(\sigma,v,D') - v + D) + J(\sigma,v) \quad \text{(II-1)}$$

where $C(\sigma,v,D')$ denotes the value of an European call on an asset currently worth $v$ with risk characteristics $\sigma$, a strike price of $D'$, and maturity $T$. ³

The net present value of the bank is equal to the sum of the asset net present value (the second term in II-1) and the government subsidy value (the first term). The value of the government subsidy is equal to the amount of deposits $D$ minus the present value of the promised amount $D'$ or asset value, whichever is smaller. We should note here that the subsidy value, and therefore the bank's objective function, for a given present value $v$, depends on the distribution of asset payoffs only over the states of nature in which the bank defaults. Proposition 1 states the well-known result that deposit insurance benefits banks.

**Proposition 1:**
The subsidy and hence the net present value $V$ is an increasing function of the amount of insured deposits, ceteris paribus (i.e. for a fixed asset level $v$ and risk parameter $\sigma$).

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² For simplicity, we assume that the payment promised to depositors at date $T$, not just the principal, is guaranteed.

³ See Merton (1977) for a statement of the isomorphic relation between deposit insurance and a call option on the bank's assets.
Additional deposits have two effects on equity value: first they allow shareholders to reduce capital by the corresponding amount, second they reduce the option value by increasing the strike price. For an additional deposit dollar, the bank commits to repay an additional \( \exp(rT) \) at date \( T \) if it is solvent. The likelihood of bankruptcy being positive, this commitment does not completely offset the one dollar reduction in capital. If the government did not guarantee deposits, depositors would require a risk adjusted return and additional deposits would not benefit shareholders. This property holds only when the asset level \( v \) is held constant. In other words, a substitution of deposit funds for bank capital increases the value of the bank. Depending on the increase in the asset present value associated with additional investment, it may or may not be in the best interest of shareholders to increase the amount invested by the full amount of additional deposits. Instead, shareholders might well prefer to reduce their own investment.

It follows from Proposition 1 that, if permitted, banks would choose to raise capital exclusively through deposits thereby maximizing the subsidy.

In order to reduce the subsidy size and to control banks' exposure, the government may impose capital constraints. Such constraints can take a variety of forms; to simplify we will assume that the bank is required not to exceed a fixed deposit-to-asset value ratio, \( \delta \). This constraint is binding under our assumptions. Hence, the amount of deposits corresponding to a given asset value is simply \( D = \delta v \). The bank's optimization program consists in choosing the asset value \( v \) and risk level \( \sigma \) which maximize the objective function

\[
V(\sigma, v) = C(\sigma, v, \delta \exp(rT)) - (1-\delta)v + J(\sigma, v). \tag{II-2}
\]
Under fairly general conditions\(^4\) on the stochastic process describing fluctuations in the asset value through time, the value of the European call is homogeneous of degree one in the underlying asset value and the strike price. We will rewrite the call value as follows

\[
C(\sigma, v, \delta \exp(\eta T)) = \delta v c(\sigma, \frac{1}{\delta}) . \tag{II-3}
\]

Where \(c(\sigma, S)\) denotes the value of a call on an asset whose risk characteristic is \(\sigma\) and whose current value is \(S\), with a strike price of \(1\). The call value is thus a function of the risk level \(\sigma\), of the investment \(v\), and of the ratio of deposits to the asset value \(\delta\).

If the assets available to the bank are all zero net present value investments \((J(\sigma, v) = 0)\), the first order conditions are:

\[
V_v = \delta [c(\sigma, \frac{1}{\delta}) + 1] - 1 > 0 \tag{II-4.1}
\]

\[
V_{\sigma} = \delta v c_1(\sigma, \frac{1}{\delta}) > 0 \tag{II-4.2}
\]

The inequality in (II-4.1) is verified because the option value strictly\(^5\) exceeds the present value of the underlying asset \(1/\delta\) minus the exercise price of \(1\). Inequality (II-4.2) follows from the positive monotonicity of the call function with respect to asset risk.

If the bank faced an unlimited set of investments with non-negative net present value, and was not constrained in the level of available deposits, it would seek to invest without bounds. As in Furlong and Keeley (1987), if the bank were

\(^4\) Namely that the return on the asset over the life of the option is independent of the current asset value \(v\), see Merton (1973), for example.

\(^5\) The inequality is strict unless the probability that the option expires without being exercised is zero, an uninteresting case since the subsidy value would then be zero as well.
allowed to invest in traded assets it would purchase the riskiest available assets in infinite quantities.\(^6\)

Bank loans are not zero net present value investments, however, and the return to loan investments is a function of the level of investment and of the riskiness of the assets chosen. Information gathering and contracting costs are a major component of lending costs for the type of real assets financed by lending institutions. Lending institutions have a competitive advantage with respect to capital markets in terms of information gathering costs, hence the competitors for loans are a set of lending institutions. The total investment \(I(\sigma, v)\) made by a lending institution is composed of two parts, the amount lent to the borrower plus the institution's lending costs. In the case of perfect competition among banks, the amount paid to the borrower is such that the bank's net present value is equal to zero which is to say the bank pays the owner of the real project the sum of the asset value and the subsidy minus the lending costs. In order to win the auction for the loan, the lending institution selects the investment level and risk index that maximizes the bid price which is equivalent to the maximization of the objective function in equation II-2. In the absence of competition among banks, the bank maximizes the same expression. In this case, lending terms are set at the borrower's reservation level. The only difference between the two cases is that the bank is able to retain the rent in the latter case. A full-fledged model of loan market competition is beyond the scope of this paper, we refer the interested reader to Gennotte (1990). Gennotte shows that the value of the subsidy can differ among lending institutions, and derives the competitive equilibrium in the loan market.

\(^6\) Or at least until the bank's purchases lead to a modified price equilibrium. The bank, being subsidized, would finally own all available traded assets.
In the following, the costs of loan initiation are assumed to be increasing and convex functions of the level of investment and of the risk index. $I(\sigma, \nu)$ depends on $\sigma$ and $\nu$ because of the lending costs incurred by the bank for a portfolio of loans characterized by $\sigma$ and $\nu$. The bank chooses first the loans with lower initiation costs, but as the size of the bank increases, the number of loans increases. To achieve this, eventually the bank extends beyond the loan market in which it has expertise and costs increase. Decreases in marginal costs may obtain locally but overall marginal costs increase. The bank selects the optimal amount of resources allocated to information gathering and contracting to improve the return characteristics of a given loan. But the resulting costs tend to be larger for higher risk projects\textsuperscript{7}. To increase the risk of the loan portfolio, the bank must invest more in the riskier assets available and less in the safer ones. By doing so the bank eventually incurs costs which increase at an increasing rate because it needs to reach further and further beyond its area of expertise for risky assets and does not get an offsetting reduction in costs from reducing its investment in the safer loans. In section IV, we analyze portfolios composed of investments in two asset classes with different risks. Even if the asset classes have identical cost functions, if both cost functions are increasing and convex in the amount invested within the asset class, total costs eventually increase as the bank shifts resources from the safer to the riskier asset.\textsuperscript{7} Consequently, as the bank increases portfolio risk (i.e. rebalances toward riskier assets), costs and marginal costs increase\textsuperscript{8}.

\textsuperscript{7} See Black, Miller and Posner (1978), p. 384 for a discussion of the "administrative costs" associated with high-risk loans.

\textsuperscript{8} Non-convexities arising from local economies of scale would slightly modify the optimal rule but lead to much more complicated derivations. It actually suffices that the function be increasing and convex for high values of $\sigma$ and $\nu$. 
The first order conditions for an interior maximum are:

\[ V_v = \delta [c(\sigma, \frac{1}{\delta}) + 1] - 1 + J_v(\sigma, v) = 0 \]  \hspace{1cm} (II-5.1)

\[ V_\sigma = \delta v c_1(\sigma, \frac{1}{\delta}) + J_\sigma(\sigma, v) = 0 \]  \hspace{1cm} (II-5.2).

The bank invests until the subsidy on the marginal dollar offsets the (negative) present value of the marginal investment\(^9\). Subsidized deposit guarantees result in inefficient investment (\(J_v < 0\)) and inefficient risk taking (\(J_\sigma < 0\)). The resulting asset net present value with the subsidy is necessarily inferior to the optimum in the no-subsidy case. These inefficiencies are sustained because the unfavorable consequences will ultimately be borne by the provider of deposit guarantees\(^10\).

**III Capital controls and economic efficiency**

Recognition of the perverse incentives of deposit guarantees has led a number of governments to impose capital controls on banking and more recently to increase capital requirements. In this section, we examine the effect of a decrease in the deposit to asset value ratio \(\delta\) (i.e. a tightening of the capital constraint) on the bank's decisions. The three factors of interest are the effect on risk-taking, the effect on the probability of bankruptcy, and the effect on the economic

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\(^9\) We assume that bank depositors have no monopsony power so that deposit guarantee subsidies are not passed on to them.

\(^10\) It is often presumed that this adverse incentive problem can be solved by imposing a risk constraint as well as a capital constraint. We assume that risk constraints of this sort do not fully eliminate the subsidy to levered risk-taking, for example because of the regulator's informational disadvantage.
efficiency of bank investment.

The bank's optimal asset portfolio is determined by the tradeoff between portfolio net present value and subsidy value (II-1 and II-5). The simplest way to model the subsidy valuation is to adopt standard option pricing assumptions or, alternatively, to assume bank risk neutrality\(^{11}\). To compare subsidy values across asset portfolios, we assume that the returns on loans in the states where the bank is insolvent have the same (risk neutral) distribution except for the standard deviation \(\sigma\). This property obtains if the bank failures are entirely determined by a downturn in the economy. The loan return distribution in the case of failure is then perfectly correlated with a market-wide index denoted \(Y\). Finally, without loss in generality, we assume that the interest rate is equal to zero.

The call value in equation II-3 is then given by

\[
c(\sigma, \frac{1}{\delta}) = E(\text{Max}[\frac{1}{\delta}(1 + \sigma Y)-1, 0]) .
\]

Denoting \(p(Y)\) as the probability density associated with a given realization \(Y\), and \(Y_0\) the default point (the lowest value of \(Y\) for which the option is exercised by the bank), we have

\[
c(\sigma, \frac{1}{\delta}) = \int_{Y_0}^{Y_{\text{Max}}} \left[\frac{1}{\delta}(1 + \sigma Y)-1\right]p(Y)\,dY, \quad \text{with} \quad Y_0 = (\delta-1)/\sigma < 0. \quad (\text{III-1})
\]

We define two parameters, \(q\) and \(q'\) by the following

\[\text{\ldots}
\]

\(^{11}\) If loan cash flows can be perfectly replicated by a (possibly dynamic) portfolio of traded assets, the subsidy value is determined by arbitrage. It is equal to the expectation of the cash flows using a modified probability measure. This modified probability measure is called the risk neutral or risk adjusted probability measure because the expectation (computed using the modified measure) of the asset return (in this case loans) is equal to the riskless return. In the following, the probability density \(p(.)\) and the expectation \(E(.)\) should be understood as risk neutral concepts. This approach does not require risk-neutrality or any other preference assumption. We refer the reader to Harrison and Kreps (1979) for a formal statement and proof of this proposition.
\[ q = \int_{Y_0}^{Y_{\text{Max}}} p(Y)\,dY, \quad \text{and} \quad q' = \int_{Y_0}^{Y_{\text{Max}}} Yp(Y)\,dY. \]

1-q is the probability of default and q' is the expectation of Y conditional on no default. The option value can then be rewritten as

\[ c(\sigma, \frac{1}{\delta}) = (\frac{1}{\delta}-1)q + \frac{\sigma}{\delta} q' = \frac{\sigma}{\delta}(q'-Y_0q), \]

and the value function becomes

\[ V(\sigma, \nu) = \sigma v(q'+Y_0(1-q)) + J(\sigma, \nu) \quad \text{(III-2)} \]

The first order conditions are\(^{12}\)

\[ vq' = -J_{\sigma}(\sigma, \nu) \quad \text{and} \quad \sigma(q'+Y_0(1-q)) = -J_{\nu}(\sigma, \nu) \quad \text{(III-3.1&2).} \]

We are interested in the effect of a tightening of the capital constraint (a decrease in \(\delta\)) on the optimal risk level, \(\sigma^*\) and on the optimal scale, \(\nu^*\) and have the following result.

**Proposition 2:**

Given an interior equilibrium, if the bank increases asset risk in response to tighter capital controls, it simultaneously decreases its scale.

As shown in the appendix, the response of the bank to a tighter capital constraint depends critically on the sign of the cross partial, \(V_{\sigma \nu}\), with a negative sign necessary for asset risk to increase. In turn, a necessary condition for this cross partial to be negative is that marginal costs increase with risk.

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\(^{12}\) The first equation is easily obtained by noting that the partial derivative of the call value with respect to the optimal exercise point \(Y_0\) is equal to \(p(Y_0)\) times the exercise value at that point which is zero by definition.
(J_v < 0). This obtains, for example, if the asset portfolio is a combination of investments in a relatively safe asset and in a risky asset, the case covered in detail in section IV. To increase risk, the bank increases the fraction invested in the riskier asset. If costs in both assets are increasing and convex, there is a point (low amount invested in the safe asset and high in the risky one) at which marginal costs increase as the bank increases the riskier asset portfolio weight. As capital controls are tightened, an increase in risk by itself leads to a decrease in the objective function (V_{\sigma_v} > 0). However, if the cross partial V_{\sigma v} is sufficiently negative and large in absolute sign, the bank does better by reducing its size and increasing its risk simultaneously because the second order cross partial term dominates the first order term in V_\sigma d\sigma which is necessarily small near the optimum.

Returning to the first order conditions, observe that their ratio yields the important equation 13:

\[
\frac{Y_0(1-q)+q'}{q'} = \frac{VJ_v}{\sigma J_\sigma}.
\]  
(III-4)

Equation (III-4) shows that the optimum is obtained by equating a function of the default point Y_0 to the ratio of the elasticities of the net present value function with respect to the control variables \sigma and v. We denote this ratio of elasticities \epsilon(\sigma,v).

Since the capital constraint requires the asset value to be strictly superior to the amount of deposits (\delta < 1), Y_0 is negative and it must be the case that in equilibrium the elasticity ratio, \epsilon(\sigma,v) is smaller than one 14. Equivalently, the

13 Note that the numerator is equal to the difference between the option value and its exercise value, so it is positive as long as default is possible.

14 It is easily verified that the numerator on the l.h.s. of III-4 is positive as long as default is possible.
elasticity of the net present value function with respect to risk \( \sigma \) is larger than the elasticity with respect to the investment scale \( (v) \).

In order to get further results, we must make assumptions on the net present value function \( J \) or on the distribution of investment return. In the following, we assume that the elasticity ratio \( \epsilon(\sigma, v) \) is a function of \( \sigma \) only. We refer the reader to section IV for a discussion of the properties of the elasticity function \( \epsilon \). This restriction simplifies the problem considerably because equation III-4 is now a restriction on \( \sigma \) only:

\[
\frac{Y_0 (1-q) + q'}{q'} = \epsilon(\sigma) \quad \text{(III-5)}
\]

Equation III-5 is a necessary condition for an interior optimum. Once the risk level \( \sigma \) is determined for an interior optimum, the investment level can be derived from III-2. The analysis of the optimum thus amounts to solving for the root(s) of III-5 and comparing the corresponding level of the objective function with its level at the boundaries of the domain for \( (\sigma, v) \). The appendix provides sufficient conditions for uniqueness provided that a technical assumption on the distribution of returns holds. We checked that condition for the lognormal distribution used in section IV as well as for other distributions used to model asset returns, including the normal. In particular, uniqueness of an interior extremum obtains if the elasticity function \( \epsilon(\sigma) \) is decreasing in \( \sigma \). We refer the reader to the appendix for the more technical details and assume in Propositions 3 and 4 that a unique interior optimum exists.

**Proposition 3:**
If there exists a unique interior optimum, a tightening in the capital requirement implies an increase in risk \( \sigma^* \) and a decrease in the investment level \( v^* \).
The effect of a tightening of the capital constraint on the probability of default by the lending institution is of considerable interest from a regulatory standpoint because of the effect of a change in this probability on the expected social cost of default by the bank. Differentiating $Y_0$ with respect to $\delta$ yields (equation III-1):

$$\frac{dY_0}{d\delta} = -\sigma^{-1} - \sigma' \frac{1-\delta}{\sigma^2}.$$ 

The probability of bankruptcy is the probability function evaluated at $Y_0$. The response of $Y_0$ to capital tightening can be decomposed into two offsetting effects. The first term of the above equation shows the lowering of the bankruptcy point due to the decrease in leverage (i.e. as $\delta$ becomes smaller) which would obtain if the risk index were left unchanged. However, the increase in portfolio risk ($\sigma'<0$) tends to offset this reduction in $Y_0$. In the constant elasticity ratio case, the two components perfectly offset one another and the bankruptcy probability is unaffected. More generally, we have:

**Proposition 4:**
If there exists a unique interior optimum, the probability of bankruptcy increases (decreases) with a tightening of the capital constraint if and only if the elasticity ratio $\epsilon(\sigma)$ is increasing (decreasing) in $\sigma$.

With a decreasing elasticity ratio, a tightening of the capital constraint achieves one of the desired objectives: the probability of bankruptcy is reduced. However, the risk adjustment simultaneously induced in the bank's loan portfolio reduces this positive effect. In the cases where the elasticity ratio is increasing in $\sigma$ and the other requirements of the equilibrium are satisfied, the
second effect dominates and the probability of bankruptcy actually increases with a tightening of the capital constraint.

IV A loan model:

In this section, we model the bank's asset as a portfolio composed of assets with different risk indices and net present value functions. Specification of the asset distribution Y then allows us to illustrate the preceding results and completely solve the optimization problem in representative cases.

The bank selects the non-negative amounts $v_A$ and $v_B$ invested in each of two asset classes characterized by the risk indices $\sigma_A$ and $\sigma_B$, $\sigma_B > \sigma_A$. The net present value functions for the two asset classes are:

$$J_A(v_A) = J_A - A v_A^k$$

and

$$J_B(v_B) = J_B - B v_B^k,$$

where $A \geq 0$, $B \geq 0$ and $k > 1$.

Total investment is $v = v_A + v_B$, and we define $\sigma$ as the average of the asset class risks weighted by the investment fractions: $\sigma = \sigma_A v_A/v + \sigma_B v_B/v$. $\sigma$ is also the portfolio risk index in the region where the bank defaults, as assumed earlier. Replacing $v_A$ and $v_B$ with their expressions in terms of $v$ and $\sigma$ yields the portfolio net present value function $J(\sigma,v)$:

$$J(\sigma,v) = J - \frac{v^k}{(\sigma_A - \sigma_B)^k} \left[ A (\sigma_B - \sigma)^k + B (\sigma - \sigma_A)^k \right]$$

The case of assets with zero net present value ("traded assets") studied in previous research is obtained by setting the cost parameters $A$ and $B$ equal to zero. In this case, III-4 has no solution, that is no interior optimum exists. The riskier asset generates higher subsidies, consequently the bank maximizes risk by
investing only in the risky asset and increasing its size, \( v \), to infinity or an upper limit set by regulation. Hence, we hence verify Furlong and Keeley’s results in the traded asset case. If one asset has decreasing net present value, the optimal choice is again a corner solution: full investment in the traded asset and the highest possible size.

In all the non-trivial cases \((A>0 \text{ and } B>0)\), the optimal investment choice depends on the capital constraint level. The net present value function \( J \) decreases in \( \sigma \) for low values of \( \sigma \), that is for portfolios with low fractions of total investment in the riskier asset. Consequently, the bank always prefers to invest a strictly positive fraction of its total assets in the riskier asset class. Because of increasing marginal costs for the risky asset, as \( \sigma \) increases \( J \) eventually increases even if the safer asset \( A \) has greater costs than the riskier one for a given investment \((A>B)\). Similarly, the bank always chooses a strictly positive and finite size \( v \). In the absence of size constraints, the only corner solution that can obtain is one in which the bank fully invests in the riskier asset \( B \). We see here that the elasticity ratio is independent of \( v \) if the ratio of marginal costs to average costs is constant and equal across asset classes. This assumption greatly simplifies the analysis and does not appear to affect its main features. The elasticity ratio \( \epsilon \) is independent of \( v \), thus if a unique interior optimum exists the optimal risk level \( \sigma^* \) increases with a tightening of the capital constraint. As the deposit to asset ratio \( \delta \) decreases, the risk index increases until the bank is fully invested in riskier asset \( B \).

To obtain finer results, we model asset returns as lognormally distributed and model loans along the lines of Black and Scholes (1973). The financial institution extends a loan to a firm and receives a claim to a fixed payment or the firm’s assets if their value is inferior to the promised payment. Our earlier assumption requires that the value of the assets of the borrowing firm be perfectly correlated
with an index in the set of states in which the bank is unsolvent -- i.e. in which the loans substantially underperform.

We derived the optimum for varying degrees of capital requirements $\delta=0.90$ to $0.98$ (that is for capital requirements ranging from 10% to 2%) and for a set of cost parameters $A$ and $B$. The relative costs of the two assets are important parameters, therefore we chose ratios $B/A$ ranging from 1 to 4. The ratio $B/A$ is the ratio of the slopes of the net present value functions of the risky and safer asset for a given investment level $v$. The other parameters are: $B=4$ in all cases ($A=1$ to 4), $k=2$ ($J(\sigma,v)$ is quadratic) and $J$, the constant, is zero. The results are given in Figure 1-4\textsuperscript{15}.

Figure 1 plots the optimal risk level as a function of $\delta$. In all cases, the bank increases the fraction of its portfolio invested in the riskier asset, and risk increases uniformly with a tightening of the capital constraint (i.e. a decrease in $\delta$) as was shown in Proposition 3. For low enough values of $\delta$ (of the order of .92), an interior optimum ceases to exist and the bank fully invests in the riskier asset. A lower cost ratio $B/A$ induces a higher investment in the risky asset and hence a higher risk level for a given capital constraint. Figure 2 plots the set of pairs $(\sigma^*,v^*)$ for capital constraints $\delta$ between 0.92 and 0.98 (i.e. capital requirements of 8% to 2%) and $B/A = 4$. As the financial institution increases its risk, its size $(v^*)$ shrinks.

\textsuperscript{15} We derived all the results from a discrete approximation of the lognormal distribution, this accounts for the apparent lack of "smoothness" of some of our graphs.
The impact of a tightening of the capital constraint on the probability of bankruptcy depends on the derivative of the elasticity ratio $\epsilon$ (Proposition 4). Figure 3 contains the plot of the ratio of the probability of bankruptcy to the probability of bankruptcy for $\delta = 0.9$ as a function of $\delta$. For low values of the capital requirement (low $\delta$'s), a tightening of the capital constraint leads to a reduced probability of bankruptcy in the case $B/A = 4$. The probability of bankruptcy reaches a minimum at $\delta = 0.9325$. At that point, the probability of bankruptcy is unaffected by a local change in the capital constraint. The bank responds to a tightening in the capital constraint (a decrease in $\delta$) by increasing the per unit risk level $\sigma$ in such a way that the two effects on the probability of bankruptcy exactly offset one another. As the capital constraint increases further, the probability of bankruptcy increases.

In all cases, the value of the subsidy to the lending institution decreases with a tightening of the capital constraint. Hence the transfer made by the government is reduced. More importantly, the net present value of the bank's portfolio increases, thereby improving the efficiency of lending.

The portfolio rebalancing triggered by the increased capital requirement, however, substantially reduces its effectiveness. Figure 4 allows a comparison between the reduction in subsidy for different capital constraint changes with
and without portfolio adjustment, the former being optimally chosen by the bank. We plotted the ratio of the subsidy received by the bank at the optimum to the subsidy it would have received if it had not changed the risk index from its level at $\delta=0.97$. Starting with a capital requirement of 3%, an increase to 8% induces a decrease in the subsidy but the portfolio rebalancing offset increases the subsidy by 35% to 120% over the base case (i.e. the optimal $\sigma$, $\sigma^*$ for $\delta=0.97$). The subsidy ratio increases with the cost ratio $B/A$ because the fraction invested in the risky asset (75%, 61% and 45%) and therefore risk (10.75%, 9.87% and 8.75%) at $\delta=0.97$, our starting point, decreases as the cost ratio $B/A$ ($B/A=1, 2$ and $4$) increases. The subsidy at $\delta=0.97$ (a capital requirement of 3%) amounts to 4.3%, 3.8% and 3.2% of asset value for $B/A$ equal to 1, 2 and 4 respectively. In all three cases, the subsidy exceeds the investment by equity holders and the asset portfolio has a negative net present value.

Figure 4 here
V. Conclusions

Our objective has been to analyze the effects of more stringent capital controls on the behavior of a bank that maximizes shareholder value where there are deposit guarantees and imperfect regulatory control of the risk of the bank's assets. In contrast to earlier work, we have taken the view that loan evaluation costs and loan monitoring costs make bank loans intrinsically different from zero NPV investments (e.g., market securities). Using a model incorporating a loan cost function that is increasing and convex in the level of investment and asset risk, we have shown that deposit guarantees lead banks to engage in inefficient investment and that there are plausible circumstances in which an increase in capital requirements will result in a decrease in the level of investment, but an offsetting increase in the per unit asset risk. Furthermore, there are situations in which the resulting asset risk increase leads to an increase in the probability of default.

The offsetting increase in asset risk reduces or even overturns the desired effect of the tighter capital constraints. In the most favorable cases, the increase in capital reduces the probability of bankruptcy, improves the economic efficiency of bank lending, and reduces the government subsidy, but not by as much as would occur without the risk offset. In other cases, bank lending inefficiency is reduced but the probability of bankruptcy increases with the tightening of the capital constraint. The expected deadweight costs associated with the transfer of assets to the regulatory agency and with financial instability therefore increase unequivocally in the latter cases.

In summary, our results suggest that required increases in bank capital are not a substitute for the monitoring and control of asset risk by the bank regulator and may imply an increased need for such surveillance.
Appendix

Proof of Proposition 1:
The asset level \( v \) being fixed, the bank shareholders' investment is equal to \( I(\sigma, v) - D \) and the derivative of \( V \) with respect to \( D \) is given by:
\[
\exp(rT)C_3(\sigma, v, D') + 1.
\]
To prove that the sign of this expression is positive, we consider the following investment strategy: buy a European call on \( v \) with a strike price of \( D' \) and shortsell a European call with a strike price of \( D' + d' \). The maximum cashflow generated by the strategy at date \( T \) is equal to the difference in the strike price \( d' \) when both call are exercised. Hence to prevent arbitrage the cost of the strategy must be strictly inferior to \( \exp(-rT)d' \):
\[
C(\sigma, v, D') - C(\sigma, v, D' + d') < \exp(-rT)d'
\]
Rearranging terms and taking the limit of \( d' \) at zero, we have:
\[
\exp(rT)C_3(r, v, D') + 1 > 0 \text{ for all } \sigma, v, \text{ and } D'.
\]

Proof of Proposition 2:
Let \( \sigma' \) and \( v' \) be the derivatives of the optimal \( \sigma^* \) and \( v^* \) with respect to \( \delta \).
\( V_\sigma \) and \( V_v \) are equal to zero at the optimum, for any level of \( \delta \). Their total derivatives with respect to \( \delta \) must therefore be equal to zero as well:
\[
V_{\sigma\sigma} \sigma' + V_{\sigma v} v' + V_{\sigma\delta} = 0 \quad \text{(A-1)}
\]
\[
V_{\sigma v} \sigma' + V_{v v} v' + V_{v\delta} = 0 \quad \text{(A-2)}
\]
where the second derivatives of \( V \) are easily shown to be equal to:
\[
V_{\sigma\sigma} = \frac{v}{\sigma} p(Y_0)Y_0^2 + J_{\sigma\sigma}, \quad V_{\sigma v} = q' + J_{\sigma v}
\]
\[
V_{v v} = J_{v v} < 0 \quad , \quad V_{\sigma\delta} = -\frac{v}{\sigma} p(Y_0)Y_0 > 0
\]
\[
V_{v\delta} = 1-q > 0.
\]
$V_{vv}$ is negative, hence a necessary and sufficient condition for the function $V$ to be concave is $\theta > 0$ where

$$\theta = V_{\sigma \sigma} V_{vv} - \frac{V^2_{\sigma v}}{V_{\sigma v}} > 0.$$  \hfill (A-3)

The solution $\sigma'$ of the system A-1 & A-2 is:

$$\theta \sigma' = -V_{\sigma \delta} V_{vv} + V_{v \delta} V_{\sigma v}.$$  

If the equilibrium is interior, $\theta$ is positive at the optimum and a necessary condition for $\sigma' < 0$ is $V_{\sigma v} < 0$. From equations (A-1) and (A-2), if $\sigma'$ and $V_{\sigma v}$ are both negative $v'$ must be positive and tighter capital controls (a decrease in $\delta$) lead to an increase in asset risk and to a decrease in the scale of investment.

**Existence and uniqueness of interior optimum:**

From the proof of Proposition 2, a necessary and sufficient condition for strict concavity is $\theta > 0$. We first show that inequality A-3 is equivalent to:

$$Y_0 Z(Y_0) + \epsilon'(\sigma^*) \; \sigma^* \; q' \; < 0, \hfill (A-4)$$

where $Y_0$ and $Z(Y_0)$ are evaluated at $\sigma^*$ and:

$$Z(Y_0) = p(Y_0) Y_0 [q' + (1-q) Y_0] + q'(1-q). \hfill (A-5)$$

Substituting the expressions for the second partial derivatives obtained above in equation A-3 yields:

$$\theta = \left[ \frac{V}{\sigma} P(Y_0) Y_0^2 + J_{\sigma \sigma} \right] J_{vv} - \left[ q' + J_{\sigma v} \right]^2. \hfill (A-6)$$

Differentiating $\epsilon(\sigma)$ with respect to $\sigma$ and $v$ respectively and making appropriate substitutions yields two expressions for $J_{vv}$:

$$J_{vv} = \frac{\epsilon_{\sigma}}{\epsilon_v} \left[ \epsilon' \sigma J_{\sigma} + \epsilon \sigma J_{\sigma} + \epsilon \sigma J_{\sigma \sigma} - J_{\sigma} \right], \hfill (A-7)$$

$$J_{vv} = \frac{\epsilon_{\sigma}}{\epsilon} \left[ J_{\sigma v} - \frac{1}{V} \right]. \hfill (A-8)$$

Substituting $q'$ with its expression obtained in the first order condition III-3
into equation A-6 yields:

$$\theta = \left[ \frac{V}{\sigma} P(Y_0) Y_0^2 + J_{\sigma\sigma} \right] J_{vv} - \left[ J_{vv} - \frac{J_{\sigma}}{V} \right]^2.$$  

Using A-8, the last term is linear in $J_{vv}$ squared. We factor $J_{vv}$ and use A-7 to obtain:

$$\theta = J_{vv} \left( \frac{V}{\sigma} P(Y_0) Y_0^2 + J_{\sigma\sigma} - \frac{J_{\sigma}}{\sigma \epsilon} \left[ \epsilon' \sigma + \epsilon - 1 \right] - J_{\sigma\sigma} \right).$$

Finally, replacing $\epsilon$ with its value in III-5 and $J_{\sigma}$ with its value from III-3, yields:

$$\theta = \frac{J_{vv} V}{\sigma \epsilon q^2} \left( Y_0 Z(Y_0) + \epsilon' \sigma q^2 \right),$$

where the function $Z$ is defined in A-5. The first term on the r.h.s. being negative, $\theta$ is positive if and only if the term in braces is negative.

Equation A-4 shows that the existence of an interior optimum depends on the characteristics of the net present value function through the derivative of the elasticity ratio $\epsilon'(\sigma)$ and on the return distribution through $Y_0 Z(Y_0)$.

$Z(Y_0)$ is positive for the lognormal distribution, the normal distribution and other distributions used to model asset returns. Consequently, we assume in the following that $Z(Y_0)>0$ for all $Y_0<0$. The derivative of the left hand side of III-5 is easily shown to be $Z(Y_0)/q^2$. Hence, the left hand side of III-5 is increasing. If $\epsilon(\sigma)$ is non-increasing in $\sigma$, III-5 has at most one root. Consequently, if there is an interior local extremum, it is unique. More generally, if the left hand side of A-4 is strictly negative everywhere, the interior optimum (if it exists) is unique.

Proof of Propositions 3 and 4:

The total derivative of equation III-5 with respect to $\delta$ yields:
\[
\frac{dY_0}{d\delta} \cdot \frac{d}{dY_0} \left[ \frac{Y_0(1-q)+q'}{q'} \right] = \varepsilon'(\sigma) \sigma'. \tag{A-9}
\]

Differentiating the term in brackets yields:

\[
\frac{dY_0}{d\delta} \left[ \frac{\bar{Z}(Y_0)}{q'^2} \right] = \varepsilon'(\sigma) \sigma'. \tag{A-10}
\]

Recalling that \(Y_0 = (\delta-1) \sigma^{-1}\) and reorganizing terms yields the equation:

\[
\sigma' = \frac{\bar{Z}(Y_0)}{Y_0 \bar{Z}(Y_0) + \varepsilon' \sigma q'^2}.
\]

The denominator has the opposite sign from \(\theta\). If an interior optimum exists, \(\theta\) is positive; hence \(\sigma'\) is negative if and only if \(\bar{Z}(Y_0)\) is positive.
References


