Technology Shocks and Asset Price Dynamics: 
The Role of Housing in General Equilibrium

Jiro Yoshida*
Haas School of Business
University of California, Berkeley

Job Market Paper
November 2006

Abstract

A simple general equilibrium model, that incorporates endogenous production and local housing markets, is developed in order to explain the price relationship among human capital, housing, and stocks. Housing serves as an asset as well as a durable consumption good. The covariation of housing and stock prices can be negative if the supply of local inputs for housing production is elastic. Several examples illustrate the way the model works, for example the housing price appreciation during the economic contraction in the U.S. after 2000, and the varying degrees of stock-market participation across countries. The model also shows that housing rent growth serves as a risk factor in the consumption-based pricing kernel, and this may mitigate the equity premium puzzle and the risk-free rate puzzle.

JEL Classification: G12, E32, R20, R30

*I would like to thank Tom Davidoff, Dwight Jaffee, John Quigley, Richard Stanton, Adam Szeidl, Johan Walden, and Nancy Wallace for their help and advice. Any mistakes are my own. Please send comments to yoshida@haas.berkeley.edu.
1 Introduction

Household wealth typically consists of human capital, housing, and financial assets. The covariance of prices among these broad asset classes is critical to portfolio choice, asset pricing and consumption behavior. For example, a high covariance of stock prices with other asset prices suggests that a low weight be given to stocks, given that holdings of human capital and housing are constrained at some positive levels. A low or negative covariance among the assets, in turn, stabilizes household wealth and consumption.¹

The actual covariance structure varies across countries as well as over time. In particular, in the U.S. housing and stock prices are negatively correlated while in Japan they are positively correlated.² However, our theoretical understanding of the covariance structure among these broad asset classes is limited. General theories of asset pricing such as the Arrow-Debreu equilibrium and the no-arbitrage pricing condition are too general to yield concrete insights into the covariance structure, while more detailed models have been either purely empirical (with a focus on a particular financial asset) or else built on simplistic assumptions regarding the production process.³

In this paper, I develop a simple general equilibrium model in order to address two questions: First, what is the covariance structure among asset prices when we incorporate endogenous responses of production sectors to technology shocks? Second, what is the role of housing in the determination of equilibrium asset prices? By relying only on straightforward economic mechanisms, I derive the direct links between primitive technology shocks and the asset price responses. Within a perfect foresight framework, asset prices in various scenarios of technology shocks are clearly shown.

The first of two main results is the finding of an equilibrium relationship among asset prices for different types of technology shocks. In particular, I show that the co-variation of housing prices and stock prices can be negative if the supply of local inputs for housing production (e.g., land) is elastic and vice versa. This finding is broadly consistent with the fact that the U.S. has a negative correlation and Japan has a positive correlation between these two assets. The result is suggestive of the housing price appreciation observed under economic contraction in the U.S. after 2000. Predictions

¹For example, it is widely believed that U.S. consumption since 2000 has been sustained in spite of depressed values of human capital and financial assets by the appreciation of housing prices.
³Empirical models such as the Fama-French three factor model for equity returns are not based on complete theories. Theoretical models often reduce production processes to simply endowments (e.g. Lucas (1978)), render them implicit to the consumption process (e.g. Breeden (1979)), or posit an exogenous return/production process (e.g. Cox et al. (1985)).
of the model about the term structure of interest rates, the capitalization rate or "cap rate", and savings are also consistent with observations. This result also implies that an economy with inelastic land supply should exhibit either more limited stock-market participation or less homeownership because of positive covariation among asset prices. This is suggestive of the variations in stock-market participation across countries.

The second result is that growth of housing rent is a component of the discount factor if utility function is non-separable in housing and other goods, and thus it serves as a risk factor in the consumption-based pricing kernel. I present the possibilities that the rent growth factor mitigates the equity premium puzzle and the risk-free rate puzzle either by magnifying consumption variation or imposing a downward bias on the estimate of the elasticity of inter-temporal substitution (EIS). The model opens an empirical opportunity to apply a new data set to the Euler equation. The risk of housing assets is also inferred from the characterization.

To derive these results, I introduce two key components: endogenous production and housing. The first component, endogenous production, characterizes asset prices and the discount factor in relation to different types of technology shocks. The discount factor is usually characterized by the consumption process without a model of endogenous production. Although real business cycle models are built on primitive technology shocks, they do not focus on asset prices but predominantly on quantity dynamics. In this paper, I analyze shocks along three dimensions: time, space and sector. On the time dimension, there are three types of shocks: 1) current, temporary shocks, 2) anticipated, temporary shocks, and 3) current, permanent shocks. Along the space dimension, shocks can occur in the "home" city or in the "foreign" city. In the sector dimension, shocks may have an effect either on consumption-goods production or housing production.

The second component of the model is housing. Housing is the major component of the household asset holdings, but it also has, at least, three unique characteristics. First, housing plays a dual role as a consumption good and as an investment asset. The portfolio choice is constrained by the consumption choice and vice versa. In particular, when the utility function is not separable in housing and other consumption goods,

\footnote{A few exceptions include Rouwenhorst (1995), Jermann (1998), and Boldrin et al. (2001) who study asset price implications of technology shocks. The current model differs from theirs in several ways, including the presence of local goods. Empirically, Cochrane (1991) and Cochrane (1996) relate marginal product of capital to discount factor.}

\footnote{Real estate accounts for 30% of measurable consumer wealth while equity holdings, including pension and mutual funds, are only 3/5 of real estate holdings based on 2002-4 Flow of Funds Accounts of the United States. Cocco (2004) reports, using PSID, that the portfolio is comprised of 60-85% human capital, 12-22% real estate, and less than 3% stocks.}
the housing choice affects consumption and asset pricing through the discount factor. Second, housing is a durable good, which introduces an inter-temporal dependence of utility within the expected utility framework. Inter-temporal dependence, which is also introduced via habit formation and through Epstein-Zin recursive utility, improves the performance of the asset pricing model. Third, housing is a local or non-traded good. Housing is supplied by combining a structure, which is capital traded nationally, and land, which is a local good. The demand for housing is also local since regionally distinct industrial structures generate regional variations in labor income. Localized housing generates important effects on the asset prices.6

To give a clearer idea about the economics of the model, I illustrate the mechanisms that transmit a technology shock throughout the economy. A country is composed of two cities, each of which is formed around a firm. The capital and goods markets are national while the labor, housing and land markets are local. Technology shocks have direct effects only on one city. For instance, suppose that a positive technology shock to goods-producing firms in a city raises the marginal products of capital and of labor, and hence changes interest rates and wages. The housing demand is affected by a higher lifetime income as well as a price change. The housing supply is also affected by the altered capital supply through the shifted portfolio choice. The other city, without the shock, is influenced through the national capital market. The capital supply to the foreign city is reduced due to the shifting portfolio choice across cities, and thus production and wages are reduced. Therefore, the responses of housing prices and the firms’ use of capital become geographically heterogeneous. The shock also affects the next period through the inter-temporal consumption choice. The saving, or the capital supply to the next period, changes depending on the elasticity of the inter-temporal substitution. In sum, a shock has effects on the whole economy through consumption substitution between goods and between periods, and through capital substitution or portfolio selection between sectors and between cities. Different effects on the economy are analyzed for different types of technology shocks, whether temporary or permanent and whether in goods production or housing production.

The paper is organized as follows. Section 2 is a review of the related literature. In section 3 the general economic environment is specified. In section 4 the equilibrium is derived in a decentralized market institution. Section 5 includes the comparative statics and analyses of the results. Section 6 concludes and details my plan for extensions.

6The elasticity of housing supply widely varies across regions. (Green et al. (2005)) The dynamics of house price and consumption are also geographically heterogeneous. (Hess and Shin (1998) and Lustig and van Nieuwerburgh (2005))
2 Related Literature

Most of models of production economies are built on the assumption of a single homogeneous good; they focus on quantities rather than asset prices. Still, a small number of recent papers introduce home production, non-tradable goods or sector-specific factors, which are all relevant in the case of housing.

In a closed economy, home production of consumption goods helps explain a high level of home investment and a high volatility of output. In these models, labor substitution between home production and market production plays an important role while in the present model, capital substitution between sectors and between cities plays an important role. The housing service sector is introduced by Davis and Heathcote (2005) and two empirical regularities are explained: 1) the higher volatility of residential investment and 2) the comovement of consumption, nonresidential investment, residential investment, and GDP. They emphasize the importance of land in housing production and the effects of productivity shocks on the intermediate good sectors. However, the authors do not examine asset prices, which are the main concern here.

In an open economy, non-traded goods are introduced in the multi-sector, two-country, dynamic, stochastic, general-equilibrium (DSGE) model. Non-traded goods in an open economy are comparable to local housing services and land in the current model. The important findings in this literature are that non-traded goods may help explain 1) the high correlation between savings and investment, 2) the low cross-country correlation of consumption growth, and 3) home bias in investment portfolio. Again, price dynamics are not considered in this literature.

The asset pricing literature typically relies on a single good by implicitly assuming the separability of the utility function. Accordingly, most empirical works put little emphasis on housing as a good, relying on a single category of good defined in terms of non-durable goods and services. Housing is often taken into account in the portfolio choice problem in partial equilibrium. Incorporating the high adjustment cost of

---


10 Exceptions include Dunn and Singleton (1986), Pakos (2003) and Yogo (2005), who take account of durable consumption. However, their durable consumption ignores housing in favor of motor vehicles, furniture, appliances, jewelry and watches.

11 The demand for housing or mortgages are considered by Henderson and Ioannides (1983), Cocco (2000), Sinai and Souleles (2004), Cocco and Campbell (2004), and Shore and Sinai (2004). The effects of housing on the portfolio of financial assets are considered by Brueckner (1997), Flavin and
housing leads to interesting results such as high risk aversion and limited stock-market participation. However, the implications of the analyses are limited in scope since covariance structures of returns are exogenously given. Others examine the lifecycle profiles of the optimal portfolio and consumption when housing is introduced. These works are complementary to the research reported in this paper since they address non-asset pricing issues in general equilibrium.

Only a few papers examine the effects of housing on asset prices. Piazzesi et al. (2004) start from the Euler equation and examine the stochastic discount factor (SDF) when the intra-period utility function has a constant elasticity of substitution (CES) form, which is non-separable in consumption goods and housing services. They show that the ratio of housing expenditure to other consumption, which they call composition risk, appears in the SDF. They then proceed to conduct an empirical study taking the observed consumption process as the outcome of a general equilibrium. Two key differences from the present model are 1) they do not include the link with technologies, and 2) their housing is not distinct from other durable goods. Lustig and van Nieuwerburgh (2004) focus on the collateralizability of housing in an endowment economy. They use the ratio of housing wealth to human capital as indicating the tightness of solvency constraints and explaining the conditional and cross-sectional variation in risk premia. Their result is complementary to those reported below, as they show that another unique feature of housing, collateralizability, is important in asset pricing. Kan et al. (2004), using a DSGE model, show that the volatility of commercial property prices is higher than residential property prices and that commercial property prices are positively correlated with the price of residential property. Although housing is distinguished from commercial properties, its locality is not considered. In addition, their focus is also not on asset pricing in general but is limited to property prices.

3 The Model

3.1 Technologies

There are two goods: a composite good ($Y_t$) and housing services ($H_t$). The latter is a quality-adjusted service flow; larger service flows are derived either from a larger house or from a higher quality house.

Composite goods are produced by combining business capital ($K_t$) and labor ($L_t$)
while housing services are produced by combining housing structures \((S_t)\) and land \((T_t)\). The production functions are both Cobb-Douglas:

\[
Y_t = Y (A_t, K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha}, \quad (1a)
\]
\[
H_t = H (B_t, S_t, T_t) = B_t S_t^\gamma T_t^{1-\gamma}, \quad (1b)
\]

where \(A_t\) and \(B_t\) are total factor productivities of goods and housing production, respectively. Parameters \(\alpha\) and \(\gamma\) are the share of capital cost in the outputs of composite goods and housing services, respectively.

The production functions exhibit a diminishing marginal product of capital (MPK) so that the return depends on production scale, unlike in the linear technology case. This property, together with changing productivities, allows the return to vary over time and across states. Note also that a technology shock to housing production can be interpreted as a preference shock in the current model. This is because produced housing services directly enter into the utility function. A higher \(B_t\) could be interpreted as implying that a greater utility is derived from the same level of structures and land and that the households are less willing to pay for housing due to their reduced marginal utility.

### 3.2 Resource Constraint

Composite goods are used either for consumption or investment. The resource constraint is

\[
Y_t = C_t + I_t + J_t, \quad (2)
\]

where \(C_t\) is consumption, \(I_t\) and \(J_t\) the investment in business capital and housing structures, respectively. The equations defining the accumulation of business capital

\[13\]The land should be interpreted as the combination of non-structural local inputs. In particular, it includes all local amenities raising the quality of housing service, such as parks. The land supply function is explained in the household section.

\[14\]With the Cobb-Douglas production function, a total factor productivity shock can be described in terms of a shock to the capital-augmenting technology or as one to the labor-augmenting technology. For example, we can rewrite the production function as

\[
Y = AK^\alpha L^{1-\alpha} = (A^{1/\alpha} K)^{\alpha} L^{1-\alpha} = K^\alpha \left( A^{1/(1-\alpha)} L \right)^{1-\alpha}.
\]

\[15\]These parameters also represent the elasticity of output with respect to capital in the Cobb-Douglas production function.
and housing structures are

\begin{align*}
K_{t+1} & = (1 - \delta K) K_t + I_t, \\
S_{t+1} & = (1 - \delta S) S_t + J_t,
\end{align*}

where \( \delta_K \) and \( \delta_S \) are the constant depreciation rate of business capital and housing structures, respectively. I assume \( \delta_K = \delta_S = \delta \) for simplicity.

Note that the inclusion of the housing structures makes housing services a durable good. Consumption of housing services is directly linked with the accumulated structures while the amount of the composite goods consumption is chosen under the constraint (2). This makes housing services different from other goods.

### 3.3 Preferences

Consumers’ preferences are expressed by the following expected utility function:

\[
U = E_0 \left[ \sum_{t=1}^{\infty} \beta^t u(C_t, H_t) \right] \tag{4}
\]

where \( E_0 \) is the conditional expectation operator given the information available at time 0, \( \beta \) is the subjective discount factor per period, \( u(\cdot) \) is the intra-period utility function over composite goods (\( C_t \)) and housing services (\( H_t \)). In a two period model with perfect foresight, the lifetime utility becomes

\[
U = u(C_1, H_1) + \beta u(C_2, H_2).
\]

The CES-CRRA (constant relative risk aversion) intra-period utility function is adopted:

\[
u(C_t, H_t) = \frac{1}{1 - \frac{1}{\rho}} \left( C_t^{\frac{1}{\rho}} + H_t^{\frac{1}{\rho}} \right)^{(1 - \frac{1}{\rho})/(1 - \frac{1}{\rho})}, \tag{5}\]

where \( \rho > 0 \) is the elasticity of intra-temporal substitution between composite goods and housing services, and \( \theta > 0 \) is the parameter for the elasticity of inter-temporal substitution. The simplest special case is that of separable log utility, \( u(C_t, H_t) = \ln C_t + \ln H_t \), which corresponds to \( \rho = \theta = 1 \).

The non-separability between composite goods and durable housing in the CES specification delinks the tight relationship between the relative risk aversion and the elasticity of inter-temporal substitution. Even though the lifetime utility function has a time-additive expected utility form, the durability of housing makes the utility function...
intertemporally dependent.\textsuperscript{16}

Other specifications that also break the link between relative risk aversion and EIS include habit formation and Epstein-Zin recursive utility. Habit formation is similar to durable consumption, but past consumption in the habit-formation model makes the agent less satisfied while past expenditure on durables makes the agent more satisfied. Both habit formation and Epstein-Zin recursive utility are known to resolve partially the equity premium puzzle.

With the non-separability of the CES function, the relative risk aversion is not simply $1/\theta$; it is defined as the curvature of the value function, which depends on durable housing. CRRA utility over a single good is a special case in which the curvature of the value function coincides with the curvature of utility function. Note also that the elasticity of inter-temporal substitution for composite goods in the continuous-time limit is defined as the weighted harmonic mean of $\rho$ and $\theta$:

$$EIS = \left(-C_t \frac{u_{CC}(C_t, H_t)}{u_C(C_t, H_t)}\right)^{-1} = \left[\frac{1}{\rho} \left(1 - \frac{C_t^{1-\frac{1}{\rho}}}{C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}}}\right) + \frac{1}{\theta} \left(\frac{C_t^{1-\frac{1}{\rho}}}{C_t^{1-\frac{1}{\rho}} + H_t^{1-\frac{1}{\rho}}}\right)\right]^{-1},$$

where the weight is the share of the composite goods component in the aggregator.\textsuperscript{17}

### 3.4 Cities

There are two cities of the same initial size, in each of which households, goods-producing firms, and real estate firms operate competitively. The variables and parameters of the city with technology shocks ("home" city) with plain characters ($C_t$, etc.) and those of the other ("foreign") city with starred characters ($C^*_t$, etc.).

Each "city" should not be interpreted literally. Instead, a "city" is understood to be a set of cities or regions that share common characteristics in their industrial structure and land supply conditions. For example, a technology shock to the IT

\textsuperscript{16}It might seem that the utility is not specified over housing as a durable but as contemporaneous housing services produced by real estate firms. However, housing services depend on the real estate firms’ past investments in the housing structure, which are analogous to the households’ expenditure on durable housing. Indeed, "real estate firms" can be characterized as the internal accounts of households. These "real estate firms" are set up just to derive explicitly the housing rent.

\textsuperscript{17}See Deaton (2002) and Flavin and Nakagawa (2004) for detailed discussions on the delinking of EIS and risk aversion. Yogo (2005) shows the importance of non-separability between durables and non-durables in explaining the equity premium. Limitations caused by homotheticity induced by the CES form are discussed in Pakos (2003).
industry mainly affects the cities whose main industry is the IT industry. A "city" in this paper represents the collection of such cities that are affected by the same technology shock.

3.5 Discount Factors and Asset Prices

Let $\phi_{t,t+1}$ denote the discount factor for time $t+1$ as of time $t$. The price of any asset is expressed as the expected return in units of the numeraire multiplied by the discount factor. For example, the ex-dividend equity price of firm $f$, $e_{f,t}$, is expressed in terms of the dividend stream $D_{f,t}$ and the discount factor as

$$e_{f,t} = E_t \left[ \sum_{j=1}^{\infty} \phi_{t,t+j} D_{f,t+j} \right].$$

The importance of the covariance between the discount factor and the return can be seen from this equation. The $j$-period risk-free discount factor as of time $t$ (i.e. the price of a bond that will deliver one dollar $j$ periods later without fail) is $E_t \left[ \phi_{t,t+j} \right]$. Equivalently, using the $j$-period risk-free rate of return, $i_{j,t}$,

$$\frac{1}{i_{j,t}} = E_t \left[ \phi_{t,t+j} \right].$$

Without uncertainty, the relationship in expectation becomes the exact relationship:

$$e_{f,t} = \sum_{j=1}^{\infty} \phi_{t,t+j} D_{f,t+j},$$

$$\frac{1}{i_{j,t}} = \phi_{t,t+j}.$$

4 Market Institutions and Equilibrium in a Two-Period Model With Perfect Foresight

I derive the decentralized market equilibrium in a two-period model with perfect foresight. Figure 1 presents the time line of economic activities.
(Goods-producing firm) Goods-producing firms competitively produce composite goods by combining capital and labor. Each goods-producing firm in the home city solves the following problem in each period, taking as given interest rates \((i_1, i_2)\), wages \((w_1, w_2)\) and total factor productivities \((A_1, A_2)\). The firms in the foreign city solve the identical problem with possibly different variables and parameters.

\[
\max_{K_t, L_t} Y (A_t, K_t, L_t) - (i_t - 1 + \delta) K_t - w_t L_t, \quad t = 1, 2.
\]

This objective function is a reduced form in which the firm’s capital investment decision does not explicitly show up and in which the firm only recognizes the periodic capital cost. (This simplification is possible because there is no stock adjustment cost.)

The first-order conditions define the factor demands of the goods-producing firm:

\[
\begin{align*}
K_t & : \quad i_t - 1 + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha}, \\
L_t & : \quad w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) A_t \left( \frac{K_t}{L_t} \right)^{\alpha}.
\end{align*}
\]

As usual, the interest rate is equal to \(1 - \delta\) plus the marginal product of capital (MPK), and the wage is equal to the marginal product of labor. In equilibrium with perfect foresight, the national market for capital implies that capital allocations are adjusted until the interest rates are equated across sectors and cities. Wages are unique to the city since the labor market is local.

(Real estate firm) Real estate firms produce housing services by combining land and structures. Each real estate firm solves the following problem in each period, taking as given the housing rent \((p_1, p_2)\), the interest rate \((i_1, i_2)\), the land rent \((r_1, r_2)\) and the total factor productivity \((B_1, B_2)\). (The firms in the foreign city solve identical problems with starred variables.)

\[
\max_{S_t, T_t} p_t H (B_t, S_t, T_t) - (i_t - 1 + \delta) S_t - r_t L_t, \quad t = 1, 2.
\]

As noted, these "real estate firms" can be also interpreted as the internal accounts of households since homeowners are not distinguished from renters. Nevertheless, I prefer describing the real estate industry in order to obtain explicitly the housing rent.
The first-order conditions define the factor demands of housing production:

\[ S_t : i_t + 1 - \delta = p_t \frac{\partial H_t}{\partial S_t} = \gamma B_t p_t \left( \frac{T_t}{S_t} \right)^{1-\gamma}, \quad (7a) \]

\[ T_t : r_t = p_t \frac{\partial H_t}{\partial T_t} = (1-\gamma) B_t p_t \left( \frac{S_t}{T_t} \right)^{\gamma}. \quad (7b) \]

The interest rate and the land rent are equal to the marginal housing product of structure (MHPS) and of land (MHPL), respectively, in units of the numeraire. Again, the interest rate will be equated across sectors and cities in equilibrium while the land rent is locally determined.

**Households** Households are endowed with initial wealth \((W_0)\) and land. They provide capital, land and labor in each period to earn financial, land and labor income, respectively, and spend income on consumption of composite goods, housing services, and savings \((W_1)\). The savings can be freely allocated among sectors and cities.

Labor is inelastically supplied and normalized at one. Households are assumed to be immobile across cities. This assumption is reasonable since most of the population does not migrate across regions. The immobility of labor will result in wage differentials across cities. The free mobility of households would make labor more like capital and render the production function linear in inputs. The costs of capital and labor would be equated across cities and the price responses would become more moderate. While the mobility would generate more moderate results on the asset price, it would not greatly change the overall results as long as homothetic CES preferences are maintained.\(^{18}\)

Land supply is assumed to be iso-elastic:

\[ T_t = r_t^\mu, \quad t = 1, 2, \]

where \(\mu\) is the price elasticity of supply. \(\mu = 0\) represents a perfectly inelastic land supply at one and \(\mu = \infty\) represents perfectly elastic land supply. By this simple form, land supply elasticity and asset prices are linked in a straightforward way. While the land supply is obviously constrained by the topographic conditions of the city, other conditions such as zoning regulations and current population densities are also critical. For example, the infill development and the conversion from agricultural to residential use make the land supply elastic. The elasticity can also be understood as reflecting short-run and long-run elasticities. For example, if eminent domain is

\(^{18}\)With CES preferences, the income elasticity of housing demand is one. Therefore, even if the housing demand per household is altered by the wage income, the offsetting change in the population will limit the effects on total housing demand.
politically hard to use in providing a local amenity or if the current landlords rarely agree on redevelopments, the housing supply process may take longer than a business cycle, in which case the land supply is more inelastic.\footnote{Many development projects in Japan take more than twenty years to complete. This is an example of an inelastic supply due to the slow development process.}

Each household solves the following problem, taking as given the housing rents, land rents, interest rates and wages.

\[
\begin{align*}
\max_{\{C_t, H_t\}} & \quad u(C_1, H_1) + \beta u(C_2, H_2) \\
\text{s.t.} & \quad C_1 + p_1 H_1 + W_1 = i_1 W_0 + r_1 T_1 + w_1 \\
& \quad C_2 + p_2 H_2 = i_2 W_1 + r_2 T_2 + w_2.
\end{align*}
\]

The above dynamic budget constraints can be rewritten as the lifetime budget constraint:

\[
\begin{align*}
C_1 + p_1 H_1 + \frac{1}{i_2} (C_2 + p_2 H_2) &= i_1 W_0 + r_1 T_1 + w_1 + \frac{1}{i_2} (r_2 T_2 + w_2) \\
&\equiv \text{Inc}.
\end{align*}
\]

The RHS of the lifetime budget constraint is defined as the lifetime income, Inc.

The first-order conditions for the CES-CRRA utility are\footnote{In the log-utility case, they reduce to \( p_t H_t = C_t \) and \( i_2 = \left( \frac{\partial u}{\partial C_2} / \frac{\partial u}{\partial C_1} \right)^{-1} = (1/\beta) (C_2/C_1) \).}

\[
\begin{align*}
p_t H_t &= C_t, \tag{8a} \\
i_2 &= \left( \beta \frac{\partial u}{\partial C_2} / \frac{\partial u}{\partial C_1} \right)^{-1} \\
&= \frac{1}{\beta} \left( \frac{C_2}{C_1} \right)^{\frac{1}{\gamma}} \left[ 1 + \frac{(H_2/C_2)^{1/\gamma}}{1 + (H_1/C_1)^{1/\gamma}} \right]^{\frac{\gamma-1}{\gamma}}. \tag{8b}
\end{align*}
\]

The interest rate is the reciprocal of the inter-temporal marginal rate of substitution (IMRS). That is, the IMRS is the discount factor in this economy. In the log utility case, the interest rate is proportional to consumption growth because of the unit elasticity of inter-temporal substitution. The inter-temporal consumption substitution expressed by this Euler equation, together with the intra-temporal substitution between two goods, is a key driver of the economy. The IMRS is discussed, in a greater detail, in the next section since it is a key to understanding the economy.
With the lifetime budget constraint, I obtain the consumption demands:\(^{21}\)

\[
C_1 = (1 + p_1^{1-\rho})^{-1} \left\{ 1 + \beta \frac{\delta}{\beta_2}^{-\beta/(1-\beta)} \left( \frac{1 + p_2^{1-\rho}}{1 + p_1^{1-\rho}} \right)^{\frac{1-\delta}{1-\rho}} \right\}^{-1} \text{Inc,} \quad (9a)
\]

\[
C_2 = \beta \frac{\delta}{\beta_2} \left( \frac{1 + p_2^{1-\rho}}{1 + p_1^{1-\rho}} \right)^{\frac{1-\delta}{1-\rho}} C_1, \quad (9b)
\]

\[
H_{t}^{\text{dem}} = \frac{C_t}{p_t}. \quad (9c)
\]

Note that the housing rents have no effect on the consumption demand in the log utility case while they do have an effect on it in general. It is also clear that the expenditure ratio of housing, \(p_t H_t/C_t\), is always 1 in the log case while it is \(p_t^{1-\rho}\) in general.

### 4.1 Discount Factors and the Role of Rent Growth

Before solving for a general equilibrium, I provide a new way of characterizing the discount factor and describe the link between rent growth and asset pricing. Small manipulations to (6a), (7a), and (8a) yield three different ways of expressing the discount factor:\(^{22}\)

\[
\phi_{1,2} = \left[ 1 + \frac{\partial Y_2}{\partial K_2} - \delta \right]^{-1} \quad \text{(Reciprocal of MPK)} \quad (10a)
\]

\[
= \left[ 1 + p_2 \frac{\partial H_2}{\partial S_2} - \delta \right]^{-1} \quad \text{(Reciprocal of MHPS)} \quad (10b)
\]

\[
= \beta \left( \frac{C_2}{C_1} \right)^{-\frac{1}{\beta}} \left[ 1 + \frac{p_2 H_2/C_2}{1 + p_1 H_1/C_1} \right]^{\frac{\rho-\delta}{\rho(1-\rho)}} \quad \text{(IMRS).} \quad (10c)
\]

These relationships hold for the foreign city as well. Indeed, the discount factor is the center piece that is common to all agents in the economy. The first equation (10a), which is empirically exploited by Cochrane (1991), is used to understand the effect of goods-sector shocks. The second equations (10b) are useful when considering housing shocks. The third equation (10c) includes the expenditure share of housing consumption, which Piazzesi et al. (2004) call the composition risk and empirically exploit.

\(^{21}\)In the log-utility case, they reduce to \(C_1 = \text{Inc}/[2(1 + \beta)]\), \(C_2 = \beta \delta_2 C_1\), and \(H_{t}^{\text{dem}} = C_t/p_t\).

\(^{22}\)For the log utility, IMRS reduces to \(\phi_{1,2} = \beta (C_2/C_1)^{-1}\).
I derive a different formula that includes only the housing rents in the second term by using (9a), (9b), and (9c).

\[ \phi_{1,2} = \beta \left\{ \left( \frac{C_2}{C_1} \right) \left[ \frac{(1 + p_2^{1-\rho})^{\frac{1}{1-\rho}}}{(1 + p_1^{1-\rho})^{\frac{1}{1-\rho}}} \right]^{\theta - \rho} \right\}^{-\frac{1}{\theta}} \]

\[ \equiv \beta \left\{ g_{c,2} \cdot g_{p,2}^{\theta - \rho} \right\}^{-\frac{1}{\theta}}, \quad (11) \]

where \( g_{c,2} \equiv C_2 / C_1 \) is the consumption growth, and \( g_{p,2} \equiv (1 + p_2^{1-\rho})^{\frac{1}{1-\rho}} / (1 + p_1^{1-\rho})^{\frac{1}{1-\rho}} \) is the growth of the CES-aggregated price index. Note that \( g_{p,2} \) is a monotonically increasing function of the rent growth. A high \( g_{p,2} \) means that the numeraire good in period 2 is relatively abundant and cheap, or equivalently that housing is relatively precious and expensive.

This equation gives a new insight about the meaning of rent growth in the context of the asset pricing. The IMRS (\( \phi_{1,2} \)), which measures how "under-satisfied" the household is in the second period, basically has the same form as in the single good CRRA case. When consumption growth is high, the household is more satisfied in that period and the marginal utility is lower. However, the level of satisfaction is not simply measured by consumption growth but by the consumption growth augmented by the growth of the aggregate price \( g_{p,2} \) raised to the \( \theta - \rho \).

When the two goods are relatively substitutable (\( \rho > \theta \)), a high growth of the aggregate price \( g_{p,2} \) (i.e., abundant composite goods) reduces the satisfaction gained from composite goods because of their abundance. A low \( g_{p,2} \), on the contrary, raises satisfaction from consuming composite goods because they are more precious. Similarly, when the two goods are relatively complementary (\( \rho < \theta \)), a low \( g_{p,2} \) (i.e., abundant housing) increases the need for composite goods. The consumption growth is adjusted downward. Conversely, a high \( g_{p,2} \) (i.e., precious housing) makes composite goods less needed, so that consumption growth is adjusted upward.

In sum, the housing rent measures the relative abundance of composite goods. This abundance affects the marginal utility of composite goods differently depending on the relative substitutability between the goods.

**Proposition 1** Housing rent growth, measured by the growth of the CES-aggregated price index \( (g_{p,2}) \), is a component of the discount factor if utility function is non-separable in housing and the numeraire good. The sign of the relationship between rent growth and the discount factor is determined by relative substitutability between the two goods.
This intuition is also confirmed by examining cross-partial derivative of the intra-period utility function: $\text{sgn } (\partial^2 u (C_t, H_t) / \partial H_t \partial C_t) = \text{sgn } (\theta - \rho)$.\textsuperscript{23} Abundant housing raises the marginal utility of consumption when $\theta > \rho$. The consumption growth, however, is not independent of the rent growth. Therefore, another partial equilibrium argument leads to the following corollary.

**Corollary 2** If the discount factor is fixed at a given level, rent growth and consumption growth have a positive (negative) relationship when the two goods are relatively substitutable (complementary):

$$\left. \frac{\partial g_{c,2}}{\partial g_{p,2}} \right|_{d\phi_{1,2}=0} > 0 \ (\ < 0) \quad \text{for } \rho > \theta \ (\rho < \theta).$$

The above arguments, however, are all based on the partial derivatives of the discount factor. The discount factor, the rent growth, and the consumption growth are determined in general equilibrium and their changes cannot be identified merely with reference to the Euler equation. Indeed, I show that the relationship between the consumption growth and the discount factor change signs depending on parameter values and the type of shock involved. The equilibrium responses to a technology shock, which will be discussed after defining the equilibrium, provide a fresh look at several related results: Tesar (1993) who considers an endowment shock to the non-tradables; and Piazzesi et al. (2004) who consider the relationship between the discount factor and the expenditure share of housing. In particular, it is shown that the rent growth component may mitigate the equity premium puzzle and the risk-free rate puzzle. The characterization of the discount factor using housing rent provides an opportunity to use different data sets in empirical analyses.\textsuperscript{24}

\textsuperscript{23}Specifically, the cross-partial derivative is

$$\frac{\partial^2 u (C_t, H_t)}{\partial H_t \partial C_t} = \left( \frac{\theta - \rho}{\rho \theta} \right) (C_t H_t)^{-\frac{1}{\rho}} \left[ C_t^{1-\frac{1}{\theta}} + H_t^{1-\frac{1}{\rho}} \right]^{-(1-\frac{1}{\theta})/(1-\frac{1}{\rho})^2}.$$

\textsuperscript{24}Housing rent data have several advantages over housing consumption data in terms of their availability and accuracy.
4.2 Definition of the Equilibrium

Markets are for composite goods, housing services, land, labor, and capital. Walras’ law guarantees market clearing in the goods market, and the market-clearing conditions are imposed for the other markets. The multi-sector structure necessitates a numerical solution. Detailed derivation of the equilibrium is shown in the appendix.

Definition 3 A competitive equilibrium in this 2-period, 2-city economy with perfect foresight is the allocation \( \{C_t, C^*_t, H_t, H^*_t, W_1, W^*_1, Y_t, Y^*_t, K_t, K^*_t, L_t, L^*_t, S_t, S^*_t, T_t, T^*_t\}_{t=1,2} \) and the prices \( \{p_t, p^*_t, w_t, w^*_t, i_t, r_t, r^*_t\}_{t=1,2} \) such that

1. optimality is achieved for households, goods-producing firms, and real estate firms, and
2. all market-clearing conditions and resource constraints are met.

5 Comparative Statics

The goals are to understand 1) the observed dynamic relationship among various asset classes, 2) the relationship between asset prices and business cycles, and 3) the role of housing in the economy. Different types of technology shocks are introduced as follows.

<table>
<thead>
<tr>
<th>Temporary, current</th>
<th>Goods Production</th>
<th>Housing Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta A_1 \geq 0 )</td>
<td>( \Delta B_1 \geq 0 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temporary, anticipated</th>
<th>Goods Production</th>
<th>Housing Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta A_2 \geq 0 )</td>
<td>( \Delta B_2 \geq 0 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Permanent, current</th>
<th>Goods Production</th>
<th>Housing Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta A_1 = \Delta A_2 \geq 0 )</td>
<td>( \Delta B_1 = \Delta B_2 \geq 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Technology shocks are given to the home city. Different parameter values are allowed for

\[ \mu : \text{Elasticity of land supply} \]
\[ \rho : \text{Elasticity of intra-temporal substitution between } C \text{ and } H \]
\[ \theta : \text{Parameter for inter-temporal substitution} \]

5.1 Mitigating the Equity Premium Puzzle and the Risk-Free Rate Puzzle

The equity premium puzzle is the fact that the historical risk premium associated with equity is too high to be explained by the observed covariance between the consumption-
based discount factor and the return under plausible levels of risk aversion. Since the puzzle arises from too little variation in the consumption growth, any factor that magnifies the variation of the consumption growth in the Euler equation helps to resolve the puzzle. A closely related issue is a low estimate of EIS, since the coefficient of relative risk aversion is the reciprocal of EIS with a single good power utility specification. A low EIS implies a much higher interest rate than the historical level. This is called the risk-free rate puzzle. Previous estimates of EIS are typically quite low and even negative.

The model generates an equilibrium relationship among consumption growth, rent growth, and the discount factor. There are two cases in which these puzzles are mitigated.

**Proposition 4** The equity premium puzzle and the risk-free rate puzzle are mitigated in the following two cases.

**Case 1:** Magnified variation of consumption growth in response to anticipated shocks to goods production when $\theta > \rho$.

**Case 2:** Estimates of EIS are biased downward by shocks to housing production.

Case 1 is based on a positive covariation of consumption growth ($g_{c,2}$) with the rent growth factor ($g_{p,2}^{\theta - \rho}$) in (11). Figure 2-a presents the variation of augmented consumption growth ($g_{c,2} g_{p,2}^{\theta - \rho}$) and its components in response to anticipated shocks to goods production ($\Delta A_2$) when $\rho = 0.2$ and $\theta = 1.8$. Augmented consumption growth exhibits much greater variation than plain consumption growth since the rent growth factor changes in the same direction. The covariation of $g_{c,2}$ and $g_{p,2}^{\theta - \rho}$ has a positive sign when the two goods are relatively complementary ($\theta > \rho$). (Figure 2-b) Suppose that a positive future shock to goods production is anticipated. ($\Delta A_2 > 0$) Both consumption ($C_2$) and rent ($p_2$) increase in the future, which drives both consumption growth ($g_{c,2}$) and rent growth higher. The rent growth factor also increases if $\theta - \rho$ is positive and vice versa. With other types of shocks, the covariation is mainly negative and variation of consumption growth is dampened. Therefore, this case applies if asset prices are mainly driven by news about future productivity shocks, and if the two goods are relatively complementary.

The condition $\theta > \rho$ is not unrealistic although previous estimates of the elasticities of substitution are mixed. Regarding intra-temporal substitution ($\rho$), most studies define durables as motor vehicles, furniture, jewelry and so on. The estimates of $\rho$ for these goods range from 0.4 to 1.2. A smaller number of studies include housing,
whose estimates range from 0.2 to 2.2.\textsuperscript{25} The estimates of EIS ($\theta$) are also mixed: Although a quite low EIS (close to zero or even negative) is usually estimated, much higher estimates (from 1 to 3) are also presented.\textsuperscript{26}

[Figure 2: Mitigating the equity premium puzzle and the risk-free rate puzzle]

Case 2 explains a bias arising from a mis-specification. In equilibrium, housing shocks lead to positive covariation of the discount factor and consumption growth. (Figure 2-c) If a model of a single good power utility, $\phi_{1,2} = \beta g_{c,2}^{-1/\theta}$, is applied to this situation, the positive covariation results in a negative estimate of $\theta$ since $\theta$ is estimated by ignoring the rent growth factor ($g_{p,2}^{\theta - \rho}$) in (11). Therefore, if housing shocks are mixed with goods production shocks, the estimate of $\theta$ is biased downward. This implies an ambiguous relationship between consumption growth and the discount factor, which in turn cautions us not to make an immediate inference about the discount factor by looking only at the consumption growth.

The discount factor ($\phi_{1,2}$) and consumption growth ($g_{c,2}$) move in the same direction since both of them move inversely with the rent growth factor, which sharply responds to housing shocks. (Figure 2-d) The inverse relationship between consumption growth and the rent growth factor is generated as follows. Suppose a positive housing shock occurs at $t = 2$ ($\Delta B_2 > 0$). Rent growth declines and the rent growth factor also declines when $\theta > \rho$ and vice versa. On the other hand, the consumption at $t = 2$, and thus consumption growth increases when $\theta > \rho$ because of the complementarity of the two goods. An analogous mechanism works with $\Delta B_1$.

\section{5.2 Effects on the Discount Factor}

Figure 3 presents selected comparative statics of the discount factor. They serve as the basis for understanding the asset price relationship. With a positive goods production shock ($\Delta A_t > 0$), the marginal product of capital becomes higher at any level of capital. The equilibrium interest rate ($i_t$) rises, or equivalently, the discount factor ($\phi_{t-1,t}$) falls although more capital ($K_t$) is allocated from the foreign city. These effects hold regardless of parameters. (Figure 3-a) The discount factor in the other period is

\textsuperscript{25}See Tesar (1993) and Yogo (2005) for non-housing durables, and Lustig and van Nieuwerburgh (2004), Piazzesi et al. (2004), and Davis and Martin (2005) for housing.

\textsuperscript{26}Among the large body of literature on the EIS estimation, Hall (1988) finds it to be negative and Yogo (2005) estimates it at 0.02 while Vissing-Jorgensen and Attanasio (2003) find it between 1 and 2 and Bansal and Yaron (2000) estimate the EIS between 1.9 and 2.7.
also affected via savings, as an increase in the lifetime income motivates households to smooth consumption by adjusting their savings ($W_1$). With $\Delta A_1 > 0$, the savings at $t = 1$ (capital supply for $t = 2$) are raised and $i_2$ falls ($\phi_{1,2}$ rises). (Figure 3-b) With $\Delta A_2 > 0$, the reduced savings at $t = 1$ allow a greater demand for goods at $t = 1$ and generally raise $i_1$ (lowers $\phi_{0,1}$) although the effects are much smaller due to the fixed capital supply.

![Figure 3-Effects of a shock on the discount rate](image)

If a positive shock is given to housing production ($\Delta B_t > 0$), the effects are much smaller. Although housing production ($H_t$) increases, expenditures ($p_t H_t$) are less affected since the rent ($p_t$) decreases. The marginal housing product may even fall if the housing rent falls enough. The effects on the contemporaneous discount factor depend on the rate of substitution between the goods. If the intra-temporal substitution ($\rho$) is low (i.e. the two goods are complements), the contemporaneous discount factor ($\phi_{t-1,t}$) rises. The reason is as follows. A low intra-temporal substitution means a low price elasticity of housing demand. The increased housing consumption necessitates a much greater reduction in housing rent ($p_t$) so that the housing expenditure ($p_t H_t$) decreases. The marginal housing product of structure also falls, which means that the discount factor rises. If the substitution is high, the opposite is true and the discount factor falls. With the log utility, $\Delta B_t$ has no effect on the discount factor. (Figure 3-c)

The other period is again affected through inter-temporal substitution. Since the effects on lifetime income are quite small, the inter-temporal substitution rather than the consumption smoothing may come into play if $\theta$ is large. Consider $\Delta B_1 > 0$. (Figure 3-d) As $\theta$ becomes large, future resources are shifted toward the current period as savings are reduced. This raises $i_2$. If $\theta$ is small, the savings are increased (for consumption smoothing) and $i_2$ falls. With $\Delta B_2 > 0$, the same mechanism affects savings although the effects on $\phi_{0,1}$ are small due to the fixed capital supply. As $\theta$ becomes large, the current capital demand is reduced by the increased savings, and $i_1$ falls. The general equilibrium effects on the discount factor are summarized in Table 1.

![Table 1: Effects on the discount factor](image)

\footnote{To be precise, $\theta$ also has a secondary effect on $\phi_{0,1}$ since the inter-temporal substitution affects capital demand. The effect of $\theta$ is more apparent when the shock is temporary.}

\footnote{To be precise, $\rho$ has a secondary effect on $\phi_{1,2}$ since intra-temporal substitution affects capital demand.}
5.3 Implications for the Risk of Housing

The risk of housing is inferred in a unique way by examining the covariation of the discount factor \((\phi_{1,2})\) and rent growth \((g_{p,2})\).\(^{29}\) Housing, unlike other assets, has a direct effect on the discount factor via rent growth, because of its dual role as a consumption good and as an asset. A negative covariation between the discount factor and rent growth indicates a positive risk premium, since housing asset generates less cashflow exactly when the household wants more wealth.

The rent responses are governed by the simple principle that a good produced by an efficient technology is relatively cheap. A positive shock to goods production in period \(t\) makes composite goods cheap in the period and makes \(p_t\) high. \((\Delta A_t > 0 \Rightarrow \Delta p_t > 0)\)

The opposite is true for a housing shock. \((\Delta B_t > 0 \Rightarrow \Delta p_t < 0)\) Rent growth increases with \(\Delta A_2 > 0\) and \(\Delta B_1 > 0\) while decreasing with \(\Delta A_1 > 0\) and \(\Delta B_2 > 0\). For \(\Delta A_1 = \Delta A_2 > 0\) and \(\Delta B_1 = \Delta B_2 > 0\), the effects on rent growth are ambiguous since the rents in both periods change in the same direction.

Table 2 summarizes the covariation of the discount factor and rent growth. For \(\Delta A_1, \Delta A_2,\) and \(\Delta B_1 = \Delta B_2\), the covariation is uniformly negative. For \(\Delta A_1 = \Delta A_2\), the covariation depends on the sign of rent growth and is generally positive except when \(\rho\) and \(\mu\) are extremely small. For \(\Delta B_2\) and \(\Delta B_2\), the covariation depends on the sign of discount factor and thus on \(\theta\) and \(\rho\). In general housing assets are riskier when \(\rho\) is smaller and \(\mu\) is smaller. (Figure 4) When the two goods are more complementary and land supply is less elastic, the rent response is larger and the negative covariation is of a larger magnitude.

**Proposition 5** Housing assets are riskier if housing services are more complementary to the numeraire good and if the land supply is less elastic.

**Table 2: Covariation of discount factor and rent growth**

**Figure 4 - Covariation of the discount factor and rent growth**

5.4 Effects on Asset Prices

Three asset classes are considered: financial assets, housing, and human capital. The prices of housing and human capital are defined as the present discounted values of

---

\(^{29}\)This analysis is distinct from the partial equilibrium analysis in (12) since consumption growth is not fixed.
housing rent and wages, respectively, for a unit amount of the asset:

\[
(Housing\ Price)_0 = \phi_{0,1}p_1 + \phi_{0,1}\phi_{1,2}p_2. \tag{13a}
\]

\[
(Human\ Capital\ Price)_0 = \phi_{0,1}w_1 + \phi_{0,1}\phi_{1,2}w_2. \tag{13b}
\]

The change in the asset price is determined by possibly competing factors on the RHS of (13a) and (13b).

The financial asset price is equivalent to the price of the installed business capital. Since the price of business capital is always one in the current model (i.e. without a stock adjustment cost), the price with adjustment cost is inferred as follows. If adjustment costs are introduced, the financial asset price would change with the equilibrium level of capital used in goods production \((K_t)\). The price of capital changes since quantity cannot immediately reach the equilibrium level. The price gradually approaches to one as capital is adjusted toward the equilibrium. Therefore, we can regard the change in equilibrium capital as a proxy for the change in capital price.

5.4.1 Effects on Housing Prices

The equilibrium housing price goes up in the following cases.

Case 1: A positive shock to goods production \((\Delta A_t > 0)\), and inelastic land supply (small \(\mu\)).

Case 2: A negative shock to goods production \((\Delta A_t < 0)\), and elastic land supply (large \(\mu\)).

Case 3: For \(\Delta A_2 < 0\); additionally, small \(\rho\) and small \(\theta\). A negative shock to goods production in the foreign city.

Case 4: For \(\Delta A_2^* < 0\); additionally, elastic land supply (large \(\mu\), small \(\rho\) and small \(\theta\). A negative shock to housing production \((\Delta B_t < 0)\).

In case 1, the housing rent \((p_t)\) rises at the time of a shock since the numeraire good becomes cheaper. The rent increase is greater if the land supply is more constrained (small \(\mu\)), since the shift in housing demand results in a greater price change.\(^{30}\) Although the discount factor \((\phi_{t-1,t})\) and rent may be lower in the other period, the overall effect on housing prices is positive because of a large positive response of rent.

\(^{30}\)The intra-temporal substitution \((\rho)\) also has a secondary effect. If the intra-temporal substitution is low, the price elasticity of housing demand is also low and the rent is more responsive to a shift in supply.
With the elasticity of land supply around 0.8 or less, a positive shock leads to the appreciation of housing prices. (Figure 5-a.) If land supply is more elastic, housing prices exhibit the opposite response, which constitutes Case 2. (Figure 5-b and 5-c) A negative shock to housing production also results in the appreciation of housing prices by increasing rent. (Figure 5-d)

Cases 2 and 3, in which a negative shock to goods production leads to housing price appreciation, provide an interesting insight into the appreciation of housing prices in the United States after 2000. This appreciation occurred in a stagnant economy and with stock prices at a low. A key driver in the model is high future rents induced by reduced housing supply in the future.

Consider a current negative shock to goods production of the home city ($\Delta A_1 < 0$) in a land-elastic economy (Case 2). There are competing forces in the housing-price equation (13a):

$$(Housing\ Price)_0 = \phi_{0,1} \; p_1 + \phi_{0,1} \; \phi_{1,2} \; p_2.$$  

The shock lowers the MPK and raises the discount factor (high $\phi_{0,1}$), which helps raise the housing price. The negative shock makes the numeraire good more precious and reduces the current housing rent (low $p_1$), but the rent reduction is relatively moderate in a supply-elastic city (large $\mu$). The households cash out part of their savings ($W_1$) in order to support their period 1 consumption (consumption smoothing motive) so that the capital supply at $t = 2$ is reduced. The reduced capital supply results in a higher interest rate or a lower discount factor at $t = 2$ (low $\phi_{1,2}$). When land supply is elastic, the housing rent is more affected by the negative supply shift than the demand shift, which leads to a rise in rent (high $p_2$). When a higher $\phi_{0,1}$ and $p_2$ surpass the other competing forces, the housing price appreciates.

In Case 2, we should observe 1) a bull-steepening of the term structure of interest rates (a lower rate at the short end of yield curve), 2) higher expected rent growth, 3) a lower current capitalization rate, or "cap rate", for housing, and 4) reduced savings (attributable to a cashing out of the investment portfolio). Case 2 is also consistent...
with the negative covariation of the housing price and the interest rate noted by Cocco (2000) and positive covariation of business investment and housing investment noted by Davis and Heathcote (2005). While standard two-sector models generate a negative covariation of investments due to the sectoral substitution of capital, the model generates a positive relationship by dint of the capital allocation across cities. A positive covariation between investments, however, means stagnation in near term construction activity after 2000, which is slightly counterfactual.

Improved results are obtained by combining an anticipated negative shock to housing production ($\Delta B_2 < 0$, Case 4) with Case 2. The negative effect of $\Delta A_1 < 0$ on housing structures is mitigated or may even be reversed. All other effects are enhanced: higher housing prices, lower financial asset prices, a steeper slope of yield curve, a higher rent growth, a lower cap rate, and lower savings. This combined case is also appealing because of a better match to a cross-regional observation that housing price appreciation is pronounced in areas with rich housing amenities such as San Diego and Miami. Housing price appreciation seems to be partly driven by a local shock to preference for housing, which is equivalent to a shock to housing production in the model.

Table 3 summarizes the model predictions for all four cases. Either Case 2 with $\Delta A_1 < 0$ (elastic land supply) or Case 3 with $\Delta A_1^* < 0$ (a negative shock in the foreign city) provides the predictions that fit best the situation after 2000. Case 3 is driven by the capital flow from the home city under recession. Case 4 is mainly driven by a higher rent due to less efficient housing production. In this case the covariation of investments is negative due to capital substitution between sectors.

[Table 3: Predictions in four cases of housing price appreciation]

5.4.2 Effects on Human Capital and Financial Assets

Table 4 presents the effects of various technology shocks on asset prices. The value of human capital rises with a positive shock to goods production ($\Delta A_t > 0$) mainly because of a large increase in wages ($w_t$) with an inelastic labor supply. (Note the second column of Table 4) A positive shock to housing production ($\Delta B_t > 0$) generates parameter-dependent effects. When the two goods are complementary (small $\rho$), the value of human capital rises because greater demand for composite goods ($Y_t$) increases wages. Inter-temporal substitution ($\theta$) also affects the value via variations in the discount factor that are discussed in Section 5.2.
The price of the financial asset exhibits very similar responses as the value of human capital. The price rises with a positive shock to goods production. (The third column of Table 4) A positive shock at \( t = 1 \) (\( \Delta A_1 > 0 \) or \( \Delta A_1 = \Delta A_2 > 0 \)), for example, will raise the price of the financial asset since the equilibrium levels of \( K_1 \) and \( K_2 \) are higher. A higher productivity leads to more capital, either due to the substitution for housing production in the same city or the substitution for foreign production. A positive shock to housing production also generates the parameter-dependent effects that are very similar to the case of human capital.

[Table 4: Effects of technology shocks on asset prices]

5.5 Covariation of Asset Prices

Now we examine the covariation of different asset prices. The covariation in response to a shock is measured in terms of the product of the percentage changes in the two prices.

5.5.1 Financial Assets and Human Capital

As seen in Table 4, most of the time the price of financial assets and the value of human capital move in the same direction. This is because a change in productivity affects both capital demand and labor demand in the same way when a shock is given at \( t = 1 \) (\( \Delta A_1 \) and \( \Delta A_1 = \Delta A_2 \)). When a shock is anticipated in the future (\( \Delta A_2 \) and \( \Delta B_2 \)), they may move in opposite directions. For example, given a positive shock to goods production in period 2 (\( \Delta A_2 > 0 \)), the household also wants to consume more at \( t = 1 \) if the inter-temporal substitution is low (small \( \theta \)). However, housing services must be produced locally while composite goods can be imported from the foreign city. Therefore, capital at \( t = 1 \) is allocated more to housing production and the amount of capital dedicated to goods production (\( K_1 \)) is reduced. Therefore, prices of financial assets and human capital may move in opposite directions when inter-temporal substitution is low.

5.5.2 Housing and Other Assets

The covariation of housing price and the value of human capital depends on the supply elasticity of land (\( \mu \)) and the elasticities in the utility function (\( \rho \) and \( \theta \)). The effect of a shock to goods production (\( \Delta A_t \)) on this covariation is determined by the sign of the change in housing prices since the response of human capital is uniform. For
example, in response to a positive shock, the human capital always appreciates due to wage increases. As seen in Figure 6-a, housing prices and human capital vary together when an inelastic land supply (small $\mu$) makes the housing rent more responsive to a positive demand shock. Conversely, the covariation is negative when relatively elastic land supply (large $\mu$) makes the rent more stable. (Figure 6-b) The critical value of $\mu$ is different for different types of shocks but is not so large for $\Delta A_1$ (Figure 6-c) and $\Delta A_1 = \Delta A_2$. ($\mu \approx 0.8$ for $\Delta A_1 > 0$ and $\mu \approx 2$ for $\Delta A_1 = \Delta A_2 > 0$)

[Figure 6-Covariation of asset prices]

With a shock to housing production ($\Delta B_t$), the link between housing prices and human capital is determined by the effect on human capital. Housing prices always depreciate with a positive shock and appreciate with a negative shock, regardless of parameters. The covariation of housing prices and human capital is generally negative when the two goods are more complementary (small $\rho$) and when the inter-temporal substitution is low (small $\theta$). (Figure 6-d) With a positive shock, for example, human capital appreciates if the two goods are complementary. This is because reduced housing expenditures lead to a lower interest rate, which stimulates production of composite goods.

The covariation between the prices of housing and the financial asset is similar to that between the housing price and the human capital. This is because of the general comovement of human capital and financial assets.

**Proposition 6** Housing assets are a hedge against human capital risk and the financial risk if

\[
\begin{align*}
1) & \quad \begin{cases}
\text{the land supply is sufficiently elastic (large $\mu$)} \\
\text{when the source of risk is a current shock to goods production, or}
\end{cases} \\
2) & \quad \begin{cases}
\text{the two goods are more complementary (small $\rho$)} \\
\text{when the source of risk is a shock to housing production.}
\end{cases}
\end{align*}
\]

A positive production shock causes declines in both the housing price and the value of human capital in the foreign city due to a lower discount factor and diminished production of both goods. A housing production shock has a very small impact on the foreign city, so that the covariation is close to zero.
5.5.3 Cross-Country Differences in Asset Price Covariation

A stylized fact, in the US, is that the correlation between the housing prices and stock prices is negative (or at least close to zero). These empirical findings suggest that housing assets provide at least a good diversification benefit and may even be a hedge against the financial risk.\(^{32}\) An illustrative sample period is after 2000, during which stock prices were depressed and housing prices appreciated. In Japan, the correlation is much higher.\(^{33}\) Illustrative periods are the 1980’s and the 90’s. In the 80’s both stock prices and housing prices appreciated, but in the 90’s both were depressed. The relationships between housing and human capital, and between human capital and stock are probably positive in both countries although the results are mixed.\(^{34}\)

This paper provides a rational foundation to explain this difference between countries in the covariation structure among the three assets. A standard explanation for a positive covariation of housing prices and stock prices in Japan relies on monetary policy. It treats both stocks and real estate the same, focusing on the nominal values of these assets. It does not explain why we observe negative covariation in the U.S. Another explanation is more "behavioral." Japanese households and investors are somehow more prone to irrational exuberance and an investment boom spreads across assets. This paper’s explanation, based on the land supply elasticity, is more natural and matches a key difference between the two economies.

It is important to understand properly the land supply in our model. The land supply is obviously most restricted by the topographic conditions and population densities. Table 5 compares the per capita habitable areas for five countries. The habitable area is the gross usable area, excluding forests and lakes. The U.S. has 25 times more habitable area per capita than Japan. The ability to supply housing, whether by land development or via infill, has been much more limited in Japan. Other supply constraints are imposed by the regulatory system and the adjustment speed of housing stock. European countries such as Germany generally impose stricter environmental and historical restrictions on new developments. Such restrictions make the land sup-

\(^{32}\) Cocco (2000) and Flavin and Yamashita (2002) among others note the negative correlation. Goetzmann and Spiegel (2000) find a negative Sharpe ratio for housing, which is consistent with the opportunity for hedging.

\(^{33}\) Quan and Titman (1999) reports a high correlation in Japan between stock and commercial real estate, which is positively correlated with housing prices. Casual observation after 1970 also confirms this.

\(^{34}\) Cocco (2000) reports a positive correlation between housing and labor income. Davidoff (2006) also obtains a positive point estimate but it is not significantly different from zero. The correlation between stock and wage income is low in the short term but the correlation is much higher for proprietary business income (Heaton and Lucas (2000)) and in the long run (Benzoni et al. (2005)).
ply more inelastic than the level implied by topographic conditions. The adjustment speed of housing stock is also affected by negotiation practices. Many Japanese redevelopment projects take more than twenty years to complete due to the prolonged negotiation process. Such slow adjustment functions as a short-run inelasticity of supply.\footnote{Quigley and Raphael (2005) and Green et al. (2005) find that population density and housing-market regulation limit housing supply in the U.S. Edelstein and Paul (2000) discuss factors that severely limit land supply in Japan.}

[Table 5: Per Capita Habitable Area]

### 5.5.4 Implications for stock-market participation and homeownership

Positive covariations among three broad asset classes have important implications for the stock-market participation and homeownership. With positive covariations, the optimal portfolio choice results in a small position (or even a short position) in the asset that can be adjusted more freely.\footnote{The partial equilibrium portfolio choice literature leads to the conclusion that less holding of stock is optimal if the exogenously given covariance is positive and vice versa. See Flavin and Yamashita (2002), Cocco (2004), and Cauley et al. (2005).} In general, there are few constraints on financial asset levels, while human capital and homeownership are constrained at some positive levels. Under these constraints, positive covariations in prices lead to less holdings of financial assets, or limited stock-market participation, as derived by Benzoni et al. (2005). They note an empirical fact that human capital and stock prices are more highly correlated in the long run, and they show that, assuming co-integrated prices of these two assets, the optimal portfolio may be even to short-sell stocks, especially for younger investors. If the rental housing market is well functioning and households are relatively free to choose their level of housing asset holdings, positive covariations lead to less homeownership. This is examined by Davidoff (2006), who shows that households with a higher correlation between labor income and housing prices own less housing.

The current model derives a positive covariation between human capital and financial assets, rather than just assuming one, for most cases, and between housing and financial assets depending on the parameters. Thus, the model provides a rationale for the limited-market participation. More interestingly, the model predicts more severe limitations in stock-market participation in a land-elastic economy, since all three assets (human capital, housing, and stock) are positively related in such an economy. In a land-elastic economy, housing assets serve as a hedge against the other assets and...
the limitation is less severe. Market participation is still limited due to the positive covariation between financial assets and human capital (the largest component of asset holdings).\textsuperscript{37}

This provides a plausible explanation for the fact that Japanese investors participate less in the stock market than the U.S. investors. The Japanese rental housing markets have not functioned well due to the tenancy law that heavily protects tenants’ rights. The government also favors homeownership through subsidized financing and tax treatments of housing.\textsuperscript{38} Households are economically encouraged to hold large housing assets, despite the positive correlations with other assets. Therefore, the optimal portfolio includes a less stock. Again, previous explanations tend to rely on the "irrationality", differences in "culture", or differences in skills and experience. In fact, based on such arguments, the Japanese government has adopted policies to encourage stock market participation, measures supported by the financial industry. Our result provides a counter argument: namely, that less participation in the stock market is a perfectly rational choice for households in the land-inelastic Japanese economy.

6 Conclusion and Discussion of Uncertainty

In this paper, I build a simple deterministic model of a production economy to study the general equilibrium relationship between the business cycles and asset prices, with an emphasis on implications to portfolio choice.

The first of my two main results is that the supply elasticity of land plays a significant role in determining the covariations of asset prices. In particular, a negative productivity shock to goods production may lead to a housing price appreciation if land supply is elastic. A key driver is a higher housing rent in the future resulting from a reduced housing supply. This implies that an economy with an inelastic housing supply is more likely to exhibit a positive price covariation between housing and other assets and thus, either less stock-market participation or less homeownership. These predictions are broadly consistent with the differences between the U.S. and Japan.

The second result is that growth of housing rent alters the marginal utility of consumption when the utility function is non-separable in housing and other goods. For example, the marginal utility will be adjusted upward (implying that consumers are less satisfied) if rent growth is lower, provided that the two goods are complementary. The rent growth factor may mitigate the well-known puzzles on the equity premium and

\textsuperscript{37}Cocco (2004), using PSID, estimates that the 60-85% of the total assets are human capital.

\textsuperscript{38}Kanemoto (1997) discusses in detail homeownership and limited rental markets in Japan.
the risk-free rate, either by magnifying consumption variation or imposing a downward bias on the estimate of the EIS. The risk of housing assets is also inferred from the characterization.

This paper suggests a rich array of opportunities for empirical analysis. First, the new characterization of the discount factor allows us to use housing rent data to estimate elasticities of substitution. Housing rent data have several advantages over housing consumption data: 1) rent data can be more accurately collected, 2) rents respond more sharply to changes in economic conditions, and 3) more detailed regional data are available for rents. Second, cross-country variations in land supply elasticity and elasticities of substitution can be used to test (or estimated jointly with) the predictions of the model on 1) the covariance between housing prices and other asset prices, 2) the level of stock holdings, 3) effects of technology shocks on housing prices, interest rates, cap rate, and savings, 4) volatility of the discount factor, and 5) risk premium of housing assets. The model also predicts cross-city variations in consumption, investment, and rent as well as cross-sector variations in production and investment although these results are not presented in detail for brevity.

The next task of this research will be to accommodate uncertainty explicitly. In a dynamic stochastic setting, the risk arising from price dynamics also affects the asset allocation, as risk-adjusted returns are equated in equilibrium. The covariance structure between asset returns and the discount factor, and thus, the risk premium with respect to different types of business cycles can be explicitly shown. The system may be solved either by second-order approximation or by numerical methods. Although the method often used in the literature is linear approximation, the certainty equivalence property resulting is not suitable for the study of asset prices. By calibration, the levels of variables can be discussed rather than just the direction and the relative magnitudes as done in the present paper. Other directions of future extension include modeling imperfect household mobility across cities and allowing for non-homothetic preferences. Household mobility will enrich the results by making the labor supply effectively more elastic and generating predictions about city size. Non-homothetic preferences will also be important when the model is calibrated to the data. There is evidence that the parameter estimates will be quite different from the CES-case. Since the relative elasticities of substitution are critical to the dynamics, this issue needs to be examined with particular care.
7 Appendix: Derivation of the equilibrium

In this appendix, I describe how to solve for the equilibrium that is defined in the paper.

(Labor markets) Labor supply is $L_{t}^{sup} = 1$. Labor demand is derived from the first order condition of a goods-producing firm (6b): $w_{t} = (1 - \alpha) A_{t} (K_{t}/L_{t})^{\alpha}$. Using the capital demand from another first order condition (6a), the equilibrium wage is derived as a function of $A_{t}$ and $i_{t}$:

$$w_{t}^{eq} (A_{t}, i_{t}) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} A_{t}^{\frac{1}{1-\alpha}} (i_{t} - 1 + \delta)^{-\frac{\alpha}{1-\alpha}}.$$

(Land markets) Land supply is $T_{t} = r_{t}^{\mu}$. Land demand is derived from the first order condition of a real estate firm (7b): $T_{t}^{dem} = \{(1 - \gamma) B_{t} p_{t} / r_{t}\}^{\frac{1}{\gamma}} S_{t}$. Using demand for housing structures from another first order condition (7a), the equilibrium land rent and the quantity of land is derived as a function of $B_{t}, p_{t}, i_{t}$:

$$r_{t}^{eq} (B_{t}, p_{t}, i_{t}) = \gamma^{\frac{\gamma}{1-\gamma}} (1 - \gamma) B_{t}^{\frac{1}{1-\gamma}} p_{t}^{\frac{1}{1-\gamma}} (i_{t} - 1 + \delta)^{-\frac{\gamma}{1-\gamma}},$$
$$T_{t}^{eq} (B_{t}, p_{t}, i_{t}) = \gamma^{\frac{\mu}{1-\mu}} (1 - \gamma)^{\mu} B_{t}^{\frac{\mu}{1-\mu}} p_{t}^{\frac{\mu}{1-\mu}} (i_{t} - 1 + \delta)^{-\frac{\mu}{1-\gamma}}.$$

Although both $r_{t}$ and $T_{t}$ depend on the housing rent ($p_{t}$), $r_{t}$ and $T_{t}$ can be written as functions of $B_{t}, A_{1}, A_{2}, i_{1}, i_{2}$ after deriving the equilibria of the other markets.

(Housing markets) Housing supply is $H_{t}^{sup} (p_{t}; B_{t}, i_{t}) = B_{t} S_{t}^{eq} (B_{t}, p_{t}, i_{t})^{\gamma} \times T_{t}^{eq} (B_{t}, p_{t}, i_{t})^{1-\gamma}$. Housing demand is derived as (9c) from the first-order conditions of the households. Analytical solution to the housing market equilibrium is available for the log case:

$$p_{1}^{eq} (i_{1}, Inc) = \{2 (1 + \beta)\}^{-\frac{\gamma}{1+\gamma}} \gamma^{-\gamma} (1 - \gamma)^{-\frac{\mu (1 - \gamma)}{1+\mu}} B_{1}^{1} (i_{1} - 1 + \delta)^{\gamma} \text{Inc}^{\frac{1+\mu}{1+\gamma}},$$
$$H_{1}^{eq} (i_{1}, Inc) = \{2 (1 + \beta)\}^{-\frac{\gamma+\mu}{1+\mu}} \gamma^{-\gamma} (1 - \gamma)^{-\frac{\mu (1 - \gamma)}{1+\mu}} B_{1} (i_{1} - 1 + \delta)^{-\gamma} \text{Inc}^{\frac{\gamma+\mu}{1+\gamma}}.$$

For the CES-CRRA case, a numerical solution must be used to derive $p_{1}, p_{2}, p_{1}^{*}, p_{2}^{*}$ jointly with $i_{1}$ an $i_{2}$.

(Capital markets) After obtaining $p_{1}^{eq} (i_{1}, Inc)$ and $H_{1}^{eq} (i_{1}, Inc)$ for the log case,
I can rewrite \( r_t^{eq}(i_t, Inc) \) and \( T_t^{eq}(i_t, Inc) \) and further derive \( Inc \) as

\[
Inc(A_1, A_2, i_1, i_2) = i_1W_0 + r_1T_1 (i_1, Inc) + w_1 (A_1, i_1) \\
+ \frac{1}{i_2} \{ r_2T_2 (i_2, Inc) + w_2 (A_2, i_2) \} \\
= 2 (1 + \gamma)^{-1} \alpha r^{1-\alpha} (1 - \alpha) \\
\times \left\{ A_1^{\frac{1}{1-\alpha}} (i_1 - 1 + \delta)^{-\frac{\alpha}{1-\alpha}} + A_2^{\frac{1}{1-\alpha}} i_2^{-1} (i_2 - 1 + \delta)^{-\frac{\alpha}{1-\alpha}} \right\}.
\]

Note that \( B_t \) does not appear in land rents or land quantity in the log-utility case while it does appear in the CES-CRRA case.

Now the capital supply for period 2, \( W_1 \), is derived. Given \( Inc(A_1, A_2, i_1, i_2) \), the consumption becomes \( C_t(A_1, A_2, i_1, i_2) \) and the households’ saving after period 1 is

\[
W_1(A_1, A_2, i_1, i_2) = i_1W_0 + r_1T_1 (A_1, A_2, i_1, i_2) + w_1 (A_1, i_1) \\
- C_1(A_1, A_2, i_1, i_2) - p_1H_1 (A_1, A_2, i_1, i_2).
\]

The market-clearing conditions in capital markets are

\[
W_0 + W_0^* = \begin{bmatrix}
K_1(A_1, i_1) + K_1^*(A_1^*, i_1) \\
+ S_1(A_1, A_2, i_1, i_2) + S_1^*(A_1^*, A_2^*, i_1, i_2)
\end{bmatrix} \quad \text{(for } t=1),
\]

\[
\begin{bmatrix}
W_1(A_1, A_2, i_1, i_2, W_0) + W_1^*(A_1^*, A_2^*, i_1, i_2, W_0)
\end{bmatrix} = \begin{bmatrix}
K_2(A_2, i_2) + K_2^*(A_2^*, i_2) \\
+ S_2(A_1, A_2, i_1, i_2) + S_2^*(A_1^*, A_2^*, i_1, i_2)
\end{bmatrix} \quad \text{(for } t=2).
\]

With these two equations, in principle two unknowns \( (i_1, i_2) \) can be solved for in terms of the exogenous variables \( (A_1, A_2, A_1^*, A_2^*, W_0, W_0^*) \). Numerical solutions must be used to obtain the actual solutions.

In the case of CES-CRRA, capital markets’ equilibria will depend additionally on the housing rents. Therefore, the housing-market equilibrium and the capital-market equilibrium are solved simultaneously.

In this paper, basic parameters are set as follows: \( \alpha = 1/3, \beta = 0.9, \gamma = 0.7, \delta = 0.5 \).

**References**


Table 1: Effects on the discount factor

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \phi_{0,1}$</th>
<th>$\Delta \phi_{1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta A_1 &gt; 0$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\Delta A_2 &gt; 0$</td>
<td>$\approx 0$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\Delta A_1 = \Delta A_2 &gt; 0$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\Delta B_1 &gt; 0$</td>
<td>+ if $\rho$ is small</td>
<td>+ if $\theta$ is small</td>
</tr>
<tr>
<td></td>
<td>- if $\rho$ is large</td>
<td>- if $\theta$ is large</td>
</tr>
<tr>
<td>$\Delta B_2 &gt; 0$</td>
<td>+ if $\theta$ is large</td>
<td>+ if $\rho$ is small</td>
</tr>
<tr>
<td></td>
<td>- if $\theta$ is small</td>
<td>- if $\rho$ is large</td>
</tr>
<tr>
<td>$\Delta B_1 = \Delta B_2 &gt; 0$</td>
<td>+ if $\rho$ is small</td>
<td>+ if $\rho$ is small</td>
</tr>
<tr>
<td></td>
<td>- if $\rho$ is large</td>
<td>- if $\rho$ is large</td>
</tr>
</tbody>
</table>

Table 1 presents general-equilibrium effects of different types of technology shock on the discount factor. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at $t$ to the production of good and housing, respectively. $\Delta \phi_{0,1}$ and $\Delta \phi_{1,2}$ refer to the response of the discount factor for the first and the second period, respectively. $\rho$ and $\theta$ are the parameters for intra- and inter-temporal substitution, respectively.
Table 2: Covariation of the discount factor and rent growth

<table>
<thead>
<tr>
<th>Covariation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔA₁</td>
</tr>
<tr>
<td>ΔA₂</td>
</tr>
<tr>
<td>ΔA₁ = ΔA₂</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ΔB₁</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ΔB₂</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ΔB₁ = ΔB₂</td>
</tr>
</tbody>
</table>

Table 2 presents covariation of the discount factor ($\phi_{1,2}$) and rent growth ($p_{2}/p_{1}$) in general equilibrium. Each row corresponds to different types of shock. $\Delta A_i > 0$ and $\Delta B_i > 0$ refer to a positive shock at $t$ to the production of good and housing, respectively. $\mu$ is elasticity of land supply. $\rho$ and $\theta$ are parameters for intra- and inter-temporal substitution, respectively.
Table 3: Predictions in four cases of housing price appreciation

Case 1: A positive shock to good production with inelastic land supply

<table>
<thead>
<tr>
<th>Term structure</th>
<th>$\Delta A_1 &gt; 0$</th>
<th>$\Delta A_1 = \Delta A_2 &gt; 0$</th>
<th>$\Delta A_2 &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future rent growth</td>
<td>bear-flattening</td>
<td>bear-parallel shift</td>
<td>bear-steepening</td>
</tr>
<tr>
<td>Current cap rate</td>
<td>+</td>
<td>+</td>
<td>+ if $\theta$ small</td>
</tr>
<tr>
<td>Savings</td>
<td>+</td>
<td>+</td>
<td>− if $\theta$ large</td>
</tr>
<tr>
<td>$\text{Cov}(K_1, S_1)$</td>
<td>+</td>
<td>+</td>
<td>≈ 0</td>
</tr>
</tbody>
</table>

Case 2: A negative shock to good production with elastic land supply

<table>
<thead>
<tr>
<th>Term structure</th>
<th>$\Delta A_1 &lt; 0$</th>
<th>$\Delta A_1 = \Delta A_2 &lt; 0$</th>
<th>$\Delta A_2 &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future rent growth</td>
<td>bull-steepening</td>
<td>bull-parallel shift</td>
<td>bull-flattening</td>
</tr>
<tr>
<td>Current cap rate</td>
<td>−</td>
<td>−</td>
<td>+ if $\theta$ large</td>
</tr>
<tr>
<td>Savings</td>
<td>−</td>
<td>−</td>
<td>− if $\theta$ small</td>
</tr>
<tr>
<td>$\text{Cov}(K_1, S_1)$</td>
<td>+</td>
<td>+</td>
<td>≈ 0</td>
</tr>
</tbody>
</table>

Case 3: A negative shock to foreign city

<table>
<thead>
<tr>
<th>Term structure</th>
<th>$\Delta A_1^* &lt; 0$</th>
<th>$\Delta A_1^* = \Delta A_2^* &lt; 0$</th>
<th>$\Delta A_2^* &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future rent growth</td>
<td>bull-steepening</td>
<td>bull-parallel shift</td>
<td>bull-flattening</td>
</tr>
<tr>
<td>Current cap rate</td>
<td>−</td>
<td>−</td>
<td>+ if $\theta$ large</td>
</tr>
<tr>
<td>Savings</td>
<td>−</td>
<td>−</td>
<td>− if $\theta$ small</td>
</tr>
<tr>
<td>$\text{Cov}(K_1, S_1)$</td>
<td>+</td>
<td>+</td>
<td>≈ 0</td>
</tr>
</tbody>
</table>

Case 4: A negative shock to housing production

<table>
<thead>
<tr>
<th>Term structure</th>
<th>$\Delta B_1 &lt; 0$</th>
<th>$\Delta B_1 = \Delta B_2 &lt; 0$</th>
<th>$\Delta B_2 &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Future rent growth</td>
<td>−</td>
<td>+ if $\rho &gt; 1$</td>
<td>− if $\rho &lt; 1$</td>
</tr>
<tr>
<td>Current cap rate</td>
<td>+</td>
<td>+ if $\rho &gt; 1$</td>
<td>− if $\rho &lt; 1$</td>
</tr>
<tr>
<td>Savings</td>
<td>+ if $\theta$ small</td>
<td>− if $\theta$ large</td>
<td>− if $\theta &lt; 1$</td>
</tr>
<tr>
<td>$\text{Cov}(K_1, S_1)$</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

Table 3 presents model predictions in the four cases of housing price appreciation. Predictions are about 1) term structure of interest rates, 2) future rent growth, 3) current cap rate, 4) savings, and 5) covariation of investments in business capital and in housing structure. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at $t$ to the production of good and housing, respectively. "Mixed" response refers to more complex comparative statics.
Table 4: Effects of technology shocks on asset prices

<table>
<thead>
<tr>
<th></th>
<th>Housing Asset</th>
<th>Human Capital</th>
<th>Financial Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta A_1 &gt; 0$</td>
<td>$+$ if $\mu$ small</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td></td>
<td>$-$ if $\mu$ large</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta A_2 &gt; 0$</td>
<td>$-$ if ${\mu$ large, $\rho$ small, $\theta$ small $}$</td>
<td>$+$ if $\theta$ large</td>
<td>$-$ if $\theta$ small</td>
</tr>
<tr>
<td></td>
<td>+ otherwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta A_1 =$</td>
<td>$+$ if $\mu$ small</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\Delta A_2 &gt; 0$</td>
<td>$-$ if $\mu$ large</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta B_1 &gt; 0$</td>
<td>$-$</td>
<td>$-$ if ${\rho$ large, $\theta$ large $}$</td>
<td>$-$ if $\rho$ large</td>
</tr>
<tr>
<td></td>
<td>+ otherwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta B_2 &gt; 0$</td>
<td>$-$</td>
<td>$-$ if ${\rho$ large, $\theta$ small $}$</td>
<td>$+$ if $\theta$ large</td>
</tr>
<tr>
<td></td>
<td>+ otherwise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta B_1 =$</td>
<td>$-$</td>
<td>if $\rho &lt; 1$</td>
<td>$+$ if $\rho &lt; 1$</td>
</tr>
<tr>
<td>$\Delta B_2 &gt; 0$</td>
<td>$-$ if $\rho &gt; 1$</td>
<td>$-$ if $\rho &gt; 1$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 presents effects of different types of technology shock on asset prices. Each row corresponds to different types of shock. $\Delta A_t > 0$ and $\Delta B_t > 0$ refer to a positive shock at $t$ to the production of good and housing, respectively. $\mu$ is elasticity of land supply. $\rho$ and $\theta$ are parameters for intra- and inter-temporal substitution, respectively.
Table 5 presents population, habitable area, and per capita habitable area for five countries. Population is an estimate for 2002 by the United Nations (Demographic Yearbook 2002). The habitable area is the total area less forests and lakes in 2001.
Figure 1: Time line

Initial wealth → Production & Consumption → Savings → Production & Consumption

Technology shock

Return for period 1

Return for period 2
2-a) Variation of consumption growth to an anticipated shock to goods production ($\Delta A_2$) ($\rho = 0.2$, $\theta = 1.8$)

2-b) Covariation of consumption growth ($g_{c,2}$) and rent growth factor ($g_{p,2}^{\rho \theta}$) to an anticipated shock to goods production ($\Delta A_2$)

2-c) Covariation of consumption growth ($g_{c,2}$) and the discount factor ($\phi_{1,2}$) to an anticipated shock to housing production ($\Delta B_2$)

2-d) Variation of the discount factor ($\phi_{1,2}$), consumption growth ($g_{c,2}$), and rent growth factor ($g_{p,2}^{\rho \theta}$) to an anticipated shock to housing production ($\Delta B_2$) ($\rho = 0.2$, $\theta = 1.8$)

Figure 2 presents two cases in which the equity premium puzzle and the risk-free rate puzzle are mitigated. Figure 2-a presents percentage changes in $g_{c,2}$, $g_{p,2}^{\rho \theta}$, and $g_{c,2}$ $g_{p,2}^{\rho \theta}$ from their baselines against different levels of $A_2$. Consumption growth is magnified by rent growth factor. Figure 2-b and 2-c show covariation of $g_{c,2}$ with $g_{p,2}^{\rho \theta}$ and $g_{c,2}$ with $\phi_{1,2}$, respectively, against different values of $\rho$ and $\theta$. Figure 2-d shows percentage changes in $g_{c,2}$, $g_{p,2}^{\rho \theta}$, and $\phi_{1,2}$ from their baselines against different levels of $A_2$. 
3-a) Response of interest rate ($\Delta i_1 = 1 / \Delta \varphi_{0,1}$) to a positive shock to goods production ($\Delta A_1 > 0$)

3-b) Response of savings ($\Delta W_1$) to a positive shock to goods production ($\Delta A_1 > 0$)

3-c) Response of interest rate ($\Delta i_1 = 1 / \Delta \varphi_{0,1}$) to a positive shock to housing production ($\Delta B_1 = \Delta B_2 > 0$)

3-d) Response of savings ($\Delta W_1$) to a positive shock to housing production ($\Delta B_1 > 0$)

Figure 3 presents selected comparative statics of the discount rate and savings. Figure 3-a and 3-b show the response of the discount rate and savings, respectively, to a positive shock to the goods production for different values of $\rho$ and $\theta$. Figure 3-c and 3-d show the response of the discount rate and savings, respectively, to a positive shock to housing production for different values of $\rho$ and $\theta$. 
4-a) Covariation of the discount factor and rent growth to a shock to goods production ($\Delta A_2$) when land supply is inelastic

4-b) Covariation of the discount factor and rent growth to a shock to goods production ($\Delta A_2$) when land supply is elastic

4-c) Covariation of the discount factor and rent growth to a shock to housing production ($\Delta B_2$)

Figure 4: Covariation of the discount factor and rent growth

Figure 4 presents selected comparative statics of the covariation between the discount factor and rent growth. Covariation is measured in terms of the product of percent changes in the discount factor and rent growth. Figure 4-a and 4-b show the covariation to a shock to goods production for different values of $\rho$ and $\theta$ when land supply is inelastic and elastic, respectively. Figure 4-c shows the covariation to a shock to housing production for different values of $\rho$ and $\theta$. 
5-a) Case 1: Response of housing prices to a positive shock to goods production (\(\Delta A_1 > 0\)) if land supply is inelastic

5-b) Case 1 and Case 2: Response of housing prices to a positive shock to goods production (\(\Delta A_1 > 0\)) for different elasticities of land supply

5-c) Case 2: Response of housing prices to a negative shock to goods production (\(\Delta A_1 < 0\)) if land supply is elastic

5-d) Case 4: Response of housing prices to a negative shock to housing production (\(\Delta B_1 < 0\)) if land supply is inelastic

Figure 5: Effects on housing prices

Figure 5 presents selected comparative statics of the response of housing prices. Figure 5-a shows the response of home prices to a positive shock to goods production for different values of \(\rho\) and \(\theta\) in an economy with \(\mu = 0\). Figure 5-b shows the response of home prices to a positive shock to goods production for different values of \(\mu\), \(\rho\) and \(\theta\). Figure 5-c shows the response of home prices to a negative shock to goods production for different values of \(\rho\) and \(\theta\) in an economy with \(\mu = 5\). Figure 5-d shows the response of home prices to a negative shock to housing production for different values of \(\rho\) and \(\theta\) in an economy with \(\mu = 0\).
6-a) Covariation of housing prices and human capital in response to a shock to goods production ($\Delta A_1$) if land supply is inelastic.

6-c) Covariation of housing prices and human capital in response to a shock to goods production ($\Delta A_1$) for different elasticities of land supply.

6-b) Covariation of housing prices and human capital in response to a shock to goods production ($\Delta A_1$) if land supply is elastic.

6-d) Covariation of housing prices and human capital in response to a shock to housing production ($\Delta B_1=\Delta B_2$) if land supply is inelastic.

Figure 6 presents covariation of home prices and human capital. Covariation is measured in terms of the product of percent changes in home prices and human capital. Figure 6-a shows the covariation in response to a shock to goods production for different values of $\rho$ and $\theta$ in an economy with $\mu = 0$. Figure 6-b shows the same case as 6-a except that $\mu = 5$. Figure 6-c shows the covariation in response to a shock to goods production for different values of $\mu$. Figure 6-d shows covariation in response to a shock to housing production for different values of $\rho$ and $\theta$ in an economy with $\mu = 0$. 