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A Theory of Corporate Capital  
Structure and Investment

by

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# A Theory of Corporate Capital Structure and Investment

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## Abstract

This paper develops a costly state verification (CSV) model which describes how financial fluctuations affect real activity in a general equilibrium setting. In an economy with differentiated lenders, the most efficient will become intermediaries (e.g. banks). Intermediation generally creates frictions which prevent banks from dominating the debt markets. In this model, firms with abundant funds avoid intermediaries, and tap the credit markets directly. Meanwhile, firms with moderate resources borrow from intermediaries. The aggregation of this model produces an economy with appealing features: aggregate investment drops with a rise in the riskless rate, and a deterioration of bank or corporate health.

## *Introduction*

It is commonly believed that financial market fluctuations have an important effect on real economic activity. Among economists, this view has been held by Keynes (1931), Fisher (1933), and Friedman (1968), for example. These scholars have argued that financial market disruptions can and have caused breakdowns in real output and employment. One link between financial markets and the real economy is monetary policy whose initial effect Friedman describes below:

*" By making nominal cash balances higher than people desire, it will tend initially to lower interest rates and in this and other ways to stimulate spending. Income and spending will start to rise. To begin with much or most of the rise in income will take the form of an increase in output and employment rather than prices. People have been expecting prices to be stable, and prices and wages have been set for some time in the future on that basis. [...] Producers will tend to react to the initial expansion in aggregate demand by increasing output, employees by working longer hours, and the unemployed by taking jobs now offered at nominal wages. This much is pretty standard doctrine " Friedman, M (1968) pp. 9-10*

Monetary policy is not the only necessary link between financial fluctuations and real activity. This paper studies how financial variables affect real investment in a non monetary economy. I am not implying, though, that monetary policy is ineffective: Irving Fisher (1933) argues that the monetary and nonmonetary links of financial and real activity are best understood in each other's light. In an illuminating study, Bernanke (1983) has shown that both non monetary and monetary variables are important in explaining output changes during the Great Depression. The nonmonetary variables that Bernanke analyzed econometrically and that I study theoretically here are corporate and bank financial health.

A central theme in this article is the study of the strengths and limitations of financial intermediaries in providing credit. This allows us to understand what intermediaries do and why they are important in propagating shocks throughout the economy. We can also understand how

firms choose different types of lenders. This paper shows that in equilibrium cash rich firms tap the bondmarket directly while firms with moderate resources borrow from intermediaries. Finally, firms with scarce funds are credit rationed. These results come from two assumptions: First, that intermediaries have a lower cost of handling financially distressed firms than bondholders. The second assumption is that intermediaries have limited internal funds and nondiversifiable risk in their loan portfolio. This implies that intermediaries need to borrow from bondholders to relend to entrepreneurs, and that they may go into default. As intermediaries expand lending, their marginal bankruptcy cost increases and eventually matches the marginal lending benefits. This paper shows that in equilibrium both bank loans and bonds coexist in the economy. Among the theoretical papers studying how firms choose their debt instruments, Diamond (1991) is the best known. His model combines costly state verification, moral hazard and adverse selection. Diamond's model arrives at different predictions from the one here. In Diamond's paper, for example, firms are sorted not according to their financial attributes but according to their reputation.

The paper stays as close as possible to "neoclassical" assumptions: it postulates perfectly competitive lending markets. It does not rely on assumptions of adverse selection or any ex-ante asymmetric information: such presuppositions cast doubt on an intermediary's ability to screen lenders, and so - to my mind - are unconvincing. Costly state verification (CSV) - which can be interpreted as the cost of financial distress - is the only market imperfection employed in this article, and is sufficient to explain why some firms borrow from intermediaries while others issue bonds directly.

One cannot fully understand the strengths and limitations of financial intermediaries without embedding them in an economy with a large number of agents with whom they do

business. This aggregation generates a macroeconomic model that explains how financial and real activity interact. In particular, I derive an aggregate investment function with intuitive properties: investment falls with a rise in the risk free rate, and with a deterioration of bank or corporate financial health. Some excellent empirical studies - such as the one by Fazzari, Hubbard and Petersen (1988) - have shown the importance of corporate cash flow in determining investment. Recent theoretical work, such as the articles by Bernake and Gertler (1989) or Greenwald and Stiglitz (1990), has studied how corporate and bank cash flow affect real activity. This research has made extensive use of adverse selection models. Since adverse selection models are fairly involved, this research has tended to use only the simplest assumptions (e.g. investment projects with only two states), and has presented partial equilibrium models. For the same reasons, these models have not studied issues of aggregation. This article presents a general equilibrium theory linking nonmonetary financial variables and real activity, and studies aggregation explicitly.

The paper is presented as follows: section .1 outlines the model's assumptions. Section 2 presents an economy without financial intermediaries. Section 3 introduces specialized lenders, who will endogenously become financial intermediaries. The general equilibrium of this economy is studied in section 4. Section 4 also presents a savings equation and studies the determination of the risk free interest rate and investment in the economy. Proofs are left to the appendix.

### ***Section 1. Assumptions.***

This model has three types of agents: (1) heterogeneous, risk neutral entrepreneurs who have a project but insufficient funds to complete it. (2) Risk neutral generic lenders (bondholders) and (3) Risk neutral specialized lenders (banks). In period  $t = 0$  all contracts are written. At time  $t = 1$  the states of the world are revealed. Bondholders and banks have imperfect information, in

the sense that it is costly for them to verify the project's returns. This setup was introduced by Townsend (1979) and used later by Gale and Hellwig (1985), and can be interpreted as the cost of financial distress. In a costly state verification framework, the optimal contract between entrepreneurs and outside investors is standard debt. This theoretical result fits well with the financing patterns for firms in industrialized countries, as described in Mayer (1990). The model's assumptions are given below:

A1. *The Project*: The project has a fixed size  $z$ , which I set to 1 without loss of generality. The realized returns  $s \in [0, \infty)$  have a density function  $f(s)$  with a finite first moment (A1.1) and an increasing hazard rate  $\rho(s)$  (A1.2). This assumption is widely used in models of incentive contracts, such as Grossman and Hart (1982). The monotone hazard rate property is satisfied by the normal, uniform, and Laplace distributions. The exponential, gamma and Weibull distributions with degrees of freedom parameter larger than one also satisfy this condition. The economic significance of this postulate is explained in A6.

$$(A1.1) \ E(s) = \mu < \infty$$

$$(A1.2) \ \rho'(s) \geq 0, \text{ where } \rho(s) = \frac{f(s)}{1-F(s)}$$

A2. *Bondholders*: There is a group of competitive generic lenders - bondholders - whose type is denoted by  $\bar{t}$ , and who can verify a project at a cost  $\alpha(s, \bar{t}) \geq 0$ . Entrepreneurs issue securities at time 0 which bondholders may buy. Bondholders have the alternative option of investing their funds in a safe asset with a return  $R_f$ . Bondholders use their own wealth to lend.

A3. *Entrepreneurs*: There are  $N$  risk neutral entrepreneurs who are identical in everything except in their internal funds  $e < 1$ . To complete their project entrepreneurs must borrow  $[1-e]$ . Entrepreneurs are indexed by their internal funds, and distributed according to a density function  $h(e)$ . Entrepreneurs have the option of investing their funds in a safe asset with a return  $R_f$ .

A4. *Banks*: There is a group of competitive specialized lenders, whose type is denoted by  $\underline{t}$ . In the paper, these lenders are called banks, because they will naturally tend to become financial intermediaries. Banks have lower verification costs than bondholders  $c(s;\underline{t}) \leq c(s;\bar{t})$ . Banks have an aggregate internal funds  $W$ .

A5. *Verification costs*:

$$(A5.1) \quad c(0;t) = 0$$

$$(A5.3) \quad E\{c(s;t)\} = \chi(t) < \infty$$

$$(A5.2) \quad c'(s;\bar{t}) \geq c'(s;\underline{t}) > 0$$

$$(A5.4) \quad c''(s;t) \geq 0$$

Assumption (A5.2) says that higher states are more costly to verify: this might be because the complexity of the project increases with size. (A5.2) may also be justified if at better states the entrepreneur tries to disinform lenders, to keep some money for himself. Assumption (A5.2) also says that verification costs rise faster for bondholders than for banks. At later stages in the model I simplify the cost structure so that it be linear  $c(s;t) = ts$ .

A6. Observe that A5.2 and A1.2 imply that  $c(s;t)p(s)$  increases in  $s^2$ . This result ensures that in expected terms, the marginal lending costs increase relative to the marginal benefits.

## ***Section 2. An Economy without Intermediaries***

This section considers only one type of lender, bondholders. An entrepreneur issues securities at time  $t = 0$  to maximize his expected payoff at  $t = 1$ . He promises to pay  $P(s)$  at time  $t = 1$  contingent on the realized return of the project. Bondholders decide to verify or not according to the realized state.  $B(s)$  is an indicator function that equals one if a firm is verified. An entrepreneur with internal funds  $e$  maximizes:

$$\text{Max } E\{s-P(s)\}$$

- Subject to
- a)  $P(s) \leq s$  (Limited liability)
  - b)  $\forall s \in B^l \Rightarrow P(s) = [1-e]\bar{r}(x)$  where  $B^l = \{s: B(s) = 0\}$
  - c)  $\forall s \in B^v \Rightarrow P(s)$  depends on  $s$  where  $B^v = \{s: B(s) = 1\}$
  - d)  $E\{P(s)\} = [1-e]R_f + E\{c(s; \bar{t})B(s)\}$  (bondholders' IR)

Gale and Hellwig (1985) show that the solution to this problem is the standard debt contract, defined in equation (1). With standard debt, the entrepreneur makes a fixed payment in good states. If unable to meet his payments, the entrepreneur is intervened by his creditors and forced to pay as much as he can. Gale and Hellwig's result justifies the existence of a very widely used contract - debt - from deep microeconomic foundations. Equation (2) displays bondholders' expected revenue, where  $m(x) \equiv [1-e]\bar{r}(x)$ . In equilibrium, equation (2) must hold as an identity since bondholders are risk neutral and competitive.

$$(1) P(s) = \begin{cases} [1-e]\bar{r}(x) & \forall s \geq [1-e]\bar{r}(x) \\ s & \forall s < [1-e]\bar{r}(x) \end{cases} \quad B(s) = \begin{cases} 1 & \forall s \geq [1-e]\bar{r}(x) \\ 0 & \forall s < [1-e]\bar{r}(x) \end{cases}$$

$$(2) E\{P(s)\} \equiv m[1-F(m)] + \int_0^m s f(s) ds = [1-e]R_f + \int_0^m c(s; \bar{t}) f(s) ds$$

***Definition of a lending equilibrium and solution technique:***

Equations (3) and (4) display  $U(x; \bar{t}, m)$  and  $V(x; m)$ , the economic profit functions for bondholders and entrepreneurs.  $\hat{\lambda}(m)$  - defined in equation (5) - is the expected bondholder revenue.  $x = [e R_f \gamma \varphi]$  is a vector of variables:  $e$  is the fraction of the project that is internally financed,  $R_f$  is the riskless rate,  $\gamma$  are the setup costs - if any - of becoming a bank and  $\varphi$  is a

measure of aggregate firm cash flow, which is explained at the end of this section. The last two variables in the  $x$  vector are included for generality, but they will not be used in section 2.

$$(3) \quad U(x; \bar{i}, m) = m[1-F(m)] + \int_0^m [s-c(s; \bar{i})]f(s)ds - [1-e]R_f = \bar{\lambda}(m) - [1-e]R_f$$

$$(4) \quad \bar{\lambda}(m) \equiv m[1-F(m)] + \int_0^m [s-c(s; \bar{i})]f(s)ds$$

$$(5) \quad V(x; m) = \int_m^\infty [s-m]f(s)ds - eR_f \equiv \phi(m) - eR_f$$

The firm defaults when a project's state falls below  $m(x; t) \equiv [1-e]r(x; t)$ . In what follows we will make the bankruptcy point  $m(x; t)$  the focus of our analysis. From the above definition there is a one to one match between the bankruptcy point and the interest rate faced by a firm.

#### *Definition (Lending Equilibrium)*

A lending equilibrium is the lowest bankruptcy point  $\bar{m}(x)$  where bondholders break even and firms make a profit  $\bar{m}(x) \equiv \min\{m: U(x; \bar{i}, m) = 0 \text{ and } V(x; m) \geq 0\}$

The equilibrium rate is typically found by solving the following problem:

$$\bar{m}(x) = \operatorname{argmax}_m V(x; m) \quad \text{s.t. } U(x; \bar{i}, m) \geq 0$$

Unfortunately, the problem can and will be unfeasible for some attributes  $x$ , since the constraint could never bind, so  $U(x; \bar{i}, m) < 0 \forall m$ . Thus, we need to solve the dual :

$$\bar{m}^d(x) = \operatorname{argmax}_m U(x; \bar{i}, m) \quad \text{s.t. } V(x; m) \geq 0 \quad (\text{Firms' IR constraint})$$

In the dual problem, the constraint can always be satisfied, provided that the project's return is higher than the risk free rate. The relation between the dual and primal solution is as

follows: if at the dual rate we have  $U(x; \bar{i}, \bar{m}^d(x)) < 0$  then bondholders can never break even by lending to a firm with attributes  $x$ . Hence at that point no equilibrium exists and no lending takes place. If at the dual rate we have  $U(x; \bar{i}, \bar{m}^d(x)) \geq 0$  then - as proposition 4 demonstrates - there exists a unique lending equilibrium  $\bar{m}(x) < \bar{m}^d(x)$ . The next subsection solves the dual problem, and studies the equilibrium between entrepreneurs and bondholders.

### *Lending Equilibrium between Bondholders and Entrepreneurs:*

The first order conditions of an unconstrained maximization of  $U(x; \bar{i}, m)$  with respect to  $m$  is given in equation (6). Assumption A6.1 implies that as  $m$  rises the marginal lending costs increase and eventually exceed the marginal benefits. Hence, bondholders cannot raise interest rates indefinitely to maximize profits. Proposition 1 states that bondholders' economic profit function has a single peak with respect to the interest rate. It is interesting to note that a CSV model generates a similar result to that of Stiglitz and Weiss (1981), but with less market imperfections.

$$(6) \quad U_m(x; \bar{i}, m) = [1-F(m)][1-c(m; \bar{i})\rho(m)]$$

#### *Proposition 1*

There exists a unique unconstrained maximizing rate  $\bar{m}^u(x) < \infty$  where  $U_m(x; \bar{i}, \bar{m}^u(x)) = 0$ .

Corollary:  $U_m(x; \bar{i}, m) > 0 \Rightarrow c(m; \bar{i})\rho(m) < 1 \quad \forall m < \bar{m}^u(x)$

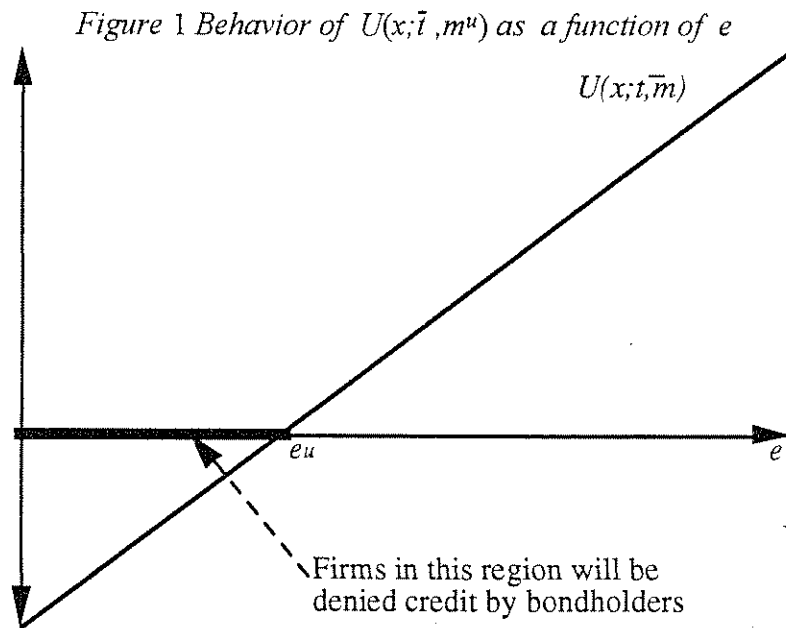
$$U_m(x; \bar{i}, m) < 0 \Rightarrow c(m; \bar{i})\rho(m) > 1 \quad \forall m > \bar{m}^u(x)$$

At the unconstrained maximizer, bondholders' economic payoff is linear in  $e$  and  $R_f$ . To see this, note that maximizing  $U(x; \bar{i}, \bar{m}^u(x))$  is the same as maximizing revenues  $\bar{\lambda}(m)$  with

respect to  $m$ . The maximizer is a constant  $\bar{m}^u(x) = \bar{k}$  (see equation 4 to verify this) so  $\bar{\lambda}(\bar{k}) > 0^3$  is a constant too. Equation (7) exhibits bondholders' economic profit evaluated at  $\bar{m}^u(x) = \bar{k}$ , and figure 1 plots it. Firms with funds below  $\bar{e}^u(x)$  - defined in equation (8) - yield no economic profits to bondholders since  $U(x; \bar{i}, \bar{m}^d(x)) \leq U(x; \bar{i}, \bar{m}^u(x)) < 0$ . Hence firms  $e \in [0, \bar{e}^u(x))$ , will be shut out of the credit markets.

$$(7) \quad U(x; \bar{i}, m^u) = \bar{\lambda}(\bar{k}) - [1-e]R_f$$

$$(8) \quad \bar{e}^u(x) = \text{Max}\left(0; 1 - \frac{\bar{\lambda}(\bar{k})}{R_f}\right)$$



We have not yet solved the dual, since we have not taken into account the entrepreneurs' participation constraint, defined in equation (5). At  $\bar{m}^u(x) = \bar{k}$  the first term of equation (5) is a constant that we define as  $\phi(\bar{k})$ . Firms with higher funds have better outside opportunities. We

define  $\bar{e}^v(x)$  in equation (9). This is the lowest entrepreneur who has a binding participation constraint so  $V(x; \bar{m}^u(x)) < 0 \forall e \in (\bar{e}^v(x), 1]$ .

$$(5) V(x; m) = \phi(m) - eR_f \geq 0$$

$$(9) \bar{e}^v(x) = \text{Min} \left( 1, \frac{\phi(\bar{k})}{R_f} \right)$$

Bondholders must lower the contractual rate they charge to firms  $e \in (\bar{e}^v(x), 1]$  to a point  $\bar{m}^v(x)$  - defined in equation (10) - so that firms are indifferent between borrowing or not. Proposition 2 studies the existence and uniqueness of  $\bar{m}^v(x)$ :

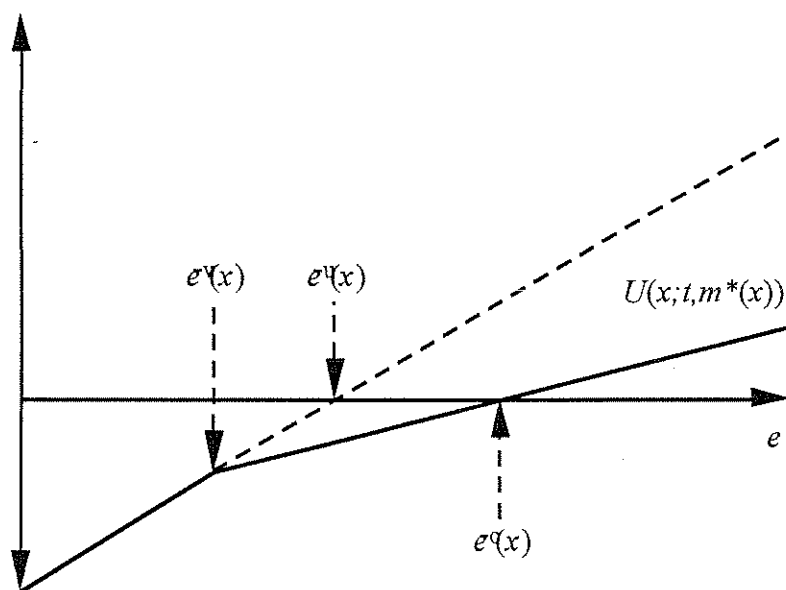
$$(10) V(x; \bar{m}^v) = \int_{\bar{m}^v}^{\infty} [s - \bar{m}^v] f(s) ds - eR_f \equiv 0$$

**Proposition 2** For  $R_f < \mu e > e^v$  there is a unique rate  $\bar{m}^v(x) \in [0, \bar{m}^u(x))$  where  $V(x; \bar{m}^v(x)) = 0$ .

The rate  $\bar{m}^d(x)$  that solves the dual is given in equation (11). Note  $c(\bar{m}^d; \bar{i}) \rho(\bar{m}^d) \leq 1$  since  $\bar{m}^d \leq \bar{m}^u = \bar{k}$ . At the dual rate, bondholders' profits rise for firms with more internal funds, so  $U_e(x; \bar{i}, \bar{m}^d) = c(\bar{m}^d, \bar{i}) \rho(\bar{m}^d) R_f > 0$ . This derivative used equations (3) and (10). Although bondholders need to satisfy a tighter participation constraint for cash rich firms, the savings from less verification outweigh the revenue loss. Bondholders' economic profit falls with an increase in the risk free interest rate, so  $U_{R_f}(x; \bar{i}, \bar{m}^d) = e c(\bar{m}^d, \bar{i}) \rho(\bar{m}^d) - 1 < 0$ .

$$(11) \bar{m}^d(x) = \begin{cases} \bar{k} & \forall e < \bar{e}^v(x) \quad \text{defined in proposition 1} \\ \bar{m}^v(x) & \forall e \geq \bar{e}^v(x) \quad \text{defined in equation (10)} \end{cases}$$

Figure 2 Bondholder Profits at the dual rate and with  $R_f > \mu - \bar{C}$



The participation constraint enlarges the range of credit rationed firms. To see this let us first define  $\bar{C} \equiv E\{c(s; t) | s < \bar{k}\} F(\bar{k})$ . When interest rates are high  $R_f > \mu - \bar{C}$ , the participation constraint is binding for all firms who are not affected by equation (7), so  $\bar{e}^v(x) < \bar{e}^u(x)$ . Figure 2 plots bondholders' payoffs at the dual rate, and shows that  $U(x; \bar{i}, \bar{m}^d) < 0$  at  $\bar{e}^u(x)$ . Proposition 3 identifies the point  $\bar{e}^o(x)$  where bondholders break even.

**Proposition 3** For  $R_f \in (\mu - \bar{C}, \mu]$  there exists a unique point  $\bar{e}^o(x) \in (\bar{e}^u(x), 1]$  where bondholders break even.  $\bar{e}^o(x)$  increases with  $R_f$  and  $\bar{e}^o(R_f = \mu) = 1$

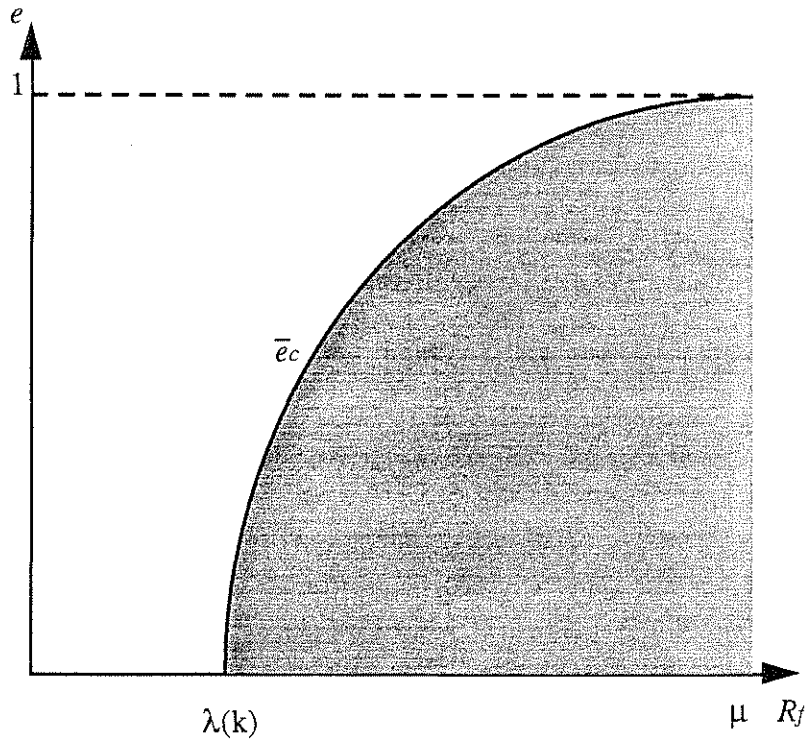
After deriving the solution to the dual problem, equation (12) defines  $\bar{e}^c(x)$ , below which firms do not receive credit, since lenders never profits:  $U(x; \bar{i}, \bar{m}^d(x)) < 0 \forall e < \bar{e}^c(x)$ . We have shown that for firms  $e \in [0, \bar{e}^c(x))$  no lending equilibrium exists. Proposition 4 shows the existence of a lending equilibrium for  $e \in [\bar{e}^c(x), 1]$ . One can show  $\bar{\lambda}(\bar{k}) < \mu - \bar{C} < \mu$ .

$$(12) \quad \bar{e}^c(x) = \begin{cases} 0 & \forall R_f \in [0; \bar{\lambda}(\bar{k})] \\ 1 - \frac{\bar{\lambda}(\bar{k})}{R_f} & \forall R_f \in [\bar{\lambda}(\bar{k}); \mu - \bar{C}] \\ \bar{e}^o(x) & \forall R_f \in [\mu - \bar{C}; \mu] \\ 1 & \forall R_f > \mu \end{cases}$$

**Proposition 4:** For  $e \in [\bar{e}^c(x), 1]$  there exists one lending equilibrium  $\bar{m}(x) \in ([1-e]R_f; \bar{m}^d(x)]$

where  $U(x; \bar{i}, \bar{m}(x)) = 0$  and  $V(x; \bar{m}(x)) \geq 0$ .

Figure 3 Cutoff rate  $\bar{e}^c(x)$



The cutoff rate  $\bar{e}^c(x)$  is shown in figure 3. To understand this figure, consider a coordinate  $(R_f, e)$ . This specifies a risk free rate and an entrepreneur  $e$ . If that coordinate is in the shaded area then the entrepreneur is credit rationed. If the coordinate is in the white area then the entrepreneur is borrowing from bondholders. No firm is rationed at low interest rates. As the riskless rate rises, more firms become rationed. The market shuts down when the riskless rate

equals the project's expected return. The reason for the steady loan extinction is that as the riskless rate rises the outside option becomes more attractive to both borrowers and lenders. This raises the verification costs for all firms and makes the projects even less desirable than before. Without verification costs all projects are financed as long as  $R_f < \mu$ .

### *Behavior of Contractual Interest Rates:*

The rate  $\bar{r}(x)$  charged by bondholders to firms is defined in equation (13). Proposition 5 studies the behavior of  $\bar{r}(x)$ . The intuition of proposition 5 is straightforward. A firm with higher internal funds has a lower default rate, and thus needs less verification. The savings in verification costs are passed on to the firm in the form of a lower contractual rate. The second result says that the contractual rate rises faster than the riskless rate, so the spread  $\bar{r}(x) - R_f$  increases with a rise in  $R_f$ . When riskless bonds pay a higher interest, entrepreneurs must offer more attractive rates to retain investors. As the contractual rate rises there are more defaults and more need for verification.

$$(13) U(x; \bar{r}, \bar{m}(x)) = \bar{m}(x) [1 - F(\bar{m}(x))] + \int_0^{\bar{m}(x)} [s - c(s; \bar{r})] f(s) ds - [1 - e] R_f \equiv 0 \quad \forall e \in [\bar{e}^c(x), 1]$$

$$\text{Proposition 5} \quad [d\bar{r}(x)/de] < 0 \quad [d\bar{r}(x)/dR_f] > 1 \quad \forall e \in [\bar{e}^c(x), 1]$$

### *Aggregate Investment in an Economy without Intermediaries*

Aggregate investment is displayed in equation (14), and is defined as the sum of the individual investment of firms with funds  $e \in [\bar{e}^c(x), 1]$ . In a model without informational problems, cash flow fluctuations are irrelevant to investment, but with costly state verification

this is not true. If a firm with funds  $e < \bar{e}^c(x)$  receives an influx  $\Delta e \geq \bar{e}^c(x) - e$ , its investment jumps from 0 to 1. An increase in a firm's funds has an analog in the aggregate economy. Let us parametrize the distribution function  $H(e, \varphi)$  of firms. A higher first order stochastic dominance parameter  $\varphi$  signifies a generalized cash increase so  $H_\varphi(e, \varphi) \leq 0$  and does not change the cutoff point  $\bar{e}^c(x)$ . Investment rises with a generalized increase in funds as seen in equation (14). Investment also falls with a rise in the risk free rate, since  $\bar{e}^c(x)$  increases with a rise in  $R_f$ .

$$(14) \quad I(R_f, \varphi) = N[1 - H(\bar{e}^c(x), \varphi)]$$

$$I_\varphi(R_f, \varphi) = -NH_\varphi(\bar{e}^c(x), \varphi) > 0 \quad I_{R_f}(R_f, \varphi) = -Nh(\bar{e}^c(x), \varphi)[d\bar{e}^c/dR_f] < 0$$

The above results show how a simple CSV framework can deliver a macroeconomic investment function where nonmonetary variables - corporate earnings and real interest rates - affect real activity. Such a model does not require taxing assumptions on its information structure, nor does it rely on the existence of financial intermediaries, who will be introduced in the next section.

### *Section 3. An economy with Financial Intermediaries*

This section introduces a new class of lenders, which I will call banks. The defining attribute of these specialized lenders is that they have lower verification costs than bondholders, so  $c(s; \bar{t}) \leq c(s; \bar{t}')$ . Empirical studies by Weiss (1990) and Gilson et al. (1990) have shown that banks are more successful than bondholders in defending their priority rights during reorganizations, a sign of their strong bargaining position. Banks are also effective promoters of private reorganizations, which are considered to be less value destroying than formal bankruptcy

proceedings. As we will see here, the assumption of low verification costs needs to be enriched to explain why both bank loans and bonds coexist in the economy. Section 3.1 assumes that banks have enough internal resources to finance all available projects. In this case bondholders are driven out of the market. Although unrealistic, the setup in section 3.1 is a helpful baseline from which to build on later.

Section 3.2 endogeneizes the level of financial intermediation. A generic lender has the choice of becoming specialized, by spending a fraction  $\gamma$  of its capital. In this setup, banks will naturally act as intermediaries, borrowing from bondholders and relending to entrepreneurs. Since the banks' portfolios are not riskless, they will sometimes go bankrupt. As a bank expands its loans, its marginal bankruptcy cost increases, and eventually matches the marginal lending benefit. I will simplify the verification cost structure to maintain a tractable model, and to make the assumption of a representative bank viable. In this section three categories of firms emerge. First, a group of well endowed companies borrows directly from the bondmarket. Second, firms with intermediate resources borrow from banks. Finally companies with very limited funds are shut out of the credit market, and are unable to undertake their projects. This section also analyzes of the interest rates charged by banks and bondholders to different firms. The aggregate investment function behaves as in section 2, with the difference that the banks' financial position will also affect real activity.

### *3.1 Solving the Dual for an overcapitalized banking sector*

This subsection studies the maximization problem of an overcapitalized banking sector with verification costs  $c(s;t) \leq c(s;\bar{t})$ . Banks have enough internal funds  $W$  to cover the economy's needs, so  $W > N[1 - E\{e\}]$ . The bank's dual problem is given below:

$$\underline{m}^d(x) = \operatorname{argmax}_m U(x; \underline{t}; m)$$

$$\text{a) } V(x; m) \geq 0 \quad \forall e \quad (\text{Individual Rationality})$$

$$\text{b) } m(x) \leq \bar{m}(x) \quad \forall e \in [\bar{e}^c(x), 1] \quad (\text{Incentive Compatibility})$$

$U(x; \underline{t}; m)$  represents the banks' economic profit. The first constraint is the entrepreneur's participation. The second constraint is an incentive compatibility condition for firms with access to the bondmarket. Below, we define the banks' economic profit and revenue functions, and proposition 6 studies some of their properties.

$$(15) \quad U(x; \underline{t}; m) = m[1-F(m)] + \int_0^m [s-c(s; \underline{t})]f(s)ds - [1-e]R_f = U(x; \bar{t}; m) + \int_0^m [c(s; \bar{t})-c(s; \underline{t})]f(s)ds$$

$$(16) \quad \underline{\lambda}(m) = m[1-F(m)] + \int_0^m [s-c(s; \underline{t})]f(s)ds = \bar{\lambda}(m) + \int_0^m [c(s; \bar{t})-c(s; \underline{t})]f(s)ds$$

**Proposition 6** A) Banks' maximizing rate has  $\underline{m}^u(x) \geq \bar{m}^u(x)$  B)  $U(x; \underline{t}; \underline{m}^u(x)) \geq U(x; \bar{t}; \bar{m}^u(x))$

The unconstrained maximizer of the bank's economic profit is a constant  $\underline{m}^u(x) = \underline{k}$ .

Following proposition 6B we can show that  $\underline{\lambda}(\underline{k}) \geq \bar{\lambda}(\bar{k}) > 0$ . There is a point

$\underline{e}^u(x)$  - defined in equation (17) - below which the bank is unwilling to lend so  $U(x; \bar{t}; \underline{m}^u(x)) < 0$

$$(17) \quad \underline{e}^u(x) = \operatorname{Max}\left(0; 1 - \frac{\underline{\lambda}(\underline{k})}{R_f}\right)$$

$$(18) \quad \underline{e}^v(x) = \operatorname{Min}\left(1; \frac{\phi(\underline{k})}{R_f}\right)$$

For  $e \in (\underline{e}^v(x), 1]$  banks need to reduce the rate to  $\underline{m}^v(x)$  - defined in equation (19) - to make the firm indifferent between borrowing or not. The existence of a unique  $\underline{m}^v(x) \in [0, \underline{m}^u(x)]$  can be shown following the arguments in proposition 2. The banker's maximizing rates subject to

the firm's individual rationality constraint are shown in equation (20), which is entirely analogous to equation (11). Note that  $c(\underline{m}^*, t)\rho(\underline{m}^*) \leq 1$  since  $\underline{m}^*(x) \leq \underline{k}$ . Equation (21) was obtained from equations (15) and (19). We can also show that at the dual rate, bank profits fall as  $R_f$  increases so  $U_{R_f}(x; t; \underline{m}^*) = ec(\underline{m}^*(x), t)\rho(\underline{m}^*) - 1 < 0$ .

$$(19) V(x; \underline{m}^v(x)) = \int_{\underline{m}^v}^{\infty} [s - \underline{m}^v(x)] f(s) ds - eR_f \equiv 0 \quad \forall e \in (e^v(x), 1]$$

$$(20) \underline{m}^*(x) = \begin{cases} \underline{k} & \forall e < e^v(t) \\ \underline{m}^v(x) & \forall e \geq e^v(t) \end{cases}$$

$$(21) U_e(x; t; \underline{m}^*) = c(\underline{m}^*(x), t)\rho(\underline{m}^*)R_f > 0$$

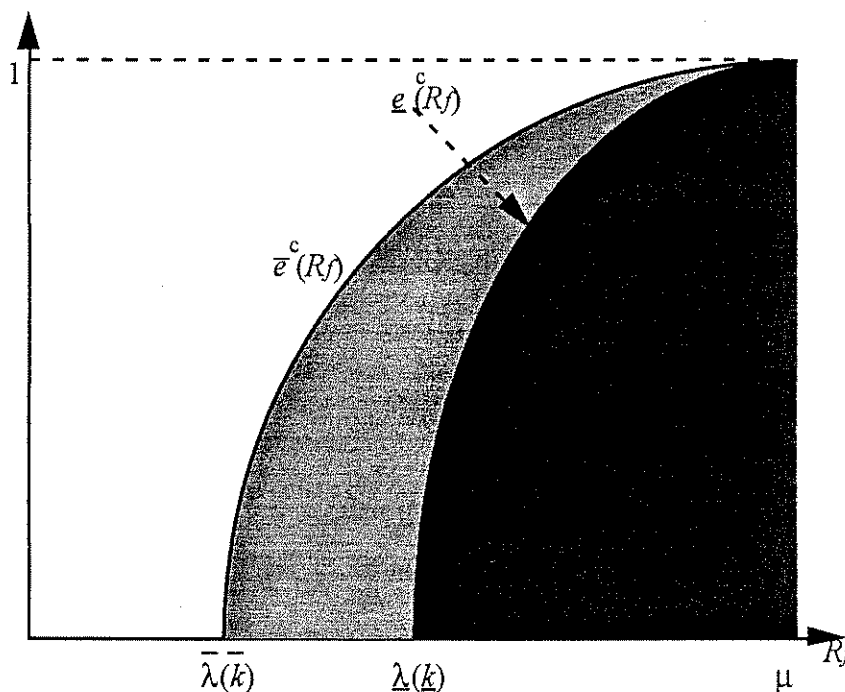
As in section 2 the individual rationality constraint enlarges the credit rationed range of firms by  $[\underline{e}^u(x), \underline{e}^o(x)]$ . First Define  $\underline{C} = E\{c(s; t) | s < \underline{k}\} F(\underline{k})$  When  $R_f$  is between  $\mu - \underline{C}$  and  $\mu$ , we can show that there is a unique point  $\underline{e}^o(x)$  increasing in  $R_f$ ,  $\underline{e}^u(x) < \underline{e}^o(x) < 1$ , where specialized lenders lose money for all firms  $e < \underline{e}^o(x)$  by the same arguments of proposition 3. We obtain the cutoff  $\underline{e}^o(x)$  analogous to the one defined in equation (12) below which firms will not borrow.

$$(22) \underline{e}^c(x) = \begin{cases} 0 & \forall R_f \in [0; \underline{\lambda}(\underline{k})] \\ 1 - \frac{\underline{\lambda}(\underline{k})}{R_f} & \forall R_f \in [\underline{\lambda}(\underline{k}); \mu - \underline{C}] \\ \underline{e}^o(x) & \forall R_f \in [\mu - \underline{C}; \mu] \\ 1 & \forall R_f > \mu \end{cases}$$

Figure 4 presents the cutoff rates for banks and bondholders. The bank lends in the white and the lightly shaded areas. Bondholders only lend in the white area. Entrepreneurs in the darkly shaded zone are credit rationed by both lenders. One can show that the bank's cutoff rate is below the bondholders cutoff so  $\underline{e}^c(x) < \bar{e}^c(x)$ . To see this, consider  $R_f > \underline{\lambda}(\underline{k})$ . At the coordinate

$(\bar{e}^c(x), R_f)$  bondholders are breaking even and firms are willing to borrow. At that same point the bank can match the rate charged by bondholders and make a profit, since it has lower verification costs: mathematically this means  $U(x; \bar{r}, \bar{m}(x)) = 0$  and  $U(x; \underline{r}, \bar{m}(x)) \geq 0$ . Since bank's profit increases in  $e$  from equation (21), the point  $\underline{e}^c(x)$  where  $U(x; \underline{r}, \bar{m}(x)) = 0$  lies below  $\bar{e}^c(x)$ .

**Figure 4 Comparison of cutoff rates for banks and bondholders**



We have ignored that firms with access to the bondmarket will not borrow from the bank if charges more than bondholders  $r(x) > \bar{r}(x)$ . This incentive compatibility constraint forces banks to change strategy as follows: for bond constrained firms  $e \in [\underline{e}^c(x), \bar{e}^c(x))$  banks keep charging their maximizing rate  $\underline{m}^*(x)$ , defined in equation (20). For firms  $e \in [\bar{e}^c(x), 1]$  banks match the rates charged by bondholders,  $\bar{m}(x)$  as defined by equation (13). With this "contestable market" strategy - shown in equation (23) - the bank drives bondholders out of the market. After some manipulation, the bank's economic profit is given in equation (24):

$$(23) \underline{m}^d(x) = \begin{cases} \underline{m}^*(x) & \forall e \in [\underline{e}^c(x); \bar{e}^c(x)] \text{ defined in equation (20)} \\ \bar{m}(x) & \forall e \in [\bar{e}^c(x); 1] \text{ defined in equation (13)} \end{cases}$$

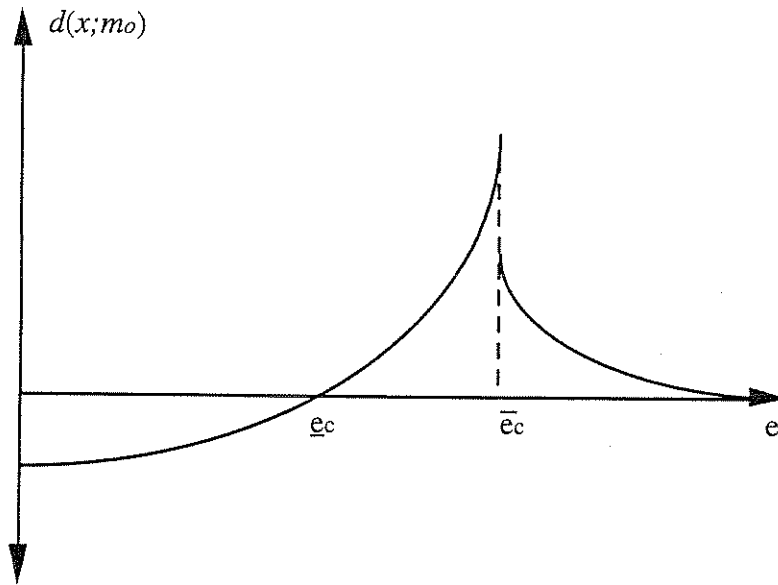
$$(24) U(x; \underline{t}, \underline{m}^d(x)) = \begin{cases} \lambda(\underline{m}^*(x)) - [1-e]R_f & \forall e \in [\underline{e}^c(x); \bar{e}^c(x)] \\ \bar{m} & \\ \int_0^{\bar{t}} [c(s; \bar{t}) - c(s; \underline{t})] f(s) ds > 0 & \forall e \in [\bar{e}^c(x); 1] \end{cases}$$

The second line in equation (24) uses equation (15) and  $U(x; \bar{t}, \bar{m}(x)) = 0$ . In the following sections the simplest paradigm to work with is an excess return function, defined in equation (25). Proposition 7 characterizes this function, and figure 5 plots it. The most noteworthy feature here is the fact that for firms with access to the bondmarket, the banks' maximal rate of return decreases in  $e$ . This occurs because of the competitive nature of the bondmarket. We have already shown that for firms  $e \in [0, \underline{e}^c(x))$  there does not exist a lending equilibrium. Following the arguments of proposition 4 we can show that there exists a unique lending equilibrium  $\underline{m}(x) < \underline{m}^d(x)$  for all firms with internal funds  $e \in [\underline{e}^c(x), 1)$ .

$$(25) \underline{d}(x; \underline{m}^d(x)) = \frac{U(x; \underline{t}, \underline{m}^d(x))}{1-e}$$

- Proposition 7:*
- |  |  |
|--|--|
| A) $\underline{d}(x; \underline{m}^d(x)) < 0$  | $e \in [0, \underline{e}^c(x))$            |
| B) $\underline{d}(x; \underline{m}^d(x)) \geq 0, \underline{d}_e(x; \underline{m}^d(x)) > 0$ | $e \in [\underline{e}^c(x), \bar{e}^c(x))$ |
| C) $\underline{d}(x; \underline{m}^d(x)) \geq 0, \underline{d}_e(x; \underline{m}^d(x)) < 0$ | $e \in [\bar{e}^c(x), 1)$                  |
| D) $\underline{d}(x; \underline{m}^d(x)) = 0$  | $e = 1$                                    |

**Figure 5** Maximal Bank excess return  $\underline{d}(x; \underline{m}^d(x))$



### 3.2 Endogenous Intermediation

So far, we have seen that introducing a specialized lender does not by itself explain why we observe intermediated and nonintermediated lending in the economy. This section solves this problem by endogeneizing the level of financial intermediation. I assume that a bondholder - a generic lender - with funds  $W_i$  can become a specialized lender, by spending a fraction  $\gamma$  of his funds. One can view this fraction as a reserve requirement, or the expenses that banks incur in setting up a monitoring technology, or simply as a liquidity pool that an intermediary sets to satisfy its depositors. At the margin, no bondholder should have an incentive to become a specialized lender. In this setup, we will see that banks use not only their own funds, but also borrow from depositors and relend to firms. Bondholders are willing to lend to such a specialized lender, since the expected return on its portfolio can be greater than  $R_f$ .

This subsection considers a profit maximizing, competitive banking sector. To carry on with this market structure we must simplify the model. We will assume that bondholders have

linear verification costs and limited liability so that  $c(s; \bar{t}) = \bar{t} s$ ,  $\bar{t} < 1$ . This simplification allows us to properly talk about a representative bank. If verification costs were not linear, then the size and distribution of banks would matter.

We prove two things in this section. First, that the optimal investment strategy for an intermediary is to set a premium  $\pi$  and to choose only those projects with a rate of return above  $R_f + \pi$ . The second result shows that there exists a unique and positive competitive premium for banks. This subsection studies the behavior of this premium  $\pi(x)$  as a function of bank's reserve requirement and risk free rate.

*Joint Distribution Function:*

In a related article, Diamond (1984) assumed that in a loan portfolio banks can diversify all risk away. Since this postulate seems unrealistic, I will assume instead that banks' loan portfolio has an undiversifiable risk component. Let us start with a generic lending strategy: consider a Borel set  $S_i \in B([0,1])$  such that the bank lends only to firms in the set  $S_i$ . Suppose that the bank charges an arbitrary rate  $r(x)$  that satisfies the firms' individual rationality and incentive constraints. The expected payoff from lending to an entrepreneur in set  $S_i$  is:

$$(16) \lambda(m(x), t) = m(x)[1 - F(m(x))] + \int_0^{m(x)} [s - c(s; t)] f(s) ds$$

where  $m(x) = [1 - e]r(x)$ . Suppose that the actual payoff of this loan is given by equation (26).  $y$  is an aggregate shock that enters multiplicatively, and  $h_e$  is a mean zero idiosyncratic random variable.  $y$  is a bounded random variable with a distribution  $g(y)$  and expected value of one so that  $y \in [0, v]$ ,  $E(y) = 1$ . If the decomposition in equation (26) is possible then, following

Al-Najjar (1994) the actual returns on a portfolio  $S_i$  is given by equation (27) and (28). Appendix 2B discusses this issue in more detail.

$$(26) \tilde{\lambda}(m(x), t) = \underline{\lambda}(m(x), t)y + h_e(m(x), t)$$

$$(27) \tilde{\Lambda}(S_i, R_f) = \Lambda(S_i, R_f)y$$

$$(28) \Lambda(S_i, R_f) \equiv \int_{e \in S_i} N \underline{\lambda}(m(x)) dH(e) de$$

$\Lambda(S_i, R_f)$  are the expected bank revenues from lending to a set  $S_i$  of firms. Equation (27) says that investors cannot diversify away the common shock, although they can completely eliminate the idiosyncratic returns. This result is a consequence of having a continuum of entrepreneurs. In reality, since loan portfolios normally include a finite number of firms, some idiosyncratic components are likely to remain. In such an environment, the banks' realized returns are not exactly matched by  $y$ . The assumption of having a continuum of entrepreneurs can be seen as an idealization of many firms with negligible weight. A representative bank lending to a set  $S_i$  of entrepreneurs has loans amounting to:

$$(29) L(S_i) = N \int_{e \in S_i} [1-e] dH(e) de$$

The bank's internal funds are  $[1-\gamma]W_i$ , which may be insufficient to cover their lending needs so  $L(S_i) > [1-\gamma]W_i$ . If the bank's state  $\Lambda(S_i)y$  is costly to verify<sup>4</sup>, the optimal contract with its depositors - with bondholders - is standard debt. The bank borrows  $L(S_i) - [1-\gamma]W_i$  from depositors, who charge a rate  $R(S_i)$  on that balance. Banks owe  $M(S_i) \equiv [L(S_i) - [1-\gamma]W_i]R(S_i)$ , and default whenever the state  $y$  drops below  $b = M/\Lambda$ . The following assumptions ensure the

existence of a lending equilibrium between the bank and its depositors: a)  $g(y)$  has an increasing hazard rate; b)  $\bar{t} [v/\Lambda] \rho(v/\Lambda) \geq 1$  and; c) At a profit maximizing rate  $k(\Lambda)$  then  $U(x; \bar{t}, k(\Lambda)) \geq 0$  and  $U(x; t, k(\Lambda)) \geq 0$ . The equilibrium is a unique value  $b_i \in (0, k(\Lambda)]$  where bondholders break even,  $U(x; \bar{t}, b_i) \equiv 0$ . The existence of a lending equilibrium can be proved using the arguments in propositions 1 and 5.

### *Equilibrium Bank Behavior*

We need to define what we mean by a bank-entrepreneur equilibrium. Lenders choose several things. First, they decide if they will become specialized investors, and thus to pay a fraction  $\gamma$  of their capital. If an investor decides to acquire the specialized lending technology, he chooses two variables: The first is the set  $S_i$  where he lends. The union of all the sets where banks lend is defined as  $S^*$ . The second choice that banks make is the rate  $\underline{m}(x)$  charged to each firm  $e \in S^*$ . I posit the following conditions that an equilibrium  $\{S^*, \underline{m}(x)\}$  should satisfy:

- E1. In equilibrium a bank has no incentive to change interest rates  $\underline{m}(x)$  to firms  $e \in S^*$ .
- E2. In equilibrium a bank have no incentive to lend to firms  $e \notin S^*$
- E3. In equilibrium a bank has no incentive to change the loan size.
- E4. In equilibrium no bondholder wants to become a bank.

These conditions are similar to those of a Nash equilibrium, where agents have no incentives to deviate from their current policies. Proposition 8 explores the nature and existence of an equilibrium bank lending in this economy. Part A of this proposition says that banks will set a premium  $\pi$ , and lend only to those projects which, evaluated at the dual rate defined in equation (23), have a rate of return greater than  $R_f + \pi$ .

**Proposition 8:** The equilibrium bank lending in this economy has :

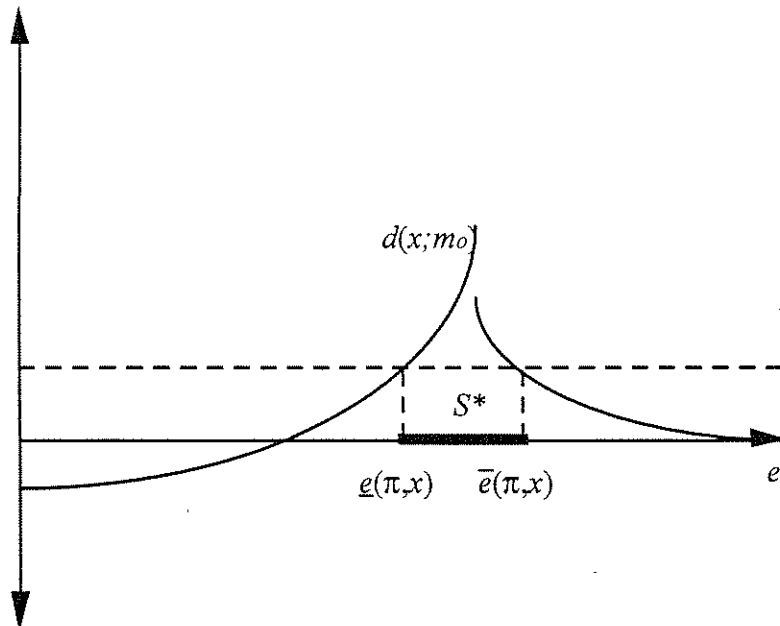
A) The optimal set  $S^*(\pi) = [e, \bar{e}]$  satisfies the condition that  $e \in S^*(\pi)$  iff  $\underline{d}(x; \underline{m}^d(x)) \geq \pi$ . The optimal pricing strategy is  $\underline{m}(x)$  such that  $\Lambda(\underline{m}(x)) = [1-e][R_f + \pi]$ .

B)  $\underline{e}_1(\pi, R_f) \geq 0$ ,  $\underline{e}_2(\pi, R_f) \geq 0$ ;  $\bar{e}_1(\pi, R_f) \leq 0$ ,  $\bar{e}_2(\pi, R_f) \geq 0$

C) There is a unique equilibrium premium  $\pi(x) > 0$ . This premium has  $\pi_{x_i} > 0$ , for  $\gamma, R_f$ .

The optimal portfolio  $S^*$  is uniquely identified with the premium  $\pi$ . Figure 6 plots what the aggregate bank portfolio  $S^*$  looks like. Banks will discard entrepreneurs with scarce resources. They will also drop firms with abundant funds, where competition with bondholders erodes economic profits. Banks will compete among themselves until the rate of return for each project  $e \in S^*$  has dropped to  $R_f + \pi$ . A positive premium implies that there are fewer bank loans than before:  $\underline{e}^c(x) \leq \underline{e}(\pi) \leq \bar{e}^c(x) \leq \bar{e}(\pi)$ . Proposition 8B signs the derivatives of  $\underline{e}(\pi)$  and  $\bar{e}(\pi)$ .

Figure 6 Optimal portfolio of bank loans



The aggregate amount of expected bank revenues and loans under the result in proposition 8 are presented in equations (28') and (29').

$$(28') \int_{\underline{e}}^{\bar{e}} \Lambda(\pi, R_f) \Lambda(e, R_f) h(e) de = [R_f + \pi] L(\pi, R_f) \quad (29') \quad L(\pi, R_f) = N \int_{\underline{e}}^{\bar{e}} [1-e] h(e) de$$

The equilibrium conditions E1-E4 imply that a) Neither bondholders nor banks are making economic profits c) Each bank with funds  $W_i$  chooses the extent of its loans  $L_i$  in a profit maximizing way. Banks and bondholders take the pricing policy  $\underline{m}(x)$  as given. This implies that neither banks nor bondholders consider the general equilibrium impact on pricing that occurs when they change their loan size. The equilibrium conditions are given in equations (30)-(33):

$$(30) \quad U(x; \bar{i}, b_i, S_i) = [\pi + R_f] L_i \left[ b_i [1 - G(b_i)] + \int_0^{b_i} y [1 - \bar{i}] g(y) dy \right] - (L_i - [1 - \gamma] W_i) R_f \equiv 0$$

$$(31) \quad U(x; \bar{i}, b_i, S_i) = [\pi + R_f] L_i \int_{b_i}^v [y - b_i] g(y) dy - W_i R_f \equiv 0 \quad \text{zero bank profits}$$

$$(32) \quad \frac{dU(x; \bar{i}, S_i)}{dL_i} = [R_f + \pi] \int_{b_i}^v [y - b_i] g(y) dy - \frac{[1 - \gamma] W_i R_f}{L_i [1 - \bar{i} b_i p(b_i)]} = 0 \quad \text{bank profit maximization}$$

$$(33) \quad \sum_i L_i = L(\pi, R_f) \quad \text{market clearing}$$

This first order condition in equation (32) uses equations (31) and (30). It considers an atomistic bank deciding how much to lend in the economy, taking the pricing policy  $m(x)$  and thus  $\pi$  as given. With these equilibrium conditions we want to find out the bankruptcy point  $b_i$  (i.e. interest rate) and leverage  $L_i$  faced by each bank. We also want to identify the equilibrium premium  $\pi$  and aggregate bank capital  $W$ . These variables are explicitly derived in the proof of proposition 8C. Equations (30) - (33) imply that the bankruptcy rate is the same across banks:  $b_i$

$= b_j = b(\gamma)$ . A rise in  $\gamma$  - which occurs because of increases in reserve requirements or because banks need to have greater liquid assets in the face of bank runs - raises the bankruptcy point, so  $b'(\gamma) > 0$ . Similarly, the equilibrium conditions imply that the leverage across banks is the same. Banks act as intermediaries, since the ratio of internal funds to loans is below one so that we have  $l(\gamma) \equiv [1-\gamma]W_i/L_i < 1$ . Leverage increases with a rise in the reserve requirement, so  $l'(\gamma) < 0$ . Equation (34) displays an expression for the premium, which was obtained by combining equations (30), (31) and (32). One can verify that  $P'(b) > 0$ ,  $P(v) = \bar{i} < 1$  and that  $P(0) = 0$ . The derivatives of this premium are given in equation (35):

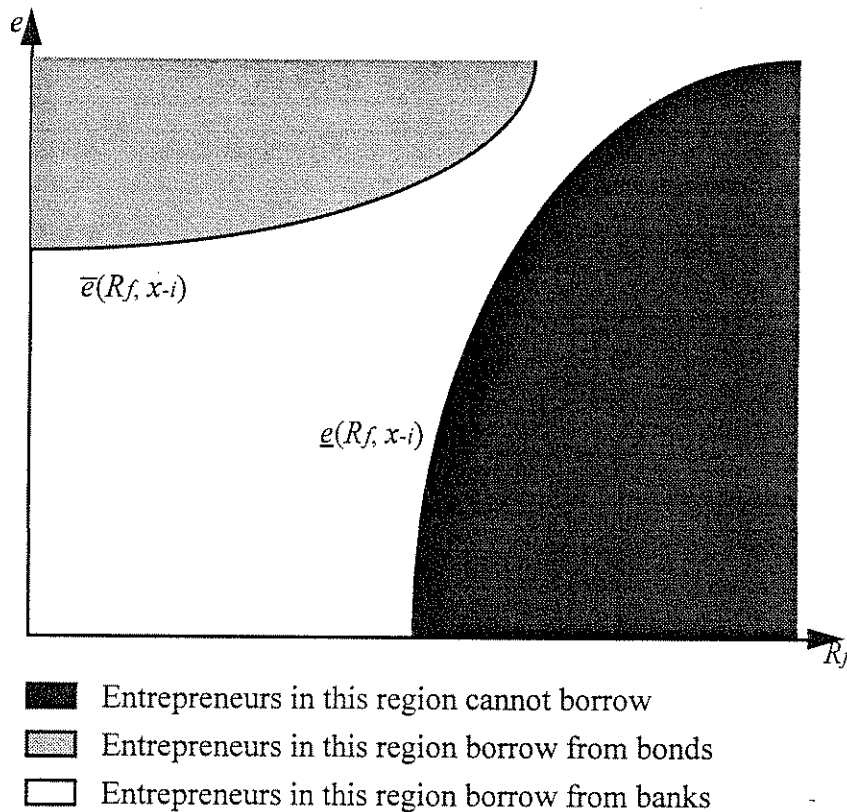
$$(34) \pi \equiv R_f \left[ \frac{P(b)}{1-P(b)} \right] > 0 \text{ where } P(b) = \bar{i} \left[ b \rho(b) \int_b^v [y-b]g(y)dy + \int_0^b yg(y)dy \right] < 1$$

$$(35) \frac{d\pi}{dR_f} = \left[ \frac{P(b)}{1-P(b)} \right] = \frac{\pi}{R_f} > 0 \quad \frac{d\pi}{d\gamma} = \frac{R_f P'(b) b'(\gamma)}{[1-P(b)]^2} > 0 \quad \frac{d^2\pi}{d\gamma dR_f} = \frac{P'(b) b'(\gamma)}{[1-P(b)]^2} > 0$$

$\pi > 0$  implies that banks will not completely dominate the debt market. As a bank expands its lending it has to borrow more from the bondmarket, making default more likely. In addition, the marginal bankruptcy cost increases with higher leverage. Banks stop lending at the point where the marginal value of the extra loan equals the marginal expected bankruptcy costs. Equation (35) shows how the premium varies with changes in reserve requirements and interest rates. With lower reserve requirements, banks need to borrow less to finance the same projects, lowering  $b$ . This reduces banks' default rate and premium, as shown in equation (34). The premium also increases with a rise in  $R_f$ . As the risk free rate rises, banks have to offer higher coupons to their depositors, becoming more prone to bankruptcy. This raises banks' premium, as defined by equation (34).

The behavior of this economy is sketched in Figure 7. To understand this figure consider a coordinate  $(R_f, e)$ . This coordinate specifies a firm with internal funds  $e$  at a prevailing interest rate  $R_f$ . If the coordinate is in the gray area, the entrepreneur borrows directly from the bondmarket. If the coordinate is in the white area the firm borrows from the bank. If the point is in the black area the firm cannot borrow.

Figure 7 Choice between public and private debt



Firms with abundant funds  $e > \bar{e}$  borrow directly from the bondmarket. Well endowed firms are seldom bankrupt, and thus need little verification. They are rather looking for lenders with a low cost of capital, which implies that bondholders can underprice intermediaries. A drop in reserve requirements lowers banks' capital cost, enabling them to steal some projects from bondholders, so  $d\bar{e}(x) / d\gamma < 0$ . Proposition 8 shows that the cutoff rate rises with a rise in the

riskless rate so  $d\bar{e}(x)/dR_f > 0$ . As  $R_f$  increases, bondholders demand a higher return from entrepreneurs. This creates more verification, and extra rents for banks to take. The bank can now capture some fringe firms: this raises  $\bar{e}$ . There is a second effect, which is the rise of the premium as the risk free rate increases, and which is outweighed by the first effect.

Firms with funds  $e \in [\underline{e}, \bar{e}]$  borrow from banks. Firms with moderate resources are more likely to go bankrupt. Banks are able to outcompete bondholders because of their lower verification costs, which are more relevant in this range of firms. The bank is unable to completely drive generic lenders out of the market since it has a higher cost of capital than bondholders. Banks expand lending when faced with lower reserve requirements, so  $d\underline{e}(x)/d\gamma > 0$  and  $d\bar{e}(x)/d\gamma < 0$ . The lower cutoff rate  $\underline{e}$  rises with an increase in the risk free rate. As  $R_f$  rises, safe bonds becomes more attractive than lending to poorly endowed firms, raising agency costs between banks and entrepreneurs. This effect pushes poorly endowed firms out of the credit markets. There is a second effect - the rise in  $\pi$  as  $R_f$  increases - which reinforces the first effect.

Firms with low cash  $e \in [0, \underline{e})$  are credit rationed. Neither banks nor bondholders can ever make money from lending to such firms. The number of firms excluded from the credit markets increases with a rise in the risk free rate or with a higher reserve requirement.

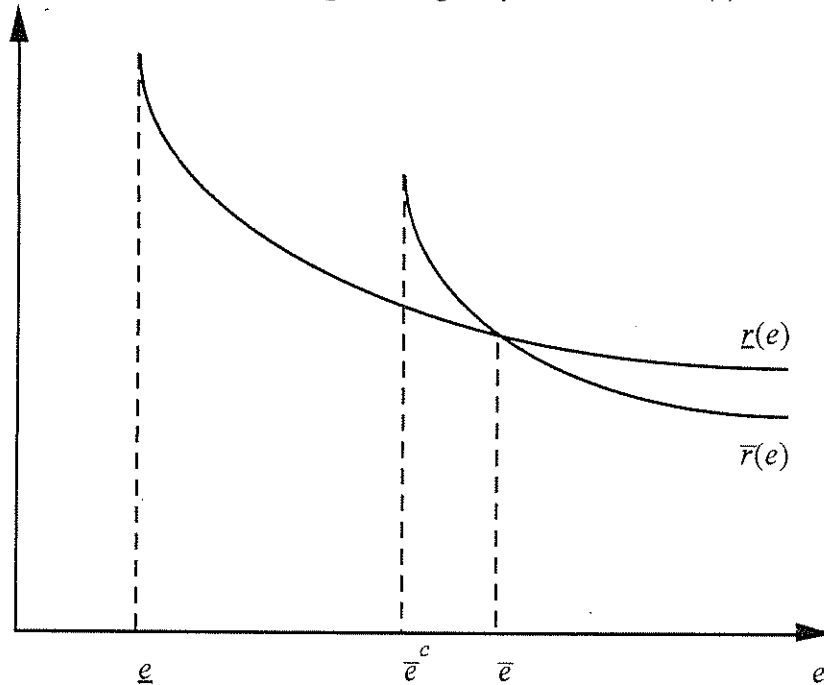
***Microeconomic Implications on contractual interest rates:***

Proposition 9 shows the behavior of bank contractual interest rates. The results of propositions 5 and 9 are presented in figure 8. The y axis of this figure plots the interest rates charged by both types of lenders to a company with internal funds  $e$ .

**Proposition 9:** Firms with funds  $e \in [\underline{e}, \bar{e}]$  face an interest rate  $\underline{r}(x)$  satisfying

$$[d\underline{r}(x)/de] < 0, \quad [d\underline{r}(x)/dR_f] > 1 \text{ and } [d\underline{r}(x)/d\gamma] > 0$$

Figure 8 Contractual rates charged by bondholders  $\bar{r}(e)$  and banks  $\underline{r}(e)$



There are three groups of firms to consider. Firms with funds  $e \in [\underline{e}(x; \pi(x)), \bar{e}^c(x)]$  can borrow only from intermediaries.

A second group of firms - those with funds  $e \in [\bar{e}^c(x), \bar{e}(x; \pi(x))]$  - can borrow from either banks or from bondholders: borrowing from the former source is less expensive. If a firm in this group borrows from the bondmarket it is for reasons outside of this model, such as agency costs. An instance of this is a manager desiring to escape bank monitoring. This could be an explanation of junk bonds.

The last group of firms, those with funds  $e \in [\bar{e}(x; \pi(x)), 1]$ , can borrow from banks or bondholders, although the latter always charge a lower rate on their loans. If a company in this

region borrows from a bank it must do so for reasons outside of our model. For example a firm in this group might not borrow bonds to avoid stringent disclosure requirements.

### ***Section 4. Aggregate Equilibrium***

We will now aggregate the microeconomic model presented thus far. First, we study aggregate corporate investment. We then analyze aggregate savings and the general equilibrium of the economy.

#### ***Aggregate Corporate Investment***

Corporate investment is the aggregation of the individual investment of those firms who can borrow from either banks or bondholders i.e. with internal funds  $e \in [\underline{e}, 1]$ . This cutoff rate is characterized in proposition 8B. Total investment is displayed in equation (36).  $\varphi$  is a first order stochastic dominance parameter such that  $H_\varphi(\underline{e}; \varphi) < 0$ . A higher value of  $\varphi$  means that there is a generalized increase in company internal funds. Proposition 10 considers how changes in the risk free rate, the reserve requirements and corporate internal funds affect investment:

$$(36) \quad I(x) = N \int_{\underline{e}}^1 h(e) de = N[1 - H(\underline{e}(\pi(x); R_f); \varphi)]$$

#### ***Proposition 10***

Investment increases with an improvement in corporate internal funds, and with a drop in the risk free rate or the reserve requirement: thus  $I_{x_i}(x) > 0$  for  $x_i = \varphi$  and  $I_{x_i}(x) < 0$  for  $x_i = \gamma, R_f$

Higher corporate internal funds raise aggregate investment. This occurs because firms that were previously excluded from the credit markets i.e.  $e < \underline{e}$  receive an inflow  $\Delta e$  which may be sufficient to bring them into the credit market, so that the firms' investment jumps from 0 to 1.

A higher interest rate lowers investment. As  $R_f$  rises, the banks cost of capital increases. This causes banks to reduce lending to firms with low funds. There is a second effect - the rise in the premium as  $R_f$  increases - which reinforces the first effect. What is the difference between this interest rate mechanism and its neoclassical counterpart? In a frictionless model with risk neutral agents, firms reduce investment because as interest rate rises, the expected profit of their projects falls below the riskless rate of return. In this model a rise in interest rates reduces investment even among companies with good projects ( $\mu > R_f$ ). This additional interest rate mechanism is difficult to disentangle with aggregate data. At a microeconomic level it could be tested running corporate investment as follows:

$$I_j = a + b_1q_j + b_2R_f + b_3\gamma + b_4C_j$$

where  $q_j$  controls for an industry's profitability,  $\gamma$  is a measure of reserve requirements, or needed liquidity, and  $C_j$  is the firm's internal funds.

The reason why a lower reserve requirement raises investment is that banks will have more internal funds and their cost of capital drops. A drop in the banks' cost of capital allows them to extend lending to previously rationed firms.

### ***Aggregate Savings and the General Equilibrium.***

This article concludes with an analysis of aggregate savings in this economy. Assume that there is a representative agent who lives in periods 0 and 1. The agent has exogenous endowments  $E_0$  and  $E_1$ . A government issues riskless bonds at  $t = 0$  worth  $B$  and promising a rate of return  $R_f$ . At time  $t = 1$  it imposes a lump sum tax of  $BR_f$  on the agent. We assume that the proceeds of the bond at time  $t = 0$  are not used for anything that the agent values, so that it does not enter into his utility function.

The agent owns shares in all companies<sup>5</sup>. We assume that those firms that do not get funding liquidate their assets at  $t = 0$ <sup>6</sup>. At  $t = 1$  all remaining companies and banks liquidate their assets in the form of a dividend. Expected dividends equal the expected equity value of the firm at time  $t = 1$ . We divide the companies in two types: Type 1 are firms that borrow from a bank, while type 2 are firms that borrow from the bondmarket. Equation (37) presents an expression for expected dividends. This equation uses the fact that banks and bondholders are competitive, i.e. equations (32) and (15). Note that at the lower cutoff we have  $D_1(\underline{e}) = \underline{e}R_f$  since at this point the firm is indifferent between undertaking the project or putting its money in a riskless asset. At the upper cutoff we have  $D_1(\bar{e}) = D_2(\bar{e})$  since at this point the firm is indifferent between borrowing from the bank or from the bondholder. The derivatives of expected dividends are shown in equations (38) and (39), which were obtained using equations (12), (13) (35), and proposition 9. Equation (38) shows that expected dividends fall as the riskless interest rate rises. Equation (39) says that firms with higher internal funds give larger expected dividends. These expected dividends grow in a concave fashion, so  $D_{ee}(x) < 0$ . This follows from propositions 5 and 9.

$$(37) D(x) = \begin{cases} D_1(x) = \mu - R_f - \int_0^{\bar{m}(x)} c(s; \bar{t}) f(s) ds - [1-e]\pi & \text{for } e \in [\underline{e}; \bar{e}] \\ D_2(x) = \mu - R_f - \int_0^{\bar{m}(x)} c(s; \bar{t}) f(s) ds & \text{for } e \in [\bar{e}; 1] \end{cases}$$

$$(38) \frac{dD(x)}{dR_f} = \begin{cases} \frac{dD_1(x)}{dR_f} = -\frac{[1-e](1+\pi_{R_f}(x))}{1-c(\bar{m}; \bar{t})\rho(\bar{m})} - e < 0 & \text{for } e \in [\underline{e}; \bar{e}] \\ \frac{dD_2(x)}{dR_f} = -\frac{1-ec(\bar{m}; \bar{t})\rho(\bar{m})}{1-c(\bar{m}; \bar{t})\rho(\bar{m})} < 0 & \text{for } e \in [\bar{e}; 1] \end{cases}$$

$$(39) \frac{dD(x)}{de} = \begin{cases} \frac{dD_1(x)}{de} = \frac{c(\underline{m}; \underline{i})\rho(\underline{m})R_f + \pi}{1 - c(\underline{m}; \underline{i})\rho(\underline{m})} > 0 & \text{for } e \in [\underline{e}; \bar{e}] \\ \frac{dD_2(x)}{de} = \frac{c(\bar{m}; \bar{i})\rho(\bar{m})R_f}{1 - c(\bar{m}; \bar{i})\rho(\bar{m})} > 0 & \text{for } e \in [\bar{e}; 1] \end{cases}$$

The representative agent is risk neutral with respect to states, but his utility function is logarithmic with time:  $u = C_0^\alpha C_1^{1-\alpha}$ . Expected consumption at  $t = 1$  - defined in equation (40) - equals the expected return on savings plus dividends plus the exogenous endowment.  $S_a$  is invested in riskless bonds and/or in bonds whose expected return is  $R_f$ .  $S_a$  can also be spent in the banking business, whose expected returns are  $R_f$ . In addition the agent has to pay taxes  $BR_f$ . Finally the agent receives dividends from active firms and his exogenous endowment. Consumption at  $t = 0$  - defined in equation (41) - equals the initial endowment less savings plus the liquidation value of noninvesting firms:

$$(40) C_1 = E(\tilde{C}_1) = (S_a - B)R_f + \int_{\underline{e}}^1 ND(e, x_{-i})h(e)de + E_1$$

$$(41) C_0 = E_0 - S_a + \int_0^{\underline{e}} Neh(e)de$$

The agent maximizes his utility with respect to  $S_a$ . Equation (42) displays the agent's first order conditions. Aggregate savings - defined in equation (43) - equals private savings plus firms' internal funds minus government dissaving. Equations (44) and (45) show how changes in the risk free rate and corporate funds affect aggregate savings. Equation (44) uses equation (38) and  $D_1(\underline{e}) = \underline{e}R_f, D_1(\bar{e}) = D_2(\bar{e})$ . It says that aggregate savings rise with an increase in the risk free rate, because of an income effect that reduces the present value of future dividends. Equation (45) uses the fact that  $D_e(x_{-i}; e=\underline{e}) = R_f$  and that  $D_{ee}(x) < 0$ . It is positive since  $H_\varphi(e, \varphi) < 0$  and  $D_e(x) < R_f$

for  $e > \underline{e}$ . It says that an increase in corporate cash flow raises aggregate savings. This result is a consequence of larger corporate savings, which are only partly offset by a decline in private saving. Finally one can show that a change in reserve requirements has no effect on savings. To see this, one integrates equation (43) by parts and uses equation (38), and the fact that  $D_1(\underline{e}) = \underline{e}R_f$ ,  $D_1(\bar{e}) = D_2(\bar{e})$ .

$$(42) [1-\alpha]C_oR_f = \alpha C_1 \Rightarrow S_a = [1-\alpha] \left[ E_0 + \int_0^{\underline{e}} N e h(e) de \right] + \alpha B - \frac{\alpha}{R_f} \left[ \int_{\underline{e}}^1 N d(e) h(e) de + E_1 \right]$$

$$(43) S = [1-\alpha] \left[ E_0 + \int_0^{\underline{e}} N e h(e) de - B \right] - \frac{\alpha}{R_f} \left[ \int_{\underline{e}}^1 N D(x) h(e) de + E_1 \right] + \int_{\underline{e}}^1 N e h(e) de$$

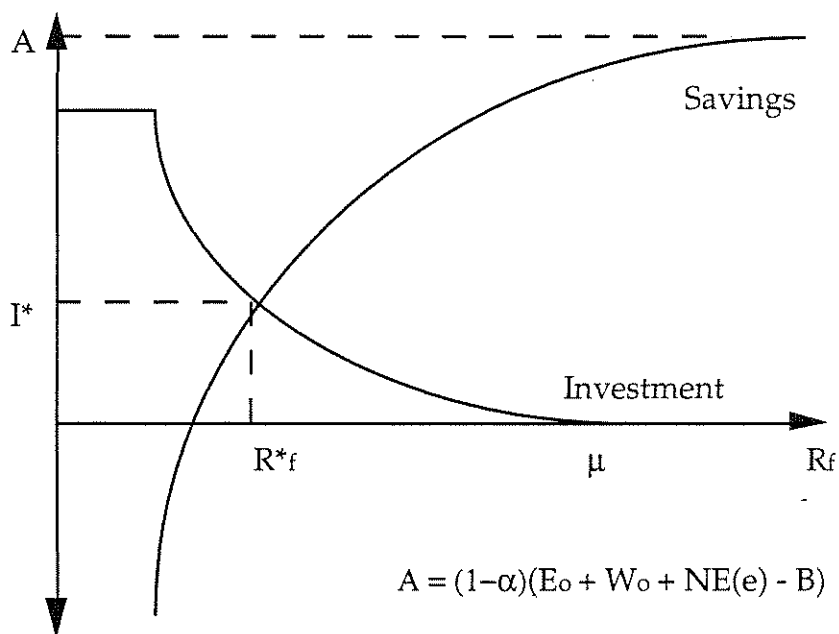
$$(44) \frac{dS}{dR_f} = \frac{\alpha}{R_f^2} \left[ \int_{\underline{e}}^1 N D(x) h(e) de + E_1 \right] - \frac{\alpha}{R_f} \left[ \int_{\underline{e}}^1 N \frac{dD(x)}{dR_f} h(e) de \right] > 0$$

$$(45) \frac{dS}{d\phi} = -[1-\alpha] \int_0^1 H_2(e, \phi) de - \frac{\alpha}{R_f} \int_{\underline{e}}^1 [R_f - D_e(x)] H_2(e, \phi) de > 0$$

When  $R_f$  is zero then  $S(0) = -\infty$ , and agents try to bring all their consumption to the first period. When the riskless rate is  $\infty$  all projects shut down, so  $\underline{e} = 1$ . At this rate savings equals  $S(\infty) = [1-\alpha][E_0 + NE\{e\} - B]$ .  $NE\{e\}$  represents all the corporate internal funds at  $t = 0$ . In equilibrium aggregate investment - defined in equation (36) - must equal savings, as given in equation (43). Proposition 10 shows that  $dI/dR_f < 0$ . At  $R_f = 0$  all firms borrow so  $I(0) = N$ . No firm borrows at rates greater than  $\mu$  so  $I(R_f) = 0$  for  $R_f \geq \mu$ . There exists an equilibrium in this economy as long as  $B < [1-\alpha][E_0 + NE(e)]$ . This equilibrium - presented in figure 9 - determines investment and the risk free interest rate. We obtain the following comparative statics:

- a) An increase in current output  $E_o$  and a decrease in government bonds  $B$  or future income  $E_1$  raises investment and reduces the risk free rate. This follows since the savings equation shifts upwards.
- b) An increase in reserve requirements lowers investment and real interest rates, given that the investment function shifts down but the savings function remains unperturbed. This result shows that different policy measures - increase in the reserve requirement vs. open market operations in (a) produce qualitatively different results in the economy.
- c) An increase in firms internal funds raises investment. The effect on the interest rate is ambiguous since both the savings and investment functions shift upwards.

*Figure 9 Equilibrium Investment*



## *Conclusions*

This paper analyzed why some firms borrow from intermediaries while others borrow directly from the bondmarket. One easily observable variable that determines this choice is corporate internal funds. Cash rich firms need not worry about costly reorganizations arising from bond defaults, since this is unlikely. The main concern of a well endowed firm is to borrow from the lender with the lowest cost of capital, i.e. directly from bondholders. On the other hand, firms with less cash do need to worry about default and its consequences. These firms prefer to borrow from intermediaries who are better able to deal with financial distress.

This article took a simple microeconomic model, and aggregated it into a macroeconomic investment function. In the model, aggregate investment drops with an increase in the risk free rate. The mechanism through which this happens is different from typical investment functions. At high risk free rates agency costs become more pronounced, reducing the number of worthwhile projects and investment. The model also shows that investment drops with a fall in corporate cash flow. This violates results emanating from models based on perfect information, which say that internal funds are irrelevant to investment. Finally, investment falls when cost of being a financial intermediary increases. These costs may rise if reserve requirements are raised, or if intermediaries need a larger pool of liquid assets to satisfy their depositors, as happened during the Great Depression, and in other episodes of bank runs in the United States. Intuitive as they are, results of this nature are among the first to come out of a general equilibrium model where aggregation is explicitly considered. The macroeconomic results of this paper transcend those of standard IS-LM models. The aggregate behavior of bonds and bank loans as a function of interest rates and corporate and bank health is studied for the first time in this model.

There are some research questions that need to be dealt more carefully in the future. First, one needs to develop an investment function where firms have more flexibility in choosing the project size. Another important area of research is to develop a microeconomic model to explicitly consider the setup costs of becoming a bank (i.e. the nature of bank monitoring, and the intermediaries' need to have pools of liquid assets). Finally, it is important to replicate these results for a dynamic economy. So far, this model has only two periods. In order to extend these results it is necessary to develop a coherent multiperiod security design model. This problem is notoriously difficult, but it is a goal worth attaining.

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<sup>2</sup> An alternative set of assumptions that allows different distributions is:

- 1)  $\rho'(s) \geq 0$  for  $s < M < \infty$ .
- 2)  $c_j(s)\rho(s)$  increases for all  $s$ .
- 3)  $c_j(M)\rho(M) > 1$

<sup>3</sup> See the proof of proposition 2.1 to see this result.

<sup>4</sup> It may seem strange that an aggregate risk factor such as  $y$  is costly to verify. If we believe, however, that the bank cannot fully diversify all idiosyncratic risks away, it is possible that its earnings are in fact costly to verify.

<sup>5</sup> It may seem strange to readers that at this point I assume that the representative agent can effortlessly get dividends from the banks or the companies, since this was exactly what the CSV framework negates. To justify this last section, one has to invoke strong aggregation theorems - such as Eisenberg's aggregation - where each agents have utility functions with special properties - properties satisfied in the present setup - and a fixed fraction of the economy's endowment. This last condition is more difficult to defend.

<sup>6</sup> This assumption is not important, since even if these firms remained active and behaved as mutual funds (i.e. place their internal cash in a riskless asset) the result remains the same.

## Appendix A Proofs

### Proposition 1

There exists a unique unconstrained profit maximizing rate  $\bar{m}^u(x) < \infty$ .

*Proof* First we show that  $\lim_{s \rightarrow \infty} \{c(s; \bar{t})\rho(s)\} = \infty$ . Consider a point  $z$  such that  $\rho(z) > 0$ . Equation (1.1) gives a first inequality, which follows from A1.2. The second inequality follows from A5.4. The result of this is given in equation (1.2)

$$(1.1) \quad c(s; \bar{t})\rho(s) > c(s; \bar{t})\rho(z) > [(s-z)c'(s; \bar{t}) + c(z; \bar{t})]\rho(z) \text{ for } z < s$$

$$(1.2) \quad \lim_{s \rightarrow \infty} \{c(s; \bar{t})\rho(s)\} = \infty$$

*Existence, Uniqueness:* Equation (6) implies that  $\text{sgn}[U_m(x; \bar{t}, m)] = \text{sgn}[1 - c(m; \bar{t})\rho(m)]$ . Equations (1.3)-(1.5) below and the intermediate value theorem (IVT) imply that there exists a unique bankruptcy rate  $\bar{m}^u(x) < \infty$  where  $c(\bar{m}^u(x); \bar{t})\rho(\bar{m}^u(x)) = 1$ . This and the fact that  $[1 - F(m)][1 - e] > 0$  insure the existence of unique unconstrained rate  $\bar{m}^u(x)$  where  $U_m(x; \bar{t}, \bar{m}^u(x)) = 0$ . A useful corollary is  $U_m(x; \bar{t}, m) > 0 \quad \forall m < \bar{m}^u(x)$  and  $U_m(x; \bar{t}, m) < 0 \quad \forall m > \bar{m}^u(x)$ .

$$(1.3) \quad c(0; \bar{t})\rho(0) = 0 \quad \text{by A5.1}$$

$$(1.4) \quad \lim_{s \rightarrow \infty} \{c(s; \bar{t})\rho(s)\} = \infty \quad \text{by equation 1.2}$$

$$(1.5) \quad d[1 - c(s; \bar{t})\rho(s)]/ds < 0 \quad \text{by A6.1}$$

*Maximality:* The second order condition is given below. It proves that the point is a maximum:

$$U_{mm}(x; \bar{t}, \bar{m}^u(x)) = -[1 - F(\bar{m}^u)] [c'(\bar{m}^u; \bar{t})\rho(\bar{m}^u) + c(\bar{m}^u; \bar{t})\rho'(\bar{m}^u)] < 0 \quad \blacklozenge$$

Claim:  $\bar{\lambda}(\bar{k}) > 0$ . Consider equations (16) -(1.10). Equation (1.10) is an exact Taylor expansion.

$$(1.6) \quad \bar{\lambda}(0) = 0$$

$$(1.7) \quad \bar{\lambda}'(m) = [1 - F(m)][1 - c(m; \bar{t})\rho(m)]$$

$$(1.8) \quad \exists \delta > 0 \exists \forall 0 < x < \delta \text{ then } c(x; \bar{t})\rho(x) < 1 \text{ by continuity of } c(m; \bar{t})\rho(m)$$

$$(1.9) \quad \bar{\lambda}'(x) > 0 \quad \forall 0 < x < \delta \text{ by equation (1.8)}$$

$$(1.10) \quad \bar{\lambda}'(\delta) = \bar{\lambda}'(0) + \delta \bar{\lambda}'(x) = \delta \bar{\lambda}'(x) > 0 \quad \blacklozenge$$

### Proposition 2

For  $R_f < \mu$  and  $e \in (\bar{e}^v(x), 1]$  there exists a unique bankruptcy rate  $\bar{m}^v(x) \in [0, \bar{m}^u(x))$  where  $V(x; \bar{m}^v(x)) = 0$ .

*Proof:* Consider equations (2.1) -(2.3). These equations and the IVT insure the existence of a unique bankruptcy rate  $\bar{m}^v(x) \in [0, \bar{m}^u(x)) \ni V(\bar{m}^v(x)) = 0$ . Equation (2.4) shows the derivatives of  $\bar{m}^v(x)$ , which come by implicitly differentiating equation (10).

$$(2.1) \quad V(x; m = 0) = \mu - eR_f > 0 \quad \text{since } R_f < \mu.$$

$$(2.2) \quad V(x; \bar{m}^u(x)) < 0 \quad \text{since we assumed } e > \bar{e}^v(x) \text{ and by equation (5)}$$

$$(2.3) \quad V_m(x; m) < 0 \quad \text{by equation (5)}$$

$$(2.4) \quad \frac{\partial \bar{m}^v(x)}{\partial R_f} = \frac{e}{[1 - F(\bar{m}^v(x))]} < 0 \quad \blacklozenge$$

### Proposition 3

For  $R_f \in (\mu - \bar{C}, \mu]$  there exists a unique  $\bar{e}^o(x) \in (\bar{e}^u(x), 1]$  where bondholders break even  $\bar{e}^o(x)$  increases with  $R_f$  and  $\bar{e}^o(R_f = \mu) = 1$ . *Proof:* We prove the following ancillary proposition:

*Lemma 4.1:* For  $R_f < \mu$  then  $U(x, i; e=1; \bar{t}, m) > 0$ . First, we establish a useful transformation of equation (3), using the fact that the entrepreneurs' participation constraint is binding, so  $V(x; m) = 0 \forall e > \bar{e}^v(x)$ . *Proof of lemma:* Equation (10) implies that at  $e = 1$  and  $R_f = \mu$  then  $\bar{m}^v(x) = 0$ . This and equation (3.0) imply that  $U(R_f = \mu; e = 1; \bar{t}, m) = 0$  and  $\bar{e}^o(R_f = \mu) = 1$ . Since  $U_{R_f}(x; \bar{t}, \bar{m}^d) = ec(\bar{m}^d, \bar{t})\rho(\bar{m}^d) - 1 < 0$  then  $U(x, i; e=1; \bar{t}, m) > 0$  for  $R_f < \mu$  ♦

$$(3.0) \quad U(x; \bar{t}, \bar{m}^v(x)) = \mu - R_f - \int_0^{\bar{m}^v} c(s; \bar{t})f(s)ds \quad \forall e \in (\bar{e}^v(x), 1], \quad R_f > \mu - \bar{C}$$

*Proof of proposition*

Equations (3.1) - (3.3) and the IVT insure the existence of a unique rate  $\bar{e}^o(x) \in (\bar{e}^u(x), 1]$  where bondholder's payoff at the dual rate is zero  $U(x, i; \bar{e}^o(x); \bar{t}, \bar{m}^d) = 0$ . Equation (3.5) shows that this cutoff rate rises as the riskless interest rate increases. Equation (3.5) implicitly differentiates equation (3.4) and uses the definition of the dual rate, as defined in equations (10), (11). Proposition 2 uses equations (3.0) and (12). Note that since  $R_f \in (\mu - \bar{C}, \mu]$  all participation constraints are binding i.e.  $V(x; m) = 0$  for all  $e > \bar{e}^u(x)$

$$(3.1) \quad U(x, i; \bar{e}^u(x); \bar{t}, m) < 0 \quad \text{since } R_f > \mu - \bar{C} \text{ and by figure 2b}$$

$$(3.2) \quad U(x, i; e=1; \bar{t}, m) > 0 \quad \text{from lemma 4.1}$$

$$(3.3) \quad U_e(x; \bar{t}, \bar{m}^d) = c(\bar{m}^d, \bar{t})\rho(\bar{m}^d)R_f > 0 \quad \text{from (3) and by implicitly differentiating equation (10).}$$

$$(3.4) \quad U(x, i; \bar{e}^o(x); \bar{t}, \bar{m}^d) = \mu - R_f - \int_0^{\bar{m}^d} c(s; \bar{t})f(s)ds \equiv 0 \quad (3.5) \quad \frac{d\bar{e}^o(x)}{dR_f} = \frac{1 - \bar{e}^o(x)c(\bar{m}^d; \bar{t})\rho(\bar{m}^d)}{R_f c(\bar{m}^d; \bar{t})\rho(\bar{m}^d)} > 0 \quad \blacklozenge$$

*Proposition 4*

For  $e \in [\bar{e}^c(x), 1]$  there exists one equilibrium  $\bar{m}(x) \in ([1-e]R_f \bar{m}^d(x))$  s.t.  $U(x; \bar{t}, \bar{m}(x)) = 0$  and  $V(x; \bar{m}(x)) \geq 0$ .

*Proof:* We first establish some ancillary lemma:

$$\text{Lemma 4.1:} \quad \forall m < [1-e]R_f \text{ then } U(x; \bar{t}, m) < 0$$

$$\text{Proof: } U(x; \bar{t}, m) = m[1-F(m)] + \int_0^m [s-c(s; \bar{t})]f(s)ds - [1-e]R_f = [1-e][1-F(m)][r-R_f] + \int_0^m [s-[1-e]R_f]f(s)ds - \int_0^m c(s; \bar{t})f(s)ds$$

The second equality follows from the economic profit in equation (3) and by simple manipulation. The first and second elements of the second identity are negative if  $r < R_f$ . The third term is always negative, so we establish lemma 4.1. ♦

*Proof of the proposition:* Consider equations (4.1)-(4.3). These and the IVT insure the existence of a unique rate  $\bar{m}(x) \in ([1-e]R_f \bar{m}^d(x))$  where  $U(x; \bar{t}, \bar{m}(x)) = 0$  and  $V(x; \bar{m}(x)) \geq 0$ .

$$(4.1) \quad U(x; \bar{t}, [1-e]R_f) < 0$$

By lemma 4.1.

$$(4.2) \quad U(x; \bar{t}, \bar{m}^d(x)) > 0$$

since  $e > \bar{e}^c(x)$  by assumption

$$(4.3) U_m(x; \bar{t}, m) > 0 \quad \forall m < \bar{m}^d(x) \leq \bar{m}^u(x) \quad \text{By proposition 1's corollary } \blacklozenge$$

$$\text{Proposition 5} \quad [d\bar{r}(x)/de] < 0 \quad [d\bar{r}(x)/dR_f] > 1 \quad \forall e \in [\bar{e}^c(x), 1]$$

$$\text{Lemma 5.1} \quad [1-e]R_f - \bar{m}[1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})] > 0.$$

*Proof:*  $U_{mm}(x; \bar{t}, m) = -f(m)[1-c(m; \bar{t})\rho(m)] + [1-F(m)][c'(m; \bar{t})\rho(m) + c(m; \bar{t})\rho'(m)] < 0$ . The concavity of the payoff function follows from the fact that  $\bar{m} < \bar{m}^u(x)$ , by proposition 1's corollary and by A6.1. We thus have inequalities (5.1) and (5.2). Inequality (5.2) establishes lemma 5.1.

$$(5.1) \quad U(x; \bar{t}, 0) < U(x; \bar{t}, \bar{m}) - \bar{m}U_m(x; \bar{t}, \bar{m}) \Rightarrow -[1-e]R_f < -[1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})]\bar{m} \Rightarrow$$

$$(5.2) \quad [1-e]R_f - [1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})]\bar{m} > 0 \quad \blacklozenge$$

*Proof of main proposition:* The derivatives of equation (13) are presented in equations (5.3) - (5.5). The signs of equations (5.3) and (5.4) follow from proposition 1's corollary and lemma 5.1. Equations (5.6) and (5.7) use implicit function theorem. Equations (5.6) and (5.7) establish proposition 5.

$$(5.3) \quad U_x(x; \bar{t}, \bar{m}) = [1-e][1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})] > 0$$

$$(5.4) \quad U_e(x; \bar{t}, \bar{m}) = R_f - \bar{r}[1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})] > 0$$

$$(5.5) \quad U_{R_f}(x; \bar{t}, \bar{m}) = -[1-e] < 0$$

$$(5.6) \quad \frac{\partial \bar{r}(x)}{\partial e} = \frac{1}{1-e} \left[ \bar{r}(x) - \frac{R_f}{[1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})]} \right] < 0 \quad \text{by lemma 5.1}$$

$$(5.7) \quad \frac{\partial \bar{r}(x)}{\partial R_f} = \frac{1}{[1-F(\bar{m})][1-c(\bar{m}; \bar{t})\rho(\bar{m})]} > 1 \quad \blacklozenge$$

#### Proposition 6

$$\text{A) At the bank profit maximizing rate we have } \underline{m}^u(x) \geq \bar{m}^u(x) \quad \text{B) } U(x; \underline{t}, \underline{m}^u(x)) \geq U(x; \bar{t}, \bar{m}^u(x))$$

*Proof:* A)  $U(x; \underline{t}, m)$  is completely analogous to  $U(x; \bar{t}, m)$ , so that the existence, uniqueness and maximality of  $\underline{m}^u(x)$  can be proven by the reader following the arguments in proposition 1. This lemma establishes the relative magnitudes of  $\bar{m}^u(x)$  and  $\underline{m}^u(x)$ . The partial derivative of  $U(x; \underline{t}, m)$  is given in equation (6.1). Equation (6.2) shows this derivative evaluated at the unconstrained rate  $\bar{m}^u(x)$  for bondholders. This equation implies that the bank can raise its rate to  $\underline{m}^u(x) > \bar{m}^u(x)$  so that the derivative equal zero.

$$(6.1) \quad U_m(x; \underline{t}, m) = [1-F(m)][1-c(m; \underline{t})\rho(m)]$$

$$(6.2) \quad U_m(x; \underline{t}, \bar{m}^u) = [1-F(\bar{m}^u)][1-c(\bar{m}^u; \underline{t})\rho(\bar{m}^u)] > [1-F(\bar{m}^u)][1-c(\bar{m}^u; \bar{t})\rho(\bar{m}^u)] = 0$$

$$\text{B) } U(x; \underline{t}, \underline{m}^u(x)) \geq U(x; \underline{t}, \bar{m}^u(x)) \geq U(x; \bar{t}, \bar{m}^u(x)) \quad \blacklozenge$$

#### Proposition 7:

$$\text{A) } d(x; \underline{m}^d(x)) < 0 \quad e \in [0, \underline{e}^c(x)] \quad \text{C) } d(x; \underline{m}^d(x)) \geq 0, \quad d_e(x; \underline{m}^d(x)) < 0 \quad e \in [\bar{e}^c(x), 1]$$

$$\text{B) } d(x; \underline{m}^d(x)) \geq 0, \quad d_e(x; \underline{m}^d(x)) > 0 \quad e \in [\underline{e}^c(x), \bar{e}^c(x)] \quad \text{D) } d(x; \underline{m}^d(x)) = 0 \quad e = 1$$

*Proof:* A) For  $e \in [0, \underline{e}^c(x)]$  then  $U(x; \underline{t}, \underline{m}^d) < 0$  which gives A

B) For  $e \in [\underline{e}^c(x), \bar{e}^c(x)]$  then  $U(x; \underline{t}, \underline{m}^d) \geq 0$ . From equation (21) we find that  $U_e(x; \underline{t}, \underline{m}^d) > 0$  and thus:

$$\underline{d}_e(x; \underline{m}^d(x)) = \frac{[1-e]U_e(x; \underline{m}^d) + U(x; \underline{m}^d)}{[1-e]^2} > 0$$

C) for  $e \in [\bar{e}^c(x), 1)$ , the derivative of the maximal return function - in equation (13) - is given in equation (7.1). Further manipulation yields equation (7.2), where  $\Lambda$  is defined in equation (7.3). The second equality in (7.3) uses equation (13). Equation (7.4) performs some tedious manipulation, after integrates  $[c(s; \bar{i}) - c(s; \bar{l})]/f(s)$  and  $c(s; \bar{i})/f(s)$  by parts.

$$(7.1) \underline{d}_e(x; \underline{m}^d(x)) = \frac{\int_0^{\bar{m}} [c(s; \bar{i}) - c(s; \bar{l})]/f(s) ds}{[1-e]^2} - \frac{[c(\bar{m}; \bar{i}) - c(\bar{m}; \bar{l})]\rho(\bar{m})R_f}{[1-e][1 - c(\bar{m}; \bar{i})\rho(\bar{m})]}$$

$$(7.2) \underline{d}_e(x; \underline{m}^d(x)) \equiv \Lambda [1-e]^2 [1 - c(\bar{m}; \bar{i})\rho(\bar{m})]$$

$$(7.3) \Lambda \equiv [1 - c(\bar{m}; \bar{i})\rho(\bar{m})] \int_0^{\bar{m}} [c(s; \bar{i}) - c(s; \bar{l})]/f(s) ds - [c(\bar{m}; \bar{i}) - c(\bar{m}; \bar{l})]\rho(\bar{m})[1-e]R_f$$

$$= [1 - c(\bar{m}; \bar{i})\rho(\bar{m})] \int_0^{\bar{m}} [c(s; \bar{i}) - c(s; \bar{l})]/f(s) ds - [c(\bar{m}; \bar{i}) - c(\bar{m}; \bar{l})]\rho(\bar{m})\{\bar{m}[1 - F(\bar{m})] + \int_0^{\bar{m}} [s - c(\bar{m}; \bar{i})]/f(s) ds\}$$

$$(7.4) \Lambda = -[1 - c(\bar{m}; \bar{i})\rho(\bar{m})] \int_0^{\bar{m}} [c'(s; \bar{i}) - c'(s; \bar{l})]F(s) ds - [c(\bar{m}; \bar{i}) - c(\bar{m}; \bar{l})]\{\rho(\bar{m}) \int_0^{\bar{m}} c'(s; \bar{i})F(s) ds + \bar{m}f'(\bar{m}) + \rho(\bar{m}) \int_0^{\bar{m}} sf(s) ds - F(\bar{m})\}$$

The first term in (7.4) is negative by A5.2. The last term is negative. To see this we take a derivative of the last three terms in the key brackets, redefined in equation (7.5). This derivative is given in equation (7.6).  $\rho'(x) > 0$  implies that  $f'(x) > -\rho(x)f(x)$ . Hence, one can derive the inequality in equation (7.7). This last equation shows that  $\xi'(\bar{m}) > \xi(0) = 0$ , and hence that  $\Lambda < 0$ . Introducing this term in equation (7.2) implies that the derivative is negative, so  $\underline{d}_e(x; \underline{m}^d(x)) < 0$  and proves the result.

$$(7.5) \xi(\bar{m}) = \bar{m}f'(\bar{m}) + \rho(\bar{m}) \int_0^{\bar{m}} sf(s) ds - F(\bar{m}) > 0$$

$$(7.6) \xi'(\bar{m}) = \bar{m}f''(\bar{m}) + \rho'(\bar{m}) \int_0^{\bar{m}} sf(s) ds + \bar{m}\rho(\bar{m})f'(\bar{m})$$

$$(7.7) \xi'(\bar{m}) > \rho'(\bar{m}) \int_0^{\bar{m}} sf(s) ds > 0.$$

D) Result D can be obtained using l'hôpital's rule.

**Proposition 8:**

The equilibrium bank lending in this economy has :

A) The optimal set  $S^*(\pi) = [e, \bar{e}]$  satisfies the condition that  $e \in S^*(\pi)$  if and only if  $\underline{d}(x; \underline{m}^d(x)) \geq \pi$ . The optimal pricing strategy is  $\underline{m}(x)$  such that  $\underline{\lambda}(\underline{m}(x)) = [1-e][R_f + \pi]$ .

B)  $\underline{e}_1(\pi, R_f) \geq 0$ ,  $\underline{e}_2(\pi, R_f) \geq 0$ ;  $\bar{e}_1(\pi, R_f) \leq 0$ ,  $\bar{e}_2(\pi, R_f) \geq 0$

C) There is a unique  $\pi(x) > 0$  where markets clear. This premium has  $\pi_{w_0} < 0$ ,  $\pi_L > 0$ .  $\pi_{R_f}(x)$  is uncertain.

Lemma 8.1:  $\partial b/\partial \Lambda < 0$

*Proof:* Consider equations (8.1) - (8.3) Inequality (8.3) follows from the fact that at equilibrium  $\bar{t}b\rho(b) < 1$ , since this rate is below the peak. Equations (8.2), (8.3) and the implicit function theorem yield equation (8.4)

$$(8.1) U_{b\Lambda}(x; \bar{t}, b) = [1-G(b)][\rho(b) - \bar{t}b\rho(b)] > 0$$

$$(8.2) U_{\Lambda}(x; \bar{t}, b) > U_{\Lambda}(x; \bar{t}, 0) = 0 \quad \text{since } b > 0$$

$$(8.3) U_b(x; \bar{t}, b) = \Lambda[1-G(b)][\rho(b) - \bar{t}b\rho(b)] > 0$$

$$(8.4) \frac{\partial b}{\partial \Lambda} = -U_{\Lambda}(x; \bar{t}, b)/U_b(x; \bar{t}, b) < 0 \quad \blacklozenge$$

*Proof of Proposition 8A:* We will prove this part of the proposition by contradiction. We will show how sets different from  $S^*$  allow for profitable deviations:

I) A set  $S$  such that  $S \cap S^*(\pi) \neq \emptyset$  and  $S \cap S^*_c(\pi) \neq \emptyset$  for some  $\pi$  admits profitable deviations. To show this consider a set  $S^d$  satisfying conditions i-iii. It is always feasible to construct  $S^d$ , given the conditions on  $S$  above. We then construct subsets  $I_1, I_2, I_3$ .

$$\begin{array}{lll} \text{i) } S^d \setminus (S^d \cap S) \in S^*(\pi) & \text{ii) } S \setminus (S^d \cap S) \in S^*_c(\pi) & \text{iii) } L(S^d) = L(S) \\ I_1 = S^d \cap S, & I_2 = S^d \setminus (S^d \cap S) \in S^*(\pi) & I_3 = S \setminus (S^d \cap S) \in S^*_c(\pi) \end{array}$$

For these sets, the banks' pricing policy is as follows:

- i) For  $e \in I_1$  charge  $m'(x) = m(x)$  so that  $\underline{\lambda}(m'(x)) = \underline{\lambda}(m(x))$
- ii) For  $e \in I_2$  charge  $m'(x)$  so that  $\underline{\lambda}(m'(x)) \geq [1-e][R_f + \pi]$ .
- iii) For  $e \in I_3$   $m(x)$  can only yield  $\underline{\lambda}(m(x)) < [1-e][R_f + \pi]$ .

Step 1: For any fixed  $L$ , we show that the expected return of the deviation portfolio is greater than the original portfolio so  $\Lambda(x; S^d) > \Lambda(x; S)$ . First we exploit the fact that the loan size is the same in both portfolios and our definition of the different sets to obtain equation (8.7). Equations (8.8) and (8.9) exhibit the revenue for both portfolios, and equation (8.10) takes the difference.

$$(8.5) L(x; S^d) = \int_{e \in S^d} N[1-e]h(e)de = \int_{e \in S} N[1-e]h(e)de = L(x; S)$$

$$(8.6) L(S^d) = N \left[ \int_{e \in I_1} [1-e]h(e)de + \int_{e \in I_2} [1-e]h(e)de \right] = N \left[ \int_{e \in I_1} [1-e]h(e)de + \int_{e \in I_3} [1-e]h(e)de \right] = L(S)$$

$$(8.7) \int_{e \in I_2} N[1-e]h(e)de = \int_{e \in I_3} N[1-e]h(e)de$$

$$(8.8) \Lambda(x; S^d) = N \left[ \int_{e \in I_1} \underline{\lambda}(m(x))h(e)de + \int_{e \in I_2} \underline{\lambda}(m(x))h(e)de \right]$$

$$(8.9) \Lambda(x; S) = N \left[ \int_{e \in I_1} \underline{\lambda}(m(x))h(e)de + \int_{e \in I_3} \underline{\lambda}(m(x))h(e)de \right]$$

$$(8.10) \Lambda(x; S^d) - \Lambda(x; S) = N \left[ \int_{e \in I_2} \underline{\lambda}(m(x))h(e)de - \int_{e \in I_3} \underline{\lambda}(m(x))h(e)de \right]$$

We obtain inequalities (8.11) and (8.12) by using the fact that for  $e \in I_2$  then  $\underline{\lambda}(m(x)) \geq [1-e](R_f + \pi)$  and for firms  $e \in I_3$  then  $\underline{\lambda}(m(x)) < [1-e][R_f + \pi]$ . We obtain the inequality in equation (8.13) by combining equations (8.7) and (8.10)-(8.12). This proves step 1 of the proposition.

$$(8.11) \int_{e \in I_2} N \underline{\lambda}(m(x)) h(e) de \geq \int_{e \in I_2} N(R_f + \pi)[1-e] h(e) de$$

$$(8.12) \int_{e \in I_3} N \underline{\lambda}(m(x)) h(e) de < \int_{e \in I_3} N(R_f + \pi)[1-e] h(e) de$$

$$(8.13) \Lambda(x; S^d) - \Lambda(x; S) > N(R_f + \pi) \left[ \int_{e \in I_2} [1-e] h(e) de - \int_{e \in I_3} [1-e] h(e) de \right] = 0$$

Step 2: We will show that the bank's expected payoff is higher under set  $S^d$  than under  $S$ . First, lemma 8.1 insures that  $b^d < b$ . Expected bank payoffs are given in equations (8.14) and (8.15), and their difference is displayed in equation (8.16), which uses the previous step i.e.  $\Lambda(x; S^d) \geq \Lambda(x; S)$  and lemma 8.1, i.e.  $b^d < b$ . This establishes that  $S$  admits profitable deviations:

$$(8.14) U(x; L S^d) = \Lambda(x; S^d) \int_{b^d}^v [y - b^d] g(y) dy - WR_f$$

$$(8.15) U(x; L S) = \Lambda(x; S) \int_b^v [y - b] g(y) dy - WR_f$$

$$(8.16) U(x; L S^d) - U(x; L S) = \Lambda(x; S^d) \int_{b^d}^b [y - b^d] g(y) dy + \int_b^v \{ [\Lambda(x; S^d) - \Lambda(x; S)] [y - b] + \Lambda(x; S^d) [b - b^d] \} g(y) dy \geq 0 \spadesuit$$

II) A portfolio  $\{m(x), S^*(\pi) = [\underline{e}, \bar{e}]\}$  where  $m(x)$  has  $\underline{\lambda}(m(x)) < [1-e](R_f + \pi)$  for some subset of  $S_o$  of  $S^*(\pi)$  is not an equilibrium. Define a constant  $c$  as below. To show this consider a set  $S^d$  satisfying conditions i-iii. It is always feasible to construct  $S^d$ , given the conditions on  $S$  above. We then construct subsets  $I_1, I_2, I_3$ .

$$c \equiv \max_{e \in S_o} \{ \underline{\lambda}(m(x)) / [1-e] \} < (R_f + \pi) \quad S_u \equiv \{ e \in S^*(\pi) : \underline{\lambda}(m'(x)) \geq [1-e]c \text{ for an incentive compatible } m'(x) \}$$

$$\begin{array}{lll} \text{i) } S^d \setminus (S^d \cap S) \in S_u(\pi) & \text{ii) } S \setminus (S^d \cap S) \in S_o(\pi) & \text{iii) } L(S^d) = L(S) \\ I_1 = S^d \cap S, & I_2 = S^d \setminus (S^d \cap S) \in S_u(\pi) & I_3 = S \setminus (S^d \cap S) \in S_o(\pi) \end{array}$$

For these subsets, the banks' pricing policy is shown below. Repeating the steps in IA, This setup allows us to show that portfolio is a profitable deviation i.e. that  $\Lambda(x; S^d) > \Lambda(x; S)$  and  $U(x; L S^d) > U(x; L S)$ :

i) For  $e \in I_1$  charge  $m'(x) = m(x)$  so that  $\underline{\lambda}(m'(x)) = \underline{\lambda}(m(x))$

ii) For  $e \in I_2$  charge  $m'(x)$  so that  $\underline{\lambda}(m'(x)) \geq [1-e]c$

iii) For  $e \in I_3$   $m(x)$  can only yield  $\underline{\lambda}(m(x)) < [1-e]c \spadesuit$

III) Suppose that we have  $\{r(e), S^*(\pi) = [\underline{e}, \bar{e}]\}$  with subsets  $S_{oa}, S_{ob}$  where  $\underline{\lambda}(m(x)) / [1-e] < c < \underline{\lambda}(m(x')) / (1-e')$  for all  $e \in S_{oa}, e' \in S_{ob}$ . Then this pricing admits profitable deviations. Consider a bank with a loan portfolio  $S$  such that  $(S \cap S_{oa}) \neq \emptyset$  and  $(S \cap S_{ob}) \neq \emptyset$ . This is always possible as long as we have more than one bank in this economy. Construct the following deviation portfolio and pricing policy  $S^d$  satisfying conditions i-iii. It is always feasible to construct  $S^d$ , given the conditions on  $S$  above. We then construct subsets  $I_1, I_2, I_3$

$$\begin{array}{lll} \text{i) } S^d \setminus (S^d \cap S) \in S_{ob} & \text{ii) } S_a \setminus (S^d \cap S) \in S_{oa} & \text{iii) } L(S^d) = L(S) \\ I_1 = S^d \cap S, & I_2 = S^d \setminus (S^d \cap S) \in S_{ob} & I_3 = S \setminus (S^d \cap S) \in S_{oa} \end{array}$$

For these subsets, the banks' pricing policy is shown below. Repeating the steps in IA, This setup allows us to show that portfolio is a profitable deviation i.e. that  $\Lambda(x; S^d) > \Lambda(x; S)$  and  $U(x; L S^d) > U(x; L S)$ . This says that

whenever there is more than one bank, there can be no price discrimination of different projects. Points I-III prove part A of proposition 8 A

- i) For  $e \in I_1$  charge  $m'(x) = m(x)$  so that  $\underline{\lambda}(m'(x)) = \underline{\lambda}(m(x))$
- ii) For  $e \in I_2$  charge  $m'(x)$  so that  $\underline{\lambda}(m'(x)) \geq [1-e]c$
- iii) For  $e \in I_3$   $m(x)$  can only yield  $\underline{\lambda}(m(x)) < [1-e]c$  ♦

*Proof of Proposition 8 B :*

*Proof:* First define sets C, C', D, and D'. From these sets, we can define the cutoff rates in equations (8.17) and (8.18). Equation (12) tells us that  $\bar{e}^c(x)$  increases with the riskless rate. The derivatives of  $\underline{e}^*(\pi, R_f)$  are shown in equations (8.19) and (8.20) by implicit differentiation of equations (8.17), (8.18) and by using equations (22) and (23). Equations (8.21) and (8.22) prove the first part of this lemma, where  $I_C$  is an indicator function.

$$C = \{(\underline{e}, \pi, R_f); U(x; \underline{L}m) - [1-e]\pi \equiv 0, \underline{e} \leq \bar{e}^c(x)\} \quad C' = \{(\underline{e}, \pi, R_f); U(x; \underline{L}m) - [1-e]\pi \neq 0, \underline{e} \leq \bar{e}^c(x)\}$$

$$D = \{(\bar{e}, \pi, R_f); U(x; \underline{L}m) - (1-\bar{e})\pi \equiv 0, \bar{e} \geq \bar{e}^c(x)\} \quad D' = \{(\bar{e}, \pi, R_f); U(x; \underline{L}m) - (1-\bar{e})\pi \neq 0, \bar{e} \geq \bar{e}^c(x)\}$$

$$(8.17) \underline{e}(\pi, R_f) = \begin{cases} \underline{e}^*(\pi, R_f) \text{ defined by } U(x; \underline{L}m) - (1-\underline{e}^*)\pi \equiv 0 & \forall (\underline{e}, \pi, R_f) \in C \\ 0 & \forall (\underline{e}, \pi, R_f) \in C' \end{cases}$$

$$(8.18) \bar{e}(\pi, R_f) = \begin{cases} \bar{e}^*(\pi, R_f) \text{ defined by } U(x; \underline{L}m) - (1-\bar{e}^*)\pi \equiv 0 & \forall (\bar{e}, \pi, R_f) \in D \\ \bar{e}^c(x) & \forall (\bar{e}, \pi, R_f) \in D' \end{cases}$$

$$(8.19) \underline{e}_1^*(\pi, R_f) = \frac{1-\underline{e}^*}{c(\underline{L}, \underline{L})\rho(\underline{L})R_f + \pi} \geq 0 \quad (8.20) \underline{e}_2^*(\pi, R_f) = \frac{1-ec(\underline{L}, \underline{L})\rho(\underline{L})}{c(\underline{L}, \underline{L})\rho(\underline{L})R_f + \pi} \geq 0 \quad \forall (\underline{e}, \pi, R_f) \in C$$

$$(8.21) \underline{e}_1(\pi, R_f) = \underline{e}_1^*(\pi, R_f)I_C \geq 0 \quad (8.22) \underline{e}_2(\pi, R_f) = \underline{e}_2^*(\pi, R_f)I_C \geq 0$$

We can obtain the derivatives of the upper cutoff by implicitly differentiating equation (8.18) Let us first remind the reader of the way in which the bank's payoff looks like, in equation (8.23). Equation (8.24) takes the derivative with respect top  $e$ . The identity in equation (8.24) uses equations (8.18), (8.23) and lemma 7C. Equations (8.25) and (8.26) represent the derivatives of the upper cutoff rate, using equations (8.24) and (18). Equations (8.27) and (8.28) establish proposition 8B

$$(8.23) U(x; \underline{L} \underline{m}^d(x)) = \int_0^{\bar{m}} [c(s; \bar{t}) - c(s; \underline{L})] f(s) ds > 0 \quad \forall e \in [\bar{e}^c(x); 1]$$

$$(8.24) U_e(x; \underline{L} \underline{m}^d(x)) + \pi = \pi - \frac{[c(\bar{m}; \bar{t}) - c(\bar{m}; \underline{L})]\rho(\bar{m})R_f}{[1 - c(\bar{m}; \bar{t})\rho(\bar{m})]} = \frac{\int_0^{\bar{m}} [c(s; \bar{t}) - c(s; \underline{L})] f(s) ds}{1-\bar{e}^*} - \frac{[c(\bar{m}; \bar{t}) - c(\bar{m}; \underline{L})]\rho(\bar{m})R_f}{1 - c(\bar{m}; \bar{t})\rho(\bar{m})}$$

$$= [1-\bar{e}^*]d_e(x; \underline{m}^d(x)) < 0$$

$$(8.25) \bar{e}_1^*(\pi, R_f) = \frac{1-\bar{e}^*}{U_e(x; \underline{L} \underline{m}^d(x)) + \pi} \leq 0 \quad \forall (\bar{e}, \pi, R_f) \in D$$

$$(8.26) \bar{e}_2^*(\pi, R_f) = -\frac{[c(\bar{m}; \bar{t}) - c(\bar{m}; \underline{L})]\rho(\bar{m})[1-\bar{e}^*]}{[1 - c(\bar{m}; \bar{t})\rho(\bar{m})][U_e(x; \underline{L} \underline{m}^d(x)) + \pi]} \geq 0 \quad \forall (\bar{e}, \pi, R_f) \in D$$

$$(8.27) \bar{e}_1(\pi, R_f) = \bar{e}_1^*(\pi, R_f)I_D \leq 0 \quad (8.28) \bar{e}_2(\pi, R_f) = \bar{e}_2^*(\pi, R_f)I_D + [d\bar{e}^c(x)/dR_f][1 - I_D] \geq 0 \quad \blacklozenge$$

*Proof of Proposition 8 C :*

The equilibrium conditions in an economy imply that a) bondholders are making no economic profits b) banks are not making economic profits: this is a no entry condition. c) Each bank with funds  $W_i$  chooses how much in loans  $L_i$  to make, such that they maximize profits. They take the pricing policy  $\underline{m}(x)$  as given, and so do bondholders. This implies that neither banks nor bondholders consider the impact on the pricing that occurs with a change in loan size. The equilibrium conditions are given in equations (8.29) - (8.32)

$$(8.29) U(x; \bar{t}, b_i, S_i) = [\pi + R_f] L_i \left[ b_i [1 - G(b_i)] + \int_0^{b_i} [1 - \bar{t}] g(v) dv \right] - (L_i - [1 - \gamma] W_i) R_f \equiv 0 \quad \forall i$$

$$(8.30) U(x; \underline{t}, b_i, S_i) = [\pi + R_f] L_i \int_{b_i}^v [v - b_i] g(v) dv - W_i R_f \equiv 0 \quad \forall i \quad \text{bank zero profits}$$

$$(8.31) \frac{dU(x; \underline{t}, S_i)}{dL_i} = [R_f + \pi] \int_{b_i}^v [v - b_i] g(v) dv - \frac{[1 - \gamma] W_i R_f}{L_i [1 - \bar{t} b_i \rho(b_i)]} = 0 \quad \text{profit maximization}$$

$$(8.32) \sum_i L_i = L(\pi, R_f) \quad \text{market clearing}$$

This first order condition in equation (8.31) used equations (8.30) and (8.29). With these equilibrium conditions we want to find out the equilibrium bankruptcy point  $b_i$  and leverage  $L_i$  faced by each bank. We also want to tie down the equilibrium premium  $\pi$  and aggregate bank capital  $W$ . Combining equations (8.30) and (8.31) we obtain equation (8.33). Thus each bank faces the same bankruptcy point. Equation (8.34) presents each bank's leverage, which was obtained by combining equations (8.29) and (8.30). In equation (8.35) we obtain an expression for the premium, after combining equations (8.29), (8.33) and (8.34). One can verify that  $P'(b) > 0$ ,  $P(v) = \bar{t} < 1$  and that  $P(0) = 0$ . The derivatives of this premium are shown in equation (8.35a)

$$(8.33) \gamma - \bar{t} b_i \rho(b_i) = 0 \Rightarrow b_i = b_j = b(\gamma) \text{ and } b'(\gamma) = \frac{1}{\bar{t} [\rho(b) + b \rho'(b)]} > 0$$

$$(8.34) \frac{W_i}{L_i} = \frac{1}{\gamma R_f} \left[ [\pi + R_f] \left( 1 - \int_0^b \bar{t} v g(v) dv \right) - R_f \right] \Rightarrow \frac{W_i}{L_i} = \frac{W_j}{L_j}$$

$$(8.35) \pi = R_f \left[ \frac{P(b)}{1 - P(b)} \right] > 0 \text{ where } P(b) = \bar{t} \left[ b \rho(b) \int_b^v [v - b] g(v) dv + \int_0^b v g(v) dv \right]$$

$$(8.35a) \frac{d\pi}{dR_f} = \left[ \frac{P(b)}{1 - P(b)} \right] > 0 \quad \frac{d\pi}{d\gamma} = \frac{R_f P'(b) b'(\gamma)}{[1 - P(b)]^2} > 0 \quad \frac{d^2\pi}{d\gamma dR_f} = \frac{P'(b) b'(\gamma)}{[1 - P(b)]^2} > 0$$

Introducing equation (8.35) in (8.34) gives a simplified expression for bank leverage, which is shown in equation (8.34'). This implies that in equilibrium, bank loans exceed bank internal funds, so that these specialized lenders are indeed acting as financial intermediaries. Equation (8.36) shows how leverage increases with rises in the reserve requirement. Finally, the aggregate amount of bank capital in this economy is given in equation (8.37) where  $L(\pi, R_f)$  is defined in equation (29'). In order to have a positive amount of bank loans, we impose that  $\pi < \pi^* \equiv \underline{d}(\bar{z}^c(x), x, \bar{t}; \underline{w}^d(x))$ , where we used the definitions in equations (12) and (25). If this condition were violated, there would be no bank loans in the economy, for banks could not find projects with the appropriate rate of return. These last equations finish the proof of proposition 8C.

$$(8.34) \frac{W_i}{L_i} = \frac{\int_0^v [y-b]g(y)dy}{1-P(b)} \Rightarrow [1-\gamma] \frac{W_i}{L_i} \equiv l(b) = 1 - \frac{b[1-G(b)] + \int_0^b y[1-\bar{i}]g(y)dy}{1-P(b)} < 1$$

$$(8.36) \frac{dl}{d\gamma} = - \frac{b'(\gamma)}{[1-P(b)]^2} \left\{ [1-G(b)][1-P(b)][1-\bar{i}b\rho(b)] + P'(b) \left[ b[1-G(b)] + \int_0^b y[1-\bar{i}]g(y)dy \right] \right\} < 0$$

$$(8.37) W = \frac{L(\pi, R_f)}{1-P(b)} \int_0^v [y-b]g(y)dy$$

*Proposition 9:* Firms with funds  $e \in [\underline{e}, \bar{e}]$  face a contractual interest rate  $r(x)$  with  $r(x) < 0$  for  $x_i = e$  and  $r(x) > 0$  for  $x_i = \gamma, R_f$ .

*Proof:* For  $e \in [\underline{e}, \bar{e}]$  the bank payoff equals zero, as defined in equation (9.1). The derivatives of the interest rate are shown in equations (9.2) - (9.4). They are obtained by implicitly differentiating (9.1). The sign of equation (9.2) is positive by similar arguments to those of proposition 5. The sign of equations (9.3) and (9.4) was found using the results of equation (35)

$$(9.1) U(x; \underline{m}) = \underline{m}[1-F(\underline{m})] + \int_0^{\underline{m}} [s-c(s; \underline{L})]f(s)ds - [1-e][\pi + R_f] \equiv 0$$

$$(9.2) \frac{\partial r(x)}{\partial e} = \frac{1}{1-e} \left[ r(x) - \frac{\pi + R_f}{[1-F(\underline{m})][1-c(\underline{m}; \underline{L})\rho(\underline{m})]} \right] < 0$$

$$(9.3) \frac{\partial r(x)}{\partial R_f} = \frac{1 + \pi R_f r(x)}{[1-F(\underline{m})][1-c(\underline{m}; \underline{L})\rho(\underline{m})]} > 0$$

$$(9.4) \frac{\partial r(x)}{\partial \gamma} = \frac{1}{[1-F(\underline{m})][1-c(\underline{m}; \underline{L})\rho(\underline{m})]} \frac{d\pi}{d\gamma} < 0 \quad \blacklozenge$$

### Proposition 10

Investment increases with an improvement in corporate internal funds, and with a drop in the risk free rate or the reserve requirement: thus  $I_{x_i}(x) > 0$  for  $x_i = \varphi$  and  $I_{x_i}(x) < 0$  for  $x_i = \gamma, R_f$

*Proof*

$$(10.1) I_\gamma(\gamma, R_f; \varphi) = -Nh(\underline{e})\underline{e}_1(\pi, R_f)\pi_\gamma(\gamma, R_f) > 0$$

Equation (10.1) uses proposition 8B and equation (8.34). To prove that  $I_{R_f}(\gamma, R_f; \varphi) < 0$  we need to show that the lower cutoff rate increases with  $R_f$ , i.e. equation (10.2). This is true by equation (8.34) and proposition 8B. Equation (10.3) shows that the upper cutoff rate increases with an increase in the risk free rate. The second line uses the derivative in equation (8.34) and in proposition 8B, equations (8.25) and (8.26). The last line uses the condition in equation (8.33) and proposition 8B, equation (8.24).

$$(10.2) \frac{d\underline{e}}{dR_f} = \underline{e}_1(\pi, R_f) \frac{d\pi}{dR_f} + \underline{e}_2(\pi, R_f) > 0$$

$$(10.3) \frac{d\bar{e}}{dR_f} = \bar{e}_1(\pi, R_f) \frac{d\pi}{dR_f} + \bar{e}_2(\pi, R_f) = \frac{1-\bar{e}^*}{U_e(x; \underline{L}) \underline{m}^d(x) + \pi} \left[ \frac{P(b)}{1-P(b)} \right] - \frac{[c(\bar{m}; \bar{i}) - c(\bar{m}; \underline{L})]\rho(\bar{m})[1-\bar{e}^*]}{[1-c(\bar{m}; \bar{i})\rho(\bar{m})][U_e(x; \underline{L}) \underline{m}^d(x) + \pi]}$$

$$= \frac{1-\bar{e}^*}{R_f[U_e(x;L) \mathbb{I}^d(x) + \pi]} \left[ \pi - \frac{[c(\bar{m};\bar{t})-c(\bar{m};\bar{t})]\rho(\bar{m})R_f}{[1-c(\bar{m};\bar{t})\rho(\bar{m})]} \right] = \frac{1-\bar{e}^*}{R_f} > 0$$

Investment behavior following a rise in the risk free rate is given in equation (10.4). We obtained this inequality with the help of equations (10.2). Equation (10.5) considers a generalized increase in company cash flow, where  $H_{\varphi}(e, \varphi) < 0$ . These two last equations prove the proposition

$$(10.4) I_{R_f}(\gamma, R_f, \varphi) = -Nh(e) \frac{de}{dR_f} < 0$$

$$(10.5) I_{\varphi}(\gamma, R_f, \varphi) = -NH_{\varphi}(e, \varphi) > 0 \spadesuit$$

## *Appendix B Joint distribution function*

In this appendix I will derive the joint distribution function of a portfolio of risky bank loans. First we need to set up some useful notation. We will work with a probability space  $([0,1], \mathcal{B}([0,1]), H)$ .  $\mathcal{B}([0,1])$  is the Borel sigma field of the interval  $[0,1]$  and  $H$  is a measure which is absolutely continuous with respect to the Lebesgue measure. We have a continuum of random variables  $s : \{s_e : e \in [0,1]\}$  which is called a process. Each random variable  $s_e$  is the return on a project with internal funds  $e$ . From the setup of our model the variables are identically, but not necessarily independently distributed. Now we will deform these random variables as follows:

$$B1. z_e(s_e, m) = \begin{cases} s_e - c^b(s_e) & \forall s_e < m \\ m & \forall s_e \geq m \end{cases}$$

This payoff basically tracks the returns of a risky loan made to a firm  $e$ , charging an amount  $m = [1-e]r$ . Note that this new variable is continuous a.e. We can construct the endogenous process  $z : \{z_e(s_e, m) : e \in [0,1]\}$ .

*Case 1: Exogenous shocks*

We will assume that the project returns  $s_e$  are mutually independent. It is clear then that the variables  $z_e$  are independent too (see Amemiya 1994 Proposition 3.5.1). The bank then is facing purely idiosyncratic risk when lending to firms (see Al-Najjar (1994), section 2.3). The random variables can then be decomposed as follows:

$$B2. z_e(s_e, m) = E[z_e(s_e, m)] + (z_e(s_e, m) - E[z_e(s_e, m)]) = \lambda(m) + \gamma_e(s_e, m)$$

Where  $\gamma_e(s_e, m)$  is a mean zero independent and bounded r.v. Al-Najjar (1994) shows in Theorem 1 of his paper that a process that can be so decomposed, then a "law of large numbers" applies. This LLN is done by an approximation of large samples (see section 2.7 in Al-Najjar for more on this). Applying his Theorem 1<sup>8</sup> gives:

$$B3. \int_{e \in S} z_e(s_e, m) dH(e) de = \int_{e \in S} N \lambda(m(x)) dH(e) de = \Lambda(S, R_f)$$

Where the first integral is a Pettis Integral, and the second integral is the standard Lebesgue integral. If the set  $S$  is compact and the function  $\Lambda$  is continuous a.e. then the Lebesgue integral is the same as the Riemann integral. The result in Al-Najjar says that the idiosyncratic shocks are diversified away in integration. To this portfolio I will attach an exogenous shock  $y \in [0, v]$  with a distribution  $g(y)$  and that  $E(y) = 1$ . One can think of the exogenous shock as an economywide interest or inflation risk. Adding an exogenous shock has then been shown to be an admissible mathematical construct.

*Case 2. Endogenous Shock*

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<sup>8</sup> See especially the proof ii)  $\rightarrow$  iv)

First let us choose  $\delta^2 > 0$ , and assume that the risk free rate  $R_f > \delta$ . We have two areas to study:

i)  $e < 1 - \delta$ . We assume the following decomposition of  $z_e$ :

$$B2. z_e(\eta_e, y, m) = \underline{\lambda}(m)y + [\underline{\lambda}(m)y \phi(\eta_e, m) - \underline{\lambda}(m)y] = g_e + h_e \quad \forall e \in [0, 1 - \delta)$$

Where  $\eta_e$  is an independent r.v. and  $y$  is an aggregate shock. We assume that  $y$  is a bounded random variable  $y \in [0, v]$  with a distribution  $g(y)$  and that  $E(y) = 1$ . In addition we assume that  $v \in (1, \partial^2/\underline{\lambda}(\delta^2)]$ .  $\phi(\eta_e, m)$  is a bounded and independent random variable with expected value of 1.  $\phi(\eta_e, m) \in [0, m/(\underline{\lambda}(m))]$  where  $m/(\underline{\lambda}(m)) > 1$ . This is true since:

$$B3. \frac{m}{\underline{\lambda}(m)} > \frac{\delta^2}{\underline{\lambda}(\delta^2)} \geq v \quad \forall m > \delta^2$$

Inequality B3. follows from the fact that  $m\underline{\lambda}(m)$  is an increasing function, since  $\underline{\lambda}'(m) > 0$ ,  $\underline{\lambda}''(m) < 0$  and  $\underline{\lambda}(0) = 0$ . We know that  $m > \delta^2$  since  $m = [1-e]r > [1-e]R_f > \delta^2$ . All these precautions were necessary to ensure that the decomposition in B.2 satisfies the properties of the debt payoff i.e  $z_e \in [0, m]$ .  $g_e$  is an aggregate risk and  $h_e$  is purely idiosyncratic risk in Al-Najjar's notation (sections 2.3 and 2.4).

ii)  $e \geq 1 - \delta$  the decomposition that we did in i) can not be replicated since the variable  $z_e$  could be greater than  $m$ . Hence we assume that for  $e \geq 1 - \delta$   $s_e$  are mutually independent. We also assume that the random variable is hit by an exogenous shock  $y \in [0, v]$  which is the same as our aggregate shock for the lower range. Hence we obtain:

$$B4. z'_e = yz_e = \underline{\lambda}(m)y + [\underline{\lambda}(m)y \phi'(\eta_e, m) - \underline{\lambda}(m)y] = g'_e + h'_e \quad \forall e \in [1 - \delta, 1]$$

Again, this variable is decomposed into an aggregate and idiosyncratic element just as in equation B2. We can now apply a Law of large numbers for the payoff of lending to risky firms. Again we apply Al-Najjar (1994) Theorem 1 and obtain:

$$B5. \int_{e \in S} z_e(s_e, m) dH(e) de = \int_{e \in S} N \underline{\lambda}(m) y dH(e) de = \Lambda(S, R_f) y$$

Where again the first integral is a Pettis integral. The second integral is a Lebesgue integral times the aggregate risk. Note that this too justifies our assumption about how the aggregate loan payoff looks like. Since we only have one source of aggregate risk in this article, we can simplify Al-Najjar's formulation. The Bochner and Lebesgue integrals are the same when there is only one source of risk. For more on this see appendix A.3 in Al-Najjar (1994). Another important point is that the Law of Large Numbers used by Al-Najjar uses a sequence of "simple" series, which is discussed in Section 2.7 of his paper.