Low Margins, Derivative Securities, and Volatility

by

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I. Introduction

Prior to 1934, margin requirements for stock purchases were set by the exchanges on which the stocks traded. In response to the market gyrations surrounding the 1929 crash, however, Congress granted regulatory authority over stock margin requirements to the Federal Reserve Board. Margins on stock index futures, in contrast, continue to be set by the exchange on which the contract is traded. Yet following the October 1987 crash, there has been a prolonged and acrimonious debate about the need to regulate stock index futures margins.

Margins are held to serve two potential roles in stabilizing markets:
(i) Assuring market integrity—that contractual obligations will be fulfilled.
(ii) Reducing price volatility which results from the actions of leveraged investors.

The role of margins in assuring market integrity has been examined in several studies, whose results are summarized in Section II below. Most evidence suggests that futures and stock margins have provided comparable degrees of assurance that contractual obligations will be met. More controversial, and the topic of this paper, is the role of margin requirements in affecting market volatility.

Critics have claimed that the low margins in stock index futures and other derivatives have added volatility, which has been passed through to the underlying stock market through index arbitrage. Some observers believe that the introduction of futures on the Nikkei 225 has added volatility to that market as well. These critics have called for futures margins to be "harmonized" with individual stock margins, by which they mean that futures margins should be raised to the level of stock margins, not vice-versa.

The debate on the role of margins in market stability far predates the existence of low-margin stock derivatives. There seems to be two strands of criticism of low margins. First, they are said to encourage destabilizing speculation by leveraged investors. Second, they may be associated with "pyramiding and de-pyramiding," the fact that as prices rise, additional wealth permits further stock purchases; but when prices fall, margin requirements may force the sale of stock which was bought on credit. This in turn could lead to further sales, further margin calls, and an eventual "meltdown." Indeed, forced margin sales were widely believed to have played an important role in
the crash of October 1929.\textsuperscript{1} And this belief was instrumental in the subsequent Securities and Exchange Act which gave the Federal Reserve Board power to regulate margin requirements.

The language of sequential "meltdown" was subsequently revived by the Chairman of the New York Stock Exchange, John Phelan, both before and after the 1987 crash. The focus of criticism in 1987 was on hedging strategies such as portfolio insurance, as well as on low margins in the derivatives markets.\textsuperscript{2}

Not surprisingly, since futures volume seems highly sensitive to margin requirements, representatives of futures markets vehemently reject the idea that low margins create volatility. They argue that exchanges have proper incentives to set margin requirements without government interference. Many of their arguments are addressed to the question of market integrity, rather than market volatility \textit{per se}. But they correctly point out that there is little empirical evidence to suggest that the introduction of derivatives raises volatility of underlying markets. Nor is their evidence that lowering margins for stocks has caused their volatility to increase. This evidence is discussed in Section III.

Academics raise a final question: Is increased volatility bad? If it reflects incremental risk, the conclusion is "yes." But if increased volatility simply reflects the earlier resolution of uncertainty, then markets are revealing information more quickly and not increasing total risk. The greater informational efficiency of markets is generally viewed as desirable rather than undesirable.\textsuperscript{3}

Given the importance of the debate, and the lack of conclusive empirical evidence, theory takes on added importance. Yet few theoretical models exist to study the impact of margin requirements. Exceptions include work by Telser

\textsuperscript{1}Garbade [1982] notes "the most famous example of forced liquidation in a declining market was the reduction in brokers' loans from $8.5 billion to $5.5 billion in 10 days, during the stock collapse that began in late October 1929." The $3 billion of sales amounts to 3.45% of the $87.1 billion value of NYSE issues in September, 1929. An equivalent forced liquidation in October 1987 would have equalled just over $100 billion; more than ten times the estimated amount of portfolio insurance sales in that crash. See the Brady Report [1988], p. VIII-13.

\textsuperscript{2}For example, the Brady Report [1988] focussed on portfolio insurance selling as a major contributor to the crash.

\textsuperscript{3}For a discussion of this, with earlier references, see Leland [1990].
[1981], who, following an earlier argument by Friedman [1953], suggests that the increased speculation which follows from lower margin requirements is stabilizing because it provides liquidity to hedgers. But other models, including De Long, Shleifer, Summers and Waldmann [1990] conclude that increased "uninformed" speculation can be destabilizing rather than stabilizing. And Telser's model does not include the pyramiding/de-pyramiding or "meltdown" effects which have concerned practitioners.

In Section IV, we develop a theoretical model which permits an examination of the effects of changing margin requirements. The model builds on work by Grossman and Stiglitz [1980], Hellwig [1980], and Gennette and Leland [1990], and provides a rich framework for analyzing the impact of margin requirements on market volatility.

We find the model supports some of the arguments of both sides:

(i) Low margins will tend to lead to more liquid markets. Markets will be more informationally efficient if speculators are informed, and less efficient if they are "noise" traders.

(ii) In the absence of pyramiding/de-pyramiding behavior, low margins will reduce total volatility if speculators are informed, and increase total volatility if they are "noise" traders.

(iii) When margins lead to the possibility of margin calls and "forced" liquidation of positions (pyramiding/de-pyramiding), low margins may lead to greater volatility. The extent of volatility increases depends critically upon the extent to which forced sales can be observed by other market participants.

(iv) In the case where no other participants are cognizant of forced margin sales, low margins can lead to the possibility of crashes. There is some evidence that this was the case in 1929, but not in 1987.

II. Margins and Market Integrity

Margins help to assure that contractual obligations will be met with a high probability. This is true for both ordinary and derivative instruments. There are two levels of margins for each type of instrument: initial margin and maintenance margin. Maintenance margins are the authority of the exchange on which the security is listed. For stocks, initial stock margins are under the authority of the Federal Reserve Board and are currently set at 50%. Minimum
maintenance margins are set by brokers, and are currently about 25-30%.\(^4\) A customer has approximately five business days to meet a margin call for a stock.\(^5\) After that grace period, failure to meet a margin call results in the forced sale of the stock.

Minimum initial and maintenance margins for futures are set by the exchanges on which they are traded.\(^6\) Hedgers' margin requirements are substantially less than speculators'. Currently, the initial margin requirement for a (non-hedged) stock index futures contract is 12-15%, and the maintenance margin is about 6-8%. A customer typically has a one-day grace period to meet a margin call for a futures position.\(^7\)

For common stocks, the margin requirement helps to assure the lender that his loan to the buyer can be repaid with high probability.\(^8\) The risk the lender faces depends on the price volatility of the underlying stock, as well as the grace period the buyer has to meet the margin call before his position is liquidated.

Several possible notions of "equality" of margins exist. If failure of market integrity implies default, then a reasonable definition of equality would be an equal probability of default across different markets. Since maintenance margins are less stringent than initial margins, the question becomes whether different maintenance margins are appropriate for the different degrees of price risk and the "grace period" over which investors can delay

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\(^4\)See Chance [1990], p. 2. Market makers and specialists are not constrained by margin requirements.

\(^5\)Regulations give up to fifteen days for investors to meet margin calls. But brokers often grant a shorter period before liquidating an account.

\(^6\)Clearing houses can (and do) demand higher margins if they wish. Increasingly, the CFTC has examined carefully the changes in the futures margin requirements.

\(^7\)The clearing houses can make intra-day margin calls. Such calls were made on October 19 and 20, 1987.

\(^8\)Futures exchanges have tried to distinguish the role of margins in ordinary securities (which they categorize as "down payment on a loan") versus margins in futures (which they categorize as a "performance bond"). We feel this distinction is semantic rather than practical; because, in both instances, the initial payment is to protect the other party of the contract against default.
payment before facing liquidation. With minor exceptions, empirical studies lend credence to the idea that margins provide equality across markets.\(^9\)

Maintenance margins in stock index futures markets can be lower because (i) the price volatility of the stock index, a portfolio of many stocks, is considerably less than the price volatility of the average stock; and (ii) the grace period is considerably shorter with futures than with stocks.

We conclude that margin requirements across markets seem consistent with respect to assuring market integrity. If further restrictions of futures margins is deemed desirable, it must be for another reason; because low margins promote undesirable volatility. We now turn to studies which have examined the relationship between level of margins and price volatility.

III. Margin Requirements and Volatility: Empirical Evidence

Margin requirements, when binding constraints, limit the level of investment of some market participants. What is less clear is whether limiting participation reduces market volatility.

Chance [1990] and Hodges [1990] provide excellent summaries of empirical studies which examine the link between margin requirements (including the introduction of low-margin derivative instruments) and volatility. With the exceptions noted below, the published evidence does not support a statistically significant relationship between low margins and high stock price volatility.

The most important exception to this conclusion is Hardouvelis [1988], who found that margin requirements are significantly and inversely related to the volatility of the S&P 500 index. Kupiec [1989] corrected for serial correlation in Hardouvelis' data, and found similar results for the full period of 1931 to 1987. But the margin effect on volatility was not significant when the period prior to 1934, plus 1987, were excluded. Hsieh and Miller [1990] further criticized Hardouvelis' methodology and rejected his conclusion of statistical significance.

Seguin [1990], looking at differences between individual stocks, finds that OTC stocks which become eligible for purchase on margin also become less risky, rather than the opposite. However, one potential problem with individual stock studies is that of sample bias: stocks which become eligible for margin are more likely to have attracted recent investor interest, perhaps

\(^9\)For example, see Figlewski [1984]; Gay, Kolb, and Hunter [1986]; Estrello [1988]; and Warshawsky [1989].
because of recent price increases. But higher prices tend to be associated with lower volatility, so this could be an alternative explanation of the empirical results.\textsuperscript{10}

Little evidence is also found that the introduction of options or futures changes the volatility of underlying stock prices. Indeed, Skinner [1989] and Gemmill [1989] find that the volatility of stocks which have newly-listed options slightly decreases. But our previous comments on sample bias are also relevant here. Edwards [1988a,b] finds no evidence that the introduction of stock index futures and interest rate futures have increased the volatility of their underlying markets. But his data includes neither 1929 nor 1987.

In sum, there is little empirical evidence to suggest a causal relationship between low margins (or the introduction of low-margin derivatives) and market volatility. But the evidence is not inconsistent with a possible relationship. And many of the results are ambiguous only when periods of very high volatility are omitted (see Kupiec's [1989] critique of Hardouvelis [1988]) or are not included in the original data sample. Given the criticism of low margins immediately following the market crashes of 1929 and 1987, it seems premature to conclude that they might not have had some effect on market volatility during these important but unique events.

Since singular events are less amenable to statistical studies, we turn now to developing a model of market behavior that will enable us to examine the link between margin requirements and market stability.

IV. A Model of Financial Markets

A satisfactory model for studying the impact of margin requirements on volatility must possess the following elements:

(i) An investor portfolio selection process which allows margin requirements to affect investor demand.

(ii) A means to study the effects of forced margin calls (i.e., the "pyramiding and de-pyramiding" resulting from low margins).

(iii) An environment in which volatility and liquidity are endogenously determined.

(iv) A rational expectations equilibrium in which investors recognize that speculators possess superior information.

\textsuperscript{10}Black [1976].
(v) An environment in which market makers (but perhaps not other investors) may observe the selling generated by margin calls. Models of the type examined by Grossman and Stiglitz [1980], Hellwig [1980], and Kyle [1985] satisfy requirements (iii) and (iv). The model of Gennette and Leland [1990] (GL hereafter) extends the environment to permit analysis of (ii) and (iv). But the GL model needs to be adapted to examine the effects of margin requirements on speculators' demands. We consider a simple extension below which will enable us to examine the impact of lower margins (or low-margin derivatives) on stock market volatility.

(i) The Basic Framework

The GL model examines single-period equilibrium in a market with a single risky asset ("the market") and a single risk-free asset. Thus it is suited to examine the impact of margins on the market price and volatility, but not the prices or volatilities of individual securities.\(^{11}\)

The market is composed of four classes of investors:

(a) Informed speculators, who receive (imperfect) information about future market prices.

(b) Uninformed investors, who receive no information but correctly recognize that current prices reflect speculators' information.

(c) Liquidity or "noise" traders, whose demand reflects exogenous needs or noise.

(d) Market makers, who receive (imperfect) information about the extent of liquidity traders' demand.

All investors (except liquidity traders, whose demands are exogenous) choose portfolios to maximize expected utility, given identical exponential utility functions. Expectations are conditional upon the information observed by each investor class.

\(^{11}\)Admati [1985] and Gennette [1985] introduce a model with multiple securities which might (with considerable effort) be adapted to examine our questions. Note that one would not necessarily expect different margins across similar securities (i.e., stocks) to have the same impact on price and volatility as changing margins for the market as a whole, since information spillovers between individual securities may importantly affect behavior and prices. Seguin and Jarrell (1991) suggest that the difference in performance of margined versus unmargined OTC securities was negligible during the 1987 crash. This observation does not preclude the possibility that market-wide changes in margin will have important effects on the overall level of volatility.
A linear rational expectations equilibrium price function can be derived which relates the current price $p_0$ to the future price $p$, and to liquidity demand $S$ observed by market makers as well as to the unobserved liquidity demand $L$:

$$p_0 = F[(p - \bar{p}) + H L + IS],$$

where $\bar{p}$ is the ex ante expected future price, and $F > 0$, $H < I < 0$ are constants determined by the model's exogenous parameters.\(^{12}\) For realistic values of these parameters\(^{13}\) the price function is given by equation (3) of Gennaioli and Leland [1990]:

**Base Case: Informed Investors = 2%**.

$$p_0 = 1.000 + 0.500[(p - \bar{p}) - 19.952L - 8.140S],$$
$$= 1.000 + 0.500(p - \bar{p}) - 9.976L - 4.070S$$

$$\text{Std}(p_0) = .2000; \text{Std}(p|p_0) = .2000$$

Note that the coefficient of $(p - \bar{p})$ can be interpreted as the responsiveness of current price to future price; it is one measure of the "informational efficiency" of the market.\(^{14}\) The coefficients of $L$ and $S$ indicate the sensitivity of current price to unobserved and observed liquidity demand, respectively. These coefficients therefore are an inverse measure of market liquidity.

Appendix B shows that, with suitable modifications, the original GL model can be adapted to include the effect of margin requirements on market equilibrium. A fall in margin requirements will lead to an increase in the

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\(^{12}\)Because there are a very large number of traders, the average signal of informed investors will converge to the true future price $p$. Thus $p$ also reflects the informed investors' mean signal about future price.

\(^{13}\)See Appendix A for details. Reflecting relative market importance, it is assumed that informed traders represent 2% of investor capital; market makers represent 0.5% of investor capital; and uninformed ('long term') investors represent the remaining 97.5%. Informed investors have relatively poor information, with a signal-to-noise ratio of 0.2. Other parameters are scaled so the market portfolio has an annual mean return of 6% above the risk-free rate, and a standard deviation of 20%.

\(^{14}\)Given the relatively low volatility of liquidity demand $S + L$, it can be shown that the coefficient $F$ will closely approximate $R^2$ of a regression of $p$ on $p_0$; another measure of the informational efficiency of the market.
size of risky asset positions held by speculative traders. An increase in speculative positions has the effect of increasing the relative importance (i.e., fraction) of speculative traders in the original GL model.

We examine the case where speculative traders are either "informed"—the usual perception of speculators—or naive. A relaxation of margin requirements from a level \( m_0 \) to a level \( m_1 \) will increase speculators' risky asset positions by a maximum factor of \( m_0/m_1 \). For example, if a speculator could take a position \( x \) when (stock) margins were 50%, s/he could take a (maximal) position of \( 4x \) in stock index futures if futures margins were 12.5%.\(^{15}\)

(ii) Case A: Informed Speculators: No Hedging

We first examine the case when speculators are informed investors; that is, they receive information about the future price which is superior to that obtained by other types of investors.\(^{16}\) If all informed investors were constrained by margins both before and after the introduction of futures, their positions would (at maximum) quadruple as argued above. Informed investors would now constitute 8% of investor "capital," or average positions, rather than 2%. If only half of the informed investors took full advantage of the increased margin, they would constitute 4% of investor capital, or double their earlier presence. In this subsection, we ignore the possible pyramiding/depymading or hedging strategies which margined investors may follow.

Using the same initial parameters as GL, but expanding informed traders' actions by factors of two and four gives the following rational expectations equilibrium price functions:

\(^{15}\)Of course this assumes all speculators would shift from stock to futures markets, and use leverage to a maximum (see Appendix B). This would clearly represent a maximal possible impact from introducing lower-margined futures.

\(^{16}\)Note that the quality of information received, even by "informed" traders, is not very precise. It has a "signal to noise" ratio of 0.2, implying that conditional on this information the volatility of future prices (given current price) is 19.1% rather than the 20% for uninformed investors.
A.1. Informed Investors = 4%:

\[ p_0 = 1.037 + .814[(p - \bar{p}) - 9.976L - 2.557S], \]
\[ = 1.037 + .814(p - \bar{p}) - 7.945L - 2.081S \]
\[ \text{Std}(p_0) = .2549; \text{Std}(p|p_0) = .1221 \]

A.2. Informed Investors = 8%:

\[ p_0 = 1.054 + .948[(p - \bar{p}) - 4.988L - 0.733S], \]
\[ = 1.054 + .948(p - \bar{p}) - 4.729L - 0.695S \]
\[ \text{Std}(p_0) = .2751; \text{Std}(p|p_0) = .0645 \]

Comparing these numbers with those of the base case, we see that the informativeness of prices—the coefficient of \((p - \bar{p})\)—has increased. The liquidity of the market—inversely related to the coefficients of \(L\) and \(S\)—has improved by 20% both for observed and unobserved liquidity trades. Average current price also rises, since the greater information efficiency of current price reduces future risks to investors.\(^{17}\)

Has market volatility increased? The answer is both "yes" and "no." In our two-period model, current volatility (the standard deviation of \(p_0\)) increases, reflecting two things: the increased informativeness of prices, but the decreased impact of liquidity trading. On the other hand, because current prices are now more informative, the future price uncertainty (given \(p_0\)) is reduced.

The average volatility, \([\text{Var}(p_0 + \text{Var}(p|p_0))/2]^{1/2}\), has fallen from .20000 to .19885 in the 4% case and to .19976 in the 8% case.\(^{18}\) The (unconditional)

\(^{17}\)We assume that the (unconditional) volatility of the future price \(p\) is fixed. In a multi-period model, the volatility of all future prices prior to the final period may rise, and the price increase in the initial period may be less.

\(^{18}\)\text{Var}(p_0)\ is the risk to an investor who purchases the risky asset prior to time 0 and resells at the price \(p_0\) which prevails in equilibrium at time 0. \text{Var}(p|p_0)\ is the risk faced by an investor who purchases the asset at time 0 and holds it until the future date. The average variance reflects the average risk of the two investors. We take the square root of the average variance in order to compare with the original standard deviation of .20.
variance of future price \( p \) has not changed, but a larger fraction of that randomness is now revealed earlier, thanks to the greater informational content of the current price. The reduction of total variance results from random liquidity trades having less current price impact due to the greater liquidity of markets.\(^{19}\) Trading volume increases as speculators (receiving different information signals) trade larger amounts amongst themselves.

(iii) Case B: Informed Speculators: Hedging

We consider now the case where margined investors must follow hedging strategies to protect margin lenders from the possibility of default. Such strategies might simply involve a stop-loss sale when a margined investor's equity falls beneath the maintenance margin requirement, the classic "forced margin sale." Alternatively, an investor might follow a dynamic hedging strategy which replicates a put option, providing protection against wealth falling beneath the minimum required level.\(^{20}\)

This dynamic hedging strategy, known as "portfolio insurance," requires that investors progressively sell their stock holdings as stock prices decline, but permits them to become more aggressive as prices rise.\(^{21}\) Thus it captures quite exactly the notion of "pyramiding/de-pyramiding" or "cascading" which has been claimed to reflect the behavior of leveraged investors facing low margin requirements. In the analysis which follows, we shall assume that investors on margin follow such a hedging strategy, and shall refer to the sales necessitated by such a strategy as "forced margin sales."

The GL model allows for the existence of possible hedging strategies, including portfolio insurance. The supply of shares or futures generated by

\(^{19}\)The small magnitude of variance reduction follows from the fact that liquidity trades create a small amount of current price uncertainty relative to information arrival. Note our model presumes that all investors are aware of the increased positions undertaken by margined speculators. The interested reader can verify that volatility will rise rather than fall if other investors are ignorant of the increased level of speculation resulting from the relaxation of margin requirements.

\(^{20}\)For an illustration of how a simple dynamic strategy can replicate a put option, see Rubinstein and Leland [1981]. Also, Cox and Huang [1988] suggest that following such a strategy is appropriate for a wealth-constrained investor.

\(^{21}\)Note that a group of investors, each following a stop-loss strategy with different stop-loss price levels, would behave similarly to a group of investors, each following an identical portfolio insurance strategy.
hedging strategies is given by an arbitrary function \( \pi(p_0) \), with \( \pi'(p_0) < 0 \). The actual function \( \pi \) will depend on the level of protection against loss, the time horizon, and the fraction of investors following such a protection strategy.

With hedging, the rational expectations equilibrium price function becomes a nonlinear correspondence:

\[
p_0 = f((p - \bar{p}) + HS + IL).
\]

GL show that the argument of \( f \) is identical to the linear rational expectations equilibrium price function when there is no hedging: the coefficients \( H \) and \( I \) are unaffected by the degree of hedging. The sensitivity of \( f \) to changes in its argument, and the possibility of discontinuities ("crashes"), will depend upon (i) the extent to which hedging activity is observed by various market participants; and (ii) the amount of hedge selling \( \pi \). We consider these two aspects below.

GL examine the impact of observability on market volatility. If investors are unaware of the amount of hedging activity, the sensitivity of \( f \) to changes in its argument will be large. Hedging sales can substantially raise price volatility and even cause crashes in realistic environments.

If, however, the extent of hedge selling is observed by even a small subset of investors (e.g., by "market makers" who can observe the origin of orders), the impact of such selling will be considerably less. This is because market makers will be willing to take the other side of such transactions, recognizing that the fall in prices was not the result of unfavorable information. The impact of hedging on volatility is even smaller if all investors are aware of its magnitude.

For any level of observability, greater hedging activity will increase the sensitivity of \( f(\bullet) \) to changes in its argument, relative to the case when there is no hedging and the sensitivity is the coefficient \( F \). Comparing \( f'(\bullet) \) with \( F \) will show the impact of hedging. Locally, the volatility (standard deviation) of current price \( p_0 \) will increase by the factor \( f'(\bullet)/F \), relative to the volatility of \( p_0 \) when there is no hedging activity.\(^{22}\)

\(^{22}\)Since the slope of \( f(\bullet) \) changes as its argument changes, we shall focus on the situation where all variables equal their expected values.
At the initial margin level of 50%, hedging strategies are assumed to protect the 2% of total market capitalization of speculators against losses exceeding 50%. In this base case, the hedging strategies have a minimal effect on both current volatility (Std(\(p_0\))) and average volatility regardless of the degree to which margin-related hedging can be observed.

<table>
<thead>
<tr>
<th>B.O. 2% Speculators; 50% Margin</th>
<th>Slope (f'(\cdot))</th>
<th>(Local) std((p_0))</th>
<th>Avg. Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (no hedging)</td>
<td>.50000</td>
<td>.20000</td>
<td>.20000</td>
</tr>
<tr>
<td>Hedging; observed by all</td>
<td>.50000</td>
<td>.20000</td>
<td>.20000</td>
</tr>
<tr>
<td>Hedging; observed by market makers</td>
<td>.50010</td>
<td>.20004</td>
<td>.20002</td>
</tr>
<tr>
<td>Hedging; not observed</td>
<td>.50024</td>
<td>.20010</td>
<td>.20005</td>
</tr>
</tbody>
</table>

The small effect of hedging on current and average volatility results from the minimal amount of hedging—even if every speculator has purchased on full margin—since the level of protection is so far beneath the current price \(p_0\).\(^{23}\) However, if prices were to fall 40%, the dynamic strategy would require larger trades, and the effect on market volatility would be more pronounced.

We now consider the effects of hedging on the equilibrium where effective margins have been reduced to 12.5% through the introduction of stock index futures. Again we consider two cases, one where the speculative demand doubles (with associated hedging), and the other where the speculative demand quadruples (with associated hedging). The former is modeled by a rise in speculative capital from 2% to 4%, with the 4% of margined investors protecting themselves against losses exceeding 12.5%. The second case is modeled by a rise in speculative capital to 8%, with all these investors protecting themselves against losses exceeding 12.5%.

Hedging will now be more aggressive for two reasons. One reason is that the equivalent of more investor capital is being hedged; and the second is that the desired level of protection is higher, necessitating more aggressive trading.

\(^{23}\)The volume of hedging as prices fall depends upon the "gamma" (at the current price) of the option being replicated. This gamma is small for a put option with strike price equal to one-half the current price.
B.1. 4% Speculators; 12.5% Margin

<table>
<thead>
<tr>
<th></th>
<th>Slope $f'(x)$</th>
<th>(Local) std($p_0$)</th>
<th>Avg. Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (no hedging)</td>
<td>.81366</td>
<td>.25488</td>
<td>.19985</td>
</tr>
<tr>
<td>Hedging; observed by all</td>
<td>.81420</td>
<td>.25505</td>
<td>.19990</td>
</tr>
<tr>
<td>Hedging; observed by market makers</td>
<td>.89604</td>
<td>.28069</td>
<td>.21644</td>
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<tr>
<td>Hedging; not observed</td>
<td>1.26886</td>
<td>.39747</td>
<td>.29409</td>
</tr>
</tbody>
</table>

B.2. 8% Speculators; 12.5% Margin

<table>
<thead>
<tr>
<th></th>
<th>Slope $f'(x)$</th>
<th>(Local) std($p_0$)</th>
<th>Avg. Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (no hedging)</td>
<td>.94791</td>
<td>.27505</td>
<td>.19977</td>
</tr>
<tr>
<td>Hedging; observed by all</td>
<td>.94823</td>
<td>.27514</td>
<td>.19983</td>
</tr>
<tr>
<td>Hedging; observed by market makers</td>
<td>1.00373</td>
<td>.29125</td>
<td>.20737</td>
</tr>
<tr>
<td>Hedging; not observed</td>
<td>1.52490</td>
<td>.44247</td>
<td>.28473</td>
</tr>
</tbody>
</table>

We conclude that, even with low margins, the hedging or "pyramiding/de-pyramiding" behavior of margined investors will have a very small impact on current and average volatility if all investors--uninformed as well as market makers--observe the amount of hedging by speculators. Volatility will increase more if market makers alone are aware of the magnitude of hedging, including forced margin sales. At this degree of observability, note that average volatility has now risen above the level in the base case, when low-margin futures were unavailable.

If no investors are cognizant of the extent of forced margin sales, then volatility can be significantly increased by the relaxation of margin requirements. Our examples show average volatility increases by 40-50% in this situation, even when only a fraction of speculators take full advantage of the lower margins.²⁴

As the current price falls, hedging activity becomes even more intense, and volatility rises even beyond the numbers reported above. Small amounts of "news" will create major price moves in these situations. Nonetheless, the equilibrium price varies continuously with respect to news or liquidity shocks.

²⁴The reader may wonder why the impact of reduced margins is not much larger when all speculators take advantage of them; the 8% case versus the 4% case. The explanation is that while more hedging is occurring in the 8% case, which potentially is more destabilizing, the liquidity of markets is improved because of the greater number of informed traders.
But the addition of merely 1% more investors following hedging strategies leads to the possibility of discontinuous "crashes." Thus the combination of a few other investors following portfolio insurance, in conjunction with the required hedging by highly-margined speculators, could jointly produce crash-like events in the unobserved case.

(iv) Case C: Uninformed Speculators; No Hedging

We now consider the case where relaxing margin requirements leads to a fraction of naive or "noise" traders (rather than informed investors) increasing their positions. In contrast with Friedman's [1953] view that speculators tend to be informed, other writers such as De Long, Shleifer, Summers and Waldmann [1990] have argued that speculators may trade on the basis of purely noisy signals, which they mistakenly interpret as information. Lower margin requirements would allow such uninformed speculators to take larger positions.

We can examine the consequences of increased noisy trading in the GL model by increasing the volatility of exogenous "liquidity" demand. We presume that other investors recognize the increased volume of purely noisy trading following the relaxation of margins.

To facilitate the comparison, we assume initially that (with 50% margins) the demand of "naive" speculators is commensurate with the demand of informed speculators. Informed speculators (with 2% of market capital) have a variance of demand of about .000065, resulting from their information signals. We therefore assume that the variance of naive speculators' (random) trading initially also is .000065, and is included as part of the total variance of liquidity demand.\(^{25}\)

Doubling the positions of uninformed speculators (consistent with half of them taking full advantage of the margin drop from 50% to 12.5%) will increase the variance of their demand by a factor of four. The variance of total liquidity demand will therefore increase from .000345 to .000540. Quadrupling their positions (the maximum if all take full advantage of the margin drop) will increase the volatility of their demand by a factor of sixteen, raising the variance of total liquidity demand to .001320.

\(^{25}\)Thus uninformed speculation accounts for about 20% of the variance of liquidity demand, which is .000345 in the GL base case.
We now examine the effects of adding these amounts of extra variance to liquidity demand. As before, we first consider the case where there is no hedging by these margined investors.

C.1. Doubling Noise Trading: No Hedging

\[ p_0 = 0.990 + .419[(p - \bar{p}) - 19.952L - 10.351S], \]
\[ = 0.990 + .419(p - \bar{p}) - 8.360L - 4.337S \]

\[ \text{Std}(p_0) = .1834; \text{ Std}(p|p_0) = .2156 \]

C.2. Quadrupling Noise Trading: No Hedging

\[ p_0 = 0.972 + .267[(p - \bar{p}) - 19.952L - 10.351S], \]
\[ = 0.972 + .267(p - \bar{p}) - 5.327L - 2.764S \]

\[ \text{Std}(p_0) = .1469; \text{ Std}(p|p_0) = .2424 \]

In contrast with the base case before the relaxation of margin requirements, we see that lower margins for noise traders will:
- lower the equilibrium price.
- reduce the informational efficiency of the market.
- increase liquidity with respect to liquidity sales (i.e., the sum of the coefficients of L and S declines).

However, since there is more liquidity trading, the average volatility of prices will marginally increase. Average volatility rises from .20000 in the base case to .20013 when noise trading doubles, and to .20042 when noise trading quadruples. While liquidity with respect to unobserved trading L increases, liquidity with respect to observed sales S is ambiguous. This is because with increased noise trading, trades are less likely to come from informed investors--leading to greater liquidity--whereas the observed noise trading is less informative to supply-informed investors--leading to lesser liquidity.
(v) Case D: Uninformed Speculators; Hedging

We finally address the case where the uninformed speculators hedge their margined positions. Consistent with our earlier parameterization, we shall look at the case when noise trading doubles (from 2% of investor capitalization to 4%) and when it quadruples (from 2% to 8%). In both cases, we shall assume that speculators hedge against losses of their capitalization exceeding 12.5%.

<table>
<thead>
<tr>
<th>D.1. Doubling Case; 12.5% Margin</th>
<th>Slope $f'(\ast)$</th>
<th>(Local) $\text{std}(p_0)$</th>
<th>Avg. Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (no hedging)</td>
<td>.41942</td>
<td>.18337</td>
<td>.20013</td>
</tr>
<tr>
<td>Hedging; observed by all</td>
<td>.42068</td>
<td>.18392</td>
<td>.20039</td>
</tr>
<tr>
<td>Hedging; observed by market makers</td>
<td>.55947</td>
<td>.24460</td>
<td>.23056</td>
</tr>
<tr>
<td>Hedging; not observed</td>
<td>.81026</td>
<td>.35424</td>
<td>.29323</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D.2. Quadrupling Case; 12.5% Margin</th>
<th>Slope $f'(\ast)$</th>
<th>(Local) $\text{std}(p_0)$</th>
<th>Avg. Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (no hedging)</td>
<td>.26715</td>
<td>.14689</td>
<td>.20042</td>
</tr>
<tr>
<td>Hedging; observed by all</td>
<td>.26914</td>
<td>.14798</td>
<td>.20082</td>
</tr>
<tr>
<td>Hedging; observed by market makers</td>
<td>.52107</td>
<td>.28651</td>
<td>.26537</td>
</tr>
<tr>
<td>Hedging; not observed</td>
<td>.81530</td>
<td>.44829</td>
<td>.36036</td>
</tr>
</tbody>
</table>

Again, we note that when all investors can observe the hedging activity of uninformed speculators, forced margin sales will have little effect. On the other hand, if only a fraction of investors (or none at all) are aware of such activity, and believe it may stem from ordinary liquidity trades or informed trades, the market can become considerably more volatile. In the quadrupling case with no observability, the market is quite volatile at $p_0 = .972$. Hedging and volatility increase as $p_0$ declines. After price falls by 13%, the hedging by margined investors will overwhelm the demand by other investors, and prices will fall discontinuously (i.e., "crash") by about 20%. This is, of course, not dissimilar to the events of October, 1929, in which the forced selling by margined investors was held culpable by some observers for the crash.

V. Conclusion

We have examined the impact of reducing margins on market volatility. If the investors utilizing margin are informed speculators, then low margins will
increase the informational efficiency and liquidity of markets. If they are uninformed or "naive" speculators, low margins will decrease the informational efficiency of markets and (slightly) raise total market volatility.

Even when speculators are informed, low margins can lead to increases in market volatility when highly levered investors are forced to sell to meet margin calls. We showed that dropping margins from 50% to 12.5% (as with the introduction of stock index futures) could dramatically increase stock market volatility, and even lead to crashes. But the latter is a possibility only if other investors are ignorant of the size of hedging activity, and instead believe they may represent actions by informed investors. When some or all investors are aware of the extent of forced margin selling, our analysis indicates such hedging will have little extra impact on market volatility.

Is there data available to alert investors to the extent of margin selling? If the market falls 5% in the next week, what volume of selling (stocks and index futures) will be forced by margin requirements? Not many investors would seem to know the answer to this question, since it requires not only current data on the extent of margin buying, but also on the initial cost bases of that buying.\textsuperscript{26}

The open interest in stock index futures markets is a measure of the maximum amount of forced selling which futures could generate in a market decline. Relative to the total value of stocks (about $3.5 trillion), the value of open interest in stock index futures is small; about $30 billion, or less than 1%. Nonetheless, since a 12.5\% or greater decline in market price could (at least theoretically) force all long holders to liquidate their positions, all $30 billion could be sold during the course of a major market fall. If 20\% of futures positions were liquidated, or $6 billion, it would approximate the amount of selling by portfolio insurers on October 19, 1987.\textsuperscript{27}

\textsuperscript{26}If the market is currently at a new high, most margin buyers will be comfortably above the level of margin sales. Only when the market has fallen a significant amount beneath the high is it likely that serious hedge selling will be required. Interestingly, both the 1929 and 1987 crash occurred at levels some 15-20\% less than newly established highs.

\textsuperscript{27}Still, the potential maximum forced margin selling in futures is far less than the 3.45\% liquidation of positions that margin selling forced in 1929. Thus we might conclude that the current danger of forced margin selling of low-margined derivatives is small relative to the 1929 situation. The Brady Report [1988] indicates that investors were not particularly concerned with forced margin sales during the crash of 1987 (pp. V-51 - V-52). However, it is not clear that respondents included forced sales of stock index futures as well as forced sales of individual stocks.
Having presented the "scary" side of the scenario, it must be pointed out that the market can in principle absorb such large selling with little price impact, if it can identify that the selling is not triggered by information.

Thus our analysis leads to a very strong policy recommendation. The introduction of low margins (or derivatives with low margins) should be accompanied by the best possible data on the potential amount of forced margin sales that could occur, for various levels of market declines. If such information can be made widely available to investors, the liquidity benefits of low margins can be realized with minimal impact on market volatility.
Appendix A: Notations and parameter values

The parameter values used in the base case are in parentheses.

Prices
\( p_0 \): Current equilibrium price.
\( p \): Realized end of period price.
\( \bar{p} \): Unconditional expected end of period price (1.06).
\( \bar{p}_i \): Investor i’s conditional expectation of end of period price.
\( \Sigma \): Unconditional variance of end of period price (0.08).
\( Z_j \): Class j investor conditional variance of \( p \).
\( Z \): Market power-weighted average conditional variance of \( p \).

Information
\( m \): Supply of shares divided by the sum of risk-tolerance coefficients, expectation \( \bar{m} \) (1.503), and variance \( \Sigma_m \) (0.00034).
\( p'_i = p + \epsilon_i \): Price signal observed by investor i in class I.
\( \epsilon_i \): Price signal noise, uncorrelated across investors, uncorrelated with other random variables, ex-ante variance \( \Sigma_\epsilon \) (0.4).
\( S \): Liquidity supply observed by investors SI, mean zero, and variance \( \Sigma_S \) (0.00017).
\( L \): Unobserved liquidity supply, mean zero, and variance \( \Sigma_L \) (0.00017); L and S are independent.

Investors
\( SI \): Supply-informed investor class, observe \( p_0 \) and S.
\( I \): Price-informed investor class, observe \( p_0 \) and \( p'_i \).
\( U \): Uninformed investor class, observe \( p_0 \).
\( j \): Investor class SI, I, or U.
\( a_j \): Investor class j risk tolerance.
\( w_j \): Number of investors in class j.
\( K_j \): Relative market power of class j: ratio of the products of \( w_j \) and \( a_j \) to the sum across classes:
\[ k_j = a_j w_j / \sum a_j w_j \] (\( k_I \): 0.02, \( k_{SI} \): 0.005, \( k_U \): 0.975).
\( \pi(p_0) \): Hedging share supply.
\( \omega \): Fraction of share total hedged (5%).
Appendix B: Equivalence of equilibria

The analysis is greatly simplified if the investors subject to margin constraints and willing to trade are actually constrained in equilibrium. If liquidity traders are subject to margin constraints, a tightening of margin constraints reduces the amount of liquidity trading proportionately.

We now turn to the case where speculators are subject to margin constraints. In order to obtain a tractable solution, we assume that speculators are risk neutral and pay transaction costs. Speculator i privately observes the signal p+ε_i, as well as the equilibrium price p_0. The expectation of the future price p conditional on the information available to investor i is denoted \( \bar{p}_i \). A share purchase (or sale) will be profitable if the expected profit exceeds the transaction cost, c. In the case of a purchase, if:

\[
\bar{p}_i - p_0 > c. \tag{B.1}
\]

A sale is profitable if:

\[
p_0 - \bar{p}_i > c. \tag{B.2}
\]

Being risk-neutral, speculators willing to trade do trade the largest possible amount. Consequently, the margin constraint is binding in equilibrium for speculators willing to trade. If speculators were risk averse, their pattern of trading would be qualitatively similar. However, aggregate speculative trading would not be normally distributed and the rational expectation equilibrium would not have the usual linear structure.

Define the random variable n(i,c) as equal to 1 if agent i faced with transaction cost c buys, -1 if agent i sells, and 0 in case of no transaction. The expectation \( \bar{p}_i \) of the future price p conditional on the information available to informed speculator i is a function of the equilibrium price p_0 and of his informative signal, p+ε_i. This function is linear in the signal p+ε_i because of the joint normality assumptions (see GL). The distribution of \( \bar{p}_i \), conditional on future and equilibrium prices, p and p_0, is therefore normal. Let q be the expectation of \( \bar{p}_i \) (conditional on p and p_0) and σ its standard deviation. q and σ are identical for all speculators. The expectation of n(i,c), conditional on p and p_0, is given by:

22
\[
E(n(i,c)) = \text{Prob}(\bar{p}_i - q > c + p_0 - q) - \text{Prob}(\bar{p}_i - q < p_0 - q - c)
\]

where the probabilities are cumulative normals with variance \( \sigma^2 \) and zero mean. Defining \( \delta \) as \( q - p_0 \) and taking advantage of the symmetry of the distribution yields:

\[
E(n(i,c)) = N(\delta - c) - N(-\delta - c),
\]

where \( N \) denotes the cumulative normal distribution with mean zero and variance \( \sigma^2 \).

The signals observed by speculators whose transaction cost is \( c \) are assumed to be independently and identically distributed. As in GL, we consider the limit of a sequence of finite economies where the relative proportion of investors in each class remains fixed and the total number of investors as well as the supply parameters grow without bound at the same rate. Hence the average demand by speculators whose transaction cost is \( c \), \( n(c) \), converges to the expectation as the number of speculators faced with transaction cost \( c \) tends to infinity (Law of Large Numbers). In the limit, the average \( n(c) \) is equal to \( E(n(i,c)) \).

Finally, we assume that the population of speculators facing transaction cost \( c \) is the same for any level of transaction cost. The number of speculators who face a transaction cost comprised between \( c \) and \( c + dc \) is thus \( \alpha dc \), where \( \alpha \) is a positive constant.

The total number of buys minus sells is then obtained by integrating over the populations with different transaction costs:

\[
\Pi = \int_0^\infty n(c) \alpha dc = \int_0^\infty [N(\delta - c) - N(-\delta - c)] \alpha dc
\]

Integrating by parts and noting that the first term is zero yields:

\[
\Pi = \int_0^\infty \alpha \ c \ [N'(\delta - c) - N'(-\delta - c)] dc,
\]

where \( N' \) denotes the normal density function (variance \( \sigma^2 \)).
Changes of variables yield:

$$\Pi = \int_{-\infty}^{\delta} \alpha N(c)(\delta-c) \, dc - \int_{-\infty}^{-\delta} \alpha N'(c)(\delta-c) \, dc,$$

which may be written as:

$$\Pi = \alpha \delta (N(\delta)+N(-\delta)) + \alpha \int_{-\delta}^{\delta} N'(c)c \, dc.$$

By symmetry of the normal density, the second term is equal to zero, hence \( \Pi \) is given by:

$$\Pi = \alpha \delta (N(\delta)+N(-\delta)) = \alpha \delta = \alpha (q-p_0).$$

At the NYSE, the collateral required for an investment in stocks is equal to a fraction of the dollar value of the position. In futures markets, however, an investor is required to actually invest a fixed amount per futures contract. We focus on the latter specification of margin constraints because speculators would prefer to trade in the futures market because of its higher liquidity, lower transactions costs, and lower margin requirements. Hence for a given level of investment capital, \( W \), an investor will be able to buy at most \( W/m \) futures contracts on the index where \( m \) is the margin requirement expressed as the dollar amount to be paid to purchase a position in futures contracts equivalent to one unit of the underlying asset.

Aggregate demand by speculators is thus given by:

$$\frac{W}{m} \alpha (q-p_0).$$  \hspace{1cm} (8.3)

In the original GL model, the demand by informed speculators is proportional to \( q-p_0 \) as well. The proportionality factor in the GL model is the product of the fraction of informed traders, \( k_i \), and the inverse of the conditional variance of the future price, \( p_0 \). In this case it is the ratio of the investment
capital controlled by speculators who face transactions costs comprised between \( c \) and \( c+1 \), \( \alpha W \), and the margin requirement, \( m \).

All the results obtained in GL therefore obtain, provided that one reinterprets the average demand by informed traders as the average demand by speculators obtained here. In the analysis, we start with a base case identical to the one in GL and study the effect of changes in margin requirements. For example, doubling the margin requirement is equivalent to halving the fraction \( k_1 \) of informed investors in the original GL model.
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