On Revelation of Private Information in Stock Market Economies

by

Marcus Berliant
and
Sankar De

April 1995
RESEARCH PROGRAM IN FINANCE AT THE
WALTER A. HAAS SCHOOL OF BUSINESS,
UNIVERSITY OF CALIFORNIA, BERKELEY

The Research Program in Finance in the Walter A. Haas School of Business at the University of California has as its purpose the conduct and encouragement of research in finance, investments, banking, securities markets, and financial institutions. The present reprint and working paper series were established in 1971 in conjunction with a grant from the Dean Witter Foundation.

INSTITUTE OF BUSINESS AND ECONOMIC RESEARCH
Carl Shapiro, Director

The Institute of Business and Economic Research is an organized research unit at the University of California, Berkeley, whose mission is to promote research by faculty and graduate students in the fields of business and economics. The Institute carries out its mission by organizing programs and activities that enrich the research environment, administering extramural research awards, publishing working papers, and making direct grants for research.
On Revelation of Private Information in Stock Market Economies

Marcus Berliant 
and 
Sankar De

April 1995

Finance Working Paper #245

The authors would like to thank Dee Dechert, Jim Friedman, David Gordon, Frank Page, and Paul Romer for helpful comments. Any errors remaining in the paper are, however, the authors' responsibility.
Abstract

The notion that an agent in a given market can infer from the market price the (non-price) information received by other agents, as embodied in the existing studies of revealing rational expectations equilibrium, requires that the agent know the correct functional relationship between the non-price information of all agents and the resulting equilibrium price. This condition is usually restrictive and unsuitable as a description of reality. In this paper we show that this condition is also unnecessary in a stock market economy where producers or firms use their private information in their own optimization programs, which include stock purchases. Interestingly, this result does not extend to the case of consumers with private information.

Marcus Berliant
Department of Economics
Washington University, Campus Box 1208
St. Louis, MO 63130-4899
and
Department of Economics
University of Rochester
Rochester, NY 14627-0156

Sankar De
Indian Institute of
Management
Calcutta, India
and
Haas School of Business
University of California
Berkeley, CA 94720-1900
1. Introduction

The notion that an economic agent in a given market can use the equilibrium market price to make inferences about the (non-price) information received by other agents regarding the exogenous states of the environment, as embodied in models of rational expectations equilibrium in markets with differential information, requires that the agent know the correct functional relationship between the non-price information received by all market participants and the resulting equilibrium price (see, for example, Grossman (1978, 1981), Kihlstrom and Mirman (1975), and Radner (1979)). Given that the information-price function is known, the question whether an individual agent can glean the initial information of all agents from market prices, that is whether the information-price function is invertible on the set of admissible prices, assumes significance. This question is the focal point in the literature on fully revealing rational expectations equilibria. Yet the condition that lends significance to it, namely that the true relationship between the information signals of all agents taken together and the market prices is known to all, has been relatively little examined. However, this condition is usually very restrictive. Two conditions that are each sufficient (but not necessary) for knowledge of this relationship are the following. First, each agent could learn this relation over time; such a justification requires a model of learning such that convergence to the appropriate relation is assured. Second, each agent could have a good deal of information about the private characteristics and plans of other agents, along with a great computational ability. In any case, each agent must know the information-price relation, and this relation could be very complex.

Interestingly, this condition is unnecessary in one important situation. We show that, in a stock market economy characterized by producers with private information which they use in their own decision-making process, equilibrium stock prices reveal all market-relevant information under very general conditions even though the agents may not know the correct information-price function. We present formal models in Sections
II and III of this paper to demonstrate this result. The process through which this comes about can broadly be sketched as follows. The producers or the firms, acting on the basis of their private information, decide to own fractions of other firms and, by retaining shares, of themselves as well. Given certain general conditions, the profits from these activities based on their private information are in equilibrium a linear function of the stock prices and other publicly observable market data. As a result, the private calculations that go into their decision-making are of no consequence to the uninformed agents. This is the main result of our study. Note that this result does not extend to the case where consumers have private information about individual endowments or wealth since, unlike firms, they cannot own each other or issue claims against each other in a standard market setting. This creates an interesting asymmetry between consumers with private information and producers with private information in a stock market economy.

In Section II of this paper, we present a formal model to demonstrate our main result. In Section III we extend our model to a general equilibrium framework and show that the result holds generically in this setting. Section IV presents some implications of our study and our conclusions.

II. A Partial Equilibrium Model of Revelation When the Information–Price Relationship is Unknown

In this section we outline an optimization model for producers with private information in a stock market economy, and discuss a necessary condition for equilibrium that results in equilibrium prices, along with other observable market data, revealing all relevant information about the firms even though their private information as well as the function mapping that information into prices remain unknown.

Suppose there are $n$ firms ($n$ integer and finite) indexed by $i$ and $j$. For simplicity, suppose that firms issue only one kind of security, namely stock. Further,
suppose each firm issues only one unit of stock (through normalization). Firms can own not only their stock, but the stock issued by other firms as well. We define $y^i_j$ to be the fraction of firm j's stock owned by firm i, a choice variable for the manager of firm i. Similarly, the variable $y^i_i$ represents the fraction of its own stock retained by firm i in its vaults. It is the quantity the firm does not sell to anybody including its own employees. Of course, it is possible that $y^i_i = 0$. Let $Y^i \equiv (y^i_1, ..., y^i_n)$. Further, let

$$Y \equiv \begin{bmatrix} y^1_1 & \cdots & y^1_n \\ \vdots & \ddots & \vdots \\ y^n_1 & \cdots & y^n_n \end{bmatrix}$$

The model has two dates in the sense that the managers of all firms in the market make their decisions about the activities of their firms at date 1. In other words, each manager solves his optimization problem at date 1. At date 1, there is uncertainty concerning the returns to the various activities. The uncertainty arises due to the stochastic nature of each firm's production set. This uncertainty is resolved only at date 2 when the firms realize the returns on their investments.

Formally, we proceed as follows. There are $k$ physical commodities and $n$ firms. The prices for the economy are denoted by $(P,Q)$, where $Q = [Q_1, ..., Q_k]'$ are prices for the physical commodities and $P = [P_1, ..., P_n]'$ are stock prices. Prices $(P,Q)$ lie in the simplex $\Delta \equiv \{ p = (P,Q) \in \mathbb{R}^{n+k} \mid P_i \geq 0 \text{ for all } i, Q_j \geq 0 \text{ for all } j, \text{ and } \sum_{i=1}^n P_i + \sum_{j=1}^k Q_j = 1 \}$. At date 1, the decision-making problem of the manager of firm i involves two vectors of choice variables. First, he chooses $y^i_j$ for $j = 1, 2, ..., n$, where $y^i_j$, as mentioned above, denotes the fraction of firm j's stock owned by firm i. Second, he chooses $Z^i$, where $Z^i$ is defined to be the vector of inputs and outputs of the firm's production process (inputs negative, outputs positive). Let $e^i_j$ be the initial endowment.
of firm j's stock held by firm i, let \( e_j = \sum_{i=1}^{n} e_{ij} \), and let \( e = [e_1^1, \ldots, e_n^1]' \). Notice that \( e_j \leq 1 \) for each \( j \). In the partial equilibrium model discussed in this section, we neglect discussion of the consumer sector, which could also have initial endowments of stock as well as stock ownership in equilibrium. We define

\[
E = \begin{bmatrix}
e_{1}^{1} & \ldots & e_{n}^{1} \\
\vdots & & \vdots \\
e_{1}^{n} & \ldots & e_{n}^{n}
\end{bmatrix}
\]

The information available to the manager of the firm i at date 1 when he makes his investment decisions is now described. First, the market value of one unit of firm j's stock \( P_j \), for \( j = 1, 2, \ldots, n \), is known to the manager of firm i. It is not necessary for the first part of our study that the stock market be perfectly competitive; it is simply assumed that the manager of firm i knows the price schedule that he faces. Our structure is general enough to include imperfectly competitive markets where some prices could be choice variables for the manager of firm i. Next, it is assumed that the matrix of stock purchases by all other firms,

\[
\begin{bmatrix}
y_1^1 & y_2^1 & \ldots & y_n^1 \\
\vdots & & & \vdots \\
y_1^{i-1} & y_2^{i-1} & \ldots & y_n^{i-1} \\
y_1^{i+1} & y_2^{i+1} & \ldots & y_n^{i+1} \\
\vdots & & & \vdots \\
y_1^n & y_2^n & \ldots & y_n^n
\end{bmatrix}
\]

as well as the matrix of initial stockholdings represented by \( E \) are known to the manager of firm i when he solves his maximization problem. Finally, the manager of firm i observes an exogenous random vector \( \epsilon^i \) that represents his personal information about the alternative states of nature. Let \( \epsilon = (\epsilon_1^1, \ldots, \epsilon_n^1) \). Further, let \( T^i(\epsilon^i) \subseteq \mathbb{R}^k \) be
the technology available to the manager of firm i at date 1. Since \( \epsilon^i \) is known only to
the manager of firm i, \( T^i(\epsilon^i) \) is private information as well. Such private information
about a firm's technology available only to the firm's manager could arise, for example,
from the outcome of research and development expenditures. The form of private
information assumed in this model is reasonable and general.

The objective function of the manager of firm i is now introduced. Of course, it
is unclear what firms maximize under incomplete markets (e.g. expected profits), and
we abstract from this issue by simply assuming that there is an objective function.
We now define the function \( V^i(Y^i; Z^i; (P, Q)) \) to be manager i's objective function.
The value of the function depends on the stock purchases of firm i, its production
activities begun at date 1, and market prices. Note that the manager of firm i does
not necessarily know \( Z^j, j \neq i \). Note that the function \( V^i \) is, by its construction, very
general in nature. It does not necessarily represent the expected value of the firm's
uncertain return distribution at date 2 conditional on the given arguments of the
function, though we could, of course, use it to mean just that.

Finally, we assume that the markets for securities are perfect; in other words,
there are no taxes, transaction costs, etc.

The consumer sector is peripheral to the results in this section of the paper; it
will be detailed in the next section, where it is needed to complete the model for
general equilibrium analysis.

---

It is possible to let \( V^i \) depend not only on \( Y^i \) but on the entire matrix of stock
purchases \( Y \), so that \( V^i = V^i(Y; Z^i; (P, Q)) \). This would allow \( V^i \) to take account of
the possible covariances between the return vectors of firms to a greater degree than
the formulation in the main body of the text permits. However, this further
generalization does not alter any of our results and is, therefore, unnecessary for our
exposition.
Given \((P, Q) \in \Delta, Y^i, \) and \(Z^i,\) we write firm \(i\)'s profits or returns per unit of stock at date 2 as \(\pi_i.\) Let \(\pi \equiv [\pi_1, ..., \pi_n]\). With the notations and the set of assumptions as described above, the manager of firm \(i\) faces the following problem at date 1 (an explanation follows immediately):

\[
\begin{array}{ll}
(1) & \text{max} \\
 & V^i(Y^i; Z^i; (P, Q)) \\
Y^i & \\
Z^i & \\
\text{(and possibly (P,Q))} & \\
\end{array}
\]

subject to

\[
\begin{array}{ll}
(b) & Z^i \in T^i(\epsilon^i) \\
(c) & \pi_j = \sum_{h=1}^{n} [y^j_h \cdot (\pi_h - P_h) + e^j_h \cdot P_h] \text{ for } j = 1, ..., n.
\end{array}
\]

The significance of the objective function itself is obvious. Apart from making production decisions, the manager of firm \(i\) must choose which stocks to buy for his company in order to maximize his objective. Note that by purchasing stock of other firms, the manager changes the return vector of his own firm. Further, in an imperfectly competitive market, \((P, Q)\) can be a decision variable for the manager.

Expression 1(b) represents the technological constraints on the firm. It simply says that input–output combinations must be in a set that depends on the realization of the random variable, where the latter is private information. As a consequence, both \(Z^i\) and the realized production set are also private information.

Expression 1(c) is a set of consistency conditions on profits or returns at date 2. Given the assumption that \(Y^j (j \neq i), \) the matrix \(E,\) and the prices \((P, Q)\) are known to the manager of firm \(i\) at date 1, condition 1(c) has a natural interpretation. Recall that both \(P_j\) and \(\pi_j\) are expressed in dollars per unit of stock. Expression 1(c) means that at date 2 the total profit of firm \(j\) is the sum of: (i) the profits earned from
speculative as well as production activities, \( \sum_{h=1}^{n} y_{h}^{j} (\pi_{h} - P_{h}) \), which represents stock purchases\(^2\) multiplied by per-share net profit (that is, final worth less cost) of the concerned firms; and (ii) \( \sum_{h=1}^{n} e_{h}^{j} P_{h} \), the value of the stock that firm \( j \) is endowed with. This is a simple accounting identity that holds in equilibrium no matter what state of the world is realized. It is an ex post condition. Note that implicit in it is the production process of firm \( j \), in that both \( Z_{j}^{i} \) and \( \epsilon_{j}^{i} \) help determine equilibrium values of \( P_{j} \) and \( \pi_{j} \), \( j = 1, \ldots, n \).

A very important point that we wish to emphasize here is that the firm manager believes that in equilibrium the consistency conditions 1(c) will be satisfied. Out of equilibrium, the manager has no reason to believe that they will be satisfied. Thus, they are a property of an anticipated equilibrium, not of demand. This is similar in spirit to the use of the information-price relationship by agents in a rational expectations model, where the relationship is anticipated to hold in equilibrium, but the relationship is used by agents to obtain information employed in formulating demand. A consequence of this condition, reflected in the results derived below, is that in equilibrium a firm's manager cannot believe that the profit or returns to his firm is higher or lower than a linear function of the equilibrium share prices.

If the constraint set is compact and \( V^{i} \) is continuous, then a solution to problem (1) exists. In fact, under the usual conditions, an explicit solution to problem (1) can

\[ y_{j}^{i} + \sum_{k=1}^{n} y_{k}^{i} y_{j}^{k} + \sum_{k=1, \ell=1}^{n} y_{k}^{i} y_{\ell}^{k} y_{j}^{\ell} + \ldots \]

\[ k \neq i \quad k \neq \ell \quad k \neq j \quad k \neq j, \ell \neq j \]

This accounts for both direct and indirect ownership. It is implicit in the interdependent set of linear equations 1(c).

\(^2\)The securities of firm \( j \) owned by firm \( i \) is actually a series
be found using Lagrangean methods. However, that is not the purpose of this paper.

Define $I$ to be the $n \times n$ identity matrix.

**Theorem 1:** Let $Y$ be an equilibrium matrix of firm shareholdings, and suppose that $I - Y$ is invertible. Then it must be true that equilibrium prices and profits satisfy:

$$
\pi = [I - Y]^{-1} \cdot [E - Y] \cdot P.
$$

(2)

**Remark.** This is a partial equilibrium result, since we use only condition 1(c) (which holds in equilibrium) but not market clearing conditions, and the consumer sector is absent. *Any agent who knows $Y$, $E$, and $P$ can figure out $\pi$ without knowledge of the relationship between private information, $\epsilon$, and prices, $P$.*

**Proof:** In vector form, 1(c) can be written as $\pi = Y \cdot [\pi - P] + E \cdot P$. Hence $[I - Y] \cdot \pi = [E - Y] \cdot P$, and since $[I - Y]$ is assumed to be invertible, $\pi = [I - Y]^{-1} \cdot [E - Y] \cdot P$.

Q.E.D.

**Corollary 1:** Suppose that each firm is endowed with all of its own stock, $E = I$, and that $Y$ is an equilibrium matrix of firm shareholdings with $I - Y$ invertible. Then a necessary condition for an equilibrium is that $\pi = P$.

**Proof:** A trivial application of Theorem 1.

**Corollary 2:** Suppose that $Y$ is an equilibrium matrix of firm shareholdings with $Y$ non-negative. Suppose further that for each firm, in equilibrium there exists a consumer who owns a fraction of that firm: $\sum_{i} y_{j}^{i} < 1$ for $j = 1, 2, \ldots, n$. Then a necessary condition for an equilibrium is that $\pi = [I - Y]^{-1} \cdot [E - Y] \cdot P$. 
Proof: Suppose that I−Y (where I is the nxn identity matrix) is not invertible. Then 1 is a characteristic root of Y.

Let \( s = \max \sum_{j=1}^{n} y_{ij} \). By assumption, \( s < 1 \). Recall that each element of Y is non-negative. Let \( r \) be the maximal characteristic root of Y, and let \( x \) be a column vector of \( n \) ones. Then, if the matrix \( Y^t \) represents the transpose of Y, \( Y^t x \leq s \cdot x \). By Debreu and Herstein (1953, Lemma*), since every element of \( x \) is strictly positive, \( r \leq s < 1 \). Hence 1 cannot be a characteristic root of Y; the hypothesis is false and I−Y is invertible.

The remainder of the proof follows from Theorem 1.

Q.E.D.

The results above indicate that if each manager faces problem (1), then stock prices in conjunction with stock purchases as well as initial holdings, which are assumed to be known to all, are sufficient to infer date 2 returns or profits anticipated by the managers acting on the basis of their private information. In the case where a firm is endowed with all of its own stock, the profits of a firm as seen by its manager are the same as their market prices.

Intuitively, the results indicate the following. If the maximization programs of the managers of the firms are executed in a consistent manner (i.e., one that satisfies 1(c)), managerial private information is of no consequence to the uninformed agents, since the equilibrium values of all relevant variables must be consistent with publicly observable data.

Of course, it is not always the case that the main assumption of Theorem 1, that I−Y is invertible, is satisfied. Hence, we show next that this assumption holds in a generic sense.
In a natural extension of the model discussed above, there would be an infinite number (continuum) of producers and hence an infinite number (continuum) of securities in a perfectly competitive stock market. The natural generalization of the matrix $Y$ is a linear operator $Y^*$ from the space of stock prices (or present values) into itself that is known to all managers. The analog of the invertibility property of the matrix $I-Y$ is the invertibility property of $I-Y^*$, where $I$ is the identity operator; see Rudin (1973, p. 98).

III. A General Equilibrium Model of Revelation When the Information–Price Relationship is Unknown

In this section, we shall complete the partial model of the previous section by adding markets for goods and a consumer sector. Properties of this general equilibrium extension of the partial equilibrium model are then examined as follows. A "completely revealing" concept of competitive equilibrium is defined, where prices reveal all market-relevant private information in the economy. Theorem 2 shows that a hypothesis of Corollary 2 holds generically. Theorem 3 demonstrates the existence of an equilibrium in the model with no uncertainty. The main result of the paper, Theorem 4, demonstrates the generic existence of a fully revealing equilibrium, and follows directly from Theorems 2 and 3. Finally, we examine the welfare properties of a fully revealing equilibrium allocation in Theorem 5.

Since the focus of this paper is clearly not on the consumer sector, we shall abbreviate the model. Naturally, in this section we assume competitive behavior on the part of all agents.

The consumption space for the model is $\mathbb{R}_{+}^{n+k}$, where the first $n$ components represent the securities. Prices reside in the $n+k-1$ dimensional simplex $\Delta$. There are $m$ consumers, $i=1,...,m$, where consumer $i$ has a continuous, quasi-concave utility function $u^i : \mathbb{R}_{+}^{n+k} \to \mathbb{R}$. The ex post utility of a consumer can depend on the
realizations of the random variables $\epsilon^j$, but since only an \textit{ex ante} equilibrium (for the consumers) is obtained, utility is not directly a function of the realizations. For example, consumer $i$ could be an expected utility maximizer. One interpretation of this framework is that $u^i$ is a reduced form utility that only depends on security ownership through a budget constraint. Consumer $i$ has endowment $\omega^i_j \in \mathbb{R}^{n+k}_+$. Hence, consumer $i$ is endowed with $\omega^i_j$ units of the stock of firm $j$ ($j = 1, \ldots, n$). Let $\omega_j \equiv \sum_{i=1}^{m} \omega^i_j$, and note that $\omega_j + e_j = 1$ for all $j$. We shall parameterize economies by $\omega = (\omega^1, \ldots, \omega^m) \in \Omega \equiv \{ \omega \in (\mathbb{R}^{n+k}_+)^m \mid \omega_j = 1 - e_j \text{ for } j = 1, \ldots, n \}$, where $0 \leq e_j \leq 1$ for $j = 1, \ldots, n$. We say that a property is \textit{generic} if it holds everywhere on $\Omega$ except for a measurable subset of Lebesgue measure zero.

Let the demand correspondence for consumer $i$, $f^i : \Delta \times \mathbb{R}^{n+k}_+ \to \mathbb{R}^{n+k}_+$ be defined by $f^i(p, \omega^i_j) \equiv \{ x \in \Omega \mid p \cdot x \leq p \cdot \omega^i_j \text{ and } u^i(x) \geq u^i(z) \text{ for all } z \in \mathbb{R}^{n+k}_+ \text{ with } p \cdot z \leq p \cdot \omega^i_j \}$. Define the aggregate excess demand correspondence by $f(p, \omega) \equiv \sum_{i=1}^{m} [f^i(p, \omega^i_j) - \omega^i_j]$. 

On the production side, we assume that shareholdings must be non-negative, and that prices are parametric. Define the supply correspondence for firm $i$ by $\beta^i(p, \epsilon^i) \equiv \{(Y^i, Z^i) \in \mathbb{R}^{n+k}_+ \times I^i(\epsilon) \mid (Y^i, Z^i) \text{ solves } (1) \}$. Let $\hat{\epsilon}$ be the vector with $\epsilon$ in the first $n$ places and 0 in the next $k$ places. The aggregate excess supply correspondence is defined by $\beta(p, \epsilon) \equiv \sum_{i=1}^{n} \beta^i(p, \epsilon^i) - \hat{\epsilon}$.

An \textit{equilibrium price} given $\omega$ and $\epsilon$ is a $p \in \Delta$ such that $0 \in \beta(p, \epsilon) + f(p, \omega)$.

\textbf{Theorem 2:} Suppose that $f^i(\cdot)$ and $\beta^i(\cdot)$ are single-valued, while $f^i(\cdot)$ and $\beta^i(\cdot, \epsilon^i)$ are continuously differentiable (the latter for each given $\epsilon^i$). For each realization of the random vector $\epsilon$, the property that $\pi = (I - Y)^{-1} \cdot [E - Y] \cdot P$ when evaluated at an equilibrium price holds generically in consumer endowments.
Remarks: 1) By assuming that the excess demand and supply correspondences are functions and \( C^1 \), we abstract from the problems associated with deriving these properties from primitives, as this is not our focus and would distract us from the main objective of this work. (Of course, this is in the tradition of Debreu (1970).) For indications of how this derivation could be accomplished, we refer to Mas-Colell (1985). We should note, however, that the structure of production employed here differs substantially from classical production theory.

2) Notice also that it might be possible to parameterize economies by producer endowments of stock. There are two drawbacks to this alternative approach. First, it would probably require a more complicated technical argument (as well as further assumptions) to show that this parameterization is regular, since producers do not have budget constraints. Second, genericity in this sort of parameterization might exclude a case of interest: the case when each producer is endowed with all of its own stock.

3) In the example where the utility of a consumer depends only on physical commodity consumption while asset holdings only affect the budget, it might be thought that asset demand might not be single-valued, particularly in the case where there is no uncertainty, so stocks are used only to store value. This problem can be solved by simply assuming "artificial" preferences over assets, for instance using an additively separable form for a global utility combining the sum of the true utility over physical commodities with the artificial utility over assets. The artificial utility would have to satisfy the usual smooth economy assumptions (for instance, CES) so that the global utility would have the desired properties, implying that demand is single-valued and \( C^1 \).

4) Of course, when \( \epsilon \) takes on only finitely many values, the conclusion of the Theorem holds generically in consumer endowments for all \( \epsilon \). This notion is closer to the standard concept of completely revealing equilibrium found in the literature,
where economies are parameterized by utility functions.

Proof: Fix $\epsilon$. That $\Omega$ is a regular parameterization is proved in Mas–Colell (1985, p. 227); the proof for the "Edgeworth box" economy there covers the case considered here. Mas–Colell (1985, Proposition 8.3.1, p. 320) implies that except for a set $B \subseteq \Omega$ of Lebesgue measure zero, $\partial f/\partial p + \partial \beta/\partial p$ has 0 as a regular value. By Mas–Colell (1975, Proposition 5.8.13, p. 229), the collection of $(p, \omega) \in \Delta x \Omega$ such that $\beta(p, \epsilon) + f(p, \omega) = 0$ for the first $n+k-1$ equations is a $C^1$ manifold of the same dimension as $\Omega$. Fix $\omega \in \Omega$, $\omega \notin B$, and let $p$ be an equilibrium price for $\omega$. Suppose that $\sum_{i=1}^{n} y_j^i = 1$ holds in equilibrium for some $j$. Then $f_j(p, \omega) + \omega_j = 0$, where the subscript $j$ refers to stock $j$ ($j = 1, \ldots, n$) in the vector belonging to $\mathbb{R}^{n+k-1}$. Suppose further that $\partial f_j/\partial \omega \neq 0$.

Hence, along the directions defined by the manifold, which is given locally by $\partial f(p, \omega)/\partial \omega = [\partial f/\partial p + \partial \beta/\partial p]^{-1}[\partial f/\partial \omega]$, there is a direction of parameter $\omega$ movement, a consequent direction of price movement, and a resulting direction of $y_j$ movement such

---

3It should be noted that there are some technical complications in this structure relative to classical economies, mainly because it is not assumed that Walras' law holds in aggregate due to the unusual maximization problem faced by producers. In this theorem, the standard technique for renormalizing prices (say, the price of the last commodity is 1), and eliminating both a price and a market clearing condition to account for the redundancy embedded in Walras' law might not work. However, we can eliminate a price and an equation regardless, as we are only looking for a necessary condition for an equilibrium. Thus, we set $p_{n+k} = 1$ (where prices are no longer in the simplex), and eliminate commodity $n+k$ from our calculations. Thus, we do not deal with an equilibrium manifold, but rather a manifold for which the first $n+k-1$ markets clear.
that nearby manifold holdings of the stock of firm \( j \) by all firms exceeds 1. This is a contradiction, as the manifold is defined by market clearance in all stocks. So \( \partial y_j / \partial \omega = 0 \) for all \( j \), and hence \( \partial \beta_j / \partial \omega = 0 \) for all \( j \). Thus, \( \partial \beta / \partial p = 0 \) for directions of price movement along the manifold. Moreover, since the consumers own no stock in firm \( j \) at this equilibrium and market clearance is maintained in all stocks on the manifold, in the same manner as for the firms, it must be the case that \( \partial f_j / \partial p = 0 \) along the directions defined by the manifold as well. Hence \( \partial f / \partial p + \partial \beta / \partial p \) is singular at the equilibrium, a contradiction. So for every \( \omega \in \omega, \omega \notin B, \sum_{i=1}^{n} y_i < 1 \) for all \( j = 1, \ldots, n \).

The remainder of the Theorem follows from Corollary 2.

Q.E.D.

In order to prove that an equilibrium exists, it is necessary to go back to the primitives rather than relying on assumptions about supply and demand. This is due to the fact that Walras' law might fail in aggregate (since producers have no budget constraint) for prices that are not equilibrium prices. We now assume that \( u^i \) is locally non-satiated, i.e. \( \forall x \in \mathbb{R}_{++}^{n+k} \) and \( \forall \delta > 0 \) there exists \( x' \in \mathbb{R}_{++}^{n+k} \), \( \|x - x'\| < \delta \) with \( u^i(x') > u^i(x) \). The endowment of consumer \( i \) is given by \( \omega^i \in \mathbb{R}_{++}^{n+k} \) (a suitable irreducibility assumption could be used in place of the interiority assumption if, for example, one wishes to endow each firm with all of its own stock). A production sector is a collection of \( n \) firms, \( i = 1, \ldots, n \). Each firm \( i \) has continuous function \( V^i \) as specified in equation (1). Let \( V_i \) be quasi-concave in \((Y^i; Z^i)\) for fixed values of its other arguments. Assume that \( T^i \) is closed and convex with \( 0 \in T^i(\epsilon^j) \) for each \( i \) and \( \epsilon^j \). In this section, we assume perfect competition, so each firm takes prices as given. The manager of firm \( i \) solves (1) taking prices and the stock purchases of other firms as given. Let \( \pi_i \) be the profit function for firm \( i \), \( \pi_i(Y^i; Z^i; p) = p \cdot (y_1^i - e_1^i, \ldots, y_n^i - e_n^i; Z^i) \).

The crucial assumption concerning \( V^i \) is that \( \pi_i(Y^i; Z^i; P) < 0 \) implies \( V^i(Y^i; Z^i; \epsilon^i; P) < \).
$V^i(e^i_1,...,e^i_n;0;p)$. That is, if profits are negative according to market prices, then a firm can do better by inaction.

Following Shafer and Sonnenschein (1975), a competitive equilibrium is

$(p;x^1,...,x^m;Y^1,...,Z^1) \in \Delta x(\mathbb{R}^n_+ + K)^m x[0,1]^2 x\mathbb{R}^{kn}$ such that $\frac{m}{i=1} x^i - \omega^i \leq \frac{n}{j=1} (Z^j; \Sigma_j^i (e^j_i - y^j_i)); p \cdot x^i = p \cdot \omega^i$ for $i = 1,...,m$, $p \cdot x \leq p \cdot \omega^i$ implies $u^i(x) \leq u^i(x^i)$ for $i = 1,...,m$, and for each $j = 1,...,n$ for any private and public information observed by agent $j$, $(Y^j_i,Z^j)$ solves (1).

**Theorem 3:** There exists a general equilibrium for a stock market model.

**Proof:** We apply Shafer and Sonnenschein (1975), where the stock purchases of firms act similar to externalities in the model. Using classical techniques, such as those in Arrow and Debreu (1954), we can bound the consumption and production sets without loss of generality. As in these papers, we add another agent, the market player, who has prices $\Delta$ as a choice set and the objective of maximizing the value of excess demand. It is easy to verify that, given our assumptions in this section, there exists an equilibrium in the sense of Shafer–Sonnenschein. To show that this equilibrium is also a competitive equilibrium, it suffices to check that $\frac{m}{i=1} x^i - \omega^i \leq \frac{n}{j=1} (\Sigma_j^i (e^j_i - y^j_i);Z^j)$. To see this, note that by local non-satiation, for each consumer $i$ $p \cdot x^i = p \cdot \omega^i$. For each producer $i$, the last assumption on $V^i$ implies that $\pi^i \geq 0$ at equilibrium, so that the value of excess demand at equilibrium is non-positive. Since the market agent maximizes the value of excess demand, it must be the case that excess demand for each commodity and stock is non-positive.

Q.E.D.

This proof might be of independent interest since it is not specific to the model with a stock market or asymmetric information, but allows general objective functions for producers. The proof can also be generalized in many standard directions, such as
the use of incomplete or intransitive preferences.

An equilibrium is called completely revealing if when each agent solves its optimization problem, it faces no uncertainty.

**Theorem 4:** Suppose that the only uncertainty relevant to any agent's optimization problem is in the profits of the firms; in other words, there is no uncertainty in the model other than $\epsilon^i$, which enters only in firm i's technology. Under the assumptions of Theorems 2 and 3, for each realization of the random vector $\epsilon$, generically there exists a completely revealing equilibrium.

**Proof:** Apply Theorem 3 to obtain a set $B \subseteq \Omega$ of measure zero such that for any economy not in $B$, all agents can derive $\pi$ from known variables. Then apply Theorem 2 to this economy in the case where there is no uncertainty (i.e. $\pi$ is known to all). This equilibrium is completely revealing.

Q.E.D.

Given $\epsilon$, a feasible allocation for the economy $\omega$ is a vector $(x^1, \ldots, x^m; Z^1, \ldots, Z^n; Y)$ such that $\sum_{i=1}^{m} x^i + \sum_{i=1}^{n} (Z^i, Y^i) \leq \omega$. Given $\epsilon$, a Pareto optimum is a feasible allocation $(x^1, \ldots, x^m; Z^1, \ldots, Z^n; Y)$ such that there is no other feasible allocation $(\bar{x}^1, \ldots, \bar{x}^m; \bar{Z}^1, \ldots, \bar{Z}^n; \bar{Y})$ with $u^i(x^i) \geq u^i(\bar{x}^i)$ for all $i$, with strict inequality holding for some $i$.

**Theorem 5:** For each consumer $i$, let $u^i$ be locally non-satiated. For each producer $j$, suppose that $V^j$ is a monotonic function of $\pi^j$ when the producer faces no uncertainty. Then any completely revealing equilibrium allocation is Pareto optimal.
The proof of this Theorem is standard.

Several remarks are in order. First, on the consumer side, this model involves no uncertainty other than in realized profits of the firms, so there is no random component in the direct utility, only in the budget constraint. This is like most of the finance literature, where there is uncertainty in (future) prices and returns only. Unlike the incomplete markets literature, multiple budget constraints are not present. An interesting example of $V^i$ fitting into our framework is the expected profit function. Finally, consumers do not learn the $\epsilon^i$'s in this model, but only learn the profits of firms in equilibrium.

IV. Implications and Conclusions

We have shown above that, in a stock market economy with producers with private information, equilibrium prices, in conjunction with certain other publicly observable variables such as stock purchases and stock endowments, reveal profits or returns anticipated by the producers acting on the basis of their private information. This result is shown to hold under general conditions and does not require that the agents know the true relationship between the initial information of all agents in the economy and the resulting market prices.

In this context, there is an interesting asymmetry between consumers with private information and producers with private information. Our result that a well-functioning stock market reveals information through equilibrium stock prices does not extend to the case where consumers have private information about individual endowments or wealth, simply because the consumers, unlike firms, cannot own each other or issue claims against each other in a standard market setting.

An implication of allowing firms to buy stock in other firms is to make their return vectors interdependent. That is, the manager of a firm can change its return...
vector (in a linear fashion) by buying stock in other firms. This provides a way to
determine the securities offered in equilibrium endogenously. Of course, it must
eventually be combined with a model explaining the entry and exit of firms.

Note that our results hold whether markets are complete or not, as long as stock
purchases as well as initial stockholdings of firms are observable. In the context of
incomplete markets or temporary equilibrium theory (see Green (1973)), the solution to
the firm's maximization problem in the presence of asymmetric information will once
again reveal some inside information that the firm might have. If consumers and other
firms observe this information and condition on it, then their expectations may be more
similar to one another than if they did not. This, in turn, might aid in establishing
the existence of an equilibrium which requires, as Hart (1974) has shown, that the
expectations of the agents must be "sufficiently similar." For further explanation, see
Page (1987, Propositions 4.3 and 5.3).
References


387–392.


GREEN, J. (1973), "Temporary General Equilibrium in a Sequential Trading Model

GROSSMAN, S. (1978), "Further Results on the Informational Efficiency of


Economic Theory*, 9, 293–311.


Differentiable Approach* (Cambridge: Cambridge University Press).


RADNER, R. (1979), "Rational Expectations Equilibrium: Generic Existence and the


SHAFER, W., and SONNENSCHEIN, H. (1975), "Equilibrium in Abstract Economies