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Welfare Economics of Financial Markets

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One of the more noteworthy developments in financial economics is the emergence of equilibrium models of the financial market. Most of these models are of the partial equilibrium variety, that is, they focus on a subset of securities and are usually based on some form of arbitrage analysis. Examples are the value conservation proposition of Modigliani and Miller (1958), the option pricing model of Black and Scholes (1973), the arbitrage pricing theory of Ross (1976b), and a multitude of applications based on the contingent claims approach (see e.g. Merton (1990, Part IV)). The principal models in the spirit of general equilibrium, much smaller in number, are those of Sharpe (1964), Lintner (1965), and Mossin (1966), and of Merton (1973) and Breeden (1979).

The primary focus of these models of the financial market has been on the structure of returns or prices and the associated risk premia, with particular attention paid to derivative securities. In contrast, relatively few studies, e.g. Borch (1968, Ch. 8), Ross (1976a), Hakansson (1982), Ohlson (1987, Ch. 5), have addressed the welfare economics of financial markets. This imbalance may seem somewhat odd in view of the proliferation of financial securities and information services over the last twenty years, which on the surface at least would appear to be welfare beneficial by moving us closer to a complete and more informed market.

Behind this asymmetry of attention, however, lurks a paradox. The power to price existing and new securities via arbitrage analysis (or contingent claims analysis) rests on the ability to replicate their payoff patterns via a portfolio of other securities. The possibilities for replication are especially potent in the standard continuous-time model, in which, for short trading intervals, only the first two moments "matter". But "new" securities whose payoff patterns can be replicated via a portfolio of existing securities are clearly redundant since they do not expand the set of feasible payoff patterns available to investors.
In short, arbitrage analysis enables us to exactly price securities we don't need, but is uncapable of pricing securities that are beneficial (except within fluid bounds).

To avoid linkage with any particular stochastic process of returns or price dynamics, the welfare economics of financial markets in this essay will be based on a discrete-time decision framework in which the stochastic process is arbitrary. The general two-period, pure exchange model of the financial market employed in the analysis is developed in Section I. Conditions under which full allocational efficiency is attained in incomplete markets are identified in Section II. Section III traces the welfare and equilibrium price effects resulting from changes in the set of securities that comprises the financial market; a discussion of the transaction cost dimension is also included. The welfare implications of public information, and the elusiveness of informational efficiency, are addressed in Section IV, while Section V compares the welfare-improving potentials of new securities to that of better public information systems.

I. THE BASIC MODEL

The earliest models systematically incorporating uncertainty in analyzing markets were those of Allais (1953), Arrow (1953), Debreu (1959, Ch. 7), and Borch (1962). They may therefore be viewed as the forerunners of more comprehensive models of the financial market, including the two-period model developed below.

Assumptions

We consider a pure exchange economy with a single commodity which lasts for two periods under the standard assumptions. That is, at the end of period 1 the economy will be in some state s where \( s = 1, \ldots, n \). There are I consumer-investors indexed by \( i \), whose probability beliefs over the states are given by the vectors \( \pi_i = (\pi_{i1}, \ldots, \pi_{in}) \), where, for simplicity, \( \pi_{is} > 0 \), all \( i, s \).
The preferences of consumer-investor $i$ are represented by the (conditional) functions $U_i(c_i, w_i)$, where $c_i$ is the consumption level in period 1 and $w_i$ is the consumption level in period 2 if the economy is in state $s$ at the beginning of that period. These functions are defined for
\[(c_i, w_i) \geq 0, \quad \text{all } i, s \quad (1)\]
and are assumed to be increasing and strictly concave.

At the beginning of period 1 (time 0), consumer-investors allocate their resources among current consumption $c_i$ and a portfolio chosen from a set $J$ of securities indexed by $j$. Security $j$ pays $a_{js} \geq 0$ per share at the end of period 1 and the total number of outstanding shares is $Z_j$. Let $z_{ij}$ denote the number of shares of security $j$ purchased by investor $i$ at time 0; his portfolio $z_i = (z_{i1}, \ldots, z_{iJ})$ then yields the payoff
\[w_i = \sum_{j \in J} z_{ij} a_{js}\]
available for consumption in period 2 if state $s$ occurs at the end of period 1. Investor endowments are denoted $(\overline{c}_i, \overline{Z}_i)$ and aggregate wealth or consumption in state $s$ is given by
\[W_s = \sum_{j \in J} Z_j a_{js}, \quad \text{all } s .\]

The financial markets, as is usual, are assumed to be competitive and perfect, that is, consumer-investors perceive prices as beyond their influence, there are no transaction costs or taxes, securities and commodities are perfectly divisible, and the full proceeds from short sales (negative holdings) can be invested. The number of securities, however, need not be large (although this is not ruled out). Since our focus is on the structure of the financial market, and
changes therein, production decisions (and hence the vector of aggregate consumption \((C, W)\)) are viewed as fixed.

If the rank of matrix \(A = [a_{js}]\) is full (equals \(n\)), the financial market will be called complete; if not, it will be called incomplete. The significance of a complete market is that any payoff pattern \(w \geq 0\) can be obtained via some portfolio \(z\) since the system \(zA = w\) will always have a solution. (In incomplete markets, in contrast, some payoff patterns \(w \geq 0\) are infeasible.) The simplest form of a complete market is that in which \(A = I\) (the identity matrix); the financial market is now said to be composed of Arrow-Debreu or primitive securities (as opposed to complex securities.) The main "advantage" of an Arrow-Debreu market is that it never requires the consumer-investor to take short positions, which is generally necessary in a complete market composed of complex securities. Finally, a financial market which contains a risk-free asset, or makes it possible to construct a risk-free portfolio, is called zero-risk compatible.

Under our assumptions, each consumer-investor \(i\) maximizes

\[
V_i \equiv \sum_s \pi_{is} u_i(c_i, \sum_{j \in J} z_{ij} a_{js})
\]

with respect to the decision vector \((c_i, z_i)\), subject to (1) and to the budget constraint

\[
c_i P_0 + \sum_{j \in J} z_{ij} P_j = \bar{c}_i P_0 + \sum_{j \in J} \bar{z}_{ij} P_j
\]

as a price-taker, where \(P_0\) is the price of a unit of period 1 consumption and \(P_j\) is the price of security \(j\).

**Equilibria and Their Properties**

In view of our assumptions, an equilibrium will exist but need not be unique (see e.g. Hart, 1974; note also that uniqueness is with
reference to the consumption allocation \((c, w)\), not allocation \((c, z)\).

The equilibrium conditions for any market structure \(A\), assuming for simplicity that the non-negativity constraints on consumption are not binding, may be written

\[
\sum_{s} \pi_{is} \frac{aU_{is}(c_i, \sum_{j \in J} z_{ij} a_{js})}{ac_i} = \lambda_i \quad \text{all } i \quad (3)
\]

\[
\sum_{s} \pi_{is} \frac{aU_{is}(c_i, \sum_{j \in J} z_{ij} a_{js})a_{js}}{aw_{is}} = \lambda_i p_j \quad \text{all } i, j \quad (4)
\]

\[
(c_i, z_i A) \geq 0 \quad \text{all } i \quad (5)
\]

\[
c_i + z_i P = c_i + z_i P \quad \text{all } i \quad (6)
\]

\[
\frac{1}{n} (c_i, z_i) = (C, Z) \quad (7)
\]

where the \(\lambda_i\) are Lagrange multipliers, \((7)\) represents the market clearing equations, and \(P_0\) has been chosen as numeraire, i.e. \(P_0 = 1\).

Any allocation \((c, z)\) which constitutes a solution to system \((3)-(7)\) (along with a price vector \(P\) and a vector \(\lambda\)) is allocationally efficient with respect to the market structure \(A\) since the marginal rates of substitution for any two securities are the same across individuals. When \((c, z)\) is allocationally efficient with respect to all conceivable allocations, whether achieved outside the existing market or not, \((c, z)\) will be said to be fully allocationally efficient (FAE).

To be more precise, define the shadow prices \(R_{is}\) by
\[
R'_{is} = \frac{1}{\lambda_i} \left[ \pi_{is} \left( \sum_{j \in J} \frac{u_i(s, \sum_j z_{ij} a_{js})}{\bar{a}w_{is}} \right) \right].
\]

It is well known that (3)-(7) plus

\[
R'_{is} = R'_{is} \quad \text{all } i \geq 2, \quad \text{all } s
\]

is a necessary and sufficient condition for the market allocation \((c, z)\) to be FAE because (8) insures that the marginal rates of substitution of wealth between any two states are the same for all investors \(i\). (4) may now be written

\[
AR'_{i} = P, \quad \text{all } i.
\]

**Implicit Prices**

The equilibrium value of a feasible second-period payoff vector \(w\) will be denoted \(V(w)\); thus if \(w\) is obtainable via portfolio \(z\), we get \(w = zA\) and hence

\[
V(w) = V(zA) = zP = wR = zAR.
\]

In the above expression, \(R = (R_1, \ldots, R_n)\) represents the not necessarily unique set of **implicit prices** of (second-period) consumption in the various states implied by \(P\) since

\[
AR = P.
\]

By Farkas' Lemma, a positive implicit price vector is always present in the absence of arbitrage and hence in equilibrium. (Arbitrage is the opportunity to obtain either a payoff \(w > 0, w = 0\), at a cost \(zP \leq 0\), or a payoff \(w = 0\) at a cost \(zP < 0\).) In view of (4') and (9), shadow prices are always implicit prices but a set of implicit prices need not be anyone's shadow prices.
II. FULL ALLOCATIONAL EFFICIENCY IN INCOMPLETE MARKETS

When the financial market $A$ is complete, systems (4') and (9) have only one solution, which insures that

$$R_i^* = R_i, \quad \text{all } i.$$ 

This condition, as noted, is necessary and sufficient to attain FAE. Complete financial markets, while a useful abstraction, are not an everyday occurrence, however. Securities number at most a few thousand, while the relevant set of states is no doubt much larger. This leads us to the question: under what circumstances is FAE attained in incomplete markets? One such case is trivial and will be dismissed quickly: the case when individuals are identical in their preferences, beliefs, and (the value of their) endowments. We now turn to three other sets of conditions when this occurs.

Diverse Endowments

Are there any conditions under which individuals with diverse endowments are as well served by a single security in the market as by many? The answer is yes; beliefs must be homogeneous and preferences e.g. of the form

$$U_{is}(c_i, w_{is}) = \begin{cases} U_i^1(c_i) + \rho_s U_i^2(w_{is}) \\ \text{or} \\ U_i^1(c_i) \rho_s U_i^2(w_{is}) \end{cases}, \quad \text{all } i, s, \quad (10)$$

(with $\rho_s > 0$), where

$$U_i^2(w_{is}) = (1/\gamma)w_{is}^\gamma, \quad \gamma < 1, \quad \text{all } i.$$ 

That is, preferences for second-period consumption must be separable, isoelastic, and homogeneous. Everyone's optimal portfolio is now of the form
\[ z_i = k_i Z, \quad \text{all } i, \]

where the \( k_i \) are fractions. In addition, the equilibrium implicit prices \( R \) are now unique and completely independent of the market structure \( A \).

**Linear Risk Tolerance**

To attain FAE with heterogeneous second-period preferences, we need at least two securities in the market. Two-fund separation occurs in every zero-risk compatible market \( A \) under homogeneous beliefs (but arbitrary return structures) when preferences are of the form (10) if and only if

\[
U_i^2(w_{is}) = \begin{cases} 
(1/\gamma)(\phi_i + w_{is})^\gamma & \gamma < 1, \quad \text{all } i \\
\text{or} & \\
-(\phi_i - w_{is})^\gamma & \gamma > 1, \phi_i \text{ large}, \quad \text{all } i \end{cases}
\]

\[
\begin{cases} 
\text{or} & \\
-\exp(\phi_i w_{is}) & \phi_i < 0, \quad \text{all } i
\end{cases}
\]

provided none of the non-negativity constraints on consumption is binding. The optimal policies are now of the form

\[ z_i = k_{i1}z' + k_{i2}z'', \quad \text{all } i, \]

where the portfolio (fund) \( z' \) is risk-free and portfolio \( z'' \) is risky (see e.g. Rubinstein, 1974). It is evident that with diverse endowments, preferences must belong to a very narrow family, even when beliefs are homogeneous, in order for FAE to be attained.

**Supershares**

Two states \( s \) and \( s' \) such that \( W_s = W_{s'} \), i.e. with equal aggregate payoffs, are said to belong to the same superstate \( t \) (Hakansson, 1977). If the financial market is complete with respect to the superstate partition \( T \), FAE is attained for arbitrary endowments if and only if
\[ \frac{\pi_{is}}{\pi_{it}} = \frac{\pi_{is'}}{\pi_{it}}, \quad \text{all } s \in t, \text{ all } i \text{ and } t \quad (11) \]

and

\[ U_{is} = U_{is'}, \quad \text{all } s \text{ and } s' \in t, \text{ all } i \text{ and } t. \quad (12) \]

Note that (11) and (12) requires only conditionally homogeneous beliefs and that preferences are insensitive to states within a superstate—beliefs and preferences with respect to superstates are unrestricted.

To complete the market with respect to superstates, three simple alternatives are available (Hakansson, 1978). The first is a full set of "supershares", each share paying $1 if and only if a given superstate occurs (superstates are readily denominated in either nominal or real terms). The second and third alternatives are a full set of (European) call options or a full set of (European) put options on the market portfolio \( \alpha Z \) or \( \alpha W \), where \( 0 < \alpha \leq 1 \).

It may be noted that a market in puts and calls on a crude approximation to the United States market portfolio, namely the Standard & Poor's 100 Index, was opened in 1983. These options are now the most actively traded of all option instruments.

III. CHANGES IN THE FINANCIAL MARKET

Changes in the set of securities available in the financial market are an everyday occurrence. Early studies on this subject include those of Borch (1968, Ch. 8), Ross (1976a) and Litzenberger and Sosin (1978). To fully trace the effects of such changes involves comparing equilibria, which is a matter of some complexity. However, using the two-period framework of this essay, it is possible to reach some
general conclusions on how changes in the market structure from A' to A'', say, affects welfare, prices, and other dimensions of interest in a pure exchange setting.

The Feasible Allocations

One of the critical determinants, not surprisingly, is the change in feasible allocations. Recall that a market structure A is any "full" set of instruments, i.e. any set of instruments capable of allocating, in some fashion, aggregate wealth \( W = (w_1, \ldots, w_n) \). The set of feasible second-period consumption allocations \( w = (w_1, \ldots, w_i) \) obtainable via market structure A will be denoted \( F(A) \), i.e.

\[
F(A) = \{ w | w_i \geq 0, \ w_i = z_i^A, \ \sum_j z_{ij} = 1, \ \text{all j} \}.
\]

In comparing two market structures A' and A'' with respect to feasible allocations, there are (since holding the market portfolio aZ is always feasible) three possibilities; either

\[
F(A') = F(A'') \quad \text{(Type I)}
\]

or

\[
F(A') \subseteq F(A'') \text{ (or the converse)} \quad \text{(Type II)}
\]

or

\[
\{F(A') \cap F(A'')\} \subseteq F(A') \quad \text{(Type III)}
\]

\[
\{F(A') \cap F(A'')\} \subseteq F(A'')
\]

These three types of changes will be referred to as feasibility preserving, feasibility expanding (or reducing), and feasibility altering.

A sure way to obtain a feasibility expanding change is to make a finer and finer breakdown of existing instruments into an ever larger set of linearly independent (or unique) securities.
Endowment Effects

Since changes in the financial market structure are generally implemented by firms or exchanges and take place when the market is closed, such changes frequently alter investors' endowments. An example would be a merger, which results in the substitution of new securities for old ones. It is useful to distinguish between three cases:

1. **Strong endowment neutrality.** This occurs if the endowed consumption patterns in the two markets are unaltered, i.e. if

   \[(\overline{c}_i', \overline{w}_i') = (\overline{c}_i'', \overline{w}_i'')\], \quad all \ i.

2. **Weak endowment neutrality.** This occurs if the values of the endowments, provided there is a common implicit equilibrium price structure \(R\), are identical in the two markets, i.e. if

   \[
   \overline{c}_i' + \overline{z}_i'P' = \overline{c}_i + \overline{w}_i'R = \overline{c}_i'' + \overline{w}_i''R = \overline{c}_i'' + \overline{z}_i''P'', \quad all \ i
   \]

   where \(R > 0\) satisfies \(A'R = P'\) and \(A''R = P''\).

3. **Non-neutral endowment changes.**

   While the first two cases are rather rare, strong endowment neutrality typically accompanies nonsynergistic (pro rata) corporate spinoffs when applicable bonds remain risk-free as well as the opening of option markets, for example.

The Welfare Dimension

As noted, in comparing different market structures, the comparison which is ultimately relevant is that which compares allocations actually attained, i.e. equilibrium allocations. Using (2), we denote investor \(i\)'s equilibrium expected utility in market structure \(A''\) by
V^n_i and his equilibrium expected utility in market structure A' by \( V'_i \). A comparison of any given equilibrium in market A'' with some equilibrium in some other market A' must then yield one of four cases:

\[
V^n_i \geq V'_i, \text{ all } i, V^n_i > V'_i, \text{ some } i \quad \text{(Pareto dominance)} \quad (i)
\]

or

\[
V^n_i = V'_i, \text{ all } i \quad \text{(Pareto equivalence)} \quad (ii)
\]

or

\[
V^n_i > V'_i, \text{ some } i, V^n_i < V'_i, \text{ some } i \quad \text{(Pareto redistribution)} \quad (iii)
\]

or

\[
V^n_i \leq V'_i, \text{ all } i, V^n_i < V'_i, \text{ some } i \quad \text{(Pareto inferiority)} \quad (iv)
\]

The task at hand, then, is to identify the conditions under which each of these cases, as well as combinations of these cases, will occur. All comparisons are contemporaneous in the sense that they compare welfare under market structure A'' to what it would be if A' were in use instead.

**Principal Results**

The principal results (Hakansson 1982) may be summarized as follows:

1. Feasibility preserving market structure changes yield either Pareto equivalence or redistributions. To preclude Pareto redistributions, we must either have efficient endowments in the first market and strong endowment neutrality, or weak endowment neutrality coupled with unique equilibria. Pareto equivalence is always accompanied by value conservation.

2. Feasibility expanding market structure changes imply either Pareto dominance, Pareto equivalence, or Pareto redistributions. To preclude redistributions, we must have efficient endowments in the first market.
and strong endowment neutrality, or weak endowment neutrality coupled with unique equilibria. Value conservation is highly unlikely.

3. Feasibility altering changes in the market structure have unpredictable value and welfare effects.

4. Value and welfare effects are relatively independent.

As noted by Hart (1975), the introduction of multiple commodities or more than two periods is a nontrivial step which may bring about additional complications, such as Pareto-dominated equilibria when feasibility is expanded.

Within the limits of the single-good, two-period model under pure exchange, certain tentative general conclusions concerning common market structure changes can be stated. Even under mild heterogeneity of preferences and/or beliefs, 100% nonsynergistic mergers tend to be welfare reducing while (nonsynergistic) spinoffs and the opening of option markets tend to be beneficial. The use of risky bonds and preferred stock tends to be virtuous as well, at least apart from bankruptcy costs. Finally, value conservation is a much rarer phenomenon than suggested by Modigliani and Miller (1958) and Nielsen (1978) among others.

From a broader perspective, it is evident that Type II, or feasibility expanding market structure changes, can be implemented either by corporations via their capital structure decisions or by intermediaries and exchanges through the creation of new types of securities. The latter two are more likely to have a greater impact in this area of financial innovation. This is because intermediaries and exchanges can more readily link payoffs to broadly based state (sub)spaces (such as the Standard & Poor’s 500 Stock Index) than the typical corporation, for which the payoff space tends to be more narrowly focussed.
Transaction Costs

As noted earlier, equilibrium is arbitrage-free and redundant securities must sell for the cost of the replicating portfolio in arbitrage-free economies. Thus, the introduction of redundant securities results in a Type I or feasibility preserving change and hence adds no social value.

Two of the most common types of new securities introduced in recent years are call and put options on individual common stocks and on stock indexes. Based on the two-period framework analyzed in this essay, such options will either be redundant or result in a Type II or feasibility expanding change in the market structure. As noted by Ross (1976a), puts are always redundant in the presence of a full set of calls and conversely.

How does one explain that many demonstrably redundant (new) securities are in fact fairly actively traded? As observed earlier, modern finance has run into a Catch 22 of sorts. By use of arbitrage arguments, we can price redundant securities, which add nothing to welfare, precisely, while the values of genuinely new payoff patterns, which potentially enhance welfare, can only be bounded via the prices of other securities. Arbitrage-based pricing is particularly prominent in the continuous-time framework when continuous trading is assumed and the underlying state variable obeys a continuous-sample-path stochastic process.

What this points out is that the perfect market assumption is only an approximation to reality. When complex securities are involved, a desired feasible payoff pattern typically requires long positions in some securities and short positions in others. For example, a $10,000 investment may call for $30,000 in long holdings and $20,000 in short positions. The merit of redundant securities is that they often simplify the construction of the desired portfolio
(payoff) pattern—in the previous example, the long and short positions might be reducible to $15,000 and $5,000, respectively. (When all or most securities are of the Arrow-Debreu variety, short positions can be avoided entirely, of course.) This typically saves transaction costs as well. In this sense, both puts and calls have a role.

IV. PUBLIC INFORMATION

So far, we have focussed on the welfare effects of different security arrangements in the financial market, taking investors’ information as given. We now address the welfare effects of different public information systems, both for a fixed set of securities as well as in a joint analysis.

Let $Y$ be an information system (such as a newspaper), which emits one of $m$ possible signals (or messages) $y$ just prior to the decision point $t = 0$ (the beginning of period 1). Signal $y$ is used by consumer-investor $i$ to update his or her prior beliefs $\pi^0_i$ (denoted $\pi_i$ previously) to the posterior beliefs $\pi^y_i = (\pi^y_{i1}, \ldots, \pi^y_{in})$ via Bayes’ Rule. That is, each consumer-investor $i$ draws on his or her information structure $\Lambda_i = [p_i(y|s)]$ (an $n$ by $m$ matrix of conditional signal probabilities) and prior probabilities to calculate the posterior probabilities $\pi^y_{is} = (p_i(y|s)\pi^0_{is})/\sum_s (p_i(y|s)\pi^0_{is})$, which also yields the signal probabilities $p_i(y)$. For the present, system $Y$ will be viewed as costless.

The solution to system (3)-(7) will now be denoted $(c^y, z^y, y^y, p^y)$ since each signal will generally generate a different equilibrium. Conditional on $\pi^y$, $(c^y, z^y)$ will (as before) be allocationally efficient with respect to $A$. The relevant question, however, is whether the set of equilibrium allocations $(c^1, z^1), \ldots, (c^m, z^m)$ are efficient in a Pareto sense. To address this,
denote consumer-investor $i$'s equilibrium expected utility given signal $y$ by

$$V_i^y = \sum_s \pi_{is}^y U_{is}(c_i^y, \sum_{j \in J} z_{ij}^y a_{js}) .$$

Consumer-investor $i$'s expected utility from information system $Y$, denoted $V_i(Y)$, then becomes

$$V_i(Y) = \sum_y p_i(y) V_i^y .$$

**Informational Efficiency**

A set of signal-contingent equilibrium allocations $(c^1, z^1), \ldots, (c^m, z^m)$ corresponding to signals 1, \ldots, $m$ will be said to be informationally efficient with respect to market $A$ and information system $Y$ if there are no other sets of allocations $(\hat{c}^1, \hat{z}^1), \ldots, (\hat{c}^m, \hat{z}^m)$ based on $A$ and $Y$ which can make some consumer-investors better off without making others worse off. A necessary and sufficient conditions for informational efficiency to be attained in equilibrium is that the endowments $(\bar{c}, \bar{z})$ are such that

$$\frac{p_i(y) \lambda_i^y}{p_i(y_1) \lambda_i^1} = F(y), \quad \text{all } i, y, \quad (13)$$

i.e., that the marginal rates of substitutions of endowments, for any two signals $y$ and $y_1$, are the same for all investors.

Finally, full informational efficiency with respect to $Y$ will be said to occur if (13) holds and the implicit prices $R_{is}^y$ satisfy

$$R_{is}^y = R_{1s}^y, \quad \text{all } i \geq 2, s, \text{ and } y . \quad (14)$$

This is because (14) insures full allocational efficiency for each signal $y$.

It should be noted that informational efficiency, as opposed to allocational efficiency, cannot be guaranteed. Endowments may simply fail to satisfy (13).
Even instituting a pre-signal round of securities trading for the purpose of aligning endowments may prove futile. This would typically be the case, for example, if the number of signals \( m \) exceeds the number of securities \( J \) since (13), in essence, gives rise to \( m \) equations in \( J \) unknowns. More specifically, what is required is that the \( J+1 \) by \( m \) matrix \( P \) of post-signal equilibrium price vectors \( [P^1, \ldots, P^m] \) be of rank \( m \) since only then does the securities market guarantee sufficient flexibility to attain condition (13) via the pre-signal trading round. \( P \), however, depends on "everything" so that one can never be certain that reliance on the regular securities market will lead to informational efficiency. This is a rather disturbing result, of course. The obvious way to guarantee informational efficiency is to conduct the pre-signal trading round in signal-contingent wealth units. While this approach is simple in principle, it does not lend itself to ready implementation in practice.

**Principal Results**

We begin by addressing a question first considered by Hirshleifer (1971), namely whether public information has any social value. This involves comparing \( V_i(Y) \) with \( V^0_i \) for all \( i \), where \( V^0_i \) (= \( V_i \) in Section III) is the null information case. Perhaps surprisingly, release of public information may give rise to Pareto inferiority in addition to Pareto dominance, Pareto equivalence, or a Pareto redistribution. The principal negative effect of public information, which may overwhelm other effects, is that it subjects consumer-investors to endowment risk--each equilibrium price vector \( p^Y \) values one's endowment differently.

More specifically, a necessary condition for public information to yield Pareto dominance over the null information case is that at least one of the following four conditions holds:
• Prior beliefs are not homogeneous
• Information structures are not homogeneous
• Utility functions are not all additive
• The financial market is less than fully allocationally efficient.

Sufficient conditions are somewhat more stringent (see Hakansson, Kunkel, and Ohlson, 1982).

An interesting, and revealing, special case occurs when consumer-investors have essentially homogeneous prior beliefs and information structures, additive utility functions, and full allocational efficiency holds. Then, if endowments are in equilibrium based on the prior beliefs, release of any signal $y$ will cause no trading—the change in equilibrium prices from $p^0$ to $p^y$ will precisely offset everyone's desire based on $m^y_1$ to trade away from portfolio $z^0$. If the information is costless, there would be no welfare effect generated by the information system $Y$.

In sum, the principal forces impinging on the value of the public information are the following:

Positive forces:

1. Heterogeneity among prior beliefs
2. Heterogeneity among information structures
3. Non-additivity among utility functions
4. Incompleteness in the financial market

Negative forces:

1. Unavailability of "endowment insurance"
2. The cost of the information system

It should be noted that the four positive forces are independent and mutually reinforcing. Each one also possesses a high degree of empirical validity. Thus,
we can be confident that at least low-cost public information systems based on equal access, such as those reporting on the money supply, inflation rates, estimated harvests, and company developments are on balance socially beneficial. Whether the current structure of periodic financial reporting, and occasional major announcements, by firms is also socially valuable is less clear. This is because the timing of releases, insider information issues, and relatively high costs complicate the analysis (see Hakansson (1990)).

**Ordering Information Structures**

Up to this point, comparisons have been between a (non-null) system $Y$ and the null system $Y^0$ based on prior beliefs only. To make comparisons between non-null systems $Y$, it will be useful to introduce a concept due to Blackwell (see Blackwell and Girshick (1954): Information system $Y''$ is more informative than $Y'$ ($Y'' \succ Y'$) if there exists a Markov matrix $M$ such that $A_i^' = A_i^" M$, for all $i$, that is, if $Y'$ is a "randomized" version of $Y"$.

The informativeness criterion induces a partial ordering of information systems $Y$. For our purposes, however, the most valuable implication is that if $Y'' \succ Y'$, then, for any financial market structure $A$, the Pareto surface corresponding to $Y''$ is at least as high as for $Y'$. Thus, the informativeness criterion provides a partial ordering of Pareto surfaces just as $F(A)$ does, as noted in Section III. This ordering is particularly "sharp" in the presence of the positive forces identified earlier.

The most informative information system (in principle) provides perfect information and will be denoted $Y_{max}$. Since a signal from this system reveals which state $s$ will occur with probability 1, the consumer-investor's decision problem, given any signal $y$, is reduced to a decision problem under certainty.
Even if market structure $A$ contains only a single security, it will be conditionally complete, that is, it will be complete conditional on any signal from $Y_{\max}$. More generally, given a market $A$, let $Y^{cc}(A)$ be the coarsest (least informative) information system $Y$ for which $A$ is conditionally complete for all signals $y$ and consumer-investors $i$. Thus, public information offers not one but two potential benefits: the value of the signals themselves and a possibly "enriched" financial market via the elimination of certain states by reducing their probabilities to zero.

V. NEW SECURITIES VS. MORE PUBLIC INFORMATION: WHICH IS BETTER?

We have now identified two distinct paths to improving welfare in the financial market (holding firms' investments fixed): one approach centers on ways to implement consecutive Type II changes in the set of securities available, the other on finding more informative public information systems. This raises the question: which path is better?

Let's return to the assumption that changes can be implemented costlessly. Denote the current market structure by $A$ and let $Y$ denote the present public information system. Holding $Y$ constant, denote consecutive Type II improvements by $A'(Y)$, $A''(Y)$, ..., so that, ceterus paribus,

$$A(Y), A'(Y), A''(Y), \ldots, I(Y)$$

represents an ordering of financial market Pareto surfaces from the present (incomplete) market $A(Y)$ to the complete market ideal $I(Y)$. Analogously, let $Y'(A)$, $Y''(A)$, ..., denote strict improvements in informativeness without changes to $A$, which gives rise to the ordering
of public information system Pareto surfaces. (Recall that, in the public information case, the competitive model does not necessarily land us on the Pareto surface.)

Putting the two sequences together gives the following picture

\[
\begin{array}{c|c}
\text{Present} & \text{Higher Pareto Surfaces} \\
\hline
A(Y), A'(Y), \ldots, I(Y) & \text{--->} \\
Y(A), Y'(A), Y''(A), \ldots, Y_\infty(A), \ldots, Y_{max}
\end{array}
\]

Clearly, A(Y) and Y(A) are identical, as are Y(I) and I(Y), and Y_\infty(A) and Y_\infty(I). Since Y_\infty(I) is associated with a higher surface than Y(I), Y_\infty(A) places consumer-investors on a higher surface than I(Y).

It is evident that while both paths are valuable, improvements in public information are potentially of greater benefit than moving (closer) to a complete market. In particular, public information has the power, via conditional completeness, to pre-empt the need for a rich set of securities. Public information systems, however, are generally more costly to implement than new securities. As the technology of communication improves, continuous progress along both paths can be expected.
References


